

# Time-Varying Wage Risk, Incomplete Markets, and Business Cycles\*

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## Abstract

Idiosyncratic earnings risk shows cyclical variation. In order to analyze its implication with respect to labor market dynamics, this paper develops an incomplete asset markets model in which individuals make consumption-saving and employment choices each period in the presence of time-varying person-specific wage risk. I measure the model's risk variation using wage data in the Panel Study of Income Dynamics. When including variation in both idiosyncratic wage risk and aggregate total factor productivity, the model produces a weakly negative correlation between total hours worked and average labor productivity close to the U.S. data. In contrast, in the absence of wage risk fluctuations, the model generates a counterfactually strong positive correlation.

*Keywords:* Idiosyncratic wage risk, uncertainty shocks, hours-productivity correlation

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# 1 Introduction

Idiosyncratic earnings risk exhibits cyclical fluctuations (Storesletten, Telmer, and Yaron (2004)). Moreover, there is empirical evidence, as well as theoretical support, for the finding that an increase in wage uncertainty increases labor supply (Parker, Belghitar, and Barmby (2005) and Flodén (2006)). However, the quantitative implication for labor market fluctuations has not been studied. Are changes in wage uncertainty relevant for aggregate fluctuations or the business cycle? The present paper examines this question using a heterogeneous-agent dynamic stochastic general equilibrium model.

The model analyzed herein is built upon incomplete asset markets models used in recent labor market analyses (e.g., Chang and Kim (2006, 2007) and Krusell, Mukoyama, Rogerson, and Sahin (2010, 2011)). Individuals face idiosyncratic wage risk because person-specific labor productivity changes stochastically. Individuals cannot fully insure against this risk because there is only one asset: physical capital. They partially self-insure by holding capital and make discrete labor supply choices each period. I introduce risk variation into this environment using uncertainty shocks in the spirit of Bloom (2009), i.e., time-varying volatility of idiosyncratic productivity shocks. Further, I calibrate these uncertainty shocks and the stochastic process for idiosyncratic productivity using individual wage data in the Panel Study of Income Dynamics (PSID). The calibrated model generates inequality in wealth and labor earnings as well as fluctuations in idiosyncratic wage risk that are similar to those in the U.S. economy.

I find that when driven by both uncertainty and aggregate total factor productivity (TFP) shocks, the model analyzed herein replicates the weakly negative correlation between total hours worked and average labor productivity found in the U.S. data ( $-0.40$  in the model compared with  $-0.32$  in the data).<sup>1</sup> In contrast, since aggregate TFP shocks primarily affect

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<sup>1</sup>Average labor productivity is output per labor hour. The data on total hours worked is taken from Cociuba, Prescott, and Ueberfeldt (2009) and as Shimer (2010) argues, it is the most comprehensive data on hours worked. The reported correlation between hours and productivity is based on the data from 1947Q3 to 2009Q3. While there is a consensus that the correlation after 1984 is weakly negative, several papers, such as Gali and Gambetti (2009), find a slightly positive correlation before 1984. In contrast, the pre-1984

labor demand, in the absence of uncertainty shocks, the model produces a strong, positive correlation of 0.83. Hence, varying idiosyncratic wage risk resolves the hours-productivity puzzle, overturning the counterfactually strong comovement of hours with productivity in equilibrium business cycle models driven solely by aggregate TFP shocks (e.g., Kydland and Prescott (1982) and Hansen (1985)). Importantly, this resolution is achieved without any loss in the model's success in explaining the variability and comovement of output, consumption, and investment.

Fluctuations in idiosyncratic wage risk resolve the hours-productivity puzzle because even a temporary increase in risk produces a persistent, negative correlation between total hours worked and average labor productivity. The main mechanism is the (ex-post) *distribution effect*. As elevated idiosyncratic wage risk materializes, the distribution of idiosyncratic productivity becomes more dispersed than before. The increased dispersion in productivity reduces the positive correlation between wealth and productivity across individuals, which is a result of the persistence in idiosyncratic productivity. Crucially, this moves the wealth-productivity distribution, generating a flow of low-productivity individuals towards larger wealth levels and a flow of high-productivity individuals towards smaller wealth levels. As a consequence, the employment of low-productivity individuals decreases, while employment increases among high-productivity groups. Total employment decreases because 1) the pre-shock wealth-productivity distribution implies a larger flow of low-productivity than high-productivity individuals and 2) the increase in the high-productivity employment pushes down the equilibrium wage rate. Thus, total hours worked decreases, while average labor productivity increases. Since the wealth-productivity distribution gradually returns to its long-run distribution, hours and productivity slowly return to their pre-shock levels, exhibiting persistent, negative comovement.

The (ex-ante) *uncertainty effect* also generates labor market fluctuations. An increase in wage uncertainty increases incentives to self-insure, especially for individuals close to their

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correlation is weakly negative ( $-0.30$ ) in the data used here. Gali and Gambetti (2009) use data on the nonfarm business sector, while the data used here includes the farm, government, and military sectors.

borrowing limit. Since such individuals with high productivity were likely to have been working, only low-productivity groups actually increase their employment. Hence, total hours worked increases, whereas average labor productivity decreases. However, quantitatively, the uncertainty effect plays a minor role in generating the negative correlation between hours and productivity in the present model. I solve a version of the model including only the uncertainty effect and excluding the distribution effect, i.e., the ex-post change in the productivity distribution, and obtain a positive correlation between hours and productivity of 0.58. This result indicates that the major impact of varying idiosyncratic risk arises from the distribution effect.

In addition to the hours-productivity correlation, the varying risk model herein is consistent with two patterns of the labor market fluctuations in the U.S. First, the U.S. labor market shows large fluctuations in the labor wedge, which is a gap between the marginal rate of substitution of leisure for consumption and the marginal product of labor.<sup>2</sup> The varying risk model can generate fluctuations in the labor wedge that are 95% of the variation in the empirical labor wedge, whereas the number is only 17% for the model without risk variation. Second, in the U.S., total hours worked lagged average labor productivity following a recession in which idiosyncratic wage risk increased, whereas hours and labor productivity recovered together after a recession in which risk remained low. The varying risk model exhibits a similar pattern. An increase in idiosyncratic wage risk delays the recovery of hours relative to labor productivity following a decline in aggregate TFP. I take these findings as additional evidence that cyclical variation in idiosyncratic wage risk has a significant impact on labor market fluctuations.

The present paper contributes to the literature on the macroeconomic impact of varying idiosyncratic earnings risk by studying its impact on labor market dynamics. While existing studies analyze how time-varying income risk affect aggregate fluctuations (Krusell and Smith (1998)), the welfare cost of business cycles (Krusell and Smith (1999), Storesletten,

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<sup>2</sup>For example, see Chari, Kehoe, and McGrattan (2007) and Shimer (2010).

Telmer, and Yaron (2001), Mukoyama and Sahin (2006), and Krusell, Mukoyama, Sahin, and Smith (2009)), and asset pricing (Krusell and Smith (1997), Pijoan-Mas (2007), and Storesletten, Telmer, and Yaron (2007)), they do not analyze labor market fluctuations, assuming exogenous earnings or inelastic labor supply. One exception is Lopez (2010), which assumes divisible labor and a different borrowing constraint from that assumed herein. Crucially, his model generates strong comovement of total hours worked with average labor productivity and leaves the hours-productivity puzzle unresolved.<sup>3</sup>

The present paper is also related to recent studies on the relationship between changes in firm-specific risk and business cycles. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) and Bachmann and Bayer (2013) investigate how uncertainty shocks interact with input adjustment costs. Arellano, Bai, and Kehoe (2012) consider financial frictions, while Schaal (2012) analyzes labor search frictions. In these two models, uncertainty shocks trigger heterogeneous changes in labor *demand* across firms, leading to negative comovement of labor input with productivity at the aggregate level. In contrast, in the present model, uncertainty shocks generate heterogeneous changes in labor *supply* among individuals with different wealth and productivity and resolve the hours-productivity puzzle.

Lastly, the present paper is related to the literature proposing labor supply shocks as a solution to the hours-productivity puzzle. While previous studies, such as Benhabib, Rogerson, and Wright (1991) and Christiano and Eichenbaum (1992), incorporate labor supply shocks into an equilibrium business cycle model with aggregate TFP shocks, they assume a representative agent. Without changes in the composition of workers with different productivities, a relatively strong positive correlation remains between hours worked and labor productivity in their models calibrated to the U.S. economy.<sup>4</sup>

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<sup>3</sup>The Lopez (2010) model produces a strong positive correlation between output and total hours worked (0.98) and a low volatility of hours relative to output (0.32). These values imply a correlation between hours and productivity of 0.96.

<sup>4</sup>Benhabib, Rogerson, and Wright (1991) include shocks to home-production technology. Their benchmark model generates a correlation of 0.49 between total hours worked and average labor productivity. Christiano and Eichenbaum (1992) introduce government spending shocks. When estimated using establishment hours data, their model with indivisible labor generates a correlation of 0.58.

