A theory of nonperforming loans and debt restructuring

Keiichiro Kobayashi[1] Tomoyuki Nakajima[2]

[1]Keio University

[2]Kyoto University

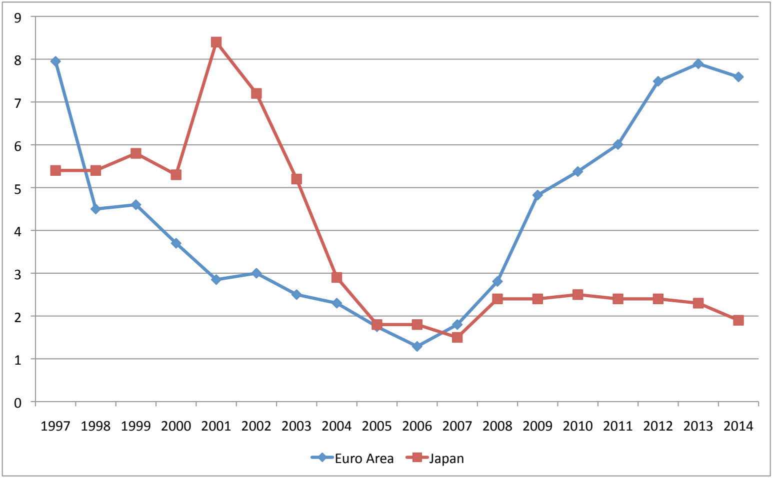
CIGS Conference

May 30, 2016

Nonperforming loans

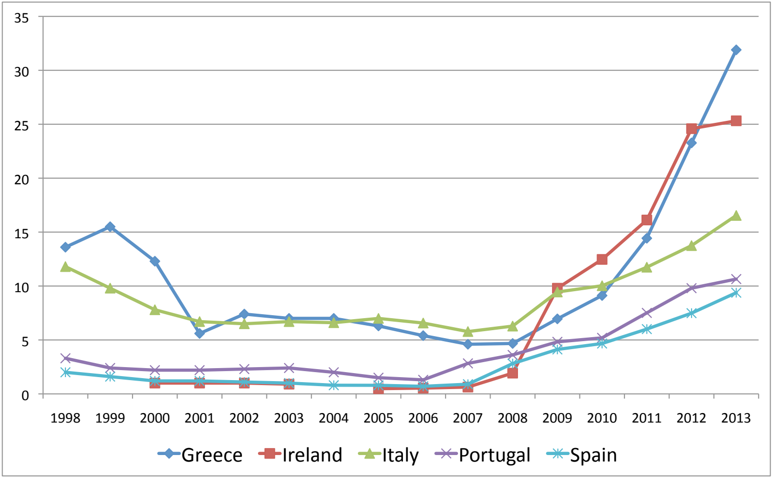
- A loan is classified as nonperforming when payments of interest and/or principal are past due by 90 days or more.

Non performing loans in Euro area and Japan



Notes: Fraction of non-performing loans in total gross loans. Source: World Bank.

Non performing loans in some European countries



Notes: Fraction of non-performing loans in total gross loans. Source: World Bank.

- Nonperforming loans can be a significant source of distortion.
- Our theory is related to but different from debt overhang.
  - Having nonperforming loans is different from just having a lot of debt.
- What is special about nonperforming loans?
  - When loans are nonperforming, the contractual value of debt is different from the present discounted value of repayments.
  - In other words, the value of debt is no longer a "state variable."

- Benchmark model: Albuquerque and Hopenhayn (2004).
  - Borrowing constraint arises because the borrower may default at any time.
  - There exists a maximum amount of debt that the borrower can repay.
- What happens if the amount of debt exceeds the repayable amount?
  - This may happen, for instance, if the borrower's productivity declines, or if the value of the collateral asset falls.
- The lender has two options:
  - rewrite the contract and reduce the amount of debt (debt restructuring);
  - retain the right to the original amount of debt (non-performing loans).

