Fiscal policy and debt management with incomplete markets

Anmol Bhandari	David Evans	Mikhail Golosov	Thomas Sargent
Minnesota	Oregon	Princeton	NYU

Classical question in macro public finance

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- Concerns that current debt levels are "too high" for rich countries...
- And "too low" (negative) for China, Norway,...

This paper

- A theory of optimal public debt management
  - Ramsey planner with distortionary taxation and incomplete markets
- Contribution: develop quadratic approximations that characterize moments of the invariant distribution in closed form
- Derive explicit formulas ("sufficient statistics") for the moments of the invariant distribution

This paper

- Most of the focus:
  - mean ("target") debt level
  - speed of reversion to the target
  - variance of debt in the invariant distribution
- Key insight: optimal debt **minimizes risk** for the gov't
- Other questions that our framework addresses
  - what is the optimal composition of portfolio of gov't debts?
  - how should gov't debt respond to shocks?
  - how should government set taxes, transfers, tax rates over the cycle?

Results

- Main formulas:

$$\text{target debt} = -\frac{cov\,(\text{returns, deficit})}{var\,(\text{returns})}$$

$$\text{speed of convergence} = \frac{1}{1+\beta^2 var\,(\text{returns})}$$

- Here:
  - returns: MU-adjusted returns on gov't portfolio of debts/assets
  - deficit: MU-adjusted present value of primary deficits
- Sufficient statistics: can be easily computed given observed data

Calibration: US 1947-2010

- Optimal debt level keeping maturity constant:
  - target debt level: -7% of GDP
  - speed of mean reversion: 250 years (half life)
  - std. deviation: 0.26
- Tax rates are peristent and smooth
- Taxes and debt have similar volatility in the data but are less persistent

Related literature

1. Complete markets: Lucas-Stokey, Chari-Christiano-Kehoe, Angeletos, Buera-Nicolini
   - any debt level is optimal, all fiscal hedging through (equivalent of) Arrow securities
   - hard to see how to achieve that with real world instruments
2. Incomplete markets: Barro, Bohn, Faraglia-Marcet-Scott, Lustig-Sleet-Yeltekin
   - mostly numerical, often for models with counterfactual returns
   - analytics (Barro): any debt level is optimal
3. Accumulate enough assets to never use taxes: Aiyagari et al (2002), Farhi (2010)
   - can get their results in the limit, knife-edge cases
4. Portfolio theory: Markowitz, Merton, ...
   - GE, benevolence, interaction of portfolio decisions with taxation
5. Nominal debt, possibility of default
   - have not studied, but our approach should work there too

The simplest model

- Continuum of identical agents with preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[c_t - \frac{1}{1+\gamma} l_t^{1+\gamma} \right]$$

- No capital + exogenous gov't expenditures

$$c_t + g_t = l_t$$

- Gov't can use proportional tax τ_t and trade with agents one-period security (in zero net supply) at price q_t with stochastic payoff p_t

$$g_t + p_t \mathrm{B}_{t-1} = \tau_t l_t + q_t \mathrm{B}_t$$

- iid shocks for (g_t, p_t) , B_t is in a compact set
- Let $B_t \equiv q_t \mathrm{B}_t$, $R_t \equiv p_t / q_{t-1}$

Characterization

Lemma

$\{c_t, l_t, R_t, B_t, \tau_t\}_{t=0}^{\infty}$ is a competitive equilibrium if and only if $\{l_t, B_t\}_{t=0}^{\infty}$ satisfies

$$\underbrace{l_t - l_t^{1+\gamma}}_{=\tau_t l_t} + B_t = R_t B_{t-1} + g_t$$

- Easier to express hours as a function of tax revenues Z

$$\begin{aligned} Z &\equiv l(Z) - l(Z)^{1+\gamma} \\ \Psi(Z) &= \frac{1}{1+\gamma} l(Z)^{1+\gamma} \end{aligned}$$

- Consumption is a residual

$$c_t = (1+\gamma)\Psi(Z_t) + R_t B_{t-1} - B_t$$

Ramsey problem in recursive form

- Bellman equation (state $s = (g, p)$) :

$$V(B) = \max_{\{Z(s), B'(s)\}} \mathbb{E}\left[RB - B' + \gamma \Psi(Z) + \beta V(B')\right]$$

subject to

$$Z(s) + B'(s) = \underbrace{R(s) B + g(s)}_{\equiv E(B,s)} \text{ for all } s$$

- Policy functions $\tilde{B}(B, s)$, $\tilde{Z}(B, s)$, $\tilde{\tau}(B, s)$ induce optimum $\left\{\tilde{B}_t, \tilde{Z}_t, \tilde{\tau}_t\right\}_t$

