Bubbly Recessions

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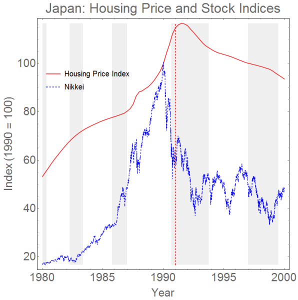
UNC Chapel Hill

CIGS Conference on Macroeconomic Theory and Policy 2017

Motivation: Japanese post-bubble recession



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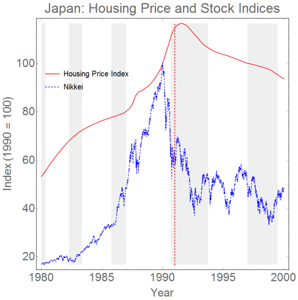

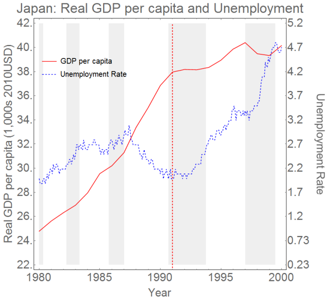

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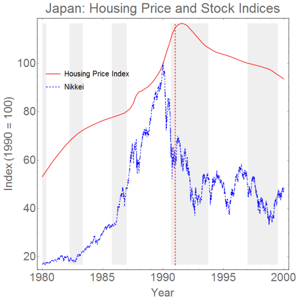

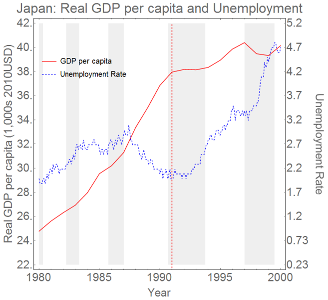

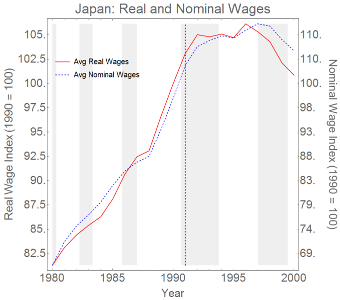

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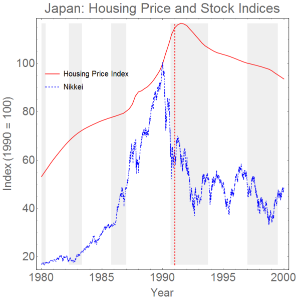
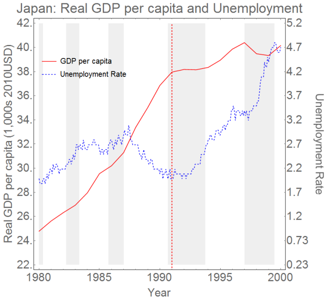
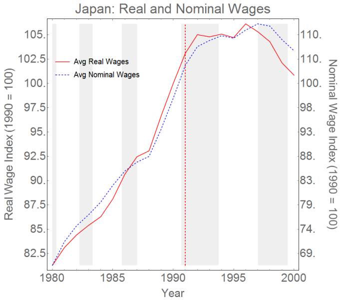
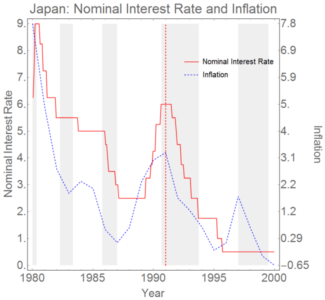
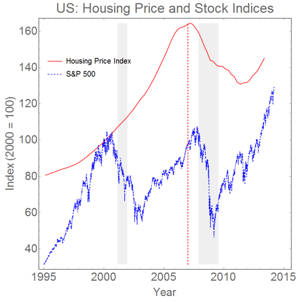