- If the bank reduces the debt, the levels of lending and output converge to their first-best levels in a finite period of time.
  - This is a kind of debt overhang, but inefficiency only lasts temporarily.
- If the bank chooses not to do so, the loans become nonperforming.
  - The PDV of repayments is lower than the contractual value of debt.
  - The equilibrium level of output is permanently lower than the first-best level.
- The value obtained by the bank is higher when the debt is restructured (reduced to a repayable amount).
  - Why would the bank choose not to do that?
- If the reduction of debt involves bargaining, the agreement may not be reached instantly, and debt restructuring could be delayed.
  - We apply the model of Abreu and Gul (2000) to illustrate this point.

- a deterministic version of Albuquerque and Hopenhayn (2004).
- A bank lends to a firm.
- r = common discount rate.
- D_0 = initial debt of the firm.
- b_t = repayment from the firm to the bank:

$$\dot{D}_t = rD_t - b_t.$$

- k_t = short-term loans (working capital) that the firm borrows from the bank:
  - $F(k_t)$ = output produced using k_t.
- x_t = dividends to the owners of the firm:

$$x_t = F(k_t) - rk_t - b_t.$$

- Limited liability:

$$x_t \geq 0.$$

Enforcement constraint

- V_t = value to the firm's owners:

$$V_t = \int_t^{\infty} e^{-r(s-t)} x_s \, ds.$$

- The firm can choose to default at any time t, after receiving working capital k_t.
  - $G(k_t)$ = the value of the outside opportunity of the firm.
  - The bank would receive none when the firm defaults.
- Enforcement constraint:

$$V_t \geq G(k_t).$$

- The liquidation value of the firm is assumed to be zero.

Plans

- At each time t, the contract between the bank and the firm specifies (D_t, r).
- Then, given (D_t, r), the bank offers a plan $\{k^t_{t+s}, b^t_{t+s}, x^t_{t+s}\}_{s\in\mathbb{R}_+}$ to the firm:
  - k^t_{t+s} = working capital provided at time $t+s$;
  - b^t_{t+s} = repayment at $t+s$;
  - $x^t_{t+s} = F(k^t_{t+s}) - rk^t_{t+s} - b^t_{t+s}$.
- The associated values for the bank and the firm are:

$$D^t_{t+s} = \int_{t+s}^{\infty} e^{-r(u-t-s)} b^t_u \, du,$$
$$V^t_{t+s} = \int_{t+s}^{\infty} e^{-r(u-t-s)} x^t_u \, du.$$

- In equilibrium, the bank's offers must be time consistent, i.e.,

$$k^t_s = k^{t'}_s, \quad b^t_s = b^{t'}_s, \quad x^t_s = x^{t'}_s, \quad \text{for all } t < t' \leq s \in \mathbb{R}_+.$$

Feasible plans

- A plan offered at time t is feasible if the limited liability and enforcement conditions are satisfied for all $s \geq 0$:

$$0 \leq x_{t+s}^{t}, \quad \text{and} \quad G(k_{t+s}^{t}) \leq V_{t+s}^{t}.$$

- Γ = the set of all feasible plans.
- $\Gamma(D)$ = the set of all feasible plans such that the value to the bank is D:

$$D = \int_0^\infty e^{-rt} b_t \, dt$$

- D_t is the state variable in this model.
  - We shall consider the "best" Markov plans under different circumstances.

First-best level of production

- $k^* =$ the first-best level of production:

$$F'(k^*) = r.$$

Associated with k^*, define:

$$V^* = G(k^*),$$
$$x* = rV^*,$$
$$b^* = F(k^*) - rk^* - x^*,$$
$$D^* = \frac{b^*}{r}.$$

If $D_0 \leq D^*$, the first-best plan with $k_t^0 = k^*$ for all t is feasible.