Optimal policy

- **Monotonicity**: $\tilde{B}, \tilde{Z}, \tilde{\tau}$ are increasing in E
- **Distortion smoothing**:

$$V'\left(\tilde{B}_t\right) = \mathbb{E}_t V'\left(\tilde{B}_{t+1}\right) + \beta cov_t\left(R_{t+1}, V'\left(\tilde{B}_{t+1}\right)\right)$$

- **Uniqueness**: $\tilde{B}_t$ converges to a unique invariant distribution

Optimal policy

- Our goal: characterize properties of the invariant distribution
- Amount of risk depends on debt level:

$$E(B, s) = R(s)\, B + g(s)$$

- Let B^* be the debt level that minimizes $var\,(E(B, \cdot))$:

$$B^* \equiv -\frac{cov\,(R, g)}{var\,(R)}$$

- Let Z^* be the level of tax revenues that satisfies budget constraint in expectation

$$Z^* \equiv \bar{g} + \frac{1-\beta}{\beta} B^*$$

Special case: p and g are perfectly correlated

- If $corr\,(p,g) = \pm 1$ then $E\,(B^*, s)$ is independent of s
  - risk is completely eliminated if $B_t = B^*$
- Monotonicity of policy rules:

$$\begin{aligned} B &< B^* \Longrightarrow cov\left(R\left(\cdot\right), V'\left(\tilde{B}\left(B,\cdot\right)\right)\right) > 0 \\ B &= B^* \Longrightarrow cov\left(R\left(\cdot\right), V'\left(\tilde{B}\left(B,\cdot\right)\right)\right) = 0 \\ B &> B^* \Longrightarrow cov\left(R\left(\cdot\right), V'\left(\tilde{B}\left(B,\cdot\right)\right)\right) < 0 \end{aligned}$$

- Euler equation and Martingale convergence theorem imply

$$\tilde{B}_t \to B^*,\ \tilde{Z}_t \to Z^*,\ var\left(\tilde{\tau}_t\right) \to 0$$

Imperfect hedging

- If shocks are imperfectly correlated, complete elimination of risk is impossible, invariant distribution of $\left\{\tilde{B}_t, \tilde{Z}_t\right\}$ is not degenerate
- Our approach: take quadratic approximation of $\tilde{B}(B, s)$ around B as variance of shocks goes to zero
- Simple linear policy rules

$$\begin{aligned}\tilde{B}(s, B) &= B + \beta[g(s) - \bar{g}] + \beta\left[R(s) - \beta^{-1}\right] \\ &\quad - \beta^2 var(R) B - \beta^2 cov(R, g) + O\left(\|s\|^3, (1-\beta)\|s\|^2\right)\end{aligned}$$

Main result: moments of invariant distribution

Proposition: the mean, variance and mean reversion of $\left\{\tilde{B}_t, \tilde{Z}_t\right\}$ satisfy, up to order $O\left(\|s\|, (1-\beta)\right)$:

- The mean of the invariant distribution

$$\mathbb{E}\tilde{B}_t = B^*, \quad \mathbb{E}\tilde{Z}_t = Z^*$$

- Speed of mean reversion

$$\frac{\mathbb{E}_{t-1}\left(\tilde{B}_t - B^*\right)}{\tilde{B}_{t-1} - B^*} = \frac{\mathbb{E}_{t-1}\left(\tilde{Z}_t - Z^*\right)}{\tilde{Z}_{t-1} - Z^*} = \frac{1}{1 + \beta^2 var\left(R\right)}$$

- Variance of the invariant distribution

$$var\left(\tilde{B}_t\right) = \frac{var\left(E\left(B^*\right)\right)}{var\left(R\right)}$$

$$var\left(\tilde{Z}_t\right) = 0$$

Intuition

- Back to Euler equation:

$$\begin{aligned} cov\left(R_{t+1}, V'\left(\tilde{B}_{t+1}\right)\right) &\propto cov\left(R_{t+1}, E_{t+1}\right) + O\left(\|s\|^3\right) \\ &\propto \frac{\partial}{\partial B} var\left(R_{t+1}, E_{t+1}\left(B, \cdot\right)\right) + O\left(\|s\|^3\right) \end{aligned}$$