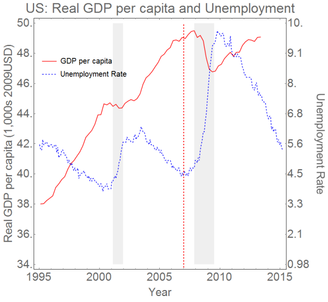


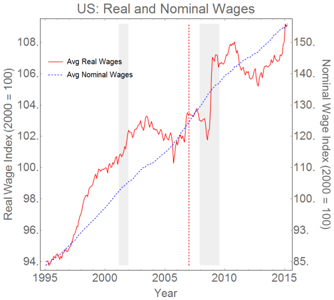


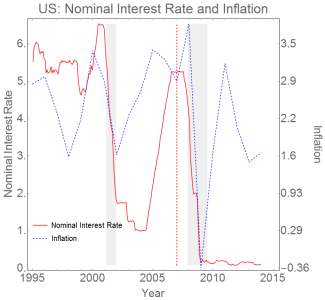


Motivation: U.S. post-bubble recession









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  - Simple model $\rightarrow$ analytical solution

Main findings/contributions

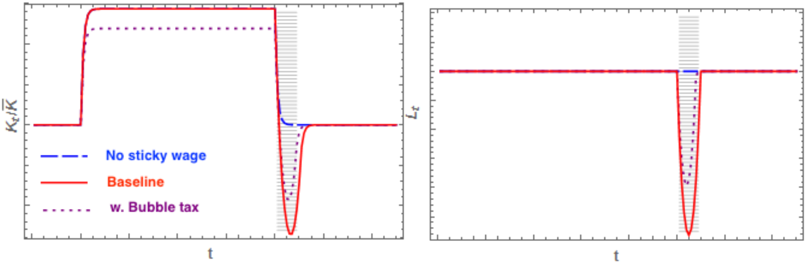

Figure: K & L before, during & after a bubble episode

1. Collapse of bubbles → "overshooting" & protracted recessions

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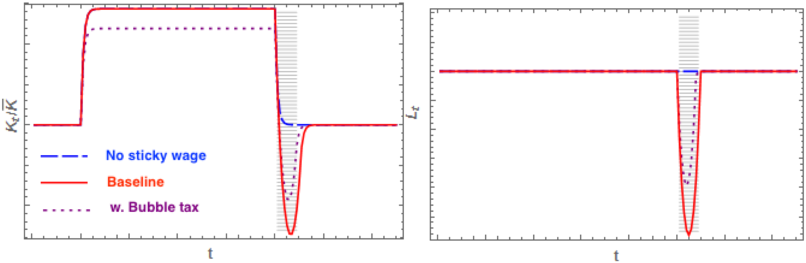

Figure: K & L before, during & after a bubble episode

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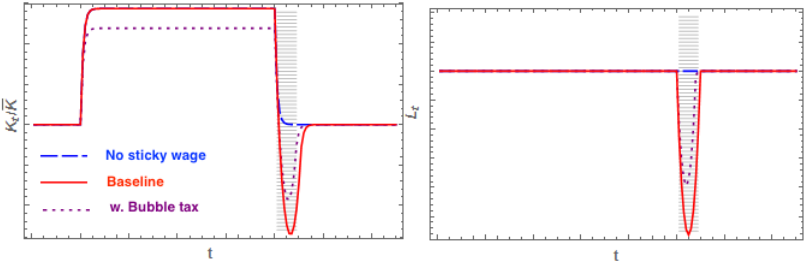

Figure: K & L before, during & after a bubble episode

1. Collapse of bubbles → "overshooting" & protracted recessions
   - "Leaning against bubbles" policies can help
2. Collapse of large bubbles → liquidity trap, which exacerbates unemployment & recession

Related literature

- Rational bubbles (esp. with infinite-lived agents)
  - Miao Wang (2011), **Hirano Yanagawa (2017),** Hirano et al. (2016), ...
  - Samuelson (1958), Diamond (1965), Tirole (1985), ...
- Rational bubbles & unemployment:
  - Kocherlakota (2011), Miao Wang Xu (2016), Hanson Phan (2017)
- Rational bubbles & sticky prices
  - Gali (2014, 2016), Asriyan Fornaro Martin Ventura (2016), Dong Miao Wang (2017), Allen Barlevy Gale (2017)
- New Keynesian models
  - Krugman (1998), Eggertsson Krugman (2012), **Schmitt-Grohe Uribe (2016)**, ...