Efficient plans

- Given $D \in \mathbb{R}_+$, the (constrained) efficient plan is a plan that solves

$$\max_{\{k_t, b_t, x_t\}_{t\in\mathbb{R}_+} \in \Gamma(D)} \int_0^\infty e^{-rt} x_t \, dt$$

- The efficient plans are expressed using the value of debt as a state variable:
  - There exists a maximum value of debt, D_{max}, which can be repaid by the firm.
  - $V_t = V_e(D_t)$, where $V_e : [0, D_{\text{max}}] \to \mathbb{R}_+$ is a strictly decreasing function.
  - k_t, x_t, and b_t are given as

$$k_e(D_t) = \begin{cases} G^{-1}\left[V_e(D_t)\right], & \text{for } D_t > D^*, \\ k^*, & \text{for } D_t \leq D^*, \end{cases}$$

$$x_e(D_t) = \begin{cases} 0, & \text{for } D_t > D^*, \\ rV_e(D_t), & \text{for } D_t \leq D^*, \end{cases}$$

$$b_e(D_t) = F\left[k_e(D_t)\right] - rk_e(D_t) - x_e(D_t),$$

Dynamics of the efficient plans

- If $D_0 \leq D^*$, the first-best is attained in the efficient plan:

$$k_{e,t} = k^*, \quad \text{for all } t \geq 0.$$

- For $D_0 \in (D^*, D_{\max}]$, the level of production is inefficiently low initially (debt overhang), but converges to the first-best level in finite time.
    - Let

$$\bar{t} \equiv \frac{1}{r} \ln \left(\frac{V^*}{V_e(D_0)} \right).$$

Then

$$V_{e,t} = \begin{cases} e^{rt} V_e(D_0), & \text{for } t < \bar{t}, \\ V^*, & \text{for } t \geq \bar{t}, \end{cases}$$

$$k_{e,t} = \begin{cases} G^{-1}(V_{e,t}), & \text{for } t < \bar{t}, \\ k^*, & \text{for } t \geq \bar{t}, \end{cases}$$

Markov perfect equilibrium

- At each point in time t, given contract (D_t, r), the bank offers a plan $\{k^t_{t+s}, b^t_{t+s}, x^t_{t+s}\}_{s\in\mathbb{R}_+}$ to the firm subject to the constraint:

$$\int_0^\infty e^{-r(s-t)} b^t_{t+s}\, ds, \le D_t, \quad \text{and} \quad x^t_{t+s} \ge 0.$$

  Then, given this offer, the firm decides whether or not to default.

- The efficient plan $\{k_e(D), x_e(D), b_e(D), V_e(D)\}$ is attained as a Markov perfect equilibrium with the following strategies:
  1. at each time t, the bank offers $\{k^t_{t+s}, b^t_{t+s}, x^t_{t+s}\}_{s\in\mathbb{R}_+}$ such that $k^t_{t+s} = k_e(D_{t+s})$, $b^t_{t+s} = b_e(D_{t+s})$, and $x^t_{t+s} = x_e(D_{t+s})$, where D_{t+s} is the solution to $\dot{D}_{t+s} = rD_{t+s} - b_e(D_{t+s})$ with initial value D_t;
  2. given $(D_t,\, b^t_t,\, k^t_t,\, x^t_t)$, the firm defaults if either (i) $V_e(D) < G(k^t_t)$, or (ii) $x^t_t < x_e(D_t)$.

Too much debt

- To analyze non-performing loans, suppose that there is an unexpected shock in period 0 so that

$$D_0 > D_{max}.$$

- Two options for the bank:
  1. rewrite the contract to reduce the amount of debt to D_{max};
  2. retain the right to D_0 with understanding the firm is never able to repay it.
- If the debt is reduced to D_{max},
  - then the efficient plan discussed in the previous section can be implemented.
  - Nonperforming loans would not arise.
- If the bank keeps the right to $D_0 > D_{max}$,
  - the PDV of future repayments to the bank would be less than the contractual value of the firm's debt.
  - The loan becomes nonperforming.