The remainder of this paper proceeds as follows. Section 2 quantifies cyclical variation in idiosyncratic wage risk using the PSID wage data. Section 3 lays out the incomplete asset markets model with varying idiosyncratic wage risk, while Section 4 determines the parameter values. Section 5 analyzes the impact of varying idiosyncratic wage risk on the model’s business cycle. Section 6 examines the implication of cyclical risk. Section 7 concludes.

## 2 Cyclical Fluctuations in Idiosyncratic Wage Risk

This section analyzes the PSID data and measures the cyclical variation in idiosyncratic wage risk in the U.S. economy.<sup>5</sup> Idiosyncratic wage risk is computed by the cross-sectional dispersion of residuals obtained from the wage regression and the cyclical variation in the identified risk is analyzed. This approach is similar to that taken by existing studies that estimate uncertainty shocks faced by firms (e.g., Bloom (2009) and Bachmann and Bayer (2013)).

First, for each person-year observation of the PSID data, I compute the hourly wage dividing the annual labor income by annual total labor hours. Next, for each year, I fit individual wages to the wage process of the model analyzed in the present paper. Specifically, an individual wage  $w_{i,t}$  ( $i$ : individual and  $t$ : time) is equal to  $w_t x_{i,t}$ , where  $w_t$  is the equilibrium wage rate per efficiency unit of labor and  $x_{i,t}$  is person-specific labor productivity:

$$\ln w_{i,t} = \ln w_t + \ln x_{i,t}. \tag{1}$$

Furthermore,  $x_{i,t}$  follows an AR(1) process:

$$\ln x_{i,t} = \rho_x \ln x_{i,t-1} + \varepsilon_{x,i,t}, \varepsilon_{x,i,t} \sim N(0, \sigma_{\varepsilon_x,t}^2). \tag{2}$$

As shown by Chang and Kim (2006), (1) and (2) imply the following wage process:

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<sup>5</sup>Appendix A1 explains the data in detail.

$$\ln w_{i,t} = \rho_x \ln w_{i,t-1} + (\ln w_t - \rho_x \ln w_{t-1}) + \varepsilon_{x,i,t}. \quad (3)$$

I conduct three types of regression to identify idiosyncratic wage risk. The first regression estimates (3) each year with ordinary least squares (OLS), replacing  $(\ln w_t - \rho_x \ln w_{t-1})$  with a constant. The regression is done for the period between 1969 and 1991.

In practice, variables such as years of education influence individual wages (e.g., Card (1999), Heathcote, Perri, and Violante (2010)), and hence individuals could forecast their wage, at least partially. In order to isolate the pure risk that individuals face, the second regression controls for demographic variables and estimates the following equation:

$$\ln w_{i,t} = \rho_{x,t} \ln w_{i,t-1} + (\ln w_t - \rho_{x,t} \ln w_{t-1}) + Z_{i,t} \beta_t + \varepsilon_{x,i,t}, \quad (4)$$

where  $Z_{i,t}$  includes education, experience (defined as age minus education minus six), and experience-squared.<sup>6</sup> I estimate (4) each year using OLS, replacing  $(\ln w_t - \rho_{x,t} \ln w_{t-1})$  with a constant. Since the data on education is discontinuous in 1974, the regression is done for the period between 1975 and 1991.

The third regression takes into account the selection effect. Specifically, following Chang and Kim (2006), I introduce the selection equation of

$$I_{i,t} = V_{i,t} \gamma_t + v_{i,t}, v_{i,t} \sim N(0, \sigma_{v,t}^2) \quad (5)$$

where  $I_{i,t} = 1$  if the individual worked in both  $t$  and  $t - 1$  (i.e., both  $w_{i,t}$  and  $w_{i,t-1}$  are available),  $V_{i,t}$  includes marital status, the number of children, education, experience, experience-squared, and a constant. I conduct Heckman-type estimation using (4) and (5). The regression is done for each year between 1975 and 1991.

Figure 1 plots the estimated idiosyncratic wage risk  $\hat{\sigma}_{\varepsilon_x,t} = std(\hat{\varepsilon}_{x,i,t})$  from the three types

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<sup>6</sup>I also controlled for occupation. The identified risk becomes slightly smaller, but its cyclical variation is virtually unchanged.

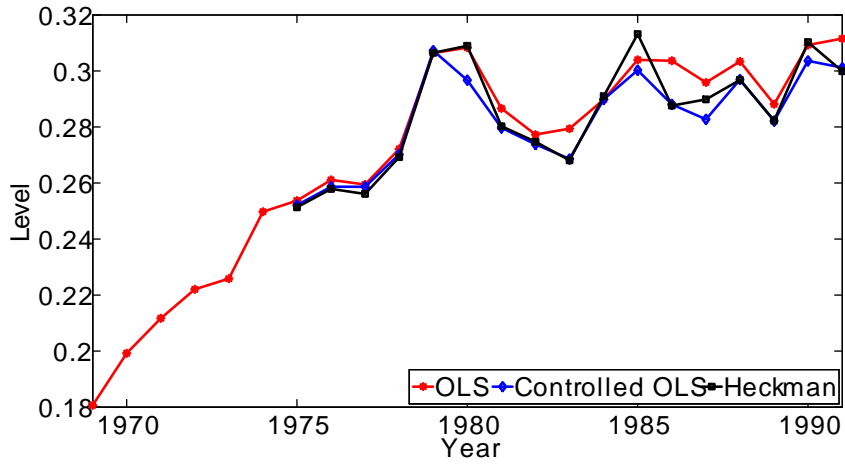


Figure 1: Estimated idiosyncratic wage risk.

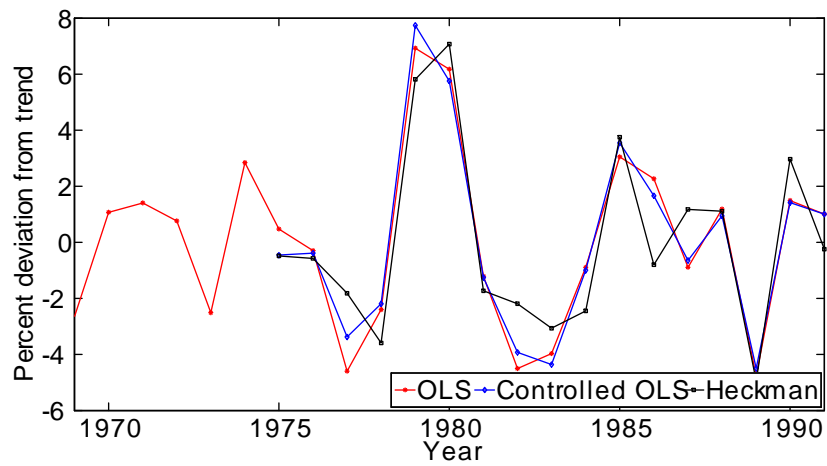


Figure 2: Cyclical components of estimated idiosyncratic wage risk.



	OLS	Controlled OLS	Heckman
$std(\hat{\sigma}_{\varepsilon_x,t})$	0.032	0.034	0.033
$corr(\hat{\sigma}_{\varepsilon_x,t}, \hat{\sigma}_{\varepsilon_x,t-1})$	0.185	0.254	0.007

Table 1: Cyclical moments of estimated idiosyncratic wage risk.

of regression. As shown, all the three cases identify similar idiosyncratic wage risk. Further, consistent with existing findings (e.g., Heathcote, Perri, and Violante (2010)), idiosyncratic wage risk exhibits an upward trend. In order to isolate its cyclical variation, I compute the percent deviation from trend using the Hodrick-Prescott filter with a smoothing parameter of 10.<sup>7</sup> Figure 2 shows this detrended result. As shown in the figure and also summarized in Table 1, four empirical regularities characterize the cyclical component of idiosyncratic wage risk. First, idiosyncratic wage risk varies over time. The largest deviation from trend is close to 8%, and 4% fluctuations are frequent. The standard deviation is 3.2–3.4%. Second, the identified idiosyncratic wage risk exhibits some persistence, typically remaining above or below trend for approximately two years. However, its first-order autocorrelation coefficient is 0.007–0.254, and the hypothesis of no autocorrelation cannot be rejected. Third, risk variation is approximately symmetric. The size and persistence of idiosyncratic wage risk are similar when it is above and below trend. Fourth, idiosyncratic wage risk exhibits neither clear procyclicality nor countercyclicality.<sup>8</sup> Idiosyncratic wage risk remained low during the 1981–1982 recession, but it increased during the 1973–1975 and 1990–1991 recessions. Section 4 uses these findings to calibrate the model with varying idiosyncratic wage risk described below.

<sup>7</sup>The result did not change substantially when using a smoothing parameter of 6.25 or 100.