Outline

1. Model
2. Equilibrium dynamics
3. Policy discussion
4. Liquidity trap (preliminary)

Model

Firms

- Single perishable good

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- Firms, entrepreneurs, workers
- Competitive firms:

$$y_t = k_t^{\alpha} l_t^{1-\alpha}$$

$$w_t = (1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha}$$

$$q_t = \alpha\left(\frac{K_t}{L_t}\right)^{\alpha-1}$$

Workers & Entrepreneurs

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- Entrepreneurs: unit measure, provide capital

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  - Throughout, assume θ small so (CC) binds for H-type

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- In equilibrium, bubble serves as savings instrument: L-type buy bubble and sell it when become H-type

Tax & Entrepreneur's Problem

- Taking prices, productivity shock, tax τ as given:

$$\max_{\{c_t^j, i_t^j, b_t^j, d_t^j\}_{t=0}^{\infty}} E_0 \left(\sum_{t=0}^{\infty} \beta^t \ln c_t^j \right) \text{ s.t.}$$

$$c_t^j + i_t^j + R_t d_{t-1}^j + (1+\tau)\tilde{p}_t^b b_t^j = q_t a_t^j i_{t-1}^j + d_t^j + \tilde{p}_t^b b_{t-1}^j \qquad \text{(BC)}$$

$$R_{t+1} d_t^j \leq \theta q_{t+1} a_t^j i_t^j \qquad \text{(CC)}$$

$$i_t^j, b_t^j \geq 0$$

- Macroprudential policy: speculation tax τ. Budget balance: $\tau \tilde{p}_t^b = T_t$

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- For now, assume fixed inflation $\frac{P_{t+1}}{P_t} \equiv \bar{\Pi} \geq 1$. Then:

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- Labor market may not clear. Workers take employment from firm as given:

$$l_t = L_t \leq 1$$
$$(1 - L_t)(w_t - \gamma w_{t-1}) = 0$$

Equilibrium

- Given τ, $k_0^j = K_0$, $d_0^j = 0$, $b_0^j = 1$, p_0^b, a competitive equilibrium consists of prices $\{w_t, q_t, R_{t+1}, p_t^b\}$, quantities $\{i_t^j, k_{t+1}^j, c_t^j\}, \{l_t, c_t^w\}, \{K_{t+1}, L_t\}$ s.t.:

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    - Bubble market clears: $hb_t^H + (1-h)b_t^L = 1$ if $\tilde{p}_t^b > 0$

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  - Labor market conditions: DWR and

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Equilibrium dynamics

1. **Bubble-less dynamics**
2. Bubble dynamics
3. Post-bubble dynamics

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- Assume K_0 small, so DWR does not bind ($L_t = 1 \forall t$)

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- From binding CC & credit market clearing:

$$K_{t+1} = \left(\frac{h(a^H - a^L)}{1 - \frac{\theta a^H}{a^L}} + a^L \right) \beta q_t K_t$$
$$R_{t+1} = a^L q_{t+1}$$

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- Bubble-less steady state:

$$K_{nb} = (\alpha \Omega)^{\frac{1}{1-\alpha}}, \qquad \Omega \equiv \left(\frac{h(a^H - a^L)}{1 - \frac{\theta a^H}{a^L}} + a^L \right) \beta$$

$$R_{nb} = a^L \alpha K_{nb}^{\alpha - 1}$$

Equilibrium dynamics

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2. **Bubble dynamics**
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Bubble equilibrium ($p_t^b > 0, \ \forall t$)

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  - Bubble is "large" if it completely crowds out L-type's k investment

$$K_{t+1} = \begin{cases} \left(\frac{h(a^H - a^L)}{1 - \frac{\theta a^H}{a^L}} + a^L \right) \beta (q_t K_t + p_t^b) - a^L (1+\tau) p_t^b & \textit{if } R_t = a^L q_{t+1} \\ a^H \beta (q_t K_t + p_t^b) - a^H (1+\tau) p_t^b & \textit{if } R_t > a^L q_{t+1} \end{cases}$$