Contractual values of debt

- D_0^c = contractual value of debt in period 0.
- If the firm repays $\{b_t\}_{t\in\mathbb{R}_+}$, then the contractual value of debt evolves as

$$D_t^c = e^{rt} D_0 - \int_0^t e^{r(t-s)} b_s \, ds.$$

- $d_t(\{b_{t+j}\}_{j\in\mathbb{R}_+})$ = PDV of repayments $\{b_{t+j}\}$ after t:

$$d_t(\{b_{t+j}\}_{j\in\mathbb{R}_+}) = \int_0^\infty e^{-rj} b_{t+j} \, dj.$$

- If $D_0^c > D_{\max}$, then for any feasible repayment plan $\{b_t\}_{t\in\mathbb{R}_+}$,

$$D_t^c > d_t(\{b_{t+j}\}_{j\in\mathbb{R}_+}).$$

Thus, the bank also suffers from an enforcement problem.

Debt is no longer a state variable

- For $D_0^c > D_{\max}$, $\Gamma(D_0^c) = \emptyset$.
- The bank can make an offer with the PDV of repayments less than D_0^c.
  - Thus, the set of feasible plans that the bank with D_0^c can offer is

$$\overline{\Gamma}(D_0^c) \equiv \bigcup_{D \le D_0^c} \Gamma(D).$$

- The set of feasible plans for the bank is independent of the value of initial debt if $D_0^c > D_{\max}$:

$$\overline{\Gamma}(D_0^c) = \Gamma(D_{\max}) = \Gamma, \quad \forall D_0^c > D_{\max}.$$

- In other words, the value of debt is no longer a state variable.

Markov plans

- With $D_0^c > D_{\max}$, there is no state variables.
  - Markov plans are constant plans.
- Let $\underline{\Gamma}$ = the set of all feasible constant plans.

$$\underline{\Gamma} \equiv \left\{ (k_t, b_t, x_t) \in \Gamma \;\middle|\; (k_t, b_t, x_t) = (k, b, x), \quad \forall t \in \mathbb{R}_+ \right\}.$$

- The highest value the bank can obtain with Markov plans is:

$$\max_{\{k_t, b_t, x_t\} \in \underline{\Gamma}} \int_0^\infty e^{-rt} b_t \, dt.$$

- The solution to this problem is given by $\{k_{\text{npl}}, b_{\text{npl}}, x_{\text{npl}}, D_{\text{npl}}, V_{\text{npl}}\}$, where k_{npl} is the solution to:

$$F'(k_{\text{npl}}) = r + rG'(k_{\text{npl}}),$$

and $b_{\text{npl}} = F(k_{\text{npl}}) - rk_{\text{npl}} - rG(k_{\text{npl}})$, $x_{\text{npl}} = F(k_{\text{npl}}) - rk_{\text{npl}} - b_{\text{npl}}$, $D_{\text{npl}} = \frac{b_{\text{npl}}}{r}$, $V_{\text{npl}} = \frac{x_{\text{npl}}}{r}$.

Markov Perfect Equilibrium

- This can be obtained as a Markov Perfect Equilibrium with the following strategies:
  1. at each time t, the bank offers $\{k_{t+s}^t, b_{t+s}^t, x_{t+s}^t\}_{s\in\mathbb{R}_+}$ such that $k_{t+s}^t = k_{\text{npl}}$, $b_{t+s}^t = b_{\text{npl}}$, and $x_{t+s}^t = x_{\text{npl}}$ for all t and s;
  2. given the offer $\{k_{t+s}^t, b_{t+s}^t, x_{t+s}^t\}_{s\in\mathbb{R}_+}$ from the bank, the firm defaults if either (i) $G(k_t^t) > V_{\text{npl}}$ or (ii) $x_t^t < x_{\text{npl}}$.

Persistence of inefficiency

- Inefficiency lasts permanently:

$$k_t = k^{\text{npl}} < k^*.$$

- Note that $D_{\text{max}} > D_{\text{npl}}$, i.e., the value to the bank is higher when debt is restructured.
  - Then why would the bank choose not to restructure debt?
  - If debt restructuring is costly, and the cost exceeds $D_{\text{max}} - D^{\text{npl}}$, then the bank would choose to hold nonperforming loans.
- But even without such costs, if debt restructuring involves bargaining, then there can be an inefficient delay in reaching an agreement.