<sup>8</sup>This result is consistent with the finding of Heathcote, Perri, and Violante (2010). They show that the cross-sectional dispersion of *hourly wages* does not exhibit clear cyclicity. Further, the result is not necessarily inconsistent with countercyclical *income* risk documented by Storesletten, Telmer, and Yaron (2004) because income and wage risk could move differently. Nevertheless, since previous macro analyses typically assume countercyclical risk, I analyze the impact of countercyclical idiosyncratic wage risk later in Section 6.

### 3 Model

The model used here is built upon that of Chang and Kim (2006, 2007), and Krusell, Mukoyama, Rogerson, and Sahin (2010, 2011). Individuals make consumption-saving and employment choices each period under the presence of idiosyncratic productivity (wage) risk. I introduce risk variation into the environment using uncertainty shocks in the sense of Bloom (2009), i.e., assuming a time-varying second moment for idiosyncratic productivity shocks. In the following, I describe individuals, firms, and equilibrium.

#### 3.1 Individuals

There is a continuum of individuals of measure one. Individuals differ in labor productivity  $x$ , which follows an AR(1) process. Individuals have the same momentary utility function  $u(c, h)$ , where  $c$  is consumption and  $h$  is labor hours. Labor is indivisible, as in Hansen (1985) and Rogerson (1988), and individuals choose whether to work for a fixed number of hours or not to work at all:  $h \in \{\bar{h}, 0\}$ . Individuals earn labor income of  $wxh$ , where  $w$  is the equilibrium wage rate per efficiency unit of labor.

Individuals face time-varying idiosyncratic wage risk because shocks to person-specific productivity  $x$  have a time-varying volatility  $\sigma_{\epsilon_x}$ , which follows a Markov process and is common to all individuals. Following the convention of the literature on uncertainty shocks, such as Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), and Bachmann and Bayer (2013), individuals learn of the size of  $\sigma_{\epsilon_x}$  one period ahead. This timing assumption captures the concept of risk. In what follows,  $\sigma_{\epsilon_x}$  represents the volatility of shocks not to  $x$ , but to  $x'$ , where a prime denotes next-period values hereinafter.

Since asset markets are incomplete, individuals are unable to insure themselves fully against varying idiosyncratic wage risk. As in Aiyagari (1994) and others, individuals partially self-insure by holding a single asset, physical capital  $k$ . Borrowing is allowed, but there is a constraint,  $k \geq \bar{k}$  ( $\bar{k} < 0$ ).

Define  $V(k, x; z, \mu, \sigma_{\epsilon_x})$  as the beginning-of-period value of an individual characterized by  $(k, x)$  under the aggregate state  $(z, \mu, \sigma_{\epsilon_x})$ , where  $z$  is aggregate TFP, which follows an AR(1) process, and  $\mu$  denotes the individual distribution over  $k$  and  $x$ . This beginning-of-period value reflects the individual's current employment choice:

$$V(k, x; z, \mu, \sigma_{\epsilon_x}) = \max \{V^E(k, x; z, \mu, \sigma_{\epsilon_x}), V^N(k, x; z, \mu, \sigma_{\epsilon_x})\}. \quad (6)$$

The individual's within-period value conditional on working is  $V^E(k, x; z, \mu, \sigma_{\epsilon_x})$ , which satisfies

$$\begin{aligned} V^E(k, x; z, \mu, \sigma_{\epsilon_x}) &= \max_{c, k'} \left\{ u(c, \bar{h}) + \beta E \left[ V(k', x'; z', \mu', \sigma'_{\epsilon_x}) | x, z, \mu, \sigma_{\epsilon_x} \right] \right\}, \quad (7) \\ \text{subject to } c &= w(z, \mu, \sigma_{\epsilon_x})x\bar{h} + [1 + r(z, \mu, \sigma_{\epsilon_x})]k - k' \\ k' &\geq \bar{k} \\ c &\geq 0 \\ \mu' &= \Gamma(z, \mu, \sigma_{\epsilon_x}), \end{aligned}$$

where  $\beta$  is the discount factor,  $E$  is the conditional expectation,  $r$  is the equilibrium rental rate of capital, and  $\Gamma$  is the law of motion for  $\mu$ .

Similarly,  $V^N(k, x; z, \mu, \sigma_{\epsilon_x})$  is the individual's within-period value conditional on not working, which satisfies

$$\begin{aligned}
V^N(k, x; z, \mu, \sigma_{\epsilon_x}) &= \max_{c, k'} \left\{ u(c, 0) + \beta E \left[ V(k', x'; z', \mu', \sigma'_{\epsilon_x}) \mid x, z, \mu, \sigma_{\epsilon_x} \right] \right\}, & (8) \\
\text{subject to } c &= [1 + r(z, \mu, \sigma_{\epsilon_x})]k - k' \\
k' &\geq \bar{k} \\
c &\geq 0 \\
\mu' &= \Gamma(z, \mu, \sigma_{\epsilon_x}).
\end{aligned}$$

### 3.2 Representative Firm

A representative firm produces the final good  $Y$  using capital  $K$  and labor  $L$ . The production function is  $Y = zF(K, L)$  and exhibits constant returns to scale. Taking  $r(z, \mu, \sigma_{\epsilon_x})$  and  $w(z, \mu, \sigma_{\epsilon_x})$  as given, the firm chooses  $K(z, \mu, \sigma_{\epsilon_x})$  and  $L(z, \mu, \sigma_{\epsilon_x})$ , and maximizes static profits. The first-order conditions are

$$r(z, \mu, \sigma_{\epsilon_x}) = zF_K(K, L) - \delta, \quad (9)$$

and

$$w(z, \mu, \sigma_{\epsilon_x}) = zF_L(K, L), \quad (10)$$

where  $\delta$  is the capital depreciation rate.

### 3.3 General Equilibrium

A recursive competitive equilibrium is a set of functions,

$$(w, r, V^E, V^N, V, c, k', h, K, L, \Gamma), \quad (11)$$

that satisfy the following conditions.

1. Individuals' Optimization:

$V(k, x; z, \mu, \sigma_{\epsilon_x})$ ,  $V^E(k, x; z, \mu, \sigma_{\epsilon_x})$ , and  $V^N(k, x; z, \mu, \sigma_{\epsilon_x})$  satisfy (6), (7), and (8), while  $c(k, x; z, \mu, \sigma_{\epsilon_x})$ ,  $k'(k, x; z, \mu, \sigma_{\epsilon_x})$ , and  $h(k, x; z, \mu, \sigma_{\epsilon_x})$  are the associated policy functions.

2. Firms' Optimization:

The representative firm chooses  $K(z, \mu, \sigma_{\epsilon_x})$  and  $L(z, \mu, \sigma_{\epsilon_x})$  to satisfy (9) and (10).

3. Labor Market Clearing:

$$L(z, \mu, \sigma_{\epsilon_x}) = \int xh(k, x; z, \mu, \sigma_{\epsilon_x})\mu([dk \times dx])$$

4. Capital Market Clearing:

$$K(z, \mu, \sigma_{\epsilon_x}) = \int k\mu([dk \times dx])$$

5. Goods Market Clearing:

$$\int \{k'(k, x; z, \mu, \sigma_{\epsilon_x}) + c(k, x; z, \mu, \sigma_{\epsilon_x})\} \mu([dk \times dx]) = zF(K(z, \mu, \sigma_{\epsilon_x}), L(z, \mu, \sigma_{\epsilon_x})) + (1 - \delta) \int k\mu([dk \times dx])$$

6. Evolution of Individual Distribution:

$\Gamma(z, \mu, \sigma_{\epsilon_x})$  is consistent with individual decisions and the laws of motion for  $x$ ,  $z$ , and  $\sigma_{\epsilon_x}$ . Specifically, for all  $B \subseteq K$ ,

$$\mu'(B, x') = \int_{\{(k,x)|k'(k,x;z,\mu,\sigma_{\epsilon_x}) \in B\}} \pi_x(x'|x, \sigma_{\epsilon_x})\mu([dk' \times dx']),$$

where  $\pi_x(x'|x, \sigma_{\epsilon_x})$  is the transition probability from  $x$  to  $x'$  under  $\sigma_{\epsilon_x}$ .

## 4 Calibration and the Steady State

This section first determines parameter values for the model described above, except for those concerning idiosyncratic productivity. I choose their values so that the model economy replicates several features of the U.S. economy. Next, I determine parameter values for

Parameter	Description	Value
$\beta$	Discount factor	0.9829
$B$	Disutility of labor	1.0203
$\bar{h}$	Working hours	1/3
$\bar{k}$	Borrowing limit	-2.0
$\alpha$	Labor share	0.64
$\delta$	Capital depreciation rate	0.025
$\rho_z$	Persistence in aggregate TFP	0.95
$\sigma_{\epsilon_z}$	Volatility of aggregate TFP shocks	0.007

Table 2: Parameters other than idiosyncratic productivity.

idiosyncratic productivity, matching moments of individual wages in the model with moments of the PSID wages. The end of this section briefly presents the steady state.

