Marketmaking Middlemen*

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Abstract

This paper develops a model in which market structure is determined endogenously by the choice of intermediation mode. We consider two representative business modes of intermediation that are widely used in real-life markets: one is a middleman mode where an intermediary holds inventories which he stocks from sellers for the purpose of reselling to buyers; the other is a market-making mode where an intermediary offers a platform for buyers and sellers to trade with each other. We show that a *marketmaking middleman*, who adopts the mixture of these two intermediation modes, can emerge in a directed search equilibrium.

Keywords: Middlemen, Marketmakers, Platform, Directed Search

JEL Classification Number:D4, G2, L1, L8, R1

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1 Introduction

This paper develops a framework in which market structure is determined by the intermediation service offered to customers. We consider two representative business modes of intermediation that are widely used in real-life markets. In one mode, an intermediary acts as a *middleman* (or a *merchant*), who is specialized in buying and selling for his own account and typically operates with inventory holdings (e.g. supermarkets, traditional brick and mortar retailers, and dealers in financial and steal markets). In the other mode, an intermediary acts as a *marketmaker*, who offers a marketplace or a platform for fees, where the participating buyers and sellers can search and trade with each other (e.g. auction site or brokers in goods and financial markets, and many real estate agencies).

In many real-life markets, however, intermediaries are not one of those extremes but operate both as a middleman and a marketmaker at the same time. This is what we call a *marketmaking middleman*. For example, the electronic intermediary Amazon, one of the largest marketmaking middlemen nowadays, started off as a pure middleman, who buys and resells products in its own name since the foundation in 1994. In the early 2000s, Amazon started to also act a marketmaker, by allowing other suppliers to participate in their marketplace (or platform) as independent sellers. In 2014, the products offered in the platform accounted for around 50% of the Amazon's total merchandise volume.

A similar pattern is observed in financial markets. Since 2006, the New York Stock Exchange (NYSE) has adopted a new hybrid trading system featuring an expanded platform sector, the "NYSE Arca", which allows investors to choose whether to trade electronically or to use traditional floor brokers and specialists (who offer trading opportunities to investors as well as take market positions with their own account).[1] The new system is further supplemented by several dark pools owned by NYSE. These strategies are also adopted by NASDAQ which has been thought of as a typical dealers' market.

Finally, while intermediaries in housing markets are mostly seen as brokers, i.e. platforms, some successful real-estate agencies often employ the same business mode as in Amazon.com or NYSE Arca. For instance, the Trump Organization (owned by Donald J. Trump and his family)

[1] In the finance literature, the following terminologies are used to classify intermediaries: brokers refer to intermediaries who do not trade for their own account, but act merely as conduits for customer orders, akin to our marketmakers; dealers refer to intermediaries who do trade for their own account, akin to our middlemen/merchants. The marketmakers (or specialists) in financial markets quote prices to buy or sell assets as well as take market positions, so they may correspond broadly to our market-making middlemen.

holds several hundred thousand square feet of prime Manhattan real estate in New York City and some more in other big cities. Besides developing and owning residential real estate, the Trump family operates a brokerage company that deals with luxury apartments, the Trump International Realty. Indeed, the Trump's business mode is a marketmaking middleman – *both* owning his own residential towers, *and* offering broker services. According to Forbes, the latter portion of Trump's empire becomes by far his largest business with a valuation of $562 million in 2006.

In this paper, we aim at understanding the occurrence and the functioning of these intermediated markets. For this purpose, we consider a model in which the intermediated-market structure is determined endogenously as a result of the strategic choice of a monopolistic intermediary. In our model, there are two markets open to agents, one is an intermediated market operated by the intermediary, and the other is a decentralized market where buyers and sellers search individually. The intermediated market combines two business modes: as a middleman, the intermediary is prepared to serve many buyers at a time by holding inventories; as a marketmaker, the intermediary offers a platform and receives fees. The intermediary can choose how to allocate the attending buyers among these two business modes.

We formulate the intermediated market as a directed search market in order to feature the intermediary's technology of spreading price and capacity information efficiently – using the search function offered in the NYSE Arca or Amazon website or in the web-based platform for house hunters, for example, one can receive instantly all relevant information such as prices and stocks of individual sellers. In addition, this approach enables us to highlight the middleman's advantage in high selling capacity that mitigates search frictions and provides customers with proximity. The decentralized market represents an individuals' outside option that determines the lower bound of their market utility.

With this set up, we consider two situations, *single-market search* versus *multiple-market search*. Under single market search, agents have to choose which market to search in advance, either the decentralized market or the intermediated market. This implies that the intermediary needs to subsidize buyers with their expected value in the decentralized market, but once they participate, the intermediated market operates without fear of competitive pressure outside. Given that the middleman mode is more efficient in realizing transactions, the intermediary uses the middleman-mode exclusively when agents search in a single market.

When agents are allowed to search in multiple markets, attracting buyers becomes less costly compared to the single-market search case — the intermediary does not need to subsidize buyers to induce participation. However, the prices/fees charged in the intermediated market must be acceptable relative to the available option in the decentralized market. Thus, under multiple-market search, the outside option creates competitive pressure to the overall intermediated market. As for the determination of selling capacity, the intermediary takes into account that a higher capacity induces more buyers to buy from the middleman, and fewer buyers to search on the platform. This has two opposing effects on its profits. On the one hand, a higher capacity of the middleman leads to more transactions in the intermediated market, and consequently to larger profits. On the other hand, sellers are less likely to be successful on a smaller-scaled platform, so that more sellers are available when a buyer attempts to search in the decentralized market. Accordingly, buyers expect a higher value from the decentralized market. This causes a cross-markets feedback that leads to a competitive pressure on the price/fees that the intermediary can charge, and a downward pressure on its profits. Hence, the intermediary trade-offs a larger quantity against lower price/fees to operate as a larger-scaled middleman. This trade-off determines the middleman's selling capacity and eventually the intermediation mode.

The single-market search may correspond to the traditional search technology for local supermarkets or brick and mortar retailers. Over the course of a shopping trip, consumers usually have to search, buy and even transport the purchased products during a fixed amount of time. Given the time constraint, they visit a limited number of shops – typically one supermarket –, and appreciate the proximity provided by its inventory. In contrast, multi-market search may be related to the advanced search technologies that are available in the digital economy. It allows the online-customers to search and compare various options easily. Multiple market search is also relevant in the market for durable goods such as housing or expensive items where customers are exposed to the market for a sufficiently long time to ponder multiple available options.

We show that a marketmaking middleman can emerge in a directed search equilibrium. The marketmaking middleman can outperform either extreme intermediation mode in two respects. Relative to a pure market-maker, its inventory holdings can reduce the out-of-stock risk, while relative to a pure middleman its platform can enlarge the surplus of intermediated trade and the profitability of middleman's selling capacities. We offer two extensions of our baseline model. One is to introduce a non-linear matching function in the decentralized market, which increases the profitability of middleman even with multi-market search, and the other is to introduce the limited production of sellers and frictions in the wholesale market, which increases the profitability of using an active platform even with single-market search. However, these extensions do not alter our main insight on the emergence of marketmaking middleman.

This paper is related to two strands of literature. One is the literature of middlemen developed by Rubinstein and Wolinsky (1987), Duffie, Garleanu, and Pedersen (2005), Lagos and Rocheteau (2009), Lagos, Rocheteau, and Weill (2011), Weill (2007), Shevichenko (2004), Johri and Leach (2002), Masters (2007), Watanabe (2010, 2013), Wong and Wright (2014), Nosal, Wong and Wright (2015) and Geromichalos and Jung (2016).[2] Using a directed search approach, Watanabe (2010, 2013) provides a model of an intermediated market operated by middlemen with high inventory holdings. The middlemen's high selling-capacity enables them to serve many buyers at a time, thus to lower the likelihood of stock-out, which generates a retail premium of inventories. This mechanism is adopted by the middleman in our model. Hence, if intermediation fees were not available, then our model would be a simplified version of Watanabe where we added an outside market. It is worth mentioning that in Watanabe (2010, 2013), the middleman's inventory is modeled as a discrete unit, i.e., a positive integer, so that the middlemen face a non-degenerate distribution of their selling units as other sellers do. In contrast, here we model the inventory as a mass, assuming more flexible inventory technologies, so that the middleman faces a degenerate distribution of sales. This simplification allows us to characterize the middleman's profit-maximizing choice of inventory holdings – in Watanabe (2010) the inventory level of middlemen is determined by aggregate demand-supply balancing, and in Watanabe (2013) it is treated as an exogenous parameter. More recently, Holzner and Watanabe (2016) study a labor market equilibrium using a directed search approach to model a job-brokering service offered by Public Employment Agencies, but the choice of intermediation mode is not the scope of their paper.

The other related strand is the two-sided market literature, e.g. Rochet and Tirole (2003, 2006), Caillaud and Jullien (2003), Rysman (2004), Armstrong (2006), Hagiu (2006) and Weyl (2010).[3] The critical feature of a platform is the presence of the cross-group externality, i.e. the

[2]Rubinstein and Wolinsky (1987) show that an intermediated market can be active under frictions, when it is operated by middlemen who have an advantage in the meeting rate over the original suppliers. Given some exogenous meeting process, two main reasons have been considered for the middlemens advantage in the rate of successful trades: a middleman may be able to guarantee the quality of goods (Biglaiser, 1993, Li, 1998), or to satisfy buyers' demand for a variety of goods (Shevichenko, 2004). While these are clearly sound reasons for the success of middlemen, the buyers' search is modeled as an undirected random matching process, implying that the middlemen's capacity cannot influence buyers' search decisions in these models.

[3]Closely related papers based on a similar spirit can be found in Baye and Morgan (2001), Rust and Hall (2003), Parker and Van Alstyne (2005), Nocke et al. (2007), Galeotti and Moraga-Gonzalez (2009), Loertscher and Niedermayer (2012) and Edelman and Wright (2015). Earlier contributions of this strand of literature are, e.g., Stahl (1988), Gehrig (1993), Yavas (1994, 1996), Spulber (1996) and Fingleton (1997). For platform studies

participants' expected gains from a platform depend positively on the number of participants on the other side of it. When such a cross-group externality exists, the marketmaker can use "divide-and-conquer" strategies, namely, subsidizing one group of participants in order to attract another group and extract the ensuing externality benefit (see, Caillaud and Jullien, 2003). A similar strategy is adopted by the intermediary in our model. Broadly speaking, if there were no middleman mode, then our model would be a directed search version of Caillaud and Jullien (2003) in combination with a decentralized market.

Finally, Rust and Hall (2003) consider two types of intermediaries, one is "middlemen" whose market requires costly search and the other is a monopolistic "market maker" who offers a frictionless market. They show that agents segment into different markets depending on heterogeneous production costs and consumption values, thus these two types of intermediaries can coexist in equilibrium. Their model is very different from ours in many respects. For instance, selling capability and inventory do not play any role in their formulation of search rule, but it is the key ingredient in our model.[4] Further, we show that a monopolistic intermediary can pursue different types of intermediation modes even faced with homogeneous agents.[5]

The rest of the paper is organized as follows. Section 2 presents the basic setup. Section 3 studies the choice of intermediation mode for single-market technologies that serves as a benchmark of our economy. Section 4 extends the analysis to allow for multiple-market technologies and gives the key finding of the paper. Section 5 discusses modeling issues. Section 6 discusses the real-life applications of our theory. Section 7 concludes. Omitted proofs are in the Appendix. Finally, an Additional Appendix contains our extension to allow for unobservable capacity and participation fees.

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emphasizing matching heterogeneity, see e.g., Block and Ryder (2000), Damiano and Li (2007) and De Fraja and Sákovics (2012).

[4]Rust and Hall (2003) say: "An important function of intermediaries is to hold inventory to provide a buffer stock that offers their customers liquidity at times when there is an imbalance between supply and demand. In the securities business, liquidity means being able to buy or sell a reasonable quantity of shares on short notice. In the steel market, liquidity is also associated with a demand for immediacy so that a customer can be guaranteed of receiving shipment of an order within a few days of placement. *Lacking inventories and stockouts, this model cannot be used to analyze the important role of intermediaries in providing liquidity* (page 401)."

[5]In the matketing literature, without considering search frictions, Hagiu and Wright (2015) study the profitability of intermediation modes as is determined by marketing activities. In their model, it is assumed that the owner of a product has private information on how effective their marketing activity will be. They show that the profit maximizing design of intermediation mode is determined, among others, by the cross-product spillovers of marketing activity, and the degree of owners' informational advantage. For each product, an intermediary only takes the preferred extreme mode instead of a hybrid one, and their theory explains which products the intermediary should offer in which mode. In contrast, by considering search frictions, we show that even for a homogeneous product, a hybrid intermediation mode can occur in equilibrium. Our theory explains how the intermediated market is structured depending on the search/competitive environment.

2 Setup

We consider a large economy with two types of agents: a mass B of buyers and a mass S of sellers. Agents of each type are homogeneous. Each buyer has unit demand of a homogeneous good, and each seller has a production technology of that good. The consumption value for the buyer is normalized to 1. The marginal production cost is constant and without loss of generality, we normalize it to zero to simplify our presentation in the first section. Sellers are able to produce as much as they want but their selling/trading capability is limited so that each seller can serve only one buyer.