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- Bubble may raise or lower interest rate

$$R_{t+1} = \max\left\{ a^L, \frac{\theta a^H (1 - (1+\tau)\phi_t)}{1 - h - (1+\tau)\phi_t} \right\} q_{t+1}$$

where $\phi_t \equiv \frac{p_t^b}{\beta(q_t K_t + p_t^b)}$ denotes bubble size (relative to agg. savings)

Proposition (Bubble existence)

A bubble steady state exists iff sufficient financial friction:

$$\theta < \frac{\beta \rho (1-h)}{1+\tau}$$

and bubble not too risky:

$$\rho > \frac{a^L - \theta a^H}{\beta (a^L - \theta a^H) + \beta h (a^H - a^L)}$$

Equilibrium dynamics

1. Bubble-less dynamics
2. Bubble dynamics
3. **Post-bubble dynamics**

What happens when bubble bursts?

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- Bubble collapses in T (i.e., $\tilde{p}_t^b = 0, \forall t \geq T$)
- If $\gamma = 0$, then K_t and w_t will $\downarrow$ towards the bubble-less SS levels

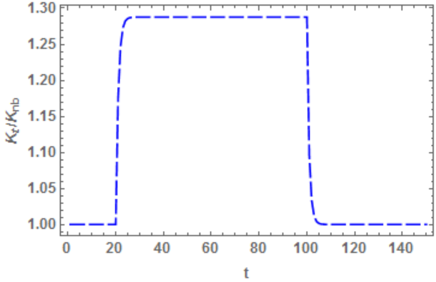

Figure: K before, during & after bubble: $\gamma = 0$

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  - and reduces capital accumulation
- Involuntary unemployment as long as rigid wage floor > market-clearing wage

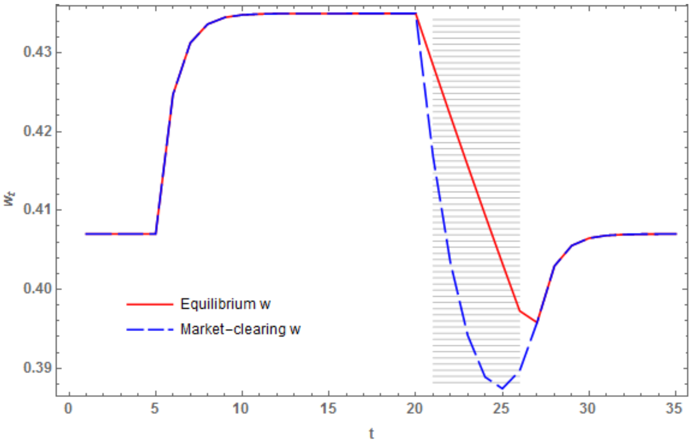

Figure: Equilibrium wage vs. market-clearing wage

Characterizing a slump

- Let $T + s^*$ be *first* post-bubble period with full employment:

$$s^* \equiv \min\{s \geq 0 | L_{T+s} = 1\}$$

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- How long and deep is the slump?

Proposition (Post-bubble slump)

1. *Slump duration:*

$$s^* = \begin{cases} 0 & \text{if } \gamma = 0 \\ \lceil \omega(\gamma) - 2\alpha \log_\gamma K_T \rceil & \text{if } \gamma \in (0,1) \\ \infty & \text{if } \gamma = 1 \end{cases}$$

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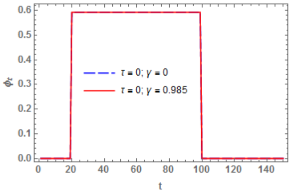
2. *During the slump:*

$$w_{T+s} = \gamma^s w_T$$

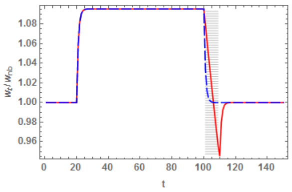
$$L_{T+s} < 1$$

$$K_{T+s+1} = \alpha\Omega \left(\frac{w_{T+s}}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} K_{T+s}$$