## 4.1 Parameters Other Than Idiosyncratic Productivity

Table 2 lists the parameter values other than those related to idiosyncratic productivity. The values are comparable to those used in existing incomplete asset markets models (e.g., Krusell and Smith (1998) and Chang and Kim (2007)). Each period is equal to one quarter. The discount factor  $\beta$  is 0.9829, which generates a one percent quarterly rental rate of capital at the steady state. The momentary utility when individuals work is  $u(c, h) = \ln c - B$ . When they do not work,  $u(c, h) = \ln c$ . The disutility parameter is  $B = 1.0203$ , producing a steady-state employment rate of 60%. The employment rate is close to the average U.S. employment-population ratio during the period of 1948Q1–2009Q3. Individuals use one third of their time when working ( $\bar{h} = 1/3$ ). The borrowing limit is  $\bar{k} = -2.0$ . Under the constraint, individuals can borrow up to 44% of the average annual income at the steady state, which is similar to the constraint set by Krusell and Smith (1998) for their model with borrowing.

As for the firm side, the production function is  $Y = zK^{1-\alpha}L^\alpha$ , and labor's share  $\alpha$  is 0.64. The capital depreciation rate  $\delta$  is 0.025. Aggregate TFP  $z$  follows an AR(1) process,  $\log z' = \rho_z \log z + \epsilon'_z$ , where  $\epsilon'_z \sim N(0, \sigma_{\epsilon_z}^2)$ . As in Cooley and Prescott (1995),  $\rho_z = 0.95$ , and  $\sigma_{\epsilon_z} = 0.007$ .

	PSID	Varying risk	Constant risk
<i>A. Moments (Annual)</i>			
$\hat{\rho}_x$	0.854	0.853	0.853
$\hat{\bar{\sigma}}_{\epsilon_x}$	0.282	0.284	0.280
$std(\hat{\sigma}_{\epsilon_x,t})$	0.032	0.031	0.008
$corr(\hat{\sigma}_{\epsilon_x,t}, \hat{\sigma}_{\epsilon_x,t-1})$	0.185	0.177	-0.269
<i>B. Parameters (Quarterly)</i>			
$\rho_x$	-	0.930	0.930
$\bar{\sigma}_{\epsilon_x}$	-	0.223	0.223
$\rho_{\sigma_{\epsilon_x}}$	-	0.900	-
$\lambda$	-	0.090	-

Table 3: Calibration moments and parameter values for idiosyncratic productivity. Panel A lists the moments of individual wages used for calibration. Panel B shows the parameter values.

## 4.2 Parameters on Idiosyncratic Productivity

Four parameters concern idiosyncratic productivity  $x$ . Since  $x$  follows an AR(1) process in (2), the first parameter is the persistence  $\rho_x$ . The other three parameters concern fluctuations in idiosyncratic wage risk  $\sigma_{\epsilon_x}$ . I assume a three-state Markov chain: high (H), middle (M), and low (L). The analysis in Section 2 finds no strong cyclicity in  $\sigma_{\epsilon_x}$ . Thus,  $\sigma_{\epsilon_x}$  evolves independently of aggregate TFP  $z$ . Furthermore, the symmetry of risk variation shown in Section 2 suggests a symmetric transition matrix: A risk state remains unchanged with a quarterly probability of  $\rho_{\sigma_{\epsilon_x}}$  and transitions to each of the other two states with a probability of  $(1 - \rho_{\sigma_{\epsilon_x}})/2$ . The risk levels also should be symmetric:  $\sigma_{\epsilon_x,H} = (1 + \lambda)\bar{\sigma}_{\epsilon_x}$ ,  $\sigma_{\epsilon_x,M} = \bar{\sigma}_{\epsilon_x}$ , and  $\sigma_{\epsilon_x,L} = (1 - \lambda)\bar{\sigma}_{\epsilon_x}$ , where  $\bar{\sigma}_{\epsilon_x}$  is the steady-state risk.

I determine the values of these four parameters  $(\rho_x, \bar{\sigma}_{\epsilon_x}, \rho_{\sigma_{\epsilon_x}}, \lambda)$  such that model wages reproduce the four moments of the PSID wages listed in Table 3A. The two moments are the persistence in individual wages  $\hat{\rho}_x$  and the long-run idiosyncratic wage risk  $\hat{\bar{\sigma}}_{\epsilon_x}$ . For both the model and the PSID data, I compute these moments by estimating (3) with year dummies using the pooled OLS.<sup>9</sup> The other two moments are the annual standard deviation  $std(\hat{\sigma}_{\epsilon_x,t})$

<sup>9</sup>I simulate the model with 10,000 individuals for 1,500 periods (discarding the first 500 periods) and generate annual panel data on hourly wages. The same sequence of aggregate TFP is used for varying and constant risk models.

and the first-order autocorrelation coefficient  $corr(\hat{\sigma}_{\varepsilon_x,t}, \hat{\sigma}_{\varepsilon_x,t-1})$  of idiosyncratic wage risk. For the model moments, I estimate (3) each year with OLS, compute idiosyncratic wage risk as  $\hat{\sigma}_{\varepsilon_x,t} = std(\hat{\varepsilon}_{x,i,t})$ , and remove trend using the Hodrick-Prescott filter with a smoothing parameter of 10. To keep the comparability, I use the PSID moments estimated with simple OLS, which were shown in the second column of Table 2.

The second column of Table 3A shows the PSID moments. The persistence in individual wages is 0.854. The long-run value of idiosyncratic wage risk is 0.282. Idiosyncratic wage risk varies by a standard deviation of 3.2% (0.032) and shows a first-order autocorrelation of 0.185.

These PSID moments pin down the model parameters as shown in the third column of Table 3B. The persistence of productivity is  $\rho_x = 0.930$ , and the steady-state risk is  $\bar{\sigma}_{\varepsilon_x} = 0.223$ . These values are comparable to those used in models assuming constant idiosyncratic wage risk. For example, Chang and Kim (2007) use  $\rho_x = 0.929$  and  $\bar{\sigma}_{\varepsilon_x} = 0.227$ . As for risk variation, the persistence is  $\rho_{\sigma_{\varepsilon_x}} = 0.90$  and the size is  $\lambda = 0.09$ , implying that idiosyncratic wage risk varies between  $\sigma_{\varepsilon_x,L} = (1-\lambda)\bar{\sigma}_{\varepsilon_x} = 0.201$  and  $\sigma_{\varepsilon_x,H} = (1+\lambda)\bar{\sigma}_{\varepsilon_x} = 0.245$ . As shown in the same column of Table 3A, the calibrated varying risk model successfully reproduces the PSID moments.

In contrast, as shown in the last column, the constant risk model fails to replicate the risk variation in the PSID. Because of endogenous employment choice, idiosyncratic wage risk exhibits some variation, even when estimated using individual wages in the constant risk model. However, the volatility and persistence are much smaller than those estimated using the PSID data. This finding provides further evidence for cyclical fluctuations in idiosyncratic wage risk.

### 4.3 Steady State

The steady state of the present model shows the inequality of wealth and labor income that is comparable to that found in the U.S. economy. The Gini coefficient of annual labor



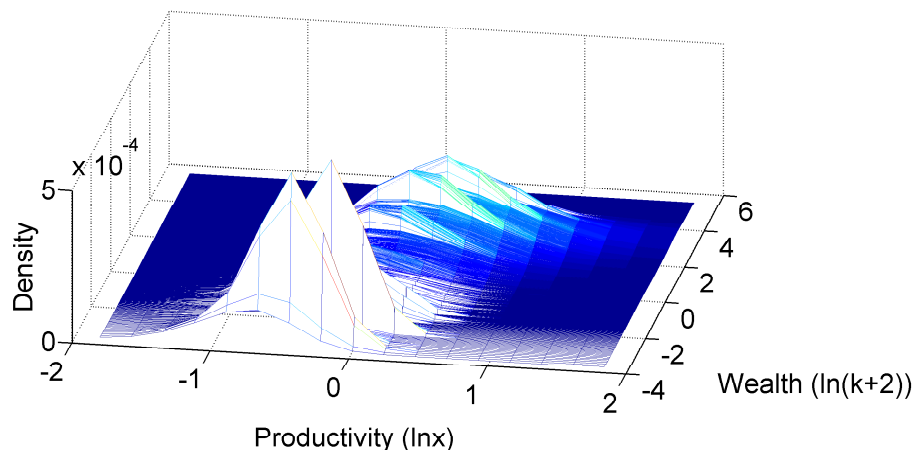


Figure 3: Steady-state distribution of wealth and productivity.

income is 0.59 in the model and 0.65 in the 1991 PSID.<sup>10</sup> As for the wealth inequality, the Gini coefficient is 0.64 in the model. Since it is difficult to define individual wealth in the actual economy, I compare this individual-level wealth inequality with the household-level inequality in the U.S. According to Díaz-Giménez, Quadrini, and Ríos-Rull (1997), the Gini coefficient is 0.78 in the 1992 Survey of Consumer Finances.