There are two retail markets, a centralized market and a decentralized market (see Figure 1). The decentralized market (hereafter D market) is featured by random matching and bilateral bargaining. We assume that the flow of contacts between sellers and buyers in the D market is given by a matching technology $M = M(B^D, S^D)$ where B^D and S^D denote the amount of buyers and sellers that actually participate in the D market. The function M is continuous, concave, nonnegative, with $M(0, S^D) = M(B^D, 0) = 0$ for all B^D, $S^D \geq 0$. Without loss of generality, we assume that for B^D, $S^D > 0$ a buyer finds a seller with probability $\lambda^b = \frac{M(B^D,S^D)}{B^D}$ and a seller finds a buyer with probability $\lambda^s = \frac{M(B^D,S^D)}{S^D}$, satisfying $B^D\lambda^b = S^D\lambda^s$. $\lambda^b, \lambda^s \in (0,1)$ is a constant. This linear matching technology is extended to general non-linear matching functions in Section 6. Matched partners follow an efficient bargaining process, which yields a linear sharing of the total surplus, with $\beta \in (0,1)$ the share for the buyer, and $1-\beta$ the share for the seller.

The centralized market (hereafter C market) is operated by a monopolistic intermediary. The intermediary can perform two different intermediation activities. As a middleman, he purchases a good from sellers in a wholesale market, and resells it to buyers. The wholesale market is operated by sellers, who have no limit in producing the good. We assume the wholesale market to be competitive so that the demand of the middleman is always satisfied at the competitive wholesale-price equal to marginal production cost (normalized to zero). We will describe later the case with a positive wholesale price. The middleman can stock the good in advance so that he is prepared to serve buyers on the retail markets. As a market-maker, he does not buy and sell but instead provides a platform where buyers and sellers can interact with each other for trade upon paying fees.

We assume that the C market is subject to coordination frictions. In a directed search environment, the prices and capacities of all the individual suppliers are publicly observable. The

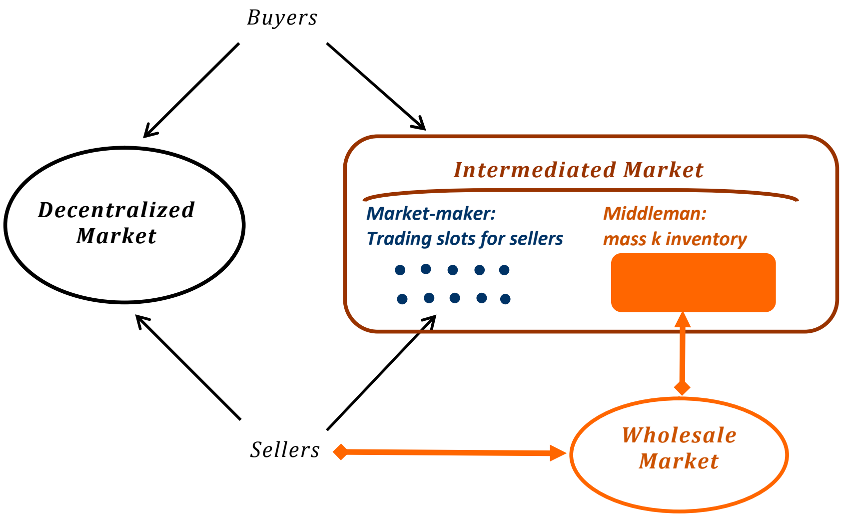

Figure 1: Overview

intermediary has technologies to spread this information. Still, given that individual buyers cannot coordinate their search activities, the limited selling capacity of individual sellers creates a possibility that some units remain unsold and some demands are not satisfied. This is the standard notion of directed search frictions, see e.g., Peters (1991, 2001), Moen (1997), Acemoglu and Shimer (1999), Burdett, Shi and Wright (2001), Albrecht, Gautier, and Vroman (2006), and Guerrieri, Shimer and Wright (2010), and in this sense, the platform in our economy is frictional. As will be detailed below, however, there is no such friction in the middleman's trade since its inventory management technologies allow for the selling capability to be comparable to the population of potential buyers in magnitude.

The timing of events is as follows.

1. Two retail markets, a C market and a D market, open. In the C market, the intermediary decides whether or not to open the platform and/or the middleman sector. Then, whenever they are open, the intermediary announces a capacity in middleman sector K, and a price for the platform service $F \equiv \left\{f^b, f^s\right\}$ and the middleman p^m, where f^b, f^s is a transaction fee charged to a buyer or a seller, respectively. [6]

2. Observing the announced capacities and price/fees, buyers and sellers simultaneously decide

[6] Allowing for participation fees/subsidy, which accrue irrespective of transactions in the C market, will not affect our main result. In the Additional Appendix, we offer such an extended model.

whether to participate in the C market. We consider two different search technologies of agents: single-market search, where agents can attend only one market, and multi-market search where agents can attend both markets.

3. In the C market, the participating buyers, sellers and middleman are engaged in a directed search game. In the D market, agents search randomly and follow the efficient sharing rule for the trade surplus.

We first derive a directed search equilibrium in the C market. Suppose that a mass of B buyers and a mass of $\tilde{S} \in \{0, S\}$ sellers have decided to participate in the C market. The C market has the following stages. In the first stage, all the suppliers, i.e., the participating sellers (if $\tilde{S} = S$) with the unit selling-capacity and the middleman with capacity K, simultaneously post a price which they are willing to sell at. Observing the prices and the capacities, all buyers simultaneously decide which supplier to visit in the second stage. As is standard in the literature, we assume that each buyer can visit one supplier. Assuming buyers cannot coordinate their actions over which supplier to visit, we investigate a symmetric equilibrium where all buyers use an identical strategy for any configuration of the announced prices. Each individual seller (if any) has a queue x^s of buyers with an equilibrium price p^s and the middleman has a queue x^m of buyers with an equilibrium price p^m. These quantities should satisfy two requirements. The first requirement is the accounting identity,

$$\tilde{S}x^s + x^m = B, \tag{1}$$

which states that the number of buyers visiting individual sellers $\tilde{S}x^s$ and the middleman x^m should sum up to the total population of participating buyers B. The second requirement is that buyers search optimally:

$$x^m = \begin{cases} B & \text{if } V^m(B) \geq V^s(0) \\ (0, B) & \text{if } V^m(x^m) = V^s(x^s) \\ 0 & \text{if } V^m(0) \leq V^s(\frac{B}{S}), \end{cases} \tag{2}$$

where $V^i(x^i)$ is the equilibrium value of buyers in the C market to visit a seller if $i = s$ and the middleman if $i = m$ (yet to be specified below). Combining (1) and (2) gives the counterpart for $x^s \in [0, \frac{B}{S}]$.

As for the intermediation mode in the C market, we shall adopt the following terminology.

Definition 1 *Suppose B buyers and $\tilde{S} \in \{0, S\}$ sellers participate in the C market. Then, given the equilibrium search conditions (1) and (2), we say that the intermediary acts as:*

- *a pure middleman if $x^m = B$;*
- *a market-making middleman if $x^m \in (0, B)$;*
- *a pure market-maker if $x^m = 0$.*

3 Single-market search

We start with the analysis for the single-market search technologies where agents can participate in only one market.

In what follows, we show that if agents have a single-market search technology, then the intermediary will not open the platform, inducing $\tilde{S} = 0$, and will act as a pure middleman with $K = B$ and $x^m = B$, serving all buyers. This leads to the following profits in the C market,

$$\Pi = Bp^m,$$

subject to the participation constraint of buyers in the C market,

$$V^m(x^m) = 1 - p^m \geq \lambda^b \beta. \tag{3}$$

The middleman sets $p^m = 1 - \lambda^b \beta$.

Note that as each retail-market is faced with two-sided participation, an issue arises for the belief-dependent multiplicity of equilibria – the participation decision of buyers (sellers) depends on their belief on the participation of sellers (buyers). For the selection of beliefs, the literature of two-sided markets assumes that agents hold pessimistic beliefs on the participation decision of agents on the other side of the C market (Caillaud and Jullien, 2003). In our setup, the middleman's capacity advantage reveals that supply K is available in the C market, irrespective of the number of sellers participating. Hence, the intermediary can induce buyers' participation under those beliefs, as long as condition (3) is satisfied. [7]

Now, we show that creating an active platform is not profitable. Suppose that the intermediary opens a platform with S and intermediation fees $f = f^b + f^s \leq 1$. Then, the platform generates

[7] When the middleman' supply is not observable in the participation stage, the intermediary may charge negative fees (give subsidy) for the participation of agents. We offer such a model in the Additional Appendix, and show that our main result is still valid.

a non-negative trade surplus $1 - f \geq 0$. The number of buyers visiting an individual seller is a random variable, denoted by N (due to coordination frictions), which follows a Poisson distribution, $\text{Prob}[N = n] = \frac{e^{-x}x^n}{n!}$, with an expected queue of buyers $x \geq 0$. With the limited selling capacity, sellers are able to serve only one buyer. A seller with an expected queue $x^s \geq 0$ has a probability $1 - e^{-x^s}$ ($= \text{Prob}[N \geq 1]$) of successfully selling, while each buyer has a probability $\eta^s(x^s) = \frac{1-e^{-x^s}}{x^s}$ of successfully buying. Hence, the expected value of a seller in the platform with a price p^s and an expected queue x^s is given by

$$W(x^s) = x^s\eta^s(x^s)(p^s - f^s),$$

while the expected value of a buyer who visits the seller is

$$V^s(x^s) = \eta^s(x^s)(1 - p^s - f^b).$$

In the presence of the platform, the middleman sector is described as follows. Suppose a middleman sets a price $p \leq 1 - \lambda^b\beta$ and has an expected queue $x = B - Sx^s$ of buyers. Then, since the middleman has a mass of capacity, the expected profit from the middleman sector is given by $\min\{K, x\}p$. The expected value of buyers visiting the middleman is

$$V^m(x) = \min\{\frac{K}{x}, 1\}(1 - p).$$

In any active platform, it must hold that $V^s(x^s) \geq \lambda^b\beta$ and $p^s \geq f^s$. These conditions imply $f = f^s + f^b < 1 - \lambda^b\beta$. Then, the expected profits of the active platform $\tilde{S} = S$ satisfy

$$\begin{aligned}
\Pi(x, f, K) &= S(1 - e^{-x^s})f + \min\{K, x\}p \\
&< Sx^sf + xp \\
&\leq (Sx^s + x)\max\{f, p\} \\
&\leq B(1 - \lambda^b\beta) = \Pi,
\end{aligned}$$

for all $x^s \in (0, \frac{B}{S}]$. Hence, opening the platform is not profitable.

The intuition behind the occurrence of a pure middleman is as follows. Given the frictions on the platform, a larger middleman sector creates more transactions. To achieve the highest possible number of transactions, the intermediary shuts down the platform. In a nutshell, the middleman's capacity is the best way to distribute the good and, if agents search within a single market, the intermediary is guaranteed with the highest possible surplus of it. The allocation characterized here serves as a benchmark for our economy.

Proposition 1 (Pure middleman) *Given single-market search technologies, the intermediary will not open the platform and will act as a pure middleman with $x^m = K = B$, serving all buyers for sure.*

4 Multi-market search

In this section, we extend our analysis to multiple-market search technologies where agents can search in both the C market and the D market. A timing issue arises on which market should open first. Below, we present the analysis of the setup that the C market opens prior to the D market. Apart from the fact that this appears to be the most natural setup in our economy, we are motivated by the first mover advantage of the intermediary: its expected profit is higher if the C market opens before, rather than after, the D market. Hence, our setup will arise endogenously if the intermediary is allowed to select the timing of the market sequence.[8] In this section, we consider explicitly constant production costs $c \in (0, 1)$ in order to guarantee the capacity choice not to be indeterminant.

4.1 C market and outside options

We work backward and first describe the directed search equilibrium of the C market. As before, any directed search equilibrium in the C market has to satisfy (1) and (2). Given the multiple-market search technology, what is new here is that agents expect a non-negative value for the D market when deciding whether or not to accept an offer in the C market. Whenever the platform is active $x^s > 0$ (and $\tilde{S} = S$), it must satisfy the incentive constraints:

$$1 - p^s - f^b \quad \geq \quad \lambda^b e^{-x^s} \beta (1 - c), \tag{4}$$

$$p^s - f^s - c \quad \geq \quad \lambda^s \xi (x^m, K) (1 - \beta) (1 - c). \tag{5}$$

The constraint of buyers (4) states that the offered price/fee in the platform is acceptable only if the offered payoff, $1 - p^s - f^b$, is no less than the expected value in the D market: the outside payoff is $\beta(1-c)$ if the buyer is matched with a seller who has failed to trade in the C market. This

[8] As a real-life correspondence of this sequence, online retailers have in principle unlimited opening hours a day, whereas such a flexible business practice is physically infeasible offline. In addition, many online retailers are enthusiastic in making their websites fast and easy, providing a wide range of information on merchandise and offering personalized service such as special offer emails tailored to a customer's interest. These efforts would enhance customer experiences and create loyalty, which may increase their chance to become a first-mover. In a recent study without intermediation, Armstrong and Zhou (2016) show that a seller often makes it harder or more expensive to buy its product later than at the first opportunity.

happens with probability $\lambda^b e^{-x^s}$. Hence, the larger the platform size x^s, the higher the chance that a seller trades in the C market, and the lower the chance that a buyer can trade successfully in the D market and the lower his expected outside payoff.

In the above formulation, all agents are supposed to stay in the D market so that some meetings are successful and others are not. This opens up the interaction between the C market and the D market. With non-linear matching functions, this assumption can be dispensed. We will clarify this point in Section 6.