- $var\left(R_{t+1}, E_{t+1}\left(B, \cdot\right)\right)$ is minimized at $B = B^*$:

$$\begin{aligned} B &< B^* \Longrightarrow cov\left(R_{t+1}, E_{t+1}\left(B, \cdot\right)\right) > 0 \\ B &= B^* \Longrightarrow cov\left(R_{t+1}, E_{t+1}\left(B, \cdot\right)\right) = 0 \\ B &> B^* \Longrightarrow cov\left(R_{t+1}, E_{t+1}\left(B, \cdot\right)\right) < 0 \end{aligned}$$

The optimal policy is to revert to risk-minimizing position

Main insights

- Target debt level: minimizes risk
  - target level is positive if $cov(R, g) < 0$
  - target level is negative (accumulate assets) if $cov(R, g) > 0$
- Speed of mean reversion is determined by $var(R)$
  - $var(R) = 0$ implies debt is random walk as in Barro (1979)
- The less hedging B^* offers, the bigger the variance of the invariant distribution is
- For β close to one, $var\left(\tilde{Z}_t\right)$ and $var(\tilde{\tau}_t)$ is close to 0 $\Longrightarrow$ all adjustment to shock is done via debt

Reliability of approximations

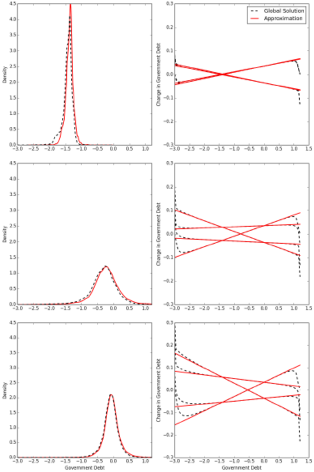

Figure 1: Using the quadratic approximation (red line) and a more accurate global approximator (black line), the top, middle, and bottom panels plot smoothed kernel densities (left side) and decision rules (right side) associated with values of $\sigma_\varepsilon = 0.001, 0.02$, and 0.04. The right panel displays policies $B(s, B\_) - B\_$ for states s that attain the extreme values for $\{g(s)\}$ and $\{p(s)\}$.

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Extensions

- Richer asset structure
- Persistence, other shocks
- Risk aversion

Extension 1: richer market structure

- Suppose there are K assets with arbitrary payoffs, duration
  - note that fixed portfolio weights are isomorphic to one security
- Notation: $\mathbf{R} = \left[R^1, ..., R^K\right]$; $\mathbb{C}\left[\mathbf{R}, \mathbf{R}\right]$ and $\mathbb{C}\left[\mathbf{R}, g\right]$ are covariances matrices
  - assume that $\mathbb{C}\left[\mathbf{R}, \mathbf{R}\right]$ is non-singular
- Risk-minizining total debt level and porfolio are

$$\begin{aligned}(B^*, \mathbf{B}^*) &\equiv \arg \min_{B = \mathbf{1}^T \mathbf{B}} var\left(\sum R^k B^k + g\right) \\ &= \left(-\mathbf{1}^T \mathbb{C}\left[\mathbf{R}, \mathbf{R}\right]^{-1} \mathbb{C}\left[\mathbf{R}, g\right], \mathbb{C}\left[\mathbf{R}, \mathbf{R}\right]^{-1} \mathbb{C}\left[\mathbf{R}, g\right]\right)\end{aligned}$$

Optimal portfolio with active debt management

- Mean debt level:

$$E\left(\tilde{B}_t\right) = B^*$$

- Mean reversion:

$$\frac{\mathbb{E}_{t-1}\left(\tilde{B}_t - B^*\right)}{\left(\tilde{B}_{t-1} - B^*\right)} = \frac{\beta^{-2}\mathbf{1}^T\mathbb{C}\left[\mathbf{R},\mathbf{R}\right]^{-1}\mathbf{1}}{1+\beta^{-2}\mathbf{1}^T\mathbb{C}\left[\mathbf{R},\mathbf{R}\right]^{-1}\mathbf{1}}$$

- Optimal portfolio:

$$\mathbf{B}_t = \mathbf{B}^* + \frac{\mathbb{C}\left[\mathbf{R},\mathbf{R}\right]^{-1}\mathbf{1}}{\mathbf{1}^T\mathbb{C}\left[\mathbf{R},\mathbf{R}\right]^{-1}\mathbf{1}}\left(\tilde{B}_t + \mathbf{1}^T\mathbb{C}\left[\mathbf{R},\mathbf{R}\right]^{-1}\mathbb{C}\left[\mathbf{R},g\right]\right)$$