Further, the present model generates a weakly positive correlation between wealth and labor income (0.30), which is close to that found by Díaz-Giménez, Quadrini, and Ríos-Rull (1997) for the U.S. economy (0.23).<sup>11</sup> The model's correlation arises from two factors. First, since idiosyncratic productivity is persistent, individuals with higher current productivity tend to hold larger wealth, as shown in Figure 3.<sup>12</sup> Second, as shown in Figure 4, individuals are more likely to work when they have higher current productivity and smaller wealth. These two factors interact in generating the weakly positive correlation between wealth and labor earnings.

<sup>10</sup>Appendix B1 explains the solution method for the steady state. I generate the distribution of annual labor income through simulation with 10,000 individuals. Appendix A2 explains the PSID data.

<sup>11</sup>The steady state of the present model is quite similar to that of Chang and Kim (2007)'s model. Table 2 of Chang and Kim (2007) provides additional evidence that their model's joint distribution of wealth and income is comparable to that in the U.S. data.

<sup>12</sup>There are a large number of individuals near the borrowing limit ( $k = -2.0$ ). The figure excludes those individuals to highlight the rest of the distribution.

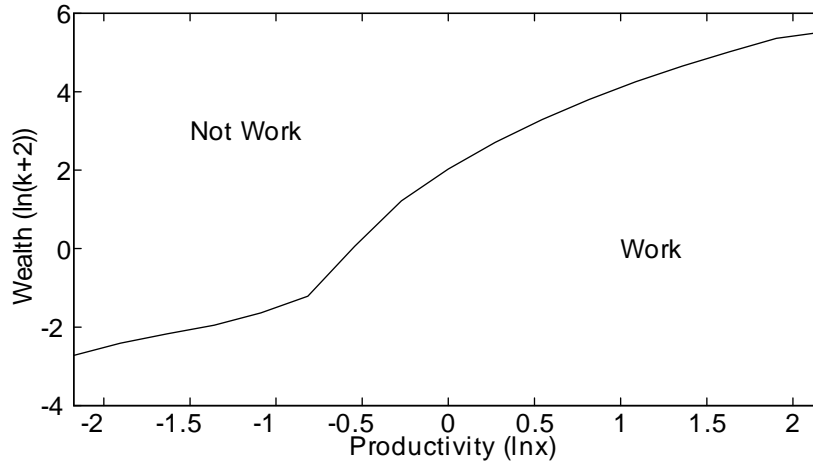


Figure 4: Individual employment decisions at the steady state.

## 5 Business Cycle Results

This section compares business cycles of the varying and constant risk models calibrated in the last section. Next, in order to understand the result, I analyze how the varying risk model responds to exogenous variation in idiosyncratic wage risk and aggregate TFP.

### 5.1 Time-Varying Idiosyncratic Wage Risk and Business Cycles

Table 4 lists business cycle statistics of the U.S. economy, the varying risk model, and the constant risk model.<sup>13</sup> I generate the two model results through simulations using the same sequence of aggregate TFP.<sup>14</sup> Hence, the difference between the two reveals the impact of variation in idiosyncratic wage risk on aggregate fluctuations.

When introducing fluctuations in idiosyncratic wage risk measured using the PSID data, the greatest improvement appears in the correlation between total hours worked and average labor productivity.<sup>15</sup> Specifically, the varying risk model generates a weakly negative correla-

<sup>13</sup>I take the U.S. macroeconomic data from the source listed in Appendix A3.

<sup>14</sup>I use the Krusell and Smith (1997, 1998) algorithm for the simulations. Appendix B2 explains the numerical method.

<sup>15</sup>Total hours worked is  $H \equiv \int h(k, x; z, \mu, \sigma_{\epsilon_x}) \mu([dk \times dx])$ , which is different from efficiency-weighted

	U.S. economy	Varying risk	Constant risk	Psych risk
$\sigma_Y$	1.69	1.43	1.37	1.37
$\sigma_C$	0.54	0.33	0.32	0.33
$\sigma_I$	2.85	3.15	3.10	3.10
$\sigma_H$	1.00	0.81	0.57	0.61
$\sigma_{Y/H}$	0.63	1.00	0.48	0.52
$\sigma_{wedge}$	1.32	1.26	0.23	0.38
$Corr(Y, C)$	0.78	0.86	0.90	0.89
$Corr(Y, I)$	0.80	0.99	0.99	0.99
$Corr(Y, H)$	0.80	0.41	0.96	0.91
$Corr(Y, Y/H)$	0.31	0.67	0.95	0.87
$Corr(H, Y/H)$	-0.32	-0.40	0.83	0.58
$Corr(H, wedge)$	0.92	0.84	0.96	0.83

Table 4: Varying idiosyncratic wage risk and business cycle moments. I take logs of all of the series and use the Hodrick-Prescott filter with a smoothing parameter of 1,600. The volatility of output  $\sigma_Y$  is the standard deviation of output (multiplied by 100). Other volatilities are their ratios with respect to  $\sigma_Y$ . *Corr* denotes a contemporaneous correlation.

tion of  $-0.40$  that is similar to the empirical value of  $-0.32$  from the U.S. data, whereas the constant risk model produces a counterfactually strong positive correlation of 0.83. Thus, the hours-productivity puzzle is resolved by shifting idiosyncratic wage risk in a manner consistent with the micro-level wage data.

Another improvement is seen in the movement of the labor wedge. The labor wedge is quite volatile and procyclical in the U.S. economy.<sup>16</sup> The varying risk model successfully reproduces these features. In particular, the volatility of the labor wedge is 95% of that in the U.S. data. In contrast, the constant risk model can account for only 17% of the volatility of the labor wedge seen in the U.S.

Furthermore, introducing the risk variation increases the volatility of hours worked and reduces the correlation between output and labor productivity, moving their values closer to the U.S. data. The volatility of productivity and the correlation between output and hours also move towards the U.S. data, although their values in the varying risk model overshoot

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total labor  $L \equiv \int xh(k, x; z, \mu, \sigma_{\epsilon_x})\mu([dk \times dx])$ .

<sup>16</sup>Following Chang and Kim (2007), the labor wedge here is the ratio of the marginal rate of substitution of leisure for consumption to labor productivity. It is computed as  $\ln wedge = \ln H^{1/\gamma}C - \ln Y/H$ , with  $\gamma = 1.5$ . Hence, the cyclical of the labor wedge is opposite to that presented in Shimer (2010).

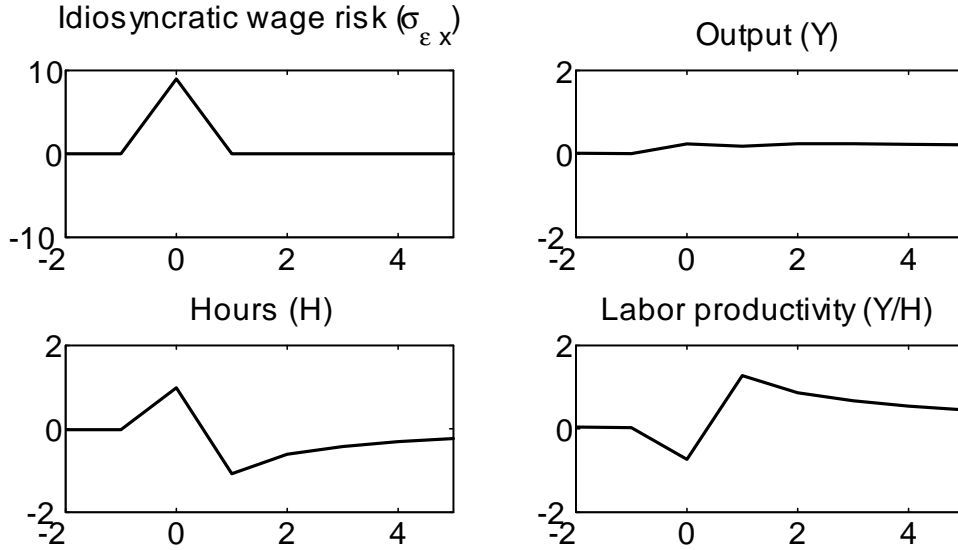


Figure 5: Increase in idiosyncratic wage risk (varying risk model). Vertical axis - period. Horizontal axis - percent deviation from the pre-shock average.

their data counterparts.

In contrast, changes in idiosyncratic wage risk have a relatively mild impact on the fluctuations in other aggregate variables. The varying and constant risk models exhibit similar volatilities and comovements of output, consumption, and investment. Thus, introducing variation in idiosyncratic wage risk strengthens the model’s ability to explain labor market fluctuations without weakening the model’s ability to account for other business cycle moments.