The constraint of sellers (5) states that the payoff in the C market $p^s - f^s - c$ should be no less than the expected payoff in the D market. This depends on a seller's chance of engaging in a trade in the D market $\lambda^s \xi(x^m, K)$, where $\xi(x^m, K)$ represents the probability that a buyers has failed to trade in the C market and is given by

$$\xi(x^m, K) \equiv 1 - \frac{1}{B}\left(\min\{K, x^m\} + S\left(1 - e^{-\frac{B - x^m}{S}}\right)\right).$$

The buyer visits the middleman sector with probability $\frac{x^m}{B}$ and is served with probability $\min\left\{\frac{K}{x^m}, 1\right\}$, or he visits the platform with probability $\frac{Sx^s}{B}$ and is served with probability $\eta^s(x^s) = \frac{1-e^{-x^s}}{x^s}$. Hence, in the above expression, the second term represents the expected chance of the buyer to trade in the C market.

We have a similar condition of buyers for the middleman sector:

$$1 - p^m \geq \lambda^b e^{-x^s} \beta (1 - c), \tag{6}$$

where the middleman's price must be acceptable for buyers relative to the expected payoff in the D market.

Given the outside option of the D market, the equilibrium value of buyers in the C market is $V = \max\{V(x^s), V(x^m)\}$, where

$$V^s(x^s) = \eta^s(x^s)\left(1 - p^s - f^b\right) + (1 - \eta^s(x^s))\, \lambda^b e^{-x^s} \beta (1 - c) \tag{7}$$

for an active platform $x^s > 0$ and

$$V^m(x^m) = \min\{\frac{K}{x^m}, 1\}(1 - p^m) + \left(1 - \min\{\frac{K}{x^m}, 1\}\right) \lambda^b e^{-x^s} \beta (1 - c) \tag{8}$$

for an active middleman sector $x^m > 0$. Here, if a buyer visits a seller (or a middleman), then he gets served with probability $\eta^s(x^s)$ (or $\eta^m(x^m)$) and his payoff is $1 - p^s - f^b$ (or $1 - p^m$). If not served in the C market, then he enters the D market and he can find an available seller with probability $\lambda^b e^{-x^s}$. Similarly, the equilibrium value of active sellers in the platform is given by

$$W(x^s) = x^s \eta^s(x^s) (p^s - f^s - c) + (1 - x^s \eta^s(x^s)) \lambda^s \xi(x^m, K) (1-\beta)(1-c). \tag{9}$$

A seller trades successfully in the C market platform with probability $x^s \eta^s(x^s)$ and receives $p^s - f^s - c$. If not successful, then he enters the D market and he can meet an available buyer with probability $\lambda^s \xi(x^m, K)$.

4.2 Intermediation Mode

Our next step is to determine the profit of each intermediation mode, denoted by $\tilde{\Pi}(x^m)$.

Pure middleman: If the intermediary does not open the platform then $x^m = B$ and any encountered seller in the D market is always available for trade. Hence, as before, the middleman selects capacity $K = B$, serves all buyers at a price $p^m = 1 - \lambda^b \beta(1-c)$, satisfying (6), and unit cost (or wholesale price) c, and makes profits

$$\tilde{\Pi}(B) = B(1 - \lambda^b \beta)(1-c). \tag{10}$$

Active platform: In an active platform ($x^s > 0$ and $\tilde{S} = S$), we now need to determine the equilibrium price p^s. We follow the standard procedure in the directed search literature. Suppose a seller deviates to a price $p \neq p^s$ that attracts an expected queue $x \neq x^s$ of buyers. Note that given the limited selling-capacity, this deviation has measure zero and does not affect the expected utility in the C market, V. Since buyers must be indifferent between visiting any seller (including the deviating seller), the equilibrium market-utility should satisfy

$$V = \eta^s(x)\left(1 - p - f^b\right) + (1 - \eta^s(x)) \lambda^b e^{-x^s} \beta(1-c), \tag{11}$$

where $\eta^s(x) \equiv \frac{1-e^{-x}}{x}$ is the probability that a buyer is served by this deviating seller. If not served, which occurs with probability $1 - \eta^s(x)$, his expected utility in the D market is $\lambda^b e^{-x^s} \beta(1-c)$. Given market utility V, (11) determines the relationship between x and p, which we denote by $x = x(p|V)$. This yields a downward sloping demand faced by the seller: when the seller raises his price p, the queue length of buyers x becomes smaller, and vice versa.

Given the search behavior of buyers described above and the market utility V, the seller's optimal price should satisfy

$$p^s(V) = \arg\max_p \left\{ \begin{array}{c} \left(1 - e^{-x(p|V)}\right)(p - f^s - c) \\ + e^{-x(p|V)} \lambda^s \xi(x^m, k)(1-\beta)(1-c) \end{array} \right\}.$$

Substituting out p using (11), the sellers' objective function can be written as

$$W(x) = \left(1 - e^{-x}\right)\left(v\left(x^m, K\right) - f\right) - x\left(V - \lambda^b e^{-x^s}\beta\left(1 - c\right)\right) + \lambda^s \xi\left(x^m, k\right)\left(1 - \beta\right)\left(1 - c\right),$$

where $x = x\left(p|V\right)$ satisfies (11) and

$$v\left(x^m, K\right) \equiv \left[1 - \lambda^b e^{-\frac{B-x^m}{S}}\beta - \lambda^s \xi\left(x^m, K\right)\left(1 - \beta\right)\right]\left(1 - c\right)$$

is the intermediated trade surplus, i.e., the total trading surplus in the C market net of the outside options. Since choosing a price is isomorphic to choosing a queue, the first order condition is

$$\frac{\partial W(x)}{\partial x} = e^{-x}\left(v\left(x^m, k\right) - f\right) - \left(V - \lambda^b e^{-x^s}\beta\left(1 - c\right)\right) = 0.$$

The second order condition can be easily verified. Arranging the first order condition using (11) and evaluating it at $x^s = x\left(p^s|V\right)$, we obtain the equilibrium price $p^s = p^s\left(V\right)$ which can be written as

$$p^s - f^s - c = \left(1 - \frac{x^s e^{-x^s}}{1 - e^{-x^s}}\right)\left(v(x^m, K) - f\right) + \lambda^s \xi\left(x^m, K\right)\left(1 - \beta\right)\left(1 - c\right). \tag{12}$$

For the platform to be active, the price and fees must satisfy the incentive constraints (4) and (5). Substituting in (12) yields

$$f \leq v(x^m, K), \tag{13}$$

which states that for the platform to be active $x^s > 0$, the total transaction fee f should not be greater than the intermediated trade surplus, $v\left(x^m, K\right)$. Whenever (4) and (5) are satisfied, (13) must hold, and whenever (13) is satisfied, (4) and (5) must hold. Hence, we can say that the market maker faces the constraint (13) for an active platform.

Observe that for $K < x^m$, we have

$$v\left(x^m, K\right) = \left[1 - \lambda^b e^{-\frac{B-x^m}{S}}\beta - \lambda^s\left(1 - \frac{K + S(1 - e^{-\frac{B-x^m}{S}})}{B}\right)\left(1 - \beta\right)\right]\left(1 - c\right),$$

which is decreasing in x^m. This occurs because a larger sized platform (i.e., a lower x^m) crowds out the D market transactions and lowers the outside value. Similarly, we have the following result.

Lemma 1 *For $K = x^m$, there exists some $K < B$ such that $v(K, K) > v(B, B)$.*

Proof. See the Appendix. ■

Hence, with multiple-market search, an active platform can enlarge the intermediation surplus.

Pure market-maker: When the middleman sector is not open, $x^s = \frac{B}{S}$. Given the equilibrium price p^s in the platform in (12), the intermediary charges a fee $f = f^b + f^s$ in order to maximize

$$S\left(1 - e^{-\frac{B}{S}}\right) f,$$

subject to the constraint (13). The constraint is binding and it yields:

$$f = v\left(0,0\right) = \left[1 - \lambda^b e^{-x^s} \beta - \lambda^s \xi\left(0,0\right)\left(1-\beta\right)\right]\left(1-c\right).$$

where $\xi\left(0,0\right) = 1 - \eta^s(x^s)$. The profit of the market-maker mode is

$$\tilde{\Pi}(0) = S(1 - e^{-\frac{B}{S}})v(0,0). \tag{14}$$

Market-making middleman: If the intermediary is a market-making middleman, then $x^m \in (0, B)$ and $x^s \in \left(0, \frac{B}{S}\right)$, satisfying $V^m\left(x^m\right) = V^s\left(x^s\right)$. Applying (7), (8), and (12), this indifference condition generates an expression for the price $p^m = p^m\left(x^m\right)$:

$$p^m = 1 - \lambda^b e^{-x^s} \beta\left(1-c\right) - \frac{x^m e^{-x^s}}{\min\left\{K, x^m\right\}}\left(v\left(x^m, K\right) - f\right). \tag{15}$$

Together with (1), this equation defines the relationship between p^m and x^m. Applying this expression, we can see that the condition (6) is eventually reduced to (13). The profit of the marketmaking middleman mode is

$$\tilde{\Pi}(x^m) = \max_{x^m, f, K} \Pi\left(x^m, f, K\right) = S(1 - e^{-x^s})f + \min\left\{K, x^m\right\} p^m - Kc$$

subject to (13) and $x^m \in (0, B)$. Note that $K > x^m$ cannot be profitable since it is a mere increase in capacity costs. The profit maximization requires the following properties.

Lemma 2 *The market-making middleman sets:* $K = x^m$ *and* $f = v(x^m, K)$.

Proof. See the Appendix. ■

The intermediary's capacity should satisfy all the forthcoming demands, and the intermediation fee should be set to extract the full intermediation surplus.

Profit-maximizing intermediation mode: To derive a profit-maximizing intermediation mode, it is important to observe that relative to the pure middleman mode, there are two benefits of using an active platform with multiple-market search. First, as shown in Lemma 1, an active platform can enlarge the intermediation surplus. Second, with $v(\cdot) = f$, the incentive constraint (6) is binding, and the middleman's equilibrium price is given by

$$p^m = 1 - \lambda^b e^{-x^s} \beta(1-c)$$

for any $x^s \geq 0$ (see (15)). This shows that p^m decreases with x^m: the outside value of buyers depends positively on the size of the middleman sector, since a larger scale of middleman crowds out the platform and increases the chance that a buyer can find an active seller in the D market (who was unsuccessful in the platform). Hence, to extend the size of the middleman sector, the intermediary has to lower the price p^m. In other words, a larger platform allows for a price increase by reducing agents' outside trade opportunities.

Proposition 2 (Market-making middleman/Pure Market-maker) *Given multi-market search technologies, there exists a unique directed search equilibrium with active intermediation. The intermediary will open a platform and act as:*

- *a market-making middleman if $\lambda^b\beta \leq \frac{1}{2}$ or if $\lambda^b\beta > \frac{1}{2}$ and $\frac{B}{S} \geq \bar{x}$, some $\bar{x} \in (0, \infty)$;*
- *a pure market-maker if $\lambda^b\beta > \frac{1}{2}$ and $\frac{B}{S} < \bar{x}$.*

Proof. See the Appendix. ■

With multiple-market search technologies, there is a cross-market feedback from the D market to the C market. That is, due to the two benefits mentioned above, using the platform as part or all of its intermediation activities will be profitable. An additional issue arises here whether the intermediary wishes to operate as a pure market maker. Our result shows that it depends on parameter values. If $\lambda^b\beta \leq \frac{1}{2}$ then the buyers' outside option value is low. In this case, the middleman sector generates good enough profits for the market-making middleman mode to be adopted for any value of $\frac{B}{S}$. If instead $\lambda^b\beta > \frac{1}{2}$ then the buyers' outside option value is high, and attracting buyers to the middleman sector is costly. In this case, the intermediary will act as a market-making middleman if $\frac{B}{S}$ is high, where the D market is tight for buyers and they expect a low value from it, and as a pure market maker if $\frac{B}{S}$ is low, where buyers expect a high value from the D market. Indeed, the same logic applies to the following comparative statics result.

Corollary 1 (Comparative statics) *Consider a parameter space in which the market-making middleman mode is profit-maximizing. Then, an increase in buyer's bargaining power β or buyer's* *meeting rate λ^b in the D market, or a decrease in the buyer-seller population ratio, $\frac{B}{S}$, leads to a smaller middleman sector x^m and a larger platform x^s.*

Proof. See the Appendix. ■

5 Extensions

This section considers two extensions of the model. As will be shown, our main insight that the profit of using a platform as part or all of the intermediation business is relatively large when agents can search multi markets, rather than single market, is robust to these extensions.[9]

5.1 Non-linear matching function

So far, we assumed the linear matching function in the D market, which leads to a constant meeting rate $\lambda^b = \frac{M(B^D,S^D)}{B^D}$ for buyers and $\lambda^s = \frac{M(B^D,S^D)}{S^D}$ for sellers. In this section, we extend it to a more general matching function. As is standard in the literature we assume that the matching function is homogeneous of degree one in B^D and S^D, $M(1, \frac{1}{x^D}) = \frac{M(B^D,S^D)}{B^D}$ and $M(x^D, 1) = \frac{M(B^D,S^D)}{S^D}$, where $x^D = \frac{B^D}{S^D}$ is the buyer-seller ratio in the D market. Then, we allow for the dependence of the individual match probabilities on the population ratio,

$$\lambda^b(x^D) = M(1, \frac{1}{x^D}) \quad \text{and} \quad \lambda^s(x^D) = M(x^D, 1) = x^D \lambda^b(x^D) \tag{16}$$

where $\lambda^b(x^D)$ is strictly concave and decreasing in x^D.[10]

With single-market search technologies, the result will not be affected by this extension. Therefore, we only consider multi-market search technologies. As mentioned before, we shall let agents exit if they have traded successfully in the C market.[11] Then, the population in the D market is given by

$$B^D = B - \min\{x^m, K\} - S(1 - e^{-x^s}) \quad \text{and} \quad S^D = Se^{-x^s}.$$

With this modification, the buyers' probability to meet an available seller changes from $\lambda^b e^{-x^s}$ to $\lambda^b(x^D)$, and the sellers' probability to meet an available buyer changes from $\lambda^s \xi(x^m, K)$ to $\lambda^s(x^D) = x^D \lambda^b(x^D)$.