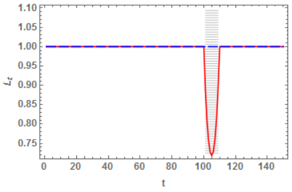
Simulation



(a) Bubble/savings ratio



(b) Real wage



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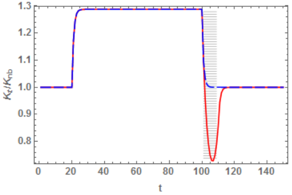

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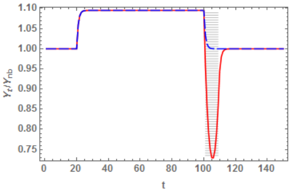
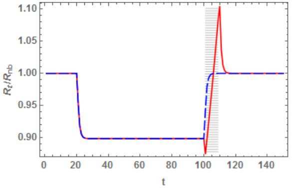


(e) Output



(f) Real interest rate

Figure: Bubbly boom-bust (relative to bubble-less SS)

"Proof": backward & forward induction

- After $T+s^*$: economy follows *full employment* bubble-less dynamics

$$w_{T+s} = w_{T+s}^{full} \equiv (1-\alpha)K_{T+s}^{\alpha}, \ \forall s \geq s^*$$

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- By definition:

$$\begin{aligned} s^* &\equiv \min\left\{s \geq 0 \mid w_{T+s}^{full} \geq \gamma^s w_b\right\} \\ &= \min\left\{s \geq 0 \mid K_{T+s}^{\alpha} \geq \gamma^s K_b^{\alpha}\right\} \\ &= \dots = \left\lceil \omega - 2\alpha \log_{\gamma} K_b \right\rceil \end{aligned}$$

Policy discussion

Proposition (Worker's expected utility in SS)

1. *Bubble-less SS:* $W_{nb}(K) \equiv \Gamma_2 + \frac{\alpha}{1-\beta\alpha}\log K$
2. *Bubble SS:*

$$W_b = \frac{\log c_b^w + \beta(1-\rho)W_{burst}(K_b)}{1-\beta\rho}$$

$$W_{burst}(K_b) \equiv \underbrace{\log[(1-\alpha)(K_b)^{\alpha}]}_{\text{contemporaneous utility}}$$

$$+\underbrace{\sum_{s=1}^{s^*-1}\beta^s\left(\Gamma_1(s)-((1-\alpha)s-\alpha)\log K_b\right)}_{\text{slump utility}}$$

$$+\underbrace{\beta^{s^*}W_{nb}\left(\gamma^{-\left(\frac{1-\sigma}{\sigma}\right)\frac{s^*(s^*+1)}{2}}\left[\alpha\Omega\cdot(K_b)^{\alpha-1}\right]^{s^*}K_b\right)}_{\text{post-slump continuation value}}$$

Proposition (Welfare-reducing bubble)

Assume bubble sufficiently risky:

$$\beta(\beta-\alpha)(1-\rho) > \alpha(1-\beta)^2$$

Then:

$$W_{nb} > \lim_{\gamma \to 1-} W_b$$

Effect of macroprudential policy

- Changing bubble tax can change how bubble affects capital accumulation

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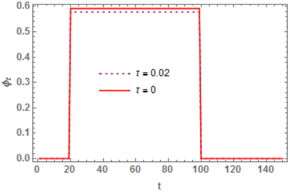
- Tradeoff: smaller economic boom vs. less severe bust

Simulation: Effects of bubble tax



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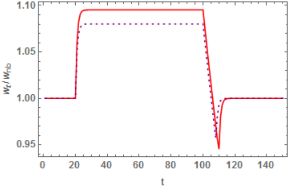

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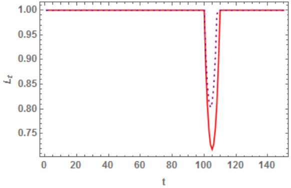

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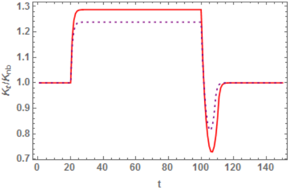

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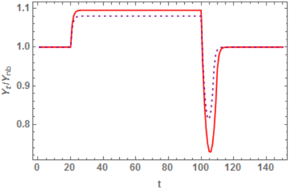