## 5.2 Responses to Changes in Idiosyncratic Wage Risk

Next, in order to identify the mechanism through which variation in idiosyncratic wage risk  $\sigma_{\varepsilon_x}$  generates labor market fluctuations, I analyze the response of the varying risk model to an increase in  $\sigma_{\varepsilon_x}$ . The simulation starts from the steady state (period  $-29$ ), and as shown in the upper-left panel of Figure 5,  $\sigma_{\varepsilon_x}$  increases by 9% for one period in period 0. In contrast, aggregate TFP is constant at its steady-state level throughout.

As the remaining panels of Figure 5 show, output, total hours worked, and average labor

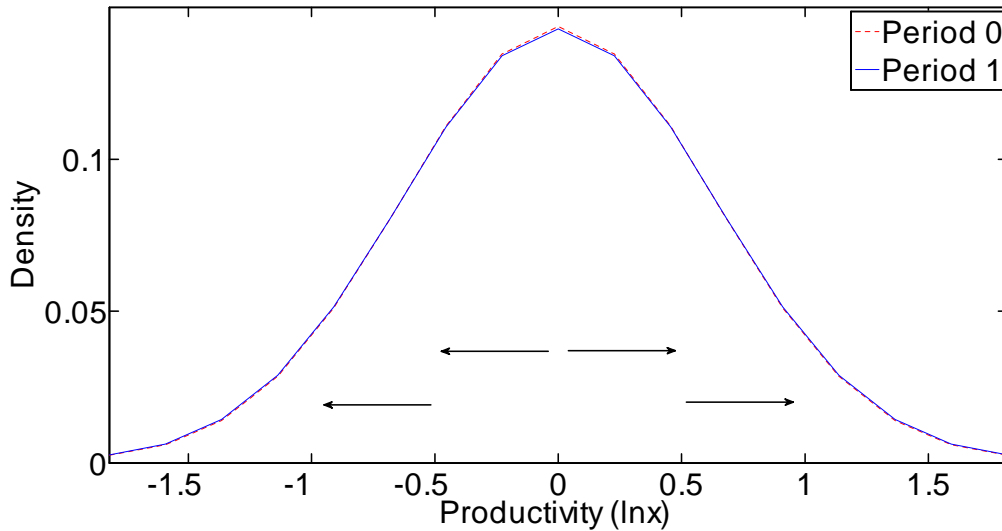


Figure 6: Shift in the productivity distribution.

productivity move in different ways. Output increases slightly and then slowly returns to the pre-shock level. Hours increases in period 0, drops below the pre-shock level in period 1, and then gradually recovers. Productivity moves in a direction exactly opposite to that of hours. Hence, a one-period increase in idiosyncratic wage risk generates a long-lasting negative comovement of hours with productivity. The result explains why the correlation between hours and productivity switches from positive to weakly negative when introducing variation in idiosyncratic wage risk.

Note also that the increase in idiosyncratic wage risk produces greater fluctuations in total hours worked and average labor productivity than in output. The greatest increase and decrease in hours are 0.98% and 1.01% respectively and the peak deviation in productivity is 1.28%. In contrast, output increases by only 0.24% at most. Hence, varying idiosyncratic wage risk increases the volatilities of hours and productivity relative to the output volatility.

The timing assumption of the present model implies that two effects underlie these responses in aggregate variables to the rise in idiosyncratic wage risk. The first is an *uncertainty effect*. In period 0, individuals become more uncertain about their future productivity. Because of the greater uncertainty on their future wages, some individuals increase labor supply

in period 0. The second effect is a *distribution effect*. As elevated risk materializes in period 1, there is an increase in the cross-sectional dispersion of idiosyncratic productivity, as shown in Figure 6. The resulting shift in the distribution of individuals over their labor productivity and wealth changes the number and composition of individuals who choose to work in period 1. In contrast, the uncertainty effect disappears in period 1 because uncertainty on future productivity returns to the pre-shock level.

First, in period 0, the uncertainty effect increases the employment of individuals with relatively low productivity and to a lesser extent decreases the employment of individuals with relatively high productivity. As shown in Figures 3 and 4, many individuals with low productivity hold small wealth close to their employment boundary. When uncertainty on their future wages increases, these individuals increase savings in order to self-insure and some of them switch from not working to working. In contrast, individuals with high productivity and near the employment boundary are wealthy and are thus well-insured. The employment of these individuals decreases slightly because the increase in the employment of low-productivity individuals lowers the equilibrium wage rate. At the aggregate level, hours worked increases in period 0. Output increases less than hours not only because aggregate TFP and capital remain unchanged, but also because employment disproportionately increases among low-productivity individuals. Hence, average labor productivity decreases in period 0.

Second, in period 1, the distribution effect decreases the employment of individuals with lower-than-average productivity and to a lesser degree increases the employment of individuals with higher-than-average productivity. As the arrows in Figure 6 indicate, there is a net flow of individuals from the middle to lower and higher levels of productivity at the beginning of period 1. Since there is a positive correlation between productivity and wealth in the pre-shock distribution (Figure 3), the flow of individuals generates a population shift from the “Work” region to the “Not work” region for lower-than-average productivity. The opposite mechanism works for higher-than-average productivity, and some of those individuals

shift from the “Not work” region to the “Work” region. However, the pre-shock distribution implies that the shift is smaller than that which occurs for those below the mean productivity. As a result of these changes in the wealth-productivity distribution, employment decreases among individuals with lower-than-average productivity in period 1, whereas the employment of higher-than-average productivity increases less. Hence, total hours worked decreases. Output increases slightly because this time the composition of workers shifts towards individuals with higher-than-average productivity, which lowers the equilibrium wage rate, further reducing hours worked. Average labor productivity increases more than output.

Crucially, even though idiosyncratic wage risk increases only for one period, it takes quite a few quarters for the wealth-productivity distribution to settle down. Hence, employment gradually recovers among individuals with lower-than-average productivity, whereas the employment of higher-than-average productivity individuals slowly decreases. As a result, output and average labor productivity gradually decrease to their pre-shock levels, whereas total hours worked recovers sluggishly.

Lastly, in order to quantify the roles of the uncertainty and distribution effects in generating the negative comovement of total hours worked with average labor productivity, I solve the model including only the uncertainty effect and shutting down the distribution effect. Specifically, I assume that individuals perceive and respond to changes in idiosyncratic wage risk, but those changes in risk do not materialize and the productivity distribution across individuals remains unchanged. Following Bachmann and Bayer (2013), I call it the psych risk model.

The last column of Table 3 shows the result of the psych risk model. As shown, the uncertainty effect alone lowers the correlation between total hours worked and average labor productivity relative to the constant risk model, but the impact is relatively small and the correlation remains positive. Figure 7 shows the response of the psych risk model to the one-period increase in  $\sigma_{\varepsilon_x}$  of 9% considered before. The response is qualitatively similar to

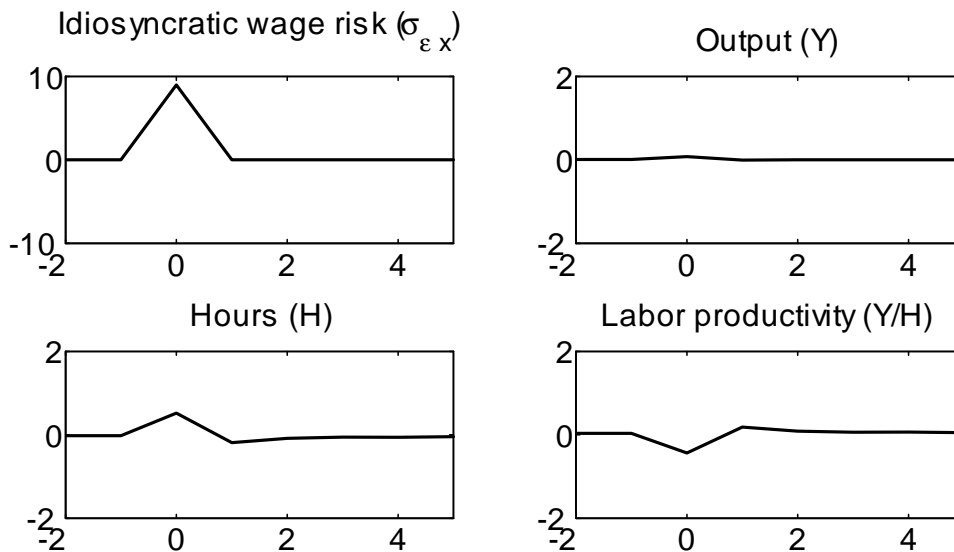


Figure 7: Increase in idiosyncratic wage risk (psych risk model). Vertical axis - period. Horizontal axis - percent deviation from the pre-shock average.

that of the varying risk model.<sup>17</sup> However, when compared to the varying risk model, the fluctuations in hours and productivity are much smaller and the negative comovement of the two disappears much more quickly in the psych risk model. The result indicates that the major impact of changes in idiosyncratic wage risk works not through the uncertainty effect, but through the distribution effect.<sup>18</sup>

### 5.3 Responses to Aggregate TFP Shocks

This subsection examines the response of the varying risk model to the other aggregate shock, namely, an exogenous change in aggregate TFP  $z$ . The simulation starts from the steady state (period -29), and as shown in the upper-left panel of Figure 8,  $z$  declines by

<sup>17</sup>Total hours worked in period 1 is slightly lower than the pre-shock level even in the psych risk model. This is because of ex-post wealth effect. Individuals accumulate unusually large savings in period 0 due to the elevated uncertainty. See Marcet, Obiols-Homs, and Weil (2007) on the ex-post wealth effect under constant idiosyncratic income risk.