[9] For expositional simplicity, we let $c = 0$ and make the tie-breaking assumption that when the middleman is indifferent between $K = x^m$ and $K > x^m$ we set $K = x^m$.

[10] See Pissarides (2000).

[11] If agents stay in the D market as in the previous section, then again the analysis would remain essentially unchanged.

In what follows, we derive a necessary condition for a pure middleman mode to be selected under multi-market search technologies. This is the case when, for example, $\lambda^{b'}(x^D) = 0$, i.e., when there is no feedback from the D-market to the intermediary's decision in the C market. We proceed with the following steps. First, note that, as before, there is no gain of having an excess capacity $K > x^m$. In addition, a pure middleman wants to avoid stockouts $K < x^m$ if

$$\frac{d\tilde{\Pi}(K)}{dK} = \frac{d}{dK}K\left(1-\lambda^b(x^D)\beta\right) = 1-\lambda^b(x^D)\beta + \frac{K}{S}\lambda^{b'}(x^D)\beta > 0,$$

for any $x^D = \frac{B-K}{S} \geq 0$, which states that the elasticity of the middleman's price $p^m = 1-\lambda^b(x^D)\beta$ should satisfy

$$z(K) \equiv -\frac{\partial p^m/\partial K}{p^m/K} = -\frac{K\lambda^{b'}(x^D)\beta}{S(1-\lambda^b(x^D)\beta)} \leq 1.$$

This condition guarantees that a pure middleman should satisfy all the forthcoming demand $K = x^m$.

Second, when all buyers are served by the middleman $x^m = K = B$, the marginal gain of allocating buyers to the platform, measured by the intermediation fee,

$$f = 1-\lambda^b(x^D)\beta - x^D\lambda^b(x^D)(1-\beta),$$

can not exceed the marginal opportunity cost, measured by the lost revenue in the middleman sector,

$$1-\lambda^b(0)\beta - K\lambda^{b'}(0)\beta\frac{dx^D(K,x^s(K))}{dK}\,|_{x^s(K)=0},$$

where $x^s(K) = \frac{B-K}{S}$ and

$$\frac{dx^D(K,x^s(K))}{dK}\,|_{x^s(K)=0}= \frac{d}{dK}\frac{B-K-S(1-e^{-x^s(K)})}{Se^{-x^s(K)}}\,|_{x^s(K)=0}= \frac{-S+(B-K-S)}{S^2e^{-x^s(K)}}\,|_{K=B}= 0.$$

Hence, the intermediary can be a pure middleman even with multiple-market search technologies.

Proposition 3 *With a non-linear matching function in the D market outlined above, a pure middleman mode can be profitable even with multi-market search technologies only if the middleman's price is inelastic at the full capacity $x^m = K = B$. Otherwise, the intermediary should be a marketmaking middleman or a pure market maker.*

Proof. See the Appendix. ■

Figure 2 plots the size of the middleman sector $\frac{x^m}{B}$ and the elasticity of middleman's price with respect to capacity, evaluated at $x^m = K = B$.[12] It shows that when a pure middleman mode is selected $\frac{x^m}{B} = 1$ the price is inelastic $z(B) < 1$, whereas when an active platform is used the price is elastic $z(B) > 1$. This confirms that given the appropriate restriction on the meeting rate $\lambda^b(x^D)$, our main conclusion in the baseline model is valid with an alternative assumption that agents exit after successful trade in the C market. It is intuitive that when the middleman's price is elastic, there is a strong enough negative feedback from the D market on the price that makes the exclusive use of the middleman sector not profitable.

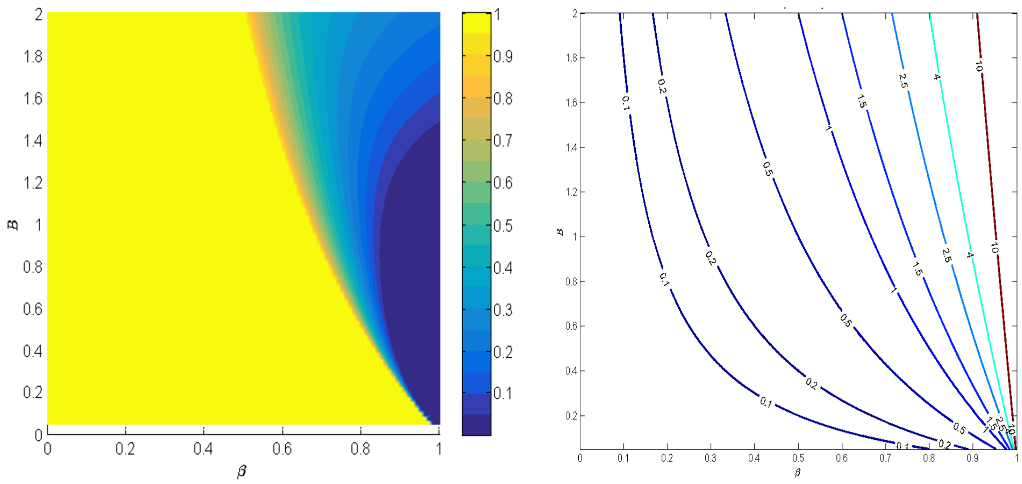

Figure 2: Size of middleman sector $\frac{x^m}{B}$ (Left) and Price elasticity $z(B)$ (Right) with Non-linear matching function

5.2 Endowment economy

In our baseline model, we simplify the middleman's inventory stocking by assuming that sellers have unlimited production capability and there are no frictions in the trade between the middleman and sellers. In this section, we study this issue in an endowment economy. Suppose that each seller owns one unit of endowment. In total, a mass of S commodities are available. In the wholesale market, the middleman can access a fraction α of sellers, where we assume that $\alpha \in (0, 1)$ is an exogenous parameter. Then, the middleman's inventory should satisfy the aggregate resource

[12]The figures is drawn with $S = 1$ and $\lambda^b(x^D) = \frac{1-e^{-x^D}}{x^D}$.

constraint,

$$K \leq \alpha S. \tag{17}$$

In a world with unlimited production capacity, sellers are willing to supply as long as the wholesale price, denoted as p^w, is enough to compensate for the marginal cost; whereas in an endowment economy, sellers are only willing to supply if p^w is high enough to compensate for trading opportunities they lose in other channels. Once contacted by the middleman, sellers choose among selling the endowment to the middleman, or joining the C market platform and/or joining the D market. To simplify the analysis, we abstract away the influence of what sellers can expect from the D market on the determination of wholesale price, and assume that sellers in the D market receive zero trade share,

$$\beta = 1.$$

Our main conclusion does not depend on this simplification. Then, the middleman's offer to buy from sellers is accepted if and only if

$$p^w \geq W(x^s), \tag{18}$$

where $W(x^s)$ is the expected value of sellers to operate in the C market platform.

Single-market search: The determination of the intermediation mode depends on the available resources. If $B \leq \alpha S$, then the middleman can stock the full inventory to cover the entire population of buyers. In this case, by closing the platform $\tilde{S} = 0$, the middleman makes the highest possible profit, $\Pi = B(1 - \lambda^b)$, with the wholesale price $p^w = 0$, just like in the baseline model. If $B > \alpha S$, then the middleman's inventory will not be enough to cover all buyers, and so the intermediary may wish to use a platform even with single-market search technologies. With the wholesale price p^w determined by the binding (18), and the fee f and the price p^m determined by the binding participation constraint of buyers, $V = \max\{V^s(x^s), V^m(x^m)\} = \lambda^b$, the intermediary's problem can be written as the choice of the size of inventory K and the allocation x^m to maximize

$$\Pi\left(x^m, f, K\right) = (S - K)(1 - e^{-x^s})f + \min\left\{K, x^m\right\} p^m - Kp^w$$

where $x^s = \frac{B - x^m}{S - K}$, subject to the resource constraint (17). To guarantee non-negative price/fees/profits, we shall assume sufficiently low values of $\lambda^b > 0$ whenever necessary (see the proof of Proposition 4).

As expected, the solution is characterized by the binding resource constraint (17) and an active platform $x^s > 0$ when $B > \alpha S$. Note that the intermediary could deactivate the platform since it would lead to the lowest wholesale price of middleman $p^w = 0$. However, it turns out that the benefit of fee revenue from the active platform outweighs the cost savings in the middleman sector. Hence, even with single-market technologies, the aggregate resource constraint can be one reason for the intermediary to open the platform sector in the endowment economy.

Proposition 4 *Consider the endowment economy outlined above with single-market search technology, and the zero trade share of sellers in the D market. The intermediation chooses to be:*

- *a pure middleman if* $B \leq \alpha S$*;*
- *a market-making middleman with* $K = \alpha S \leq x^m$ *if* $B > \alpha S$*.*

Proof. See the Appendix. ■

The result $x^m \geq K$ occurs because, in line with the previous setup, an excess inventory means extra costs in the middleman sector and lost revenues in the platform. Figure 3 demonstrates that when $B > \alpha S$, it is possible that the intermediary attracts an excessive number of buyers to the middleman sector $x^m > K$, resulting in stockouts, in order to lower the wholesale price in the middleman sector.[13] When this occurs, the resource constraint is tight and the outside value of agents is high so that economizing on stocking costs is relatively important.

Multi-market search: With multi-market search technologies, the participation constraint of agents is not the issue but the intermediation fee and the middleman's price should be acceptable relative to the outside value. Hence, the intermediary faces the incentive constraints, (4) − (6) with an appropriate modification of the match probability in the D market (see the details in the proof of Proposition 5). As before, these conditions are reduced to $f \leq v(x^m, K)$.

To be consistent, we shall maintain the assumption of zero trade share of sellers $\beta = 1$ in the D market. This assumption now implies that sellers are fully exploited in the C market, thus $p^w = W(x^s) = 0$ for any $x^s \geq 0$.

With the multi-market search setup, the buyers' outside option value depends negatively on the number of sellers available in the D market. This appears to have the following consequences.

[13] The figures in this subsection are drawn with $B = 0.8$ and $S = 1$. We cut out the region where negative profits result, for high values of λ^b.

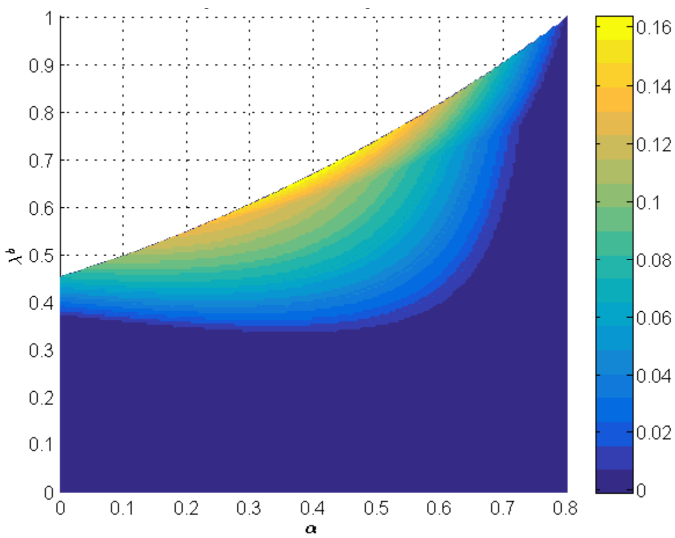

Figure 3: Values of $x^m - K$ with single-market search in endowment economy

First, just like in the baseline setup, a pure middleman mode can never be profit maximizing. Second, in our endowment economy, the intermediary may wish to stock more inventories than the number of buyers visiting the middleman sector. This is because a larger K will crowd out the supply available in the D market, which will eventually lower the outside value of buyers and increase the profit. Therefore, unlike in all the previous setups, the solution here allows for an excess inventory in the middleman sector.

Proposition 5 *Consider the endowment economy outlined above with multi-market search technology, and the zero trade share of sellers in the D market. The intermediation chooses to be a market-making middleman or a pure market-maker with $x^m \leq K = \alpha S$.*

Proof. See the Appendix. ■

Figure 4 shows the occurrence of excess inventory holdings in the middleman sector with high values of λ^b and α. This confirms our intuition that the crowd-out effect of excess inventory is stronger when the agents outside value in the D market is higher.

Comparing Proposition 4 and 5, we summarize the implication of the search frictions in wholesale markets represented by α and the agents' search technologies in retail markets on the determination of intermediation mode in our endowment economy.

- For $\alpha S \geq B$, the middleman can stock the full inventory that satisfies all the buyers' demand.

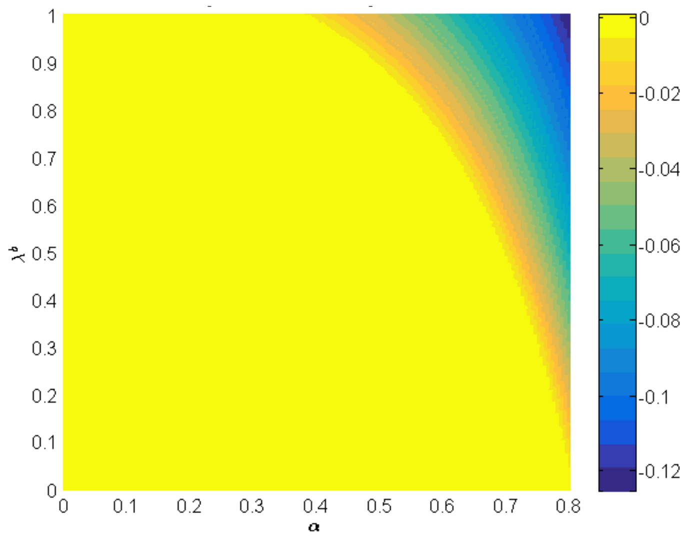

Figure 4: Values of $x^m - K$ with multi-market search in endowment economy

As in the benchmark setup, the intermediary chooses to be a pure middleman with single-market search, but uses an active platform with multi-market search. Unlike in the previous setups, the middleman holds an excessive amount of inventory.