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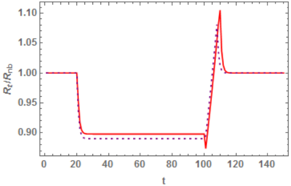

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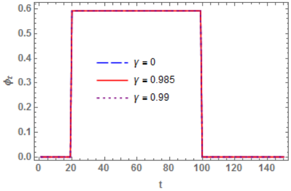
Simulation: Effects of changing inflation target



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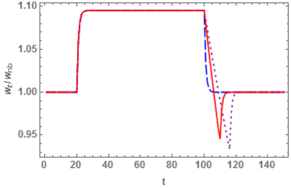

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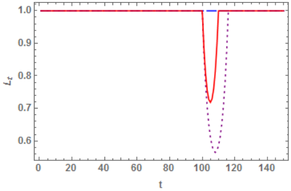

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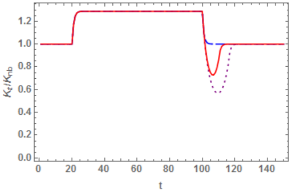

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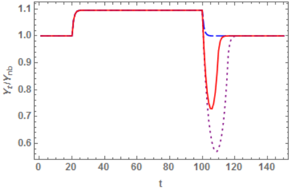

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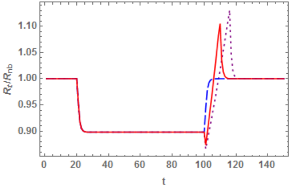

(l) Real interest rate

Summary

- Embed DWR in rational bubble model

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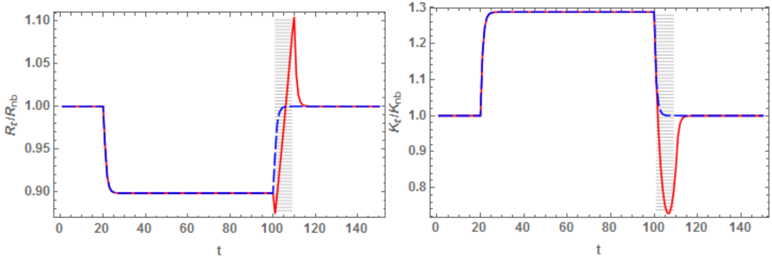
- Find: Collapse of bubble can → persistent & inefficient slump. Warrants policy interventions

Liquidity trap (preliminary)

Collapse of large bubble & overshooting R



Proposition (Post-bubble interest rate)

Suppose economy reaches steady state with large expansionary bubble; then bubble collapses in T.
If $K_{lb} >$ some threshold $\bar{K}$, then post-bubble nominal interest rate is negative:

$$R_{T+1}\bar{\Pi} < 1.$$

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  - Note: overinvestment is endogenous here due to bubble (exogenous in Rognlie Shleifer Simsek, 2017)

Introducing money holding

- To microfound ZLB, assume entrepreneurs choose cash holding:

$$c_t^j + i_t^j + (1+\tau)\tilde{p}_t^b b_t^j + \frac{M_t^j - M_{t-1}^j}{P_t} = q_t k_t^j + d_t^j - R_{t-1,t} d_{t-1}^j + \tilde{p}_t^b b_{t-1}^j \qquad \text{(BC)}$$

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- Assume no liquidity trap in steady states:

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  - Liquidity trap in $T+1$,

$$R_{T+1} \frac{P_{T+1}}{P_T} = 1 \qquad \text{(ZLB binds)}$$

Liquidity trap $\rightarrow$ deflated price level

- Focus on parameters s.t. liquidity trap lasts for only one period (as in Krugman 1998, Eggertsson Krugman 2012)

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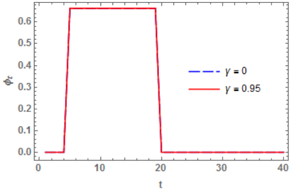
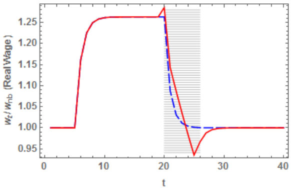
- So $\downarrow R_{T+1}$ (due to bubble bursting) must be associated with $\downarrow P_T$ (deflated price level)
- $\downarrow P_T$ exacerbates DWR:

$$w_T \geq \frac{\gamma}{P_T / P^*_{T-1}} w_b > \frac{\gamma}{\bar{\bar{\Pi}}} w_b$$