Inefficient delays in bargaining

- Rubinstein (1982): a complete information model of bargaining.
  - The unique SPE is efficient (the agreement is reached immediately).
- Inefficient delay may occur with asymmetric information:
  - Abreu and Gul (2000), Feinberg and Skrzypacz (2005), Fuchs and Skrzypacz (2010), etc.
- Here we apply the model of Abreu and Gul (2000).
  - Debt restructuring is inefficiently delayed, and loans become nonperforming.

Abreu and Gul (2000)

- Two agents bargain over their shares of a pie.
- Each agent may be either "rational" or "irrational."
  - An irrational type is identified by a fixed offer and acceptance rule.
- Independence-from-procedures result:
  - Regardless of the details of the bargaining protocol, the equilibrium distribution of outcomes in discrete-time bargaining games converge to the same limit.
- This limit corresponds to the (unique) equilibrium in the continuous-time bargaining game with a war of attrition structure.
  - The rational type of each agent pretends to be their irrational type.
  - Their strategy is described by a distribution over the time to concede.
- The equilibrium exhibits inefficient delay.
  - As the probability of irrationality goes to zero, delay and inefficiency disappear.

Two banks

- Continue to consider the case where $D_0^c > D_{\max}$.
- Bank i holds a share $\omega^i \in (0,1)$ of D_0^c.
  - Before debt restructuring, if the firm repays $\hat{b}_t$, then bank i receives $\omega^i \hat{b}_t$.
- Two banks bargain over their shares of the value of the debt after it is reduced to $D_{\max}$.
- Simplifying assumptions:
  - When the two banks bargain over their shares of $D_{\max}$, they take as given the repayments $\{\hat{b}_t\}$ that the firm makes before debt restructuring.
  - On the other hand, the repayments before debt restructuring, $\{\hat{b}_t\}$, are determined to maximize the joint surplus of the banks taking as given the equilibrium in the bargaining game.

Bargaining between the two banks

- The irrational type of bank i is identified by a number $\alpha^i \in (0,1)$.
  - It always demands $\alpha^i D_{\max}$ and would accept the offer from the other bank if and only if its share is greater than or equal to α^i.
  - $z^i =$ initial probability that bank i is irrational.
- Each bank's strategy is described by a cdf function $\Phi^i(t)$, i.e., the probability that lender i concedes to the other lender by time t (inclusive).
- In equilibrium, there exists a time $T^0 > 0$ such that
  - $\Phi^i(t)$ is continuous for all $t > 0$ and $i = 1, 2$;
  - $\Phi^i(t)$ is constant for $t \geq T^0$ and $i = 1, 2$;
  - $\Phi^i(t)$ is strictly increasing for $t \in [0, T^0)$ and $i = 1, 2$.

- Given $\{b_t\}$ and $\Phi^j(t)$, the expected value of bank i when it concedes to bank j at time t is:

$$u_t^i = \int_{s=0}^{t} \left\{ \int_{w=0}^{s} e^{-rw} \omega^i \hat{b}_w \, dw + e^{-rs} \alpha^i D_{\max} \right\} d\Phi^j(s) + \left[1 - \Phi^j(t)\right] \left\{ \int_{s=0}^{t} e^{-rs} \omega^i \hat{b}_s \, ds + e^{-rt} (1 - \alpha^j) D_{\max} \right\}$$

- Using the condition that $\frac{du_t^i}{dt} = 0$ for $t \in (0, T^0)$, the equilibrium is given by:

$$\Phi^j(t) = \begin{cases} 1 - c^j \exp\left(-\int_0^t \lambda^j(s)\, ds \right), & t < T^0, \\ 1 - z^j, & t \geq T^0. \end{cases}$$

where

$$\lambda^j(t) \equiv \frac{(1 - \alpha^j) r - \beta^i(t)}{\alpha^1 + \alpha^2 - 1},$$

with $\beta^i(t) \equiv \frac{\omega^i \hat{b}_t}{D_{\max}}$.