- For $\alpha S < B$, the full inventory is not possible due to the aggregate resource constraint. The intermediary uses a platform irrespective of whether agents search single or multiple markets. Our main insight is still valid. Namely, *the intermediation mode is further away from the pure middleman when agents search multiple, rather than single, markets:* the size of middleman sector, measured by x^m, is smaller with multi-market search than with single-market search technologies.

6 Applications

Our analysis shows that a marketmaking middleman can outperform either extreme intermediation mode in two respects. Relative to a pure market-maker, its inventory holdings can reduce the out-of-stock risk, while relative to a pure middleman its platform can enlarge the surplus of intermediated trade and the profitability of middleman's selling capacities. In this section, we offer real markets examples to see how this simple intuition is practiced.

Online Retailers The electronic commerce company Amazon.com is traditionally an online retailer, who mainly aims at selling its inventories to customers. In the late 1990s, Amazon was facing fierce competition from local brick and mortar rivals, as well as chain stores such as Walmart, Sears etc., and especially from eBay. According to the book, *The Everything Store: Jeff Bezos and the Age of Amazon*, Jeff Bezos worried that eBay may become the leading online retailer who attracts the majority of customers. In the summer of 1998, he invited eBay's management team and suggested the possibility of a joint venture or even of buying out their business. This is perhaps the Amazon's first trail to set up an online marketplace. In the end, this trial failed. After several more trials and errors, however, Amazon finally launched their own marketplace in the early 2000s.

Amazon's launch of the platform business influenced significantly the book industry. On the one hand, Amazon attracts many of its competitors to join their platform. Indeed, Amazon drove physical book and record stores out of business, and many bookstore owners re-launched their business on the Amazon-website platform. On the other hand, Amazon lowers the chance of buyers to trade outside. As local bookstores disappeared, it became the habit for most book buyers to start their everyday online-shopping using Amzon as the prime site (De los Santos, Hortacsu, and Wildenbeest, 2012). Overall, these observed phenomena are in line with our theory.[14] Not surprisingly, Amazon promoted this shopping pattern to customers in other product categories. Similar business strategies are used by many other e-commerce companies such as Rakuten (Japan), Bol.com (the Netherlands) and JD.com (China).

Specialist Markets The New York Stock Exchange (NYSE) is considered as a specialist market, which is defined as a hybrid market that includes an auction component (e.g., a floor auction or a limit order book) together with one or more specialists (also called designated market makers). The specialists have some responsibility for the market: as brokers, they pair executable customer orders against each other; and as dealers, they post quotes with reasonable depth (Conroy and Winkler, 1986).

As for their role as dealers in the exchanges, our model suggests that at least for less active securities (represented by smaller outside option values in the model), the specialists' market

[14]Nowadays, most buyers and sellers use Amazon as the main website (the first one to visit). On the seller side, according to a survey on Amazon sellers conducted in 2016, more than three-quarters of participants sell through multiple channels, online marketplaces, webstores and bricks-and-mortar stores. The second most popular channel, after Amazon, is eBay, with 73% selling through this marketplace. On the buyer side, according to a recent Reuters/Ipsos poll, 51 percent of consumers plan to do most of their shopping on the Amazon.com.

can provide predictable immediacy and increase the trading volume and liquidity. In real-life markets, this result features the following trading patterns observed in many financial markets. First, "large/mid cap" securities are mostly traded on the platform, while the trade of "small cap" securities is usually supported by middlemen. Second, there is a trend over the past two decades to adopt hybrid markets in derivatives exchanges and stock markets around the world, especially for thinly-traded securities. For example, several European stock exchanges implemented a program which gives less active stocks an option of accompanying a designated dealer in the auction market. These initiatives were effective not only in enhancing the creation of hybrid specialist markets, but also in increasing trade volumes and reducing liquidity risks (Nimalendran and Petrella, 2003; Anand et al, 2005; Menkveld and Wang, 2008; Venkataraman and Waisburd, 2007).

Another prediction from our analysis is related to the changing competitive environment faced by securities exchanges. As a broader implication, our result that the increased outside pressure goes hand in hand with more decentralized trades, captures the background trend in general: the market for NYSE-listed stocks was highly centralized in the year of 2007 with the NYSE executing 79% of volume in its listings; in 2009, this share dropped to 25% (SEC, 2010); today, the order-flow in NYSE-listed stocks is divided among many trading venues – 11 exchanges, more than 40 alternative trading systems, and more than 250 broker-dealers in the U.S. (Tuttle, 2014). As a more specific implication, we show that the increased pressure from outside markets will scale up the platform component. This is indeed the case. Starting from 2006, the NYSE adopted the new hybrid trading system featuring an expanded platform sector "NYSE Arca", which allows investors to choose whether to trade electronically or by using traditional floor brokers and specialists. The new system is further supplemented by several dark pools, akin to platforms, owned by the NYSE. In addition, the use of fees is widely adopted, as is consistent with our theory. For instance, in 2014, the NYSE offered banks a discount of trading costs by more than 80% conditional on their agreement to be away from the outside dark pools and other off-exchange venues.[15]

Real estate agencies As mentioned in the Introduction, while intermediaries in housing markets are mostly thought of as brokers, i.e. platforms, the business mode employed by the Trump family is a marketmaking middleman. Another example is Thor Equities, a large-scaled real estate company, which owns and redevelops retail properties in Soho, Madison Avenue, and Fifth

[15]See a recent report in the Wall Street Journal, http://www.wsj.com/articles/intercontinental-exchange-proposing-major-stock-market-overhaul-1418844900.

Avenue, and also runs brokerage agencies, Thor Retail Advisors and Town Residential.

In the endowment economy version of our model, we show that the marketmaking middleman over-invests in inventory with multi-market search, up to the point where the resource constraint is binding. Perhaps, the real estate market in New York City (NYC) is an appropriate example of this since it is well known to be competitive and tight for house/apartment hunters. In addition, most new developments in big cities are renovations of old houses, and so we can roughly regard the total supply as fixed.

Notably, top real estate firms in NYC attempt to expand their business by being engaged in many new joint projects with developers. Mapped into our model, these efforts are aimed at relaxing their resource constraint and increasing their inventory.

For example, Nest Seekers, a real estate brokerage and marketing firm in NYC, works tightly with constructors on new developments. They work together from the very early stage of layout design and fund raising (in some cases Nest Seekers offers their own capital) to the later marketing stage. Nest Seekers provides qualified sales and administrative staff to the sales office, prepares pricing schedules, manages all contracts with the brokerage community, and is eventually in change of the entire marketing process. This co-development business is one step beyond the middleman mode formulated in our theory, but is considered as an alternative way to secure their inventory.[16] This business mode is adopted in many other big real-estate companies in NYC, such as Douglas Elliman, Stribling, and Corcoran.

A final note is that some intermediaries not only help market new developments, but also manage apartment complex, which constitutes another source of "inventory". For example, Brown Harris Stevens provides the residential management service for it since cooperative apartments were first introduced to NYC. These cooperative apartments usually contain hundred of units in one building, and Brown Harris Stevens is then in charge of listing these properties when they are for rent or on sale.

[16] Strictly speaking, Nest Seekers does not own properties, but becomes the exclusive agent of projects. So far, they have co-developed/marketed more than 30 projects. See https://www.nestseekers.com/NewDevelopments. A report titled "Inside the fight for Manhattans most valuable new development exclusives" by The Real Deal introduces more detailed information on how brokers cooperate with developers, which is available in http://therealdeal.com/2016/03/15/inside-the-fight-for-manhattans-most-valuable-new-development-exclusives/ (visited on July 15, 2016).

7 Conclusion

This paper developed a model in which market structure is determined endogenously by the choice of intermediation mode. We considered two representative business modes of intermediation that are widely used in real-life markets, a market-making mode and a middleman mode. We showed that the mixture of the two modes, a *marketmaking middleman*, can emerge. The marketmaking middleman can outperform either extreme intermediation mode in two respects. Relative to a pure market-maker, its inventory holdings can reduce the out-of-stock risk, while relative to a pure middleman its platform can enlarge the surplus of intermediated trade and the profitability of middleman's selling capacities.

For future research, it would be interesting to examine whether competition among intermediaries, rather than outside option, can shape the emergence of market-making middlemen. We believe that the model can be also extended to analyze the market-making behaviors of intermediaries in dynamic financial markets settings.

Appendix

Proof for Lemma 1

Applying $K = x^m$, we have

$$v(K,K) = \left[1 - \lambda^b e^{-\frac{B-K}{S}}\beta - \lambda^s\left(1 - \frac{K + S(1-e^{-\frac{B-K}{S}})}{B}\right)(1-\beta)\right](1-c),$$

and

$$\frac{\partial v(K,K)}{\partial K} = \frac{\lambda^b}{S}\left[1 - \beta - e^{\frac{B-K}{S}}\right](1-c) < 0$$

in the neighbourhood of $K = B$. Hence, there exists some $K < B$ such that $v(K,K) > v(B,B)$. This completed the proof of Lemma 1. ■

Proof for Lemma 2

Using $K \le x^m$ and (15), the intermediary's problem can be written as

$$\begin{aligned}
\max_{x^m,f,K} \Pi(x^m,f,K) &= S(1-e^{-x^s})f + \min\{K,x^m\}p^m - Kc \\
&= S(1-e^{-\frac{B-x^m}{S}})f + K(1-\lambda^b e^{-\frac{B-x^m}{S}}\beta)(1-c) - x^m e^{-\frac{B-x^m}{S}}(v(x^m,K)-f)
\end{aligned}$$

subject to (13) and

$$0 < K \le x^m < B.$$

Observe that: $\lim_{x^m\to B}\Pi(x^m,f,K) = \tilde{\Pi}(B)$ and $\lim_{x^m\to 0}\Pi(x^m,f,K) = \tilde{\Pi}(0)$, where $\tilde{\Pi}(B) = B(1-\lambda^b\beta)(1-c)$ is the profit of pure middleman mode (10) and $\tilde{\Pi}(0) = S(1-e^{-\frac{B}{S}})f$ is the profit of pure market-maker mode (14). Hence, we can compactify the constraint set and set up a general problem to pin down a profit-maximizing intermediation mode using the following Lagrangian:

$$\mathcal{L} = \Pi(x^m,f,K) + \mu_k(x^m - K) + \mu_b(B - x^m) + \mu_v(v(x^m,K) - f) + \mu_0 K,$$

where the μ's ≥ 0 are the lagrange multiplier of each constraint. In the proof of Proposition 2, we show that the following first order conditions are necessary and sufficient:

$$\frac{\partial \mathcal{L}}{\partial x^m} = \frac{\partial \Pi(x^m,f,K)}{\partial x^m} + \mu_k - \mu_b + \mu_v\frac{\partial v(x^m,K)}{\partial x^m} = 0, \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial f} = \frac{\partial \Pi(x^m,f,K)}{\partial f} - \mu_v = 0, \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial K} = \frac{\partial \Pi(x^m,f,K)}{\partial K} - \mu_k + \mu_0 + \mu_v\frac{\partial v(x^m,K)}{\partial K} = 0. \tag{21}$$

The solution is characterized by these and the complementary slackness conditions of the four constraints.

We now prove the claims in the lemma. First, (20) implies that we must have

$$\mu_v = S(1-e^{-x^s}) + x^m e^{-x^s} > 0,$$

which implies the binding constraint (13),

$$f = v(x^m,K) = \left[1 - \lambda^b e^{-\frac{B-x^m}{S}}\beta - \lambda^s\left\{1 - \frac{K}{B} - \frac{S}{B}(1-e^{-\frac{B-x^m}{S}})\right\}(1-\beta)\right](1-c).$$

Second, applying μ_v from (20) into (21) gives

$$\mu_k = \left[1 - \lambda^b e^{-\frac{B-x^m}{S}}\beta + \lambda^b\left(1 - e^{-\frac{B-x^m}{S}}\right)(1-\beta)\right](1-c) + \mu_0 > 0,$$

which implies that $K = x^m$. This completes the proof of Lemma 1. ■

Proof for Proposition 2

$\odot$ Active platform. First of all, we show that the platform will always be active (i.e., $x^m < B$) in equilibrium. Substituting μ_k, μ_v into (19),

$$\begin{aligned}(1-c)^{-1}(\mu_b - \mu_0) &= -e^{-\frac{B-x^m}{S}}\left[1 - \lambda^b e^{-\frac{B-x^m}{S}}\beta - \lambda^b\left\{\frac{B}{S} - (1 - e^{-\frac{B-x^m}{S}})\right\}(1-\beta)\right] \\ &\quad -\lambda^b \frac{x^m}{S} e^{-\frac{B-x^m}{S}} + 1 - \lambda^b\beta + \lambda^b(1 - e^{-\frac{B-x^m}{S}})^2 \\ &\equiv \phi(x^m \mid B, S, \beta, \lambda^b).\end{aligned} \tag{22}$$

Suppose that the solution is $x^m = B$. Then, (22) yields $\phi(B \mid \cdot) = (1-c)^{-1}\mu_b = -\frac{B}{S}\lambda^b\beta < 0$, which contradicts $\mu_b \geq 0$. Hence, the solution must satisfy $x^m < B$ (which implies $\mu_b = 0$).