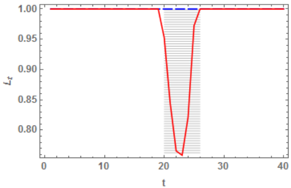
Simulation: ZLB



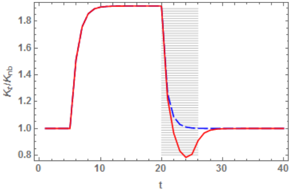
(m) Bubble/savings ratio



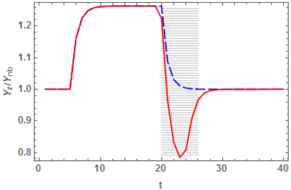
(n) Real wage



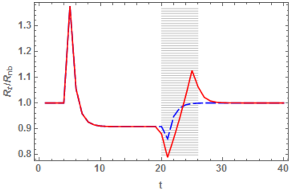
(o) Employment



(p) Capital



(q) Output



(r) Real interest rate

Figure: Post-bubble liquidity trap.

U.S. data

Taking stock: Bubble → ZLB → slump

- Bursting bubble in T

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- Bursting bubble in T

→ Overshooting R_{T+1}

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→ Liquidity trap

Taking stock: Bubble $\rightarrow$ ZLB $\rightarrow$ slump

- Bursting bubble in T

$\rightarrow$ Overshooting R_{T+1}

$\rightarrow$ Liquidity trap

$\rightarrow$ Deflated price level

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$\rightarrow$ Exacerbated wage rigidity: $\Pi_T \downarrow \Rightarrow \frac{\gamma}{\Pi_T} \uparrow$

Taking stock: Bubble $\rightarrow$ ZLB $\rightarrow$ slump

- Bursting bubble in T

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$\rightarrow$ Liquidity trap

$\rightarrow$ Deflated price level

$\rightarrow$ Exacerbated wage rigidity: $\Pi_T \downarrow \Rightarrow \frac{\gamma}{\Pi_T} \uparrow$

$\rightarrow$ Sufficient deflation ($\frac{\gamma}{\Pi_T} > 1$) causes unemployment ($L_T < 1$)

Conclusion

Collapse of large bubbles can trigger persistent slump and liquidity trap

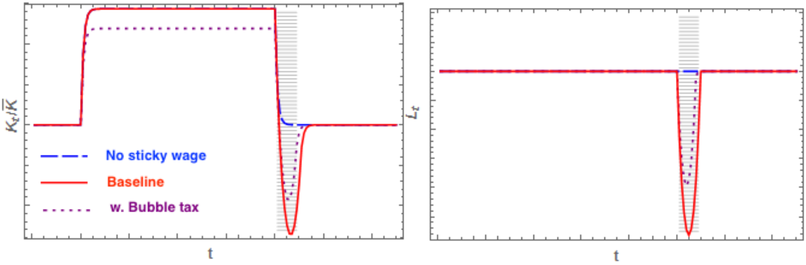
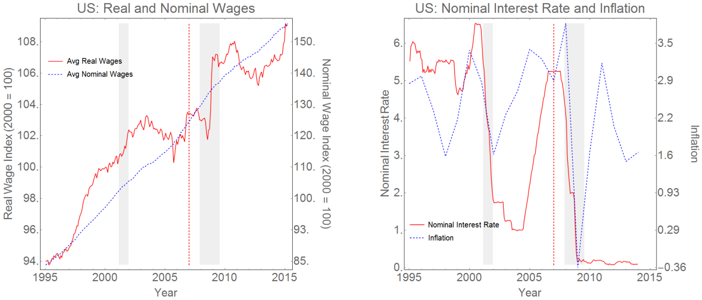


Figure: K & L before, during & after a bubble episode

U.S. post-bubble ZLB & deflation



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