Some Pleasant Development Economics Arithmetic

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Introduction

- Recent literature emphasis on inter-firm distortions in the allocation of inputs
- Firm-specific wedges
- Understand mapping between distortions and aggregate productivity?
- On the inference from data
- The role of firm dynamics

Understanding firm level distortions: The undistorted economy

- Simplified Lucas style model
- Production function: $y_i = e_i n_i^{\alpha}$
- N total labor endowment
- Optimal allocation:
  - $\ln n_i = a + \left(\frac{1}{1-\alpha}\right) e_i$
  - y_i / n_i should be equated across firms (TFPR in HK jargon)

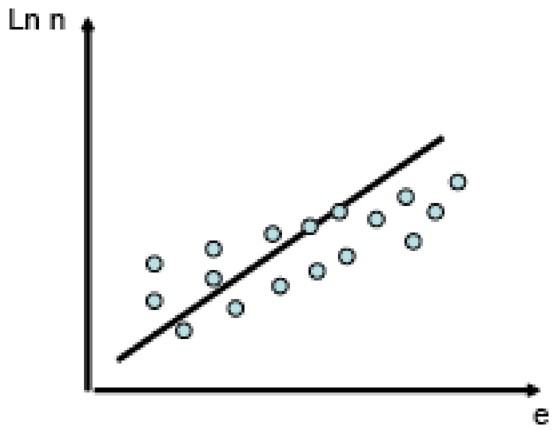
Aggregation

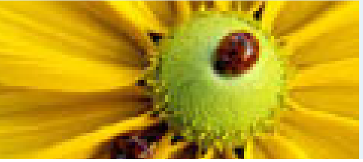
- Aggregate production function
- Homogeneous of degree one in firms (given distribution) and labor

$$
\begin{aligned}
y &= AM^{1-\alpha}N^{\alpha} \\
A &= \left(Ee_i^{\frac{1}{1-\alpha}}\right)^{1-\alpha}
\end{aligned}
$$

The Distorted Economy

- y_i / n_i not equated across firms
- Two types of distortions:
  - n_i not equal for all firms with same e_i (uncorrelated distortion)
  - average $\ln n_i(e) \neq a + \frac{1}{1-\alpha} \ln e$ (correlated distortion)





Restuccia-Rogerson

- Let a firm's profits be $(1 - \tau_i)\, y_i - wl_i - rk_i$,where τ_i denotes a sales tax
- variance and covariance

% Estab. taxed	Uncorrelated		Correlated	
	τ_t		τ_t	
	0.2	0.4	0.2	0.4
90%	0.84	0.74	0.66	0.51
50%	0.96	0.92	0.80	0.69
10%	0.99	0.99	0.92	0.86

- potentially large effects
- More when correlated

A measure of distortions

- Distortions result in deviations of output frome optimal:

$$n(\tau, e) = (1-\tau)^{\frac{1}{1-\eta}} n(e)$$

- Using $\theta = (1-\tau)^{\frac{1}{1-\eta}}$ distorted employment is $\theta n(e)$.

Definition 1. A feasible distortion is a conditional probability distribution $P(\theta|e)$ such that $N = \int n(e)\, \theta dP(\theta|e)\, dG(e)$.

- Using a change of variable on order of integration, can define $Q(n|\theta)$ and $F(d\theta)$ define measure $N(d\theta)$ as follows:

$$dN(\theta) = dF(\theta) \int n dQ(n|\theta)$$

A measure of distortions

- $N(\theta)$ is the measure of total *original* employment that was distorted with some $\theta' \leq \theta$.
- It is silent about the productivity of the firms underlying these distortions.
  - Example: $n_1 = 10$, $n_2 = 100$. Suppose equal number (e.g. 20) of each.
  - Distorting by θ half of the firms of type 1 and by $1/\theta$ the remaing half gives the same measure as distorting 1/20th of the type 2 firms with θ and 1/20th with $1/\theta$.
  - $N(\theta) : \{(\theta, 100), (1/\theta, 100), (1, 2000)\}$
- It integrates to total employment