Some insights

- Optimal portfolio chosen to **minimize risk**
  - unlike Merton's investor's, no risk-return trade-off
  - gov't benevolent + general equilibrium implies that not optimal to chase returns for gov't
- Speed of mean reversion is slower with more asset: can hedge risks better when $B_t \neq B^*$
- Higher debt $B_t \implies$ higher weight of securities with small $var\left(R^k\right)$

Extension 2: persistent shocks

- Suppose that shocks are first order Markov + TFP shocks θ + discount factor shocks
- For any random variable x let

$$PV(x; s) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t x_t \middle| s_0 = s\right].$$

Optimal policy with persistent shocks

- Optimal debt satisfies

$$V_t'\left(\tilde{B}_t\right) = \mathbb{E}_t V_{t+1}'\left(\tilde{B}_{t+1}\right) + \beta cov_t\left(R_{t+1}, V_{t+1}'\left(\tilde{B}_{t+1}\right)\right)$$

- Our quadratic approximations imply that in invariant distribution

$$\mathbb{E}\tilde{B}_t = \frac{cov\left(R, PV\left(g\right)\right) - \bar{g}\,cov\left(R, PV\left(\theta^{\frac{1+\gamma}{\gamma}}\right)\right)}{var\left(R\right)}$$

mean reversion: $$\frac{1}{1+\beta^2 var\left(R\right)}$$

Intuition: risk minimization

- Planner wants to minimize fluctuations in τ_t
- Primary deficit, holding τ constant is

$$X_\tau \equiv g - \theta^{\frac{1+\gamma}{\gamma}} Z_\tau = g - \theta^{\frac{1+\gamma}{\gamma}} \tau (1-\tau)^{\frac{1}{\gamma}}$$

- Mean level of debt B and τ related through budget constraint:

$$\frac{1-\beta}{\beta} B = \bar{g} - \tau (1-\tau)^{\frac{1}{\gamma}} \mathbb{E}\theta^{\frac{1+\gamma}{\gamma}}$$

- The mean of invariant distribution is risk-minimizing debt:

$$B^* \equiv \arg\min_B var\left(RB + PV\left(X_{\tau(B)}\right)\right)$$

- Effect from $\tau(B)$ is second order:

$$B^* \approx -\frac{cov\left(R, X_{\tau(B)}\right)}{var(R)} \text{ for any } B$$

Extension 3: Risk aversion

- Same environment as extension 1 but utility is

$$\frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$$

- New implementability constraint

$$U_{c,t}\mathrm{B}_t + U_{c,t}\left[l_t + \frac{U_{l,t}}{U_{c,t}} l_t - g_t\right] = \frac{p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}} U_{c,t-1}\mathrm{B}_{t-1}$$

Effective debt and return

- Define
  - effective debt: $\mathcal{B}_t = U_{c,t}\mathrm{B}_t$
  - effective return: $\mathcal{R}_t = \frac{p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}}$
  - effective primary deficit: $\mathcal{X}_t = U_{c,t} X_t$
- All can be written as functions of c_t

Recursive problem

- Bellman equation

$$V(\mathcal{B}, s\_) = \max_{\{c(s), X'(s)\}} \mathbb{E}\left[U\left(c(s), \frac{c(s)+g(s)}{\theta(s)} \right) + \beta V(\mathcal{B}, s) | s\_ \right]$$

subject to

$$\mathcal{B}'(s) = \mathcal{R}(s)\mathcal{B} + \mathcal{X}(s) \text{ for all } s$$

- Similar to recursive formulation in quasi-linear case, same optimality condition for *effective* debt:

$$V_t(\tilde{\mathcal{B}}_t) = \mathbb{E}_t V'_{t+1}(\tilde{\mathcal{B}}_{t+1}) + \beta cov_t(\mathcal{R}_{t+1}, V'_{t+1}(\tilde{\mathcal{B}}_{t+1}))$$

Risk-minimizing effective debt

- Planner wants to minimize fluctuations in τ
- The risk-minizing effective debt is

$$\tilde{\mathcal{B}}^* = -\frac{cov\left(\mathcal{R}, PV\left(\mathcal{X}\right)\right)}{var\left(\mathcal{R}\right)}$$