$\odot$ Market-making middleman or pure market-maker. Second, we derive the condition for a pure market-maker $x^m = 0$ or a market-making middleman $x^m > 0$. Since $\phi(B \mid \cdot) < 0$, if $\phi(0 \mid \cdot) > 0$ then there exists $x^m \in (0, B)$ that satisfies $\phi(x^m \mid \cdot) = 0$, i.e. a market-making middleman. Further,

$$\frac{\partial \phi(x^m \mid \cdot)}{\partial x^m}\Big|_{\phi=0} = -\frac{1}{S}\left[1 - \lambda^b\beta + \lambda^b(1 - e^{-\frac{B-x^m}{S}})2\lambda^b(1 - e^{-\frac{B-x^m}{S}})e^{-\frac{B-x^m}{S}}\right] - \frac{\lambda^b}{S}e^{-\frac{B-x^m}{S}}(1 - e^{-\frac{B-x^m}{S}}) < 0.$$

This implies that the allocation of middleman sector $x^m \in (0, B)$ is unique (if it exists), and that if $\phi(0 \mid \cdot) < 0$ then $\phi(x^m \mid \cdot) < 0$ for all $x^m \in [0, B]$ and the solution must be a pure market maker, $x^m = 0$.

Now, we need to investigate the sign of it:

$$\begin{aligned}\phi(0 \mid B, S, \beta, \lambda^b) &= -e^{-x}\left[1 - \lambda^b e^{-x}\beta - \lambda^b\left(x - 1 + e^{-x}\right)(1-\beta)\right] + 1 - \lambda^b\beta + \lambda^b(1 - e^{-x})^2 \\ &\equiv \Theta(x),\end{aligned}$$

where $x \equiv \frac{B}{S}$. Observe that:

$$\Theta(0) = 0 < 1 - \lambda^b\beta + \lambda^b = \Theta(\infty),$$

and

$$\frac{\partial \Theta(x)}{\partial x} = e^{-x}\left[1 - \lambda^b x + \lambda^b\beta(x-2) + 4\lambda^b(1 - e^{-x})\right].$$

This derivative has the following properties: $\frac{\partial \Theta(x)}{\partial x}\big|_{x=0} = 1 - 2\lambda^b\beta$;

$$\frac{\partial \Theta(x)}{\partial x}\Big|_{\Theta(x)=0} = 1 - \lambda^b\beta(1 + e^{-x}) + \lambda^b(1 - e^{-x})(1 + 2e^{-x}) \equiv \Upsilon(x).$$

There are two cases.

- When $\lambda^b\beta \leq \frac{1}{2}$, we have $\frac{\partial \Theta(x)}{\partial x}\big|_{x=0} \geq 0$ and $\frac{\partial \Theta(x)}{\partial x}\big|_{\Theta(x)=0} > 0$, implying that no $x \in (0, \infty)$ exists such that $\Theta(x) = 0$. Hence, $\Theta(x) = \phi(0 \mid \cdot) > 0$ for all $x \in (0, \infty)$.
- When $\lambda^b\beta > \frac{1}{2}$, we have $\frac{\partial \Theta(x)}{\partial x}\big|_{x=0} < 0$. Hence, there exists at least one $\bar{x} \in (0, \infty)$ such that $\Theta(x) < 0$ for $x < \bar{x}$ and $\Theta(x) \geq 0$ for $x \geq \bar{x}$. Below we show that such a value has to be unique. For this purpose, observe that:

$$\Upsilon(0) = 1 - 2\lambda^b\beta < 0 < 1 + \lambda^b(1-\beta) = \Upsilon(\infty), \frac{\partial \Upsilon(x)}{\partial x} = \lambda^b e^{-x}(4e^{-x} - 1 + \beta),$$
$$\frac{\partial \Upsilon(x)}{\partial x}\Big|_{x=0} = \lambda^b(3+\beta) > 0, \frac{\partial^2 \Upsilon(x)}{\partial x^2}\Big|_{\frac{\partial \Upsilon(x)}{\partial x}=0} = -4e^{-x}\lambda^b e^{-x} < 0.$$

These properties imply that there exists an $x' \in (0, \infty)$ such that $\Upsilon(x) <$ for all $x < x'$ and $\Upsilon(x) \geq 0$ for all $x \geq x'$. This further implies that the critical value defined above $\bar{x}$ is unique.

To summarize, we have shown that if $\lambda^b \beta \leq \frac{1}{2}$ then the solution is a market-making middleman $x^m \in (0, B)$ for all $x = \frac{B}{S} \in (0, \infty)$. If $\lambda^b \beta > \frac{1}{2}$ then there exists a unique critical value $\bar{x} \in (0, \infty)$ such that the solution is a market-making middleman for $x \geq \bar{x}$ and is a pure market-maker $x^m = 0$ for $x < \bar{x}$.

$\odot$ Second order condition. Finally, we verify the second order condition. Define $\mathbf{X} \equiv [x^m, f, K]$ and write the binding constraints as

$$h_1(\mathbf{X}) = v(x^m, K) - f, \; h_2(\mathbf{X}) = x^m - K.$$

The solution characterized above is a maximum if the Hessian of $\mathcal{L}$ with respect to $\mathbf{X}$ at the solution denoted by $(\mathbf{X}^*, \mu^*)$ is negative definite on the constraint set $\{\mathbf{w} : \mathbf{Dh}(\mathbf{X}^*)\,\mathbf{w} = 0\}$ with $\mathbf{h} \equiv [h_1(\mathbf{X}), h_2(\mathbf{X})]$. This can be verified by using the bordered Hessian matrix, denoted as H.

$$\begin{aligned}
H &\equiv \begin{bmatrix} 0 & D\mathbf{h}(\mathbf{X}^*) \\ D\mathbf{h}(\mathbf{X}^*)^T & D^2_{\mathbf{X}}\mathcal{L}(\mathbf{X}^*, \mu^*) \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & \frac{\partial h_1}{\partial x^m} & \frac{\partial h_1}{\partial f} & \frac{\partial h_1}{\partial K} \\ 0 & 0 & \frac{\partial h_2}{\partial x^m} & \frac{\partial h_2}{\partial f} & \frac{\partial h_2}{\partial K} \\ \frac{\partial h_1}{\partial x^m} & \frac{\partial h_2}{\partial x^m} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial f \partial x^m} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial K \partial x^m} \\ \frac{\partial h_1}{\partial f} & \frac{\partial h_2}{\partial f} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m \partial f} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial f^2} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial K \partial f} \\ \frac{\partial h_1}{\partial K} & \frac{\partial h_2}{\partial K} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial f \partial K} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial K^2} \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & -\frac{\lambda^b}{S} e^{-x^s}(1-c) & -1 & \frac{\lambda^s}{B}(1-\beta)(1-c) \\ 0 & 0 & 1 & 0 & -1 \\ -\frac{\lambda^b}{S} e^{-x^s}(1-c) & 1 & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{x^m}{S} e^{-x^s} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} \\ -1 & 0 & \frac{x^m}{S} e^{-x^s} & 0 & 0 \\ \frac{\lambda^s}{B}(1-\beta)(1-c) & -1 & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} & 0 & 0 \end{bmatrix}
\end{aligned}$$

with

$$\begin{aligned}
\frac{\partial^2 \mathcal{L}(\mathbf{X}^*, \mu^*)}{\partial x^{m2}} &= -\frac{1}{S} e^{-x^s} v + \left(-\frac{1}{S} \frac{x^m}{S} \lambda^b e^{-x^s} \beta + 2 \left(1 + \frac{x^m}{S}\right) e^{-x^s} \frac{\lambda^b}{S} e^{-x^s} - \frac{\lambda^b}{S} \left(1 - e^{-x^s}\right) e^{-x^s} \right)(1-c), \\
\frac{\partial^2 \mathcal{L}(\mathbf{X}^*, \mu^*)}{\partial x^m \partial K} &= -\left(\frac{\lambda^b}{S} e^{-x^s} \beta + \left(1 + \frac{x^m}{S}\right) e^{-x^s} \frac{\lambda^s}{B}(1-\beta) \right)(1-c).
\end{aligned}$$

The determinant is given by $|H| = -\frac{1}{S}\left[e^{-x^s} v(x^m, K^*) + \frac{x^m}{S} \lambda^b e^{-x^s} \beta (1-c) + 3\lambda^b e^{-x^s}\left(1 - e^{-x^s}\right)(1-c)\right] < 0$. Thus, the sufficient condition is satisfied. This completed the proof of Proposition 2. ■

Proof of Corollary 1

In (22), we have:

$$\begin{aligned}
\frac{\partial \phi(x^m \mid ., ., \beta, .)}{\partial \beta} \big|_{(\phi(x^m|\cdot)=0)} &= -\lambda^b(1 - e^{-2x^s}) - \lambda^b e^{-x^s}\left(\frac{B}{S} - 1 + e^{-x^s}\right) < 0, \\
\frac{\partial \phi(x^m \mid B, ., ., .)}{\partial B} \big|_{(\phi(x^m|\cdot)=0)} &= \frac{1}{S}\left[1 + \lambda^b(1-\beta) - \lambda^b e^{-2x^s} + \lambda^b e^{-x^s}(1 - e^{-x^s})\right] > 0 \\
\frac{\partial \phi(x^m \mid ., S, ., .)}{\partial S} \big|_{(\phi(x^m|\cdot)=0)} &= -\frac{x^s}{S}\left[1 + \lambda^b(1-\beta) - \lambda^b e^{-2x^s} + \lambda^b e^{-x^s}(\frac{B}{x^s} - e^{-x^s})\right] < 0 \\
\frac{\partial \phi(x^m \mid ., ., ., \lambda^b)}{\partial \lambda^b} \big|_{(\phi(x^m|\cdot)=0)} &= -\frac{1 - e^{-x^s}}{\lambda^b} < 0.
\end{aligned}$$

Hence, since $\frac{\partial \phi(x^m|\cdot)}{\partial x^m} \big|_{(\phi(x^m|\cdot)=0)} < 0$ (see the proof of Proposition 2), it follows that: $\frac{\partial x^m}{\partial \beta} < 0$; $\frac{\partial x^m}{\partial B} < 0$; $\frac{\partial x^m}{\partial S} > 0$; $\frac{\partial x^m}{\partial \lambda^b} < 0$. This completes the proof of Corollary 1. ■

Proof of Proposition 3

The proof takes the steps very similar to the ones shown in the proof of Lemma 1 and Proposition 1. With the non-linear matching function, the intermediary's profit function is modified to

$$\begin{aligned}\Pi\left(x^m, f, K\right) &= S(1-e^{-x^s})f + \min\left\{K, x^m\right\} p^m \\ &= S(1-e^{-\frac{B-x^m}{S}})f + K(1-\lambda^b(x^D)\beta) - x^m e^{-\frac{B-x^m}{S}}\left(v(x^m,K)-f\right),\end{aligned}$$

where $x^D = \frac{\max\{B-\min\{x^m,K\}-S(1-e^{-x^s}),0\}}{Se^{-x^s}}$, and the surplus function to

$$v\left(x^m, K\right) = 1-\lambda^b(x^D)\beta - \lambda^s(x^D)(1-\beta).$$

With these profit and surplus functions, the constraints and the Lagrangian remain unchanged, and the first orders are given by (19) – (21) (see below the second order conditions). As before, (20) implies that we must have

$$\mu_v = S(1-e^{-x^s}) + x^m e^{-x^s} > 0,$$

and the binding constraint (13). Further, applying μ_v from (20) into (21) gives

$$\mu_k = \mu_0 + 1 - \lambda^b(x^D)\beta + \frac{K}{Se^{-x^s}}\lambda^{b\prime}(x^D)\beta + \frac{1-e^{-x^S}}{e^{-x^s}}\left(\lambda^{b\prime}(x^D)\beta + (\lambda^b(x^D) + x^D\lambda^{b\prime}(x^D))(1-\beta)\right). \quad (23)$$

Substituting μ_k, μ_v into (19),

$$\begin{aligned}\mu_b &= \mu_0 - e^{-x^s}\left(1-\lambda^b(x^D)\beta - \lambda^b(x^D)x^D(1-\beta)\right) + 1 - \lambda^b(x^D)\beta + \frac{B-K}{S}\frac{K}{Se^{-x^s}}\lambda^{b\prime}(x^D)\beta \\ &\quad + \frac{B-K}{S}\frac{1-e^{-x^S}}{e^{-x^s}}\left(\lambda^{b\prime}(x^D)\beta + (\lambda^b(x^D)+x^D\lambda^{b\prime}(x^D))(1-\beta)\right). \end{aligned} \quad (24)$$

Suppose now that $x^m = B$ and $K > 0$. Then, $\mu_k > 0$ in (23) if and only if

$$1-\lambda^b(x^D)\beta + \frac{K}{S}\lambda^{b\prime}(x^D)\beta > 0,$$

and $\mu_b \geq 0$ in (24) if and only if

$$\frac{B-K}{S}\left[(1-\beta)\lambda^b(x^D) + \frac{K}{S}\lambda^{b\prime}(x^D)\beta\right] \geq 0,$$

with $x^D = \frac{B-K}{S}$. Both of these conditions are satisfied only when $K = B$ (which implies $x^D = 0$, satisfying the latter condition) and

$$1-\lambda^b(0)\beta + \frac{B}{S}\lambda^{b\prime}(0)\beta > 0 \quad (25)$$

(satisfying the former condition with $x^D = 0$). Under this condition, the solution is unique, $K = B = x^m$, $x^s = 0$ and $f = v(B,B)$. Hence, we have shown that the solution can be a pure middleman $x^s = 0$ only if (25) holds true, and otherwise the solution has to be $x^s > 0$ (either a marketmaking middleman or a pure marketmaker).