<sup>18</sup>Including unemployment benefits strengthens the distribution effect for low-productivity individuals and weakens the distribution effect for high-productivity individuals and the uncertainty effect. Since the distribution effect of low-productivity groups is dominant in the present model, the inclusion of unemployment benefits is unlikely to change the main result of the present paper.



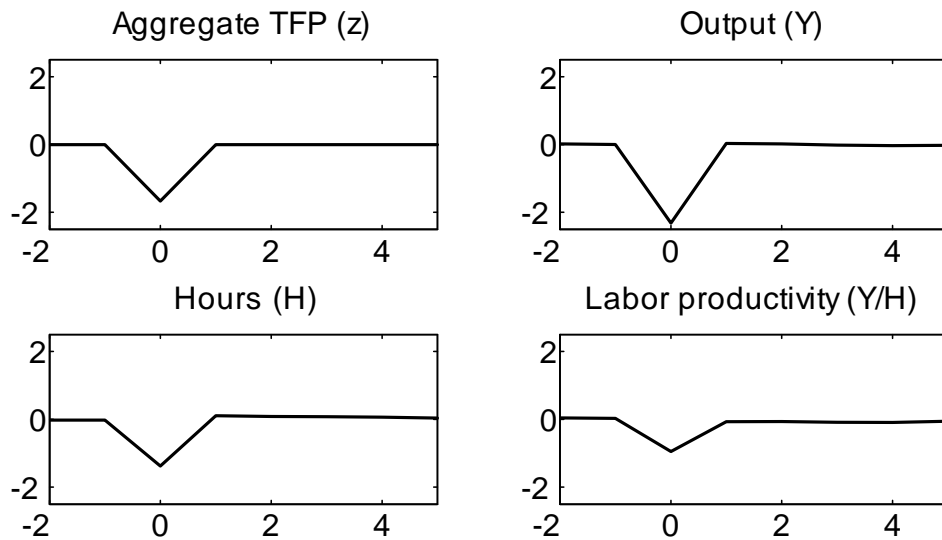


Figure 8: Decrease in aggregate TFP (varying risk model). Vertical axis - period. Horizontal axis - percent deviation from the pre-shock average.

1.67% in period 0. Idiosyncratic wage risk remains constant.

As the other panels indicate, output, total hours worked, and average labor productivity all decrease following the decrease in aggregate TFP. In this model economy, as in the prototype equilibrium business cycle model, a decline in aggregate TFP reduces labor demand, without significantly affecting labor supply. In equilibrium, the wage rate falls, and employment decreases across all productivity groups. Since aggregate TFP decreases, output decreases more substantially than hours, lowering average labor productivity. Furthermore, since the wealth-productivity distribution shifts only slightly, output, hours, and productivity recover quickly. Hence, a temporary decrease in aggregate TFP generates a short-lived positive correlation between hours and productivity.

Including only aggregate TFP shocks, the constant risk model generates a strong positive correlation between total hours worked and average labor productivity.<sup>19</sup> In contrast, since fluctuations in idiosyncratic wage risk generate a persistent negative correlation, the varying

<sup>19</sup>Chang and Kim (2007) report a weakly positive correlation of 0.23 in the constant risk model with slightly different parameter values from those used herein, but as Takahashi (2014) shows, their finding is a result of computational errors.

risk model exhibits a weakly negative correlation between hours and productivity.

## 6 Countercyclical Idiosyncratic Wage Risk

Up to this point, I have assumed no correlation between idiosyncratic wage risk and aggregate TFP. Although the finding in Section 2 rationalizes the assumption, countercyclical risk is worth examining. First, idiosyncratic wage risk actually increased in the 1973–1975 and the early 1990s recessions, and hence it would be interesting to examine the impact of elevated risk during a recession. Second, the seminal paper by Storesletten, Telmer, and Yaron (2004) documents countercyclical income risk and existing macro models typically assume countercyclical income risk. Although labor income and wage risk could move differently, in order to analyze how cyclical risk affects aggregate fluctuations, I introduce countercyclical idiosyncratic wage risk into the model herein and compute business cycle moments.

### 6.1 Increase in Idiosyncratic Wage Risk During Recessions

In order to analyze a smooth change in idiosyncratic wage risk  $\sigma_{\varepsilon_x}$ , as seen in the actual economy, I conduct 500 model simulations and compute the average response. Each simulation starts from the steady state (in period  $-250$ ) and  $\sigma_{\varepsilon_x}$  increases by one state in period 0. At the same time, aggregate TFP  $z$  decreases by one grid point.<sup>20</sup> At other times,  $\sigma_{\varepsilon_x}$  and  $z$  evolve according to their independent stochastic processes. As shown in the upper two panels of Figure 9,  $\sigma_{\varepsilon_x}$  increases by 7.09% on average relative to the pre-shock level in period 0 and slowly decreases, whereas  $z$  decreases by 1.53% initially and gradually recovers. I compare the results of this experiment with those when only  $z$  decreases.

The lower panels show the movements of total hours worked and average labor productivity in these two cases. Compared with the case in which only aggregate TFP decreases, labor

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<sup>20</sup>If  $\sigma_{\varepsilon_x}$  was in the high state in period  $-1$ , then  $\sigma_{\varepsilon_x}$  remains there in period 0. Similarly, if  $z$  was the lowest in period  $-1$ , then  $z$  remains unchanged in period 0.

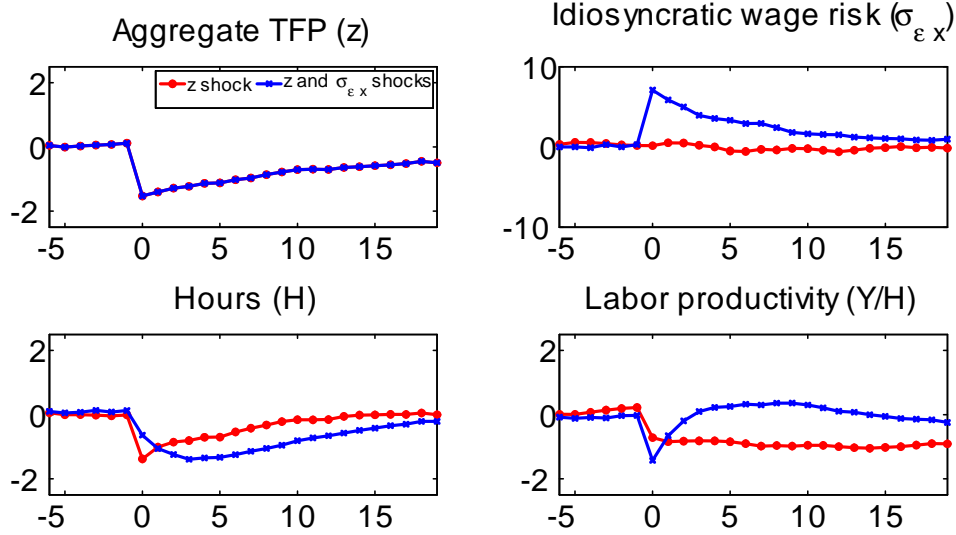


Figure 9: Increase in idiosyncratic wage risk during a recession. Vertical axis - period. Horizontal axis - percent deviation from the pre-shock average.

productivity recovers faster when idiosyncratic wage risk increases simultaneously. However, total hours worked recovers more slowly.

This result is similar to the U.S. experience. Figure 10 shows the cyclical components of total hours worked and average labor productivity between 1947Q3 and 2009Q3. The recovery of hours relative to labor productivity is slower after the 1973–1975 and the 1990–1991 recessions, during which idiosyncratic wage risk rose, than after the 1981–1982 recession, during which risk remained low. This finding provides additional evidence that the movement of idiosyncratic wage risk plays an important role in labor market dynamics over the business cycle.