- Terms on the r.h.s. are endogenous but, up to the second order, do not depend on τ
- Can be easily computed without doing dynamic programing
- Risk-free $R \implies \mathcal{R}$ is when $\mathcal{X}$ is high $\implies$ optimal to hold negative quantity of risk-free debt
- Easy to generalize to K asset

Quantitative exercise

- Apply our analysis to the U.S. economy
- Since formulas are approximation, also evaluate how well they do

Model specification

- Preferences

$$\ln c - \frac{1}{3} l^3$$

- 1 asset, return are matched to returns of the U.S. gov't portfolio
- 3 shock process:

$$\begin{aligned}
\ln \theta_t &= \rho_\theta \theta_{t-1} + \sigma_\theta \epsilon_{\theta,t} \\
\ln g_t &= \ln \bar{g} + \chi_g \epsilon_{\theta,t} + \sigma_g \epsilon_{g,t} \\
\ln p_t &= \chi_p \epsilon_{\theta,t} + \sigma_p \epsilon_{p,t}
\end{aligned}$$

Calibration

- Target statistics:
  - dynamics of GDP
  - dynamics of returns to U.S. gov't portfolio
- Returns computed from budget constraint:

$$\begin{aligned}(q_t + p_t)\, B_{t-1} &= X_t + q_t B_t \\ &\Longrightarrow \\ R_t &= \frac{\text{market value of debt}_t + \text{primary deficit}_t}{\text{market value of debt}_{t-1}}\end{aligned}$$

- GDP and returns are endogenous, depend on tax policy. We estimate

$$\tau_t = (1-\rho_\tau)\,\tau_{t-1} + \rho_\tau \bar{\tau} + \rho_Y \ln Y_t + \rho_{Y_-} \ln Y_{t-1}$$

Model fit

Param	Value	Moment	Model	Data
		Log Output		
σ_θ	0.020	std. dev	1.7%	1.70%
ρ_θ	0.160	auto corr	0.28	0.28
		Returns		
σ_p	0.05	std. dev	5.1%	5.02%
χ_p	0.650	corr with $\log Y_t$	-0.06	-0.08
		G/Y		
$\bar{g}$	0.230	mean	23%	23%
σ_g	0.040	std. dev	4.7%	4.7%
χ_g	-0.150	corr with $\log Y_t$	-0.42	-0.41

Optimal policy: computed and analytical

Effective debt: X_t	Using simulation	Using formula
Mean	-0.07	-0.06
Half life (years)	250	257
Std. deviation	0.26	0.26

Table 4: Ergodic moments and comparison with formula

- Correlation of returns and output is close to 0:
  - correlation with effective returns is negative
  - accumulate assets
- Variability of effective returns is quite low, provides bad hedge
  - slow convergence to the mean
  - large variance of debt

Simple back of envelope

- Run VAR

$$\begin{pmatrix} \mathcal{X}_t \\ Y_t \end{pmatrix} = A \begin{pmatrix} \mathcal{X}_{t-1} \\ Y_{t-1} \end{pmatrix} + \varepsilon_t$$

- Let

$$\begin{pmatrix} \alpha_X \\ \alpha_Y \end{pmatrix} = \left(I - \beta^{-1} A\right)^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- Then

$$PV_t\left(\mathcal{X}\right) = \alpha_X \mathcal{X}_t + \alpha_Y Y_t$$

- Risk minimizing effective debt

$$\mathcal{B}^* = -\frac{cov\left(\mathcal{R}_t, PV_t\left(\mathcal{X}\right)\right)}{var\left(\mathcal{R}_t\right)} = -\frac{\alpha_Y cov\left(\mathcal{R}_t, Y_t\right) + \alpha_X cov\left(\mathcal{R}_t, \mathcal{X}_t\right)}{cov\left(\mathcal{R}_t\right)}$$

- Applying to the U.S. data

$$\mathcal{B}^* = -0.08$$

Comparison to the U.S. policy

Moments	Benchmark	Comparison to U.S. Simulated	Comparison to U.S. Data
Tax Rate			
std. dev	0.2%	0.2%	0.7%
auto corr	0.97	0.31	0.24
Log Debt			
std. dev	10%	1.8%	3.3%
auto corr	0.95	0.31	0.33

- Similar orders of magnitude
- Debt in the U.S. too smooth, reverts to the mean too quickly

Conclusion

- Portfolio theory for government assets
  - general equilibrium effects
  - benevolence
- Easily extend to other countries
  - open economy and accumulating foreign debt (e.g. China)
  - investing in stocks (e.g. Norway, sovereign funds)