- (c^1, c^2, T^0) is determined as follows:

$$T^0 \equiv \min(T^1, T^2),$$

where T^i is defined implicitly by

$$1 - \exp\left(\int_0^{T^i} \lambda^i(s)\, ds\right) = 1 - z^i,$$

and c^i is determined by

$$1 - c^i \exp\left(\int_0^{T^0} \lambda^i(s)\, ds\right) = 1 - z^i.$$

Repayments before debt restructuring

- Define $\Phi(t)$ = the probability that either one of the two lenders concede by time t:

$$\Phi(t) = \begin{cases} 1 - c \exp\left(-\int_0^t \lambda(s)\, ds\right), & t < T^0, \\ 1 - z, \quad t \geq T^0, \end{cases}$$

  where $c \equiv c^1 c^2$, $\lambda(s) \equiv \lambda^1(s) + \lambda^2(s)$, and $z = z^1 z^2$.

- Given Φ and T^0, we consider a Markov plan that maximizes the joint value of the two banks.
- Because of T^0, time t is a payoff-relevant state variable in this problem.

- Let $\{\hat{k}_t, \hat{b}_t, \hat{x}_t\}$ is the plan before the debt reduction.
- Conditional on the event that debt restructuring has not been done by t, the values to the banks and the firm are:

$$\hat{D}_t = \int_t^{T^0} \left\{ \int_t^s e^{-r(w-t)} \hat{b}_w \, dw + e^{-r(s-t)} D_{\max} \right\} \frac{d\Phi(s)}{1-\Phi(t)} + \frac{1-\Phi(T^0)}{1-\Phi(t)} e^{-r(T^0-t)} D_{\text{npl}},$$

$$\hat{V}_t = \int_t^{T^0} \left\{ \int_t^s e^{-r(w-t)} \hat{x}_w \, dw + e^{-r(s-t)} V_{\min} \right\} \frac{d\Phi(s)}{1-\Phi(t)} + \frac{1-\Phi(T^0)}{1-\Phi(t)} e^{-r(T^0-t)} V_{\text{npl}}.$$

- Given Φ and T^0, let $\hat{\Gamma}(t)$ be the set of all plans $\{\hat{k}_t, \hat{b}_t, \hat{x}_t\}_{t\in[0,T^0)}$ such that

$$\hat{V}_t \geq G(\hat{k}_t), \quad \text{and} \quad \hat{x}_t = F(\hat{k}_t) - r\hat{k}_t - \hat{b}_t \geq 0,$$

where $\hat{V}_t$ is given as above.

- The Markov plan that maximizes the joint surplus of the two banks is:

$$\max_{\{\hat{k}_t,\hat{b}_t,\hat{x}_t\}\in\hat{\Gamma}(0)} \hat{D}_0.$$

- In this solution, $\hat{x}_t = 0$ for all $t < T^0$ and

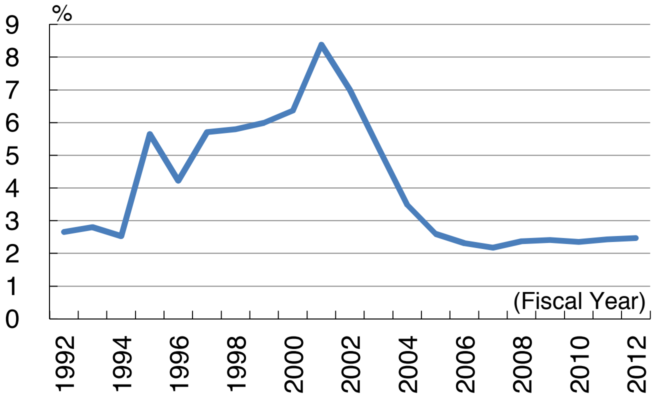
$$\hat{V}_t = V_{\min}\int_t^{T^0} \lambda(s)\exp\left(-\int_t^s [\lambda(w)+r]\,dw\right)\,ds + V_{\text{npl}}\exp\left(-\int_t^{T^0}[\lambda(s)+r]\,ds\right).$$