$$N = \int dN(\theta).$$

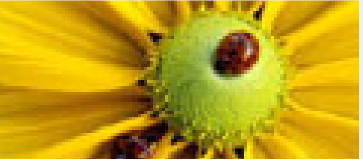
The measure of distortions and TFP

- First define total output: $y = \int e\left(\theta n\left(e\right)\right)^{\eta} dP\left(\theta|e\right) dG\left(e\right)$
- Using $n\left(e\right) = ae^{\frac{1}{1-\eta}}$ for some constant a

$$\begin{aligned} y &= \int e\left(\theta a e^{\frac{1}{1-\eta}}\right)^{\eta} dP\left(\theta|e\right) dG\left(e\right) \\ &= a^{\eta} \int e^{\frac{1}{1-\eta}} \theta^{\eta} dP\left(\theta|e\right) dG\left(e\right) \\ &= a^{\eta-1} \int n\left(e\right) \theta^{\eta} dP\left(\theta|e\right) dG\left(e\right). \end{aligned}$$

- Since it is linear in $n\left(e\right)$ we can use our measure:

$$y = a^{\eta-1} \int \theta^{\eta} dN\left(\theta\right).$$



TFP and the measure of distortions

- We obtained our formula:

$$y = a^{\eta - 1} \int \theta^{\eta} dN(\theta).$$

- Undistorted economy has $N(\theta)$ mass point at one.

$$y_{eff} = a^{\eta - 1} N$$

- It follows that:

$$\frac{TFP}{TFP_{eff}} = \frac{y}{y_{eff}} = \frac{1}{N} \int \theta^{\eta} dN(\theta)$$

- The effect of distortions depends on η and the distribution of distortions.

TFP and the concentration of distortions

$$\frac{TFP}{TFP_{eff}} = \frac{y}{y_{eff}} = \frac{1}{N}\int \theta^{\eta} dN(\theta)$$

- $dN(\theta)/N$ is a probability measure
- Mean preserving spreads in this measure reduce TFP/TFP_{eff}
- And mean preserving spreads give rise to the same aggregate employment!

$$N = \int \theta dN(\theta)$$

- Mean preserving spreads $\Longleftrightarrow$ more concentrated distortions.
- Lower η implies more risk aversion so larger effect of a mean preserving spread.

Examples of mean preserving spreads

Uncorrelated taxes to larger firms are worse for productivity than for smaller firms (holding the number of firms affected constant)

- Increasing the variance of the θ's for large firms will put more employment at the tails than if done for small firms
- But might not be so when taking into account that there are more small firms.
- It all depends on the share of employment of small vs. large firms.