Finally, we verify the second order condition. With the modified profit and surplus functions, as before, the bordered Hessian matrix is computed as

$$
\begin{aligned}
H &\equiv \begin{bmatrix} 0 & D\mathbf{h}(\mathbf{X}^*) \\ D\mathbf{h}(\mathbf{X}^*)^T & D^2_{\mathbf{X}} L(\mathbf{X}^*, \mu^*) \end{bmatrix} \\
&= \begin{bmatrix}
0 & 0 & \frac{\partial h_1}{\partial x^m} & \frac{\partial h_1}{\partial f} & \frac{\partial h_1}{\partial K} \\
0 & 0 & \frac{\partial h_2}{\partial x^m} & \frac{\partial h_2}{\partial f} & \frac{\partial h_2}{\partial K} \\
\frac{\partial h_1}{\partial x^m} & \frac{\partial h_2}{\partial x^m} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial f \partial x^m} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial K \partial x^m} \\
\frac{\partial h_1}{\partial f} & \frac{\partial h_2}{\partial f} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial f} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial f^2} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial K \partial f} \\
\frac{\partial h_1}{\partial K} & \frac{\partial h_2}{\partial K} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial f \partial K} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial K^2}
\end{bmatrix} \\
&= \begin{bmatrix}
0 & 0 & \frac{\partial v(\mathbf{X}^*)}{\partial x^m} & -1 & \frac{\partial v(\mathbf{X}^*)}{\partial K} \\
0 & 0 & 1 & 0 & -1 \\
\frac{\partial v(\mathbf{X}^*)}{\partial x^m} & 1 & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{B}{S} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} \\
-1 & 0 & \frac{B}{S} & 0 & 0 \\
\frac{\partial v(\mathbf{X}^*)}{\partial K} & -1 & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} & 0 & 0
\end{bmatrix}
\end{aligned}
$$

with

$$
\begin{aligned}
\frac{\partial^2 L(\mathbf{X}^*, \mu^*)}{\partial x^{m2}} &= -\frac{1}{S} f - B \frac{\partial^2 \lambda^b}{\partial x^{m2}}(\mathbf{X}^*)\beta - (2 + \frac{B}{S})\frac{\partial v(\mathbf{X}^*)}{\partial x^m}, \\
\frac{\partial^2 L(\mathbf{X}^*, \mu^*)}{\partial x^m \partial K} &= -\frac{\partial \lambda^b}{\partial x^m}\beta - B\frac{\partial^2 \lambda^b}{\partial x^m \partial K}\beta - \frac{B}{S}\frac{\partial v(\mathbf{X}^*)}{\partial K}.
\end{aligned}
$$

The determinant is $|H| = -\frac{1}{S}(1 - \lambda^b(0)\beta) - \frac{B}{S^2}\lambda^{b\prime}(0)\beta < 0$. This completes the proof of Proposition 3. ■

Proof of Proposition 4

As stated in the main text, for $\alpha S \geq B$ the intermediary can achieve the highest possible profit by choosing to be a pure middleman. What remains here is to prove the proposition for $\alpha S < B$. Applying the analysis in the previous section, we get the value of sellers, $W(x^s) = (1 - e^{-x^s} - x^s e^{-x^s})(1 - f)$ and the indifferent condition of buyers, $V^m(x^m) = V^s(x^s)$ where $V^m(x^m) = \min\{\frac{K}{x^m}, 1\}(1 - p^m)$ and $V^s(x^s) = e^{-x^s}(1 - p^s)$. The binding participation constraint for buyers implies that $p^m = 1 - \frac{\lambda^b}{\min\{\frac{K}{x^m}, 1\}}$ and $f = 1 - \frac{\lambda^b}{e^{-x^s}}$, and the binding (18) implies $p^w = (1 - e^{-x^s} - x^s e^{-x^s})\frac{\lambda^b}{e^{-x^s}}$.

To guarantee $f \geq 0$, it is sufficient to assume that

$$\lambda^b \leq e^{-\frac{B - \alpha S}{S}}.$$

This also guarantees $p^m - p^w = 1 - \lambda^b(1 - \frac{1 - e^{-x^s} - x^s e^{-x^s}}{e^{-x^s}}) > 0$ and non negative profits.

Using all these expressions of prices and fee, we can write the profit function as

$$\Pi(x^m, K) = (S - K)(1 - e^{-x^s})(1 - \frac{\lambda^b}{e^{-x^s}}) + \min\{K, x^m\} - x^m \lambda^b - K(1 - e^{-x^s} - x^s e^{-x^s})\frac{\lambda^b}{e^{-x^s}},$$

where $x^s = \frac{B - x^m}{S - K}$. Differentiation yields

$$\frac{\partial \Pi(x^m, K)}{\partial x^m} = \frac{S}{S - K}\frac{1 - e^{-x^s}}{e^{-x^s}}\lambda^b + \frac{\partial \min\{K, x^m\}}{\partial x^m} - e^{-x^s}, \tag{26}$$

which is positive if $\min\{K, x^m\} = x^m$. Hence, the solution has to satisfy $x^m \geq K$.

Observe that: $\lim_{x^m \to B} \Pi(x^m, K) = \Pi$ and $\lim_{x^m \to 0} \Pi(x^m, K) = \tilde{\Pi}(0)$, where $\Pi = \alpha S - B\lambda^b$ is the profit of pure middleman mode and $\tilde{\Pi}(0) = S(1 - e^{-\frac{B}{S}})(1 - \frac{\lambda^b}{e^{-\frac{B}{S}}})$ is the profit of pure market-maker mode. Hence, as before, we can find a profit-maximizing intermediation mode using the following Lagrangian:

$$\mathcal{L} = \Pi(x^m, K) + \mu_k(x^m - K) + \mu_b(B - x^m) + \mu_0 K + \mu_s(\alpha S - K).$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x^m} = \frac{\partial \Pi(x^m, K)}{\partial x^m} + \mu_k - \mu_b = 0, \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial K} = \frac{\partial \Pi(x^m, K)}{\partial K} - \mu_k + \mu_0 - \mu_s = 0, \tag{28}$$

where

$$\frac{\partial \Pi(x^m, K)}{\partial K} = e^{-x^s} + x^s e^{-x^s} - \frac{S}{S-K} \frac{1-e^{-x^s}}{e^{-x^s}} \lambda^b x^s.$$

Suppose $x^m = B$. Then, we must have $\mu_k = 0$ (since $B > \alpha S \geq K$) and so (27) implies we also must have $\frac{\partial \Pi(x^m,K)}{\partial x^m} |_{(x^m=B)} = \mu_b \geq 0$. However, $\frac{\partial \Pi(x^m,K)}{\partial x^m} |_{(x^m=B)} = -1 < 0$, a contradiction. Hence, the solution must satisfy $x^m < B$ (and $\mu_b = 0$), i.e., an active platform.

Summing up the two first order conditions with $\mu_b = 0$,

$$\begin{aligned} \mu_s - \mu_0 &= \frac{\partial \Pi(x^m, K)}{\partial K} + \frac{\partial \Pi(x^m, K)}{\partial x^m} \\ &= x^s e^{-x^s} + (1 - x^s) \frac{S}{S-K} \frac{1-e^{-x^s}}{e^{-x^s}} \lambda^b \\ &= -x^s \frac{\partial \Pi(x^m, K)}{\partial x^m} + \frac{S}{S-K} \frac{1-e^{-x^s}}{e^{-x^s}} \lambda^b > 0 \end{aligned}$$

where the last inequality follows from (27) and $\mu_b = 0$ that implies $\frac{\partial \Pi(x^m,K)}{\partial x^m} = -\mu_k \leq 0$. This implies $\mu_s > 0$, i.e., the binding resource constraint (17), which implies $K = \alpha S$. This completes the proof of Proposition 4. ■

Proof of Proposition 5

In our endowment economy, the middleman's inventory purchase influences the market tightness not only in the C market platform, but also in the D market. Given all sellers are in the D market, the probability that a buyer meets a seller available for trade in the D market changes from $\lambda^b e^{-x^s}$ to $\lambda^b \frac{S-K}{S} e^{-x^s}$. With this change and using the analysis of multi-market search shown in the previous section, we get the value of sellers, $W(x^s) = (1 - e^{-x^s} - x^s e^{-x^s})(v(x^m, K) - f)$ and the middleman's price, $p^m = 1 - \lambda^b \frac{S-K}{S} e^{-x^s} - \frac{x^m e^{-x^s}}{\min\{x^m,K\}} (v(x^m, K) - f)$, where $v(x^m, K) = 1 - \lambda^b \frac{S-K}{S} e^{-x^s}$. Applying these expressions to the profit function, it is immediate that the profit is strictly increasing in the fee f. Hence, the incentive constraints are binding, $f = v(x^m, K)$. Using this result, we can write the profit function as

$$\Pi(x^m, K) = (S-K)(1-e^{-x^s})(1 - \lambda^b \frac{S-K}{S} e^{-x^s}) + \min\{K, x^m\}(1 - \lambda^b \frac{S-K}{S} e^{-x^s}),$$

where $x^s = \frac{B-x^m}{S-K}$. Differentiation yields

$$\begin{aligned} \frac{\partial \Pi(x^m, K)}{\partial x^m} &= -e^{-x^s} \left(1 - \lambda^b \frac{S-K}{S} e^{-x^s} - \lambda^b \frac{S-K}{S} (1 - e^{-x^s})\right) - \frac{\min\{K, x^m\}}{S} \lambda^b e^{-x^s} \\ &\quad + \frac{\partial \min\{K, x^m\}}{\partial x^m} \left(1 - \lambda^b \frac{S-K}{S} e^{-x^s}\right), \end{aligned}$$

which is negative if $\min\{K, x^m\} = K$. Hence, the solution has to satisfy $x^m \leq K$.

Suppose $x^m = B$. Then,

$$\frac{\partial \Pi(x^m, K)}{\partial x^m} |_{x^m=B} = -\frac{B}{S} \lambda^b < 0.$$

Hence, the solution has to be $x^m < B$, i.e., an active platform.

We set the Lagrangian,

$$\mathcal{L} = \Pi(x^m, K) + \mu_0 x^m + \mu_k (K - x^m) + \mu_s (\alpha S - K).$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x^m} = \frac{\partial \Pi(x^m, K)}{\partial x^m} + \mu_0 - \mu_k = 0, \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial K} = \frac{\partial \Pi(x^m, K)}{\partial K} + \mu_k - \mu_s = 0, \tag{30}$$

where

$$\begin{aligned}\frac{\partial \Pi(x^m, K)}{\partial K} &= x^s e^{-x^s}\left(1 - \lambda^b \frac{S-K}{S} e^{-x^s} + x^s \lambda^b \frac{S-K}{S}(1 - e^{-x^s})\right) + \frac{x^s x^m}{S}\lambda^b e^{-x^s} \\ &\quad + \left(1 - e^{-x^s}\right)\left(1 - 2\lambda^b \frac{S-K}{S} e^{-x^s}\right) + \frac{x^m}{S}\lambda^b e^{-x^s}.\end{aligned}$$

Combining (29) and (30),

$$\frac{\partial \Pi(x^m, K)}{\partial x^m} + \frac{\partial \Pi(x^m, K)}{\partial K} = x^s e^{-x^s}\left[\left(1 - \lambda^b \frac{S-K}{S} e^{-x^s}\right) + \frac{S-K}{S}(1 - e^{-x^s})\lambda^b + \frac{x^m}{S}\lambda^b\right] = \mu_s - \mu_0,$$

which implies $\mu_s > 0$ and $K = \alpha S$. This completes the proof of Proposition 5. ■

Additional Appendix

Participation fees

In this Additional Appendix, we show that our main result does not change in a version of our model where the middleman's supply is not observable in the participation stage, but instead the intermediary can use participation fees/subsidy. Suppose now that in the first stage the intermediary announces a set of fees $F \equiv \{f^b, f^s, g^b, g^s\}$ for the platform, where $f^b,\ f^s \in [0,1]$ is a transaction fee charged to a buyer or a seller, respectively, and $g^b,\ g^s \in [-1,1]$ is a registration fee charged to a buyer or a seller, respectively.

As is consistent with the main analysis, we follow the literature of two-sided markets and assume that agents hold pessimistic beliefs on the participation decision of agents on the other side of the market (Caillaud and Jullien, 2003). Agents believe that the intermediary would never supply anything at all unless the C market attracts some buyers. This is the worst situation for the intermediary, and (3) is not the right participation constraint. A pessimistic belief of sellers means that sellers believe the number of buyers participating in the C market is zero whenever

$$\lambda^b \beta > -g^b,$$

where $\lambda^b \beta$ is the expected payoff of buyers in the D market and $-g^b$ is the payoff buyers receive in the C market (it is a participation subsidy when $g^b < 0$).

Single-market search: To induce the participation of agents under those beliefs, the best the intermediary can do is to use a divide-and-conquer strategy, denoted by h. To divide buyers and conquer sellers, referred to as $h = D_b C_s$, it is required that

$$D_b \quad : \quad -g^b \geq \lambda^b \beta, \tag{31}$$

$$C_s \quad : \quad W - g^s \geq 0. \tag{32}$$

The divide-condition D_b tells us that the intermediary should subsidize the participating buyers so that they receive at least what they would get in the D market, *even if the C market is empty.* This makes sure the participation of buyers to the C market whatever happens to the other side of the market. The conquer-condition C_s guarantees the participation of sellers, by giving them a nonnegative payoff – the participation fee $g^s \geq 0$ should be no greater than the expected value of sellers in the C market, $W = W(x^s)$. Observing that the intermediary offers buyers enough to participate, sellers understand that all buyers are in the C market, the D market is empty, and so the expected payoff from the D market is zero. Here, the expected value of sellers in the C market W is defined under the sellers' belief that the intermediary will select the capacity level optimally given the full participation of buyers.