  Given $\hat{V}_t$ for $t < T^0$,

$$\hat{k}_t = G^{-1}(\hat{V}_t), \quad \hat{b}_t = F(k_t) - r\hat{k}_t.$$

- The equilibrium as a whole is given by $(\{\hat{b}_t, \hat{k}_t, \hat{x}_t\}_{t\in[0,T^0)}, \Phi)$ that jointly solves these two sets of the conditions.

Nonperforming loans in Japan (relative to GDP)



Notes: Outstanding loans are measured as a fraction of GDP.
Sources: Financial Services Agency, The Japanese Government, *Status of Non-Performing Loans*.

Interpretation of Japan's lost decades

- Evidence on evergreening and "zombie firms" in Japan:
  - Peek and Rosengren (2005), Caballero, Hoshi, and Kashyap (2008), etc.
  - Fukuda and Nakamura (2011): Most firms which are identified as zombies by Caballero, Hoshi and Kashyap (2008) did recover substantially in the 2000s.
- In the 1990s, nonperforming loans piled up and evergreening was widespread.
  - It created zombie firms as discussed by Caballero, Hoshi, and Kashyap (2008).
- In the 2000s, the bankruptcy and reorganization procedures were reformed.
  - The Civil Rehabilitation Law was enacted in 2000 and the Alternative Dispute Resolution Law followed in 2004.
  - Outstanding debt decreased rapidly, and most zombie firms recovered as shown by Fukuda and Nakaumara (2011).

Summary: Debt restructuring

- Suppose that $D_0^c > D_{\max}$.
- Debt restructuring:
  - The bank reduces D_0^c to $D_{\max}$.
- The contractual value of debt is used as a state variable.
- The efficient plan is the solution to

$$\max_{\{k_t, b_t, x_t\}_{t \in \mathbb{R}_+} \in \Gamma(D_{\max})} \int_0^\infty e^{-rt} x_t \, dt$$

  which has a Markovian form: $\{k_e(D), x_e(D), b_e(D)\}$.
- Inefficiency (debt overhang) only lasts temporariliy.
  - The first best allocation is attained in a finite period of time.

Summary: Nonperforming loans

- Suppose that the bank does not reduce D_0^c.
  - The loan becomes nonperforming.
  - The PDV of repayments is less than the value of debt.
- The contractual value of debt is no longer a state variable.
  - The set of feasible plans does not depend on the value of debt on the contract.
- Markov plans are constant plans.
  - Let $\bar{\Gamma}$ = the set of constant feasible plans.
- The most profitable Markov plan for the bank is the solution to

$$\max_{\{k_t, b_t, x_t\}_{t \in \mathbb{R}_+} \in \bar{\Gamma}} \int_0^\infty e^{-rt} b_t \, dt$$

  which is given as $\{k_{\text{npl}}, x_{\text{npl}}, b_{\text{npl}}\}$.
- Inefficiency lasts permanently.
- Note, however, that $D_{\max} > D_{\text{npl}}$.

Summary: Bargaining with two banks

- Apply Abreu and Gul's model of bargaining with asymmetric information.
  - Each bank may be irrational with a fixed offer and acceptance rule.
- The equilibrium in the bargaining game between the two banks exhibits an inefficient delay.
  - Debt restructuring is not done immediately, and the loan becomes nonperforming.
- The time t becomes a state variable.
  - $\hat{\Gamma}(t) =$ the set of all feasible plans after t (before debt restructuring).
- The Markov plan that maximizes the joint surplus for the banks is the solution to:

$$\max_{\{\hat{k}_t, \hat{b}_t, \hat{x}_t\} \in \hat{\Gamma}(0)} \hat{D}_0,$$

which has a Markovian form: $\{\hat{k}(t), \hat{x}(t), \hat{b}(t)\}$.