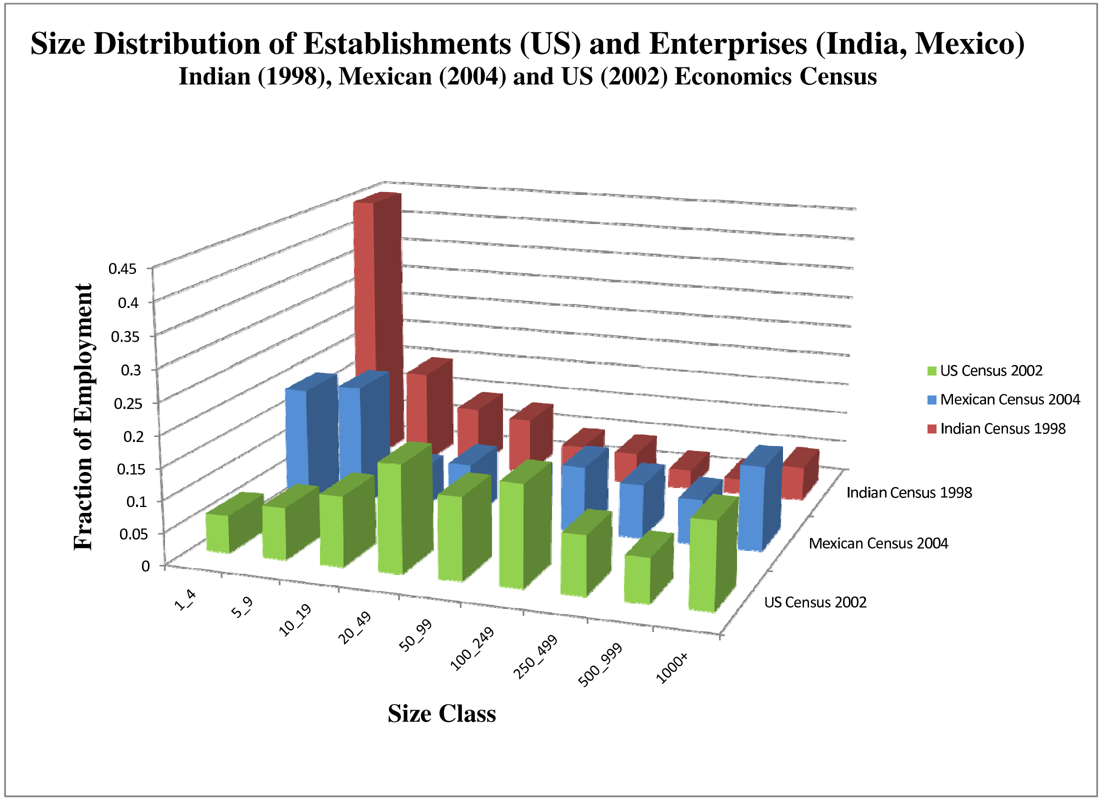
Examples of mean preserving spreads

Increasing the share of firms taxed (while subsidizing others to keep employment constant)

- Let a share s of firms have $\theta_t < 1$
- Then share $(1 - s)$ must have $\theta_s = \frac{s}{1-s}\theta_t$
- This corresponds to a mean preserving spread of the employment that was subsidized initially at a lower rate

Inference from the size distribution of firms

- Size distribution of firms vary a lot across economies
- "missing middle" of underdeveloped economies



Size distribution and distortions

- What inferences can be made by comparing size distributions?
- Nothing in general: distortions might not be revealed in size distribution
  - Can generate the efficient distribution by taxing all efficient firms out of the market and a distribution of taxes across the most inefficient ones that replicates the efficient size distribution.
- Special case:
  - Both economies with same underlying $G(de)$
  - One of the economies with no distortions
  - Can easily calculate lower bound on distortions for other economy.

A lower bound on distortions

- Take efficient distribution of firm sizes with cdf $F(n)$
- Distorted economy with cdf $D(n)$
- Class of candidate distortions $P(\theta, n)$ such that:

$$D(n) = \int_{\theta n' \leq n} P(\theta, n') \, dF(n)$$

- This is a large class

Lower bound on distortions

- Objectivie: minimize spread
- Two key principles:
  - $P(\theta|n)$ should be concentrated at one point for each n
  - Preserve ordering: $n_2 \geq n_1 \; iff \; \theta_2 n_2 \geq \theta n_1$
- Identifies unique solution function $\theta(n)$ defined implictly by:

$$F(n) = D(\theta n)$$

$$\frac{TFP}{TFP_{eff}} = \frac{1}{\bar{n}} \int \theta(n)^{\eta} \, ndF(n)$$

- TFP India/TFP US = 0.4 (Hsieh-Klenow report 0.38)

- HK use $\eta = 0.5$ others $\eta = 0.85$. How does curvature affect the impact of distortions?
- Relationship subtle
- Curvature affects the underlying efficient employment at different levels of e

$$TFP_0 = \frac{\sum e_i^{\frac{1}{1-\eta}} \left(1 - \tau_i\right)^{\frac{\eta}{1-\eta}}}{\left[\sum e_i^{\frac{1}{1-\eta}} \left(1 - \tau_i\right)^{\frac{1}{1-\eta}}\right]^{\eta}}$$

- If $\eta = 0$, interfirm elasticity of substitution is zero, no effect.
- If $\eta = 1$, interfirm elasticity is infinite: no effect for uncorrelated taxes, large for correlated.
- Non-monotonic relationship.