Similarly, a strategy to divide sellers and conquer buyers, referred to as $h = D_s C_b$, requires that

$$D_s \quad : \quad -g^s \geq \lambda^s (1-\beta), \tag{33}$$

$$C_b \quad : \quad V - g^b \geq 0. \tag{34}$$

where $V = \max\{V^s(x^s), V^m(x^m)\}$ is the expected value of buyers in the C market.

Given the participation decision of agents described above, the intermediary's problem of determining the intermediation fees $F = \{f^b, f^s, g^b, g^s\}$ for $h = \{D_b C_s, D_s C_b\}$ is described as

$$\Pi = \max_{F,h} \{B g^b + S g^s + \max_{p^m, K} \Pi(p^m, f, K)\},$$

subject to (31) and (32) if $h = D_b C_s$, or (33) and (34) if $h = D_s C_b$. Here, Bg^b and Sg^s are participation fees from buyers and sellers, respectively, and $\Pi(\cdot)$ is the expected profit in the C market described above. Under either of the divide-and-conquer strategies, the choice of participation fees g^i, $i = b, s$, does not influence anyone's behaviors in the C market. The choice of transaction fees affects the expected value of agents and thus the participation fees and intermediary's profits. However, it does not alter the original solution, a pure middleman, remains optimal.

Proposition 6 *With unobservable capacity and with participation fees, the intermediary sets $f > 1$, $p^m = 1$ and $K = B$. All the buyers buy from the middleman, $x^m = B$, and the platform is inactive, $x^s = 0$. The intermediary makes profits,*

$$\Pi = B - \min\{B\lambda^b\beta, S\lambda^s(1-\beta)\},$$

guaranteeing the participation of agents by $h = D_bC_s$ if $\beta < \frac{1}{2}$ and $h = D_sC_b$ if $\beta > \frac{1}{2}$.

Proof. Consider first $h = D_bC_s$. Then, by (31) and (32), $g^b = -\lambda^b\beta$ and $g^s = W$. For $f > 1$, no buyers go to the platform $x^s = 0$ and all buyers are in the middleman sector $x^m = B$, yielding $g^s = W = 0$. By selecting $K = B$ and $p^m = 1$, the intermediary makes profits,

$$\Pi = -B\lambda^b\beta + \Pi(p^m, 1, B) = (-\lambda^b\beta + 1)B.$$

To show that this is indeed the maximum profit, we have to check two possible cases. Suppose $f = f^b + f^s \leq 1$ and $K = 0$. Then, $x^s = \frac{B}{S}$ and $x^m = 0$, and $g^s = W(B/S) \geq 0$, if there is a non-negative surplus in the platform for buyers, $f^b + p^s \leq 1$, and for sellers, $f^s \leq p^s$. The resulting profit satisfies

$$\begin{aligned} Bg^b + Sg^s + \Pi(p^m, f, 0) &= -B\lambda^b\beta + S(1 - e^{-\frac{B}{S}})(p^s - f^s) + S(1 - e^{-\frac{B}{S}})f \\ &= -B\lambda^b\beta + S(1 - e^{-\frac{B}{S}})(f^b + p^s) \\ &< -B\lambda^b\beta + B = \Pi \end{aligned}$$

for all $f^b + p^s \leq 1$. Hence, this is not profitable.

Suppose $f = f^b + f^s \leq 1$ and $K \in (0, B]$, and both sectors have a non-negative surplus to buyers, i.e., $p^m \leq 1$ and $f^b + p^s \leq 1$. This leads to $x^m \in (0, B)$ and $x^s \in (0, \frac{B}{S})$ that satisfy the add-up requirement (1) and the indifferent condition (2). Then, $g^s = W(x^s) \geq 0$, and the resulting profit is

$$\begin{aligned} &Bg^b + Sg^s + \Pi(p^m, f, K) \\ &\quad = -B\lambda^b\beta + S(1 - e^{-x^s})(p^s - f^s) + S(1 - e^{-x^s})f + \min\{K, x^m\}p^m \\ &\quad < -B\lambda^b\beta + Sx^s(f^b + p^s) + x^m p^m \\ &\quad \leq -B\lambda^b\beta + (Sx^s + x^m)\max\{f^b + p^s, p^m\} \\ &\quad \leq -B\lambda^b\beta + B = \Pi \end{aligned}$$

for all $f^b + p^s \leq 1$ and $p^m \leq 1$. Hence, this is not profitable either. All in all, no deviation is profitable for $h = D_bC_s$.

Consider next $h = D_sC_b$. Then, by (33) and (34), $g^s = -\lambda^s(1-\beta)$ and $g^b = V$. When $f > 1$, no one go to the platform $x^s = 0$ and all buyers are in the middleman sector $x^m = B$ as long as $p^m \leq 1$. This yields $g^b = V = V^m(B) \geq 0$ and $\Pi(p^m, f, B) = Bp^m$ with $K = B$. The profits are

$$\Pi = -S\lambda^s(1-\beta) + B(1 - p^m) + \Pi(p^m, f, K) = -S\lambda^s(1-\beta) + B.$$

To show that this is indeed the maximum profit, we have to check two possible cases. Suppose $f = f^b + f^s \leq 1$ and $K = 0$. Then, $x^s = \frac{B}{S}$ and $x^m = 0$, and $g^b = V = V^s(B/S) \geq 0$, if there is a non-negative surplus in the platform for buyers, $f^b + p^s \leq 1$, and for sellers, $f^s \leq p^s$. This leads to

$$\begin{aligned} Sg^s + Bg^b + \Pi(p^m, f, 0) &= -S\lambda^s(1-\beta) + B\frac{1 - e^{-\frac{B}{S}}}{\frac{B}{S}}(1 - p^s - f^b) + S(1 - e^{-\frac{B}{S}})f \\ &= -S\lambda^s(1-\beta) + S(1 - e^{-\frac{B}{S}})(1 - p^s + f^s) \\ &< -S\lambda^s(1-\beta) + B = \Pi \end{aligned}$$

for all $f^s \leq p^s$. Hence, this is not profitable.

Suppose $f = f^b + f^s \leq 1$ and $K \in (0, B]$, and both sectors have a non-negative surplus to buyers, i.e., $p^m \leq 1$ and $f^b + p^s \leq 1$. This leads to $x^m \in (0, B)$ and $x^s \in (0, \frac{B}{S})$ that satisfy the add-up constraint (1),

$Sx^s + x^m = B$, and the indifferent condition (2), $V^s(x^s) = V^m(x^m)$. Then, $g^b = V = V^s(x^s)$, and the resulting profit is

$$
\begin{aligned}
& Sg^s + Bg^b + \Pi(p^m, f, K) \\
&= -S\lambda^s(1-\beta) + B\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + \min\{K, x^m\}p^m.
\end{aligned}
$$

There are two cases. Suppose $K \geq x^m$. Then, the indifferent condition (2) implies that

$$
p^m = 1 - \frac{1-e^{-x^s}}{x^s}(1-p^s-f^b).
$$

Applying this expression to the profits, we get

$$
\begin{aligned}
& Sg^s + Bg^b + \Pi(p^m, f, K) \\
&= -S\lambda^s(1-\beta) + B\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + x^m\left(1 - \frac{1-e^{-x^s}}{x^s}(1-p^s-f^b)\right) \\
&= -S\lambda^s(1-\beta) + (B-x^m)\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + x^m \\
&= -S\lambda^s(1-\beta) + S(1-e^{-x^s})(1-p^s+f^s) + x^m \\
&< -S\lambda^s(1-\beta) + B
\end{aligned}
$$

for all $f^s \leq p^s$. Suppose $K < x^m$. Then, the indifferent condition implies that

$$
p^m = 1 - \frac{x^m}{K}\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b).
$$

Applying this expression to the profits, we get

$$
\begin{aligned}
& Sg^s + Bg^b + \Pi(p^m, f, K) \\
&= -S\lambda^s(1-\beta) + B\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + K\left(1 - \frac{x^m}{K}\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b)\right) \\
&= -S\lambda^s(1-\beta) + (B-x^m)\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + K \\
&= -S\lambda^s(1-\beta) + S(1-e^{-x^s})(1-p^s+f^s) + K \\
&< -S\lambda^s(1-\beta) + B
\end{aligned}
$$

for all $f^s \leq p^s$. Hence, any deviation is not profitable for $h = D_sC_b$.

Finally, since the intermediary makes the maximum revenue B for either h, which side should be subsidized is determined by the required costs: noting $B\lambda^b = S\lambda^s$, we have $B\lambda^b\beta \gtreqless S\lambda^s(1-\beta) \iff \beta \gtreqless \frac{1}{2}$. This completes the proof of Proposition 6. ■

Multi-market search: With multiple-market search, any non-positive registration fee can ensure that agents are in the C market, since the participation to the C market is not exclusive. Hence, attracting one side of the market becomes less costly. By contrast, conquering the other side becomes more costly, since the conquered side still holds the trading opportunity in the D market. The D_sC_b condition with multiple-market search is

$$
\begin{aligned}
D_s:&\quad -g^s \geq 0, \\
C_b:&\quad \max\{V^s(x^s), V^m(x^m)\} - g^b \geq \lambda^b e^{-x^s}\beta(1-c).
\end{aligned}
$$

The divide-condition D_s tells that now a non-positive fee is sufficient to convince one side to participate. The conquer-condition C_b now needs to compensate for the outside option in the D market. Similarly, the D_bC_s condition becomes

$$
\begin{aligned}
D_b:&\quad -g^b \geq 0, \\
C_s:&\quad W(x^s) - g^s \geq \lambda^s\xi(x^s, x^m)(1-\beta)(1-c).
\end{aligned}
$$

Participation fees are designed to induce buyers and sellers' participation. Once agents join the C market, the participation fees become sunk costs, and will not influence their trading decision.

The intermediary's problem of choosing $F = \{f^b, f^s, g^b, g^s\}$ together with $h = \{D_bC_s, D_sC_b\}$ and $p^m, K \in [0, B]$ are described as

$$\Pi = \max_{F,h,K} \left\{ Bg^b + Sg^s + \max_{p^m} \Pi(p^m, f, K) \right\}, \tag{35}$$

where $\Pi(p^m, f, K) = S(1 - e^{-x^s})f + \min\{K, x^m\} p^m - Kc$. Besides the divide-and-conquer constraints, this maximization problem is also subject to the incentive constraints as described in the main text.

Proposition 7 *In the extended problem described in (35) with unobservable capacity, participation fees and multiple-market search, the determination of the profit-maximizing intermediation mode is identical to the one described in Proposition 2, with $g^i = 0$, $i = s, b$.*

Proof. It suffices to prove that the solution is $g^i = 0$, $i = s, b$ for each intermediation mode, since then the problem (35) will become identical to the one we have already solved in the main text. For a pure middleman mode ($x^m = B$), the intermediary sets $g^b = 0$ to divide buyers, with $p^m = 1 - \lambda^b \beta(1 - c)$ satisfying (6). For a pure market-maker mode ($x^s = 0$), either with D_bC_s or D_sC_b, the intermediary sets the transaction fee to satisfy the binding incentive constraint (13), $f = v(0, 0) = \left[1 - \lambda^b e^{-B/S} - \lambda^s \xi(0, 0)\right](1 - c)$, and $g^b = g^s = 0$.

For a hybrid mode, the intermediary's problem is subject to the incentive constraint (13), and p^m satisfying (15) so that buyers are indifferent between the two modes. We can rewrite the maximization problem (35) as a two-stage problem over a vector $\mathbf{X} \equiv (x^m, f, K) \in \mathbb{X}$, where $\mathbb{X} \equiv [0, B] \times [0, 1] \times [0, K]$:

$$\begin{aligned} \text{Stage 1: } & \max_{(f,K)} Bg^b(\mathbf{X}) + Sg^s(\mathbf{X}) + \Pi(x^m(f, K), f, K) \\ & s.t.\ 0 \le f \le v(x^m(f, K), K),\ 0 \le K \le B. \\ \text{Stage 2: } & \max_{x^m} \Pi(x^m, f, K) \\ & s.t.\ f \le v(x^m, K),\ 0 \le x^m \le B, \end{aligned} \tag{$\mathcal{A}$}$$

where $g^b(\mathbf{X})$ and $g^s(\mathbf{X})$ are given by the binding divide-and-conquer conditions,

$$g^b(\mathbf{X}) = 0,\ g^s(\mathbf{X}) = \left(1 - e^{-x^s} - x^s e^{-x^s}\right)(v(x^m, K) - f),$$

if $h = D_bC_s$, or

$$g^s(\mathbf{X}) = 0,\ g^b(\mathbf{X}) = e^{-x^s}(v(x^m, K) - f).$$

if $h = D_sC_b$. As our objective is to prove $g^i(\mathbf{X}) = 0$, $i = s, b$, all that remains here is to show that $f = v(x^m, K)$ at the solution. However, it is immediate that the objective function in ($\mathcal{A}$) is strictly increasing in f and any change in f ($< v(x^m, K)$) does not influence the other constraints. Hence, as in the original problem, we must have $f = v(x^m, K)$. This completes the proof of Proposition 7. ■

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