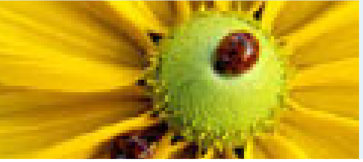
Measuring distortions (H-K)

- Measure y_i, n_i, k_i
- Compute e_i and wedges.
- Counterfactual experiments.
- TFP gains of 30-50% in China and 40-60% in India

Dispersion in ln MP

	US (97)		China (98)			India (94)		
	ln AP	$\frac{1-\tau_{25}}{1-\tau_{75}}$	ln AP	$\frac{1-\tau_{25}}{1-\tau_{75}}$	China/US	ln AP	$\frac{1-\tau_{25}}{1-\tau_{75}}$	India/US
SD	0.49		0.74			0.67		
75-25	0.53	1.7	0.97	2.6	55%	0.81	2.2	32%
90-10	1.19	3.3	1.87	6.5	97%	1.60	5.5	51%

ratio $(1-\tau_{25})$ / $(1-\tau_{75})$. : e.g. assuming the decile 75 corresponded to no taxes in China, decile 25 would have a subsidy of 160%



The role of curvature in HK

- Distribution of productivities depends on η
- Implicit distortions also vary with η
- Data: $(n_1, y_1, n_2, y_2, ..., n_M y_M)$
- Production function $y_i = e_i n_i^\eta$
- Given parameter η, solve for e_i and do counterfactuals.

TFP gains

- Aggregate TFP in economy: $TFP = \frac{y}{n^\eta}$

- Efficient: $TFP_e = \sum \left(e_i^{\frac{1}{1-\eta}} \right)^{1-\eta}$

Substitute measured e_i

$$TFP_e = \left(\sum \left(\frac{y_i}{n_i^\eta} \right)^{\frac{1}{1-\eta}} \right)^{1-\eta}$$

$$\frac{TFP_e}{TFP} = \left(\sum \left(\frac{\frac{y_i}{n_i^\eta}}{\frac{y}{n^\eta}} \right)^{\frac{1}{1-\eta}} \right)^{1-\eta} = \left(\sum \frac{n_i}{n} \left(\frac{\frac{y_i}{n_i}}{\frac{y}{n}} \right)^{\frac{1}{1-\eta}} \right)^{1-\eta}$$

TFP gains

$$\frac{TFP_e}{TFP} = \left(\frac{n_i}{n} \left(LPR_i \right)^{\frac{1}{1-\eta}} \right)^{1-\eta}$$

$$\text{where } LPR_i = \frac{y_i / n_i}{y / n}$$

$$\left(\frac{TFP_e}{TFP} \right)^{\frac{1}{1-\eta}} = \sum \frac{n_i}{n} \left(LPR_{1i} \right)^{\frac{1}{1-\eta}}$$

Proposition. TFP_e / TFP is the *certainty equivalent* of the lottery $\left\{ \frac{n_i}{N}, LPR_i \right\}$ with CRRA $\frac{-\eta}{1-\eta}$. It is thus incresing in η.

- Extreme: equal to one when $\eta = 0$.

Example: sensitivity to curvature

- Suppose $n_1/n = n_2/n = 1/2$

relative y_i/n_i \ η	0.2	0.5	0.8	0.95
0.2	1.09	1.28	1.57	1.74
0.4	1.05	1.17	1.39	1.55
0.6	1.02	1.08	1.22	1.35
0.8	1.01	1.02	1.07	1.16
1	1	1	1	1

.

The role of entry and firm dynamics

- These models abstract from entry/exit margin
- How do results change when this margin can adjust?
- Constrained social planner and entry
- Potential role of distortions to entry

The dynamic economy

- firms productivitiesl cdf $F(ds'; s)$. Exogenous death rate $1 - \delta$.
- sequence of entries of firms $\{m_0, ..., m_t\}$

$$M_t = \delta^t m_0 + \delta^{t-1} m_1 + ...\delta m_{t-1} + m_t$$

- firms producing in period t with probability distribution

$$\mu_t = M_t^{-1} \left(m_t \tilde{\mu}_0 + \delta m_{t-1}^{t-1} \tilde{\mu}_1 + ... + \delta^t m_0 \tilde{\mu}_t \right). \tag{1}$$

- aggregation$y_t = \left(\int e^{\frac{1}{1-\eta}} d\mu_t(e) \right)^{1-\eta} M_t^{1-\eta} N^\eta$.

- $v_t\left(e;w\right)$ value for a firm at time t for a given sequence of wages $w=\left\{w_s\right\}_{s=0}^{\infty}.v_t\left(e;w\right)=\max_n en^{\eta}-w_t n+\beta\delta E v_{t+1}\left(e';w|e\right).$
- $v_t^e=\int v_t\left(e;w\right)dG\left(e\right)-w_t c_e$ expected value for an entrant.

Definition 2. A competitive equilibrium is a sequence $\left\{m_t,n_t\left(e\right),v_t\right\}$ and wages $\left\{w_t\right\}$ that satisfy the following conditions:

1. Employment decisions are optimal given wages
2. The value functions are as defined above
3. $v_t^e\leq 0$ and $m_t v_t^e=0$
4. $m_t c_e+\int n_t\left(e\right)\mu_t\left(de\right)=N$

The distorted economy

- Firm specific output taxes τ_i
- From firm's point of view, same as productivity shock $(1-\tau_i)\, e_i$

$$
\begin{aligned}
r_i &= (1-\tau_i)\, y_i = (1-\tau_i)\, e_i n_i^{\eta} \\
&= \alpha \left(e_i \left(1-\tau_i\right)\right)^{\frac{1}{1-\eta}},
\end{aligned}
$$

- Joint distribution of (e, τ) for each age cohort: $\mu_s(e, \tau)$

$$
\tilde{r}_s = \int e^{\frac{1}{1-\eta}} \left(1-\tau\right)^{\frac{1}{1-\eta}} d\tilde{\mu}_s(e, \tau)
$$

- ("distorted" social planner) revenue-value of sequence $[m_0, ..., m_t, m_{t+1}, ...]$,

$$
\sum_{t=0}^{\infty} \beta^t \left(N - c_e m_t\right)^{\eta} \left(\sum_{s=0}^{t} \delta^{t-s} m_s \tilde{r}_s\right)^{1-\eta}
$$

Equilibrium entry

- First order conditions for m_t at steady state m:

$$N - c_e m = \frac{\eta c_e}{(1-\eta)} \frac{\sum_{s=0}^{\infty} \delta^s \tilde{r}_s}{\sum_{s=0}^{\infty} \beta^s \delta^s \tilde{r}_s}$$

- m is increasing in:

$$\frac{\sum_{s=0}^{\infty} \beta^s \delta^s \tilde{r}_s}{\sum_{s=0}^{\infty} \delta^s \tilde{r}_s}$$

- equilibrium m depends on the age-structure of distortions (Fattal-Jaef, Hopenhayn)
  - m is independent of distortions either if $\delta = 0$ or $\beta = 1$.
  - m is independent of distortions if $\tilde{r}_s / r_s$ is independent of s

Equilibrium entry - comparison

Definition 3. The sequence $\{\delta^s r_s\}$ dominates (is dominated by) the sequence $\{\delta^s \tilde{r}_s\}$ if and only if

$$\frac{\sum_{s=0}^{t} \delta^s r_s}{\sum_{s=0}^{\infty} \delta^s r_s} \leq (\geq) \frac{\sum_{s=0}^{t} \delta^s \tilde{r}_s}{\sum_{s=0}^{\infty} \delta^s \tilde{r}_s}$$

for all t.

Proposition 4. *Suppose* $\{\delta^s r_s\}$ *dominates (is dominated by)* $\{\delta^s \tilde{r}_s\}$*Then* $m \geq (\leq) m_0$.

- d*ominates* if distortions tend to be higher for older firms
    - sufficient condition $\tilde{r}_{s\,/}\,r_s$ decreasing in s
- Fattal-Jaef (2010) shows accounting for the response of entry to wedges can lead to substantively different values.

Equilibrium and Optimal entry

- private vs. social value of a cohort:

$$\begin{aligned} \tilde{r}_s &= \int e^{\frac{1}{1-\eta}} (1-\tau)^{\frac{1}{1-\eta}} \, d\tilde{\mu}_s(e,\tau) \\ \tilde{y}_s &= \int e^{\frac{1}{1-\eta}} (1-\tau)^{\frac{\eta}{1-\eta}} \, d\tilde{\mu}_s(e,\tau) \end{aligned}$$

- Social planner's objective:

$$\max_{m_t} \sum \beta^t \underbrace{\left[\left(\sum_{s=0}^{t} m_{t-s} \delta^s \tilde{r}_s \right)^{1-\eta} (N - c_e m_t)^{\eta} \right]}_{P_t(m_0, m_1, \ldots)} \underbrace{\left(\frac{\sum_{s=0}^{t} m_{t-s} \delta^s \tilde{y}_s}{\sum_{s=0}^{t} m_{t-s} \delta^s \tilde{r}_s} \right)}_{D_t(m_0, m_1, \ldots)}$$

- P_t stands for *private return* and D_t for *distortion*
- First order conditions: $\sum_{s \geq t} \beta^s \frac{\partial P_s}{\partial m_t} D_s + \sum \beta^s \frac{\partial D_s}{\partial m_t} P_s$
- Note: first term is zero at steady state (D_s is constant)

The planner and distortions

$$\sum_{s \geq t} \beta^s \frac{\partial P_s}{\partial m_t} D_s + \sum \beta^s \frac{\partial D_s}{\partial m_t} P_s$$

Proposition 5. *In a steady state* $\sum \beta^s \frac{\partial D_s}{\partial m_t} P_s$ *has sign of:* $\sum_{s=0}^{\infty} \beta^s \left(\frac{\delta^s \tilde{y}_s}{\sum \delta^s \tilde{y}_s} - \frac{\delta^s \tilde{r}_s}{\sum \delta^s \tilde{r}_s} \right)$

- Negative if $\delta^s \tilde{r}_s$ is dominated by $\delta^s \tilde{y}_s$

- Sufficient condition $\tilde{r}_s / \tilde{y}_s$ decreasing in s

$$\frac{\tilde{r}_s}{\tilde{y}_s} = \left(\frac{\int (1 - \tau)\, y\,(e, \tau)\, d\tilde{\mu}_s\,(e, \tau)}{y\,(e, \tau)\, d\tilde{\mu}_s\,(e, \tau)} \right)$$

- Larger in less distorted cohorts

- If distortions are higher (lower) or more (less) correlated with output for younger cohorts then planner wants more (less) entry than equilibrium.

Intuition

- Take economy where older firms are more distored than younger firms (e.g.taxes positively correlated with age/size)
- In that economy, there will be more entry: entrants are "less taxed" than typical incument
- Entry will be excessive: at the equilibrium, the marginal social value of an entrant is less than the marginal social value of an incumbent.
- Conjecture: If distortions are size/age related and older firms are larger, planner would want less entry than in the undistored equilibrium.
- Optimal entry at distorted economy $<$ Optimal entry at undistorted economy $<$ equilibrium entry at distored economy.

Final remarks

- What matters for aggregate TFP is the concentration of distortions
- Correlation with size/efficiency not as important
- Effects of distortions sensitive to curvature
- In HK effects increase with curvature
- Dichotomy between distortions and the productivity of firms
- Distortions and entry/exit