## 6.2 Cyclicity of Risk and Business Cycles

Next, in order to evaluate the impact of countercyclical idiosyncratic wage risk on overall business cycles, I introduce negative comovement of idiosyncratic wage risk  $\sigma_{\varepsilon_x}$  with aggregate TFP  $z$  as follows. When  $z$  is approximately equal to the mean (approximately  $\pm 1.7\%$  relative to the steady-state level),  $\sigma_{\varepsilon_x} = \sigma_{\varepsilon_x, M} = \bar{\sigma}_{\varepsilon_x}$ . When  $z$  is lower than this range,

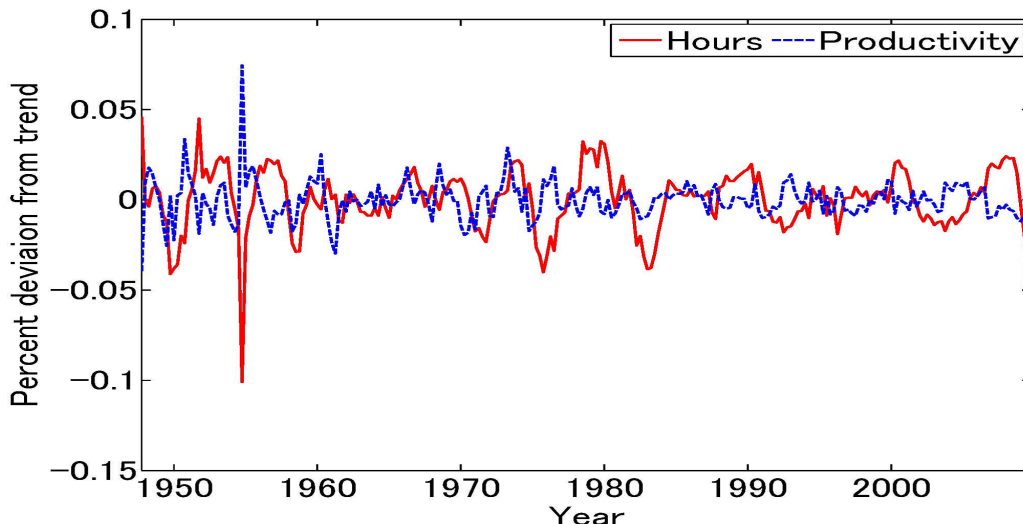


Figure 10: Total hours worked and average labor productivity in the U.S. I take logs of the variables and use the Hodrick-Prescott filter with a smoothing parameter of 1,600.

$\sigma_{\varepsilon_x} = \sigma_{\varepsilon_x,H} = (1 + \lambda)\bar{\sigma}_{\varepsilon_x}$ . When  $z$  exceeds the range,  $\sigma_{\varepsilon_x} = \sigma_{\varepsilon_x,L} = (1 - \lambda)\bar{\sigma}_{\varepsilon_x}$ . All of the other parameters maintain their values as determined in Section 4, including  $\bar{\sigma}_{\varepsilon_x} = 0.223$  and  $\lambda = 0.09$ , except that  $\rho_{\sigma_{\varepsilon_x}}$  is determined by the law of motion for  $z$ .

Importantly, the results for the countercyclical risk model are not directly comparable with those for the varying risk model in Section 5. This is because the fluctuations of idiosyncratic wage risk  $\sigma_{\varepsilon_x}$  are different in these two models. Specifically,  $\sigma_{\varepsilon_x}$  in the countercyclical risk model is substantially less volatile and slightly more persistent than  $\sigma_{\varepsilon_x}$  in the varying risk model calibrated to reproduce the risk fluctuations in the PSID data. In order to isolate the impact of the cyclicity of risk from the impact of the volatility and persistence of risk, I reset  $\lambda$  and  $\rho_{\sigma_{\varepsilon_x}}$  of the varying risk model, targeting the standard deviation and the first-order autocorrelation of  $\sigma_{\varepsilon_x}$  in the countercyclical risk model. The results are  $\lambda = 0.058$  and  $\rho_{\sigma_{\varepsilon_x}} = 0.925$ . All of the other parameters inherit their original values.

Table 5 lists the results for the countercyclical risk and recalibrated varying risk models along with the results for the constant risk model for comparison.<sup>21</sup> As shown, acyclical

<sup>21</sup>Note that these results should not be compared with the U.S. data moments because these models are not calibrated to reproduce the risk fluctuations seen in the PSID data.

	Countercyclical risk	Recalibrated varying risk	Constant risk
$\sigma_Y$	1.32	1.38	1.37
$\sigma_C$	0.37	0.33	0.32
$\sigma_I$	2.98	3.11	3.10
$\sigma_H$	0.56	0.68	0.57
$\sigma_{Y/H}$	0.61	0.74	0.48
$\sigma_{wedge}$	0.60	0.83	0.23
$Corr(Y, C)$	0.91	0.88	0.90
$Corr(Y, I)$	0.99	0.99	0.99
$Corr(Y, H)$	0.84	0.68	0.96
$Corr(Y, Y/H)$	0.87	0.73	0.95
$Corr(H, Y/H)$	0.46	0.00	0.83
$Corr(H, wedge)$	0.67	0.77	0.96

Table 5: Cyclicity of idiosyncratic wage risk and business cycle moments. I take logs of all of the series and use the Hodrick-Prescott filter with a smoothing parameter of 1,600. The volatility of output  $\sigma_Y$  is the standard deviation of output (multiplied by 100). Other volatilities are their ratios with respect to  $\sigma_Y$ . *Corr* denotes a contemporaneous correlation.

and countercyclical idiosyncratic wage risk move the model’s labor market statistics in the same direction, although countercyclical risk has a smaller impact. In particular, while both varying risk models generate a lower correlation between total hours worked and average labor productivity, as compared to the constant risk model, the reduction in the correlation under acyclical risk is about twice as large as that under countercyclical risk. Hence, variation in idiosyncratic wage risk and its independence from aggregate TFP play roughly equal roles in the resolution of the hours-productivity puzzle.

## 7 Conclusion

This paper analyzed how cyclical variation in idiosyncratic wage risk affects business cycles using an incomplete asset markets model with endogenous employment choice. I found that changes in wage uncertainty lead to heterogeneous employment responses among individuals with different wealth and productivity. At the aggregate level, total hours worked and average labor productivity move persistently in opposite directions. When introducing uncertainty shocks measured with the PSID wage data, on top of aggregate TFP shocks,

the hours-productivity correlation in the model switches from positive to weakly negative, matching the U.S. data. Introducing uncertainty shocks also helps the model reproduce the volatile labor wedge seen in the U.S. economy.

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## 8 Appendix A: Data

### 8.1 A1: Individual Wage Data

I take data for heads of households from the family-level file of the PSID. Individual wages are the ratios of annual labor income (1975: V3863–1992: V21484) to annual hours worked (1975: V3823–1992: V20344), converted to real wages in terms of 1983 dollars using the CPI data.<sup>22,23</sup> I only use interview years of 1975–1992 because data on years of education are discontinuous in 1974.<sup>24</sup>

I also exclude the following observations.

- Observations whose heads change in the year (1975: V4114–1992: V21388).
- Observations with major assignments assigned to the labor income and/or hours.<sup>25</sup>
- Observations with wages of less than one dollar (in 1983 dollars) or higher than 500 dollars (in 1983 dollars).
- The most recent Latino sample and the Survey of Economic Opportunity sample.
- Observations with fewer than 100 annual hours.
- Self-employment observations (1975: V3970–1992: V20696).
- Observations in the agricultural sectors (1975: V3968–1992: V20701).
- Top-coded observations for income.

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<sup>22</sup>Numbers in parentheses are variable labels of the PSID.

<sup>23</sup>I take Monthly “Consumer Price Index for All Urban Consumers: All Items” from the FRED database at the Federal Reserve Bank of St. Louis. The annual CPI in a year is the average monthly CPI in the year.

<sup>24</sup>I exclude the interview year of 1993 because data on major assignment for labor income are not available.

<sup>25</sup>See Swanson (2007) for details. For labor hours, I use the total hour accuracy (1975: V3824–1984: V10038) until 1984, and after 1984, the main job hour accuracy (1985: V11141–1992: V20339), the overtime hour accuracy (1985: V11143–1992: V20341), and the extra job accuracy (1985: V11145–1992: V20343). For labor income, I use the accuracy code for wages and salaries (1975: V3859–1992: V20430) and the accuracy code for labor income except wages and salaries (1975: V3864–1992: V20435).

Other variables are age (1975: V3921–1992: V20651), marital status (1975: V4053–1992: V21522), and the number of children (1975: V3924–1992: V20654). For years of education, I select “Grade Completed” (1975: ER30169–1992: ER30748) from the individual-level file and define experience as age minus education minus sex.

## **8.2 A2: Individual Labor Income Data**

The 1992 PSID individual-level file provides annual labor income data in 1991 for individuals, including those other than heads of households. I take total labor income (ER30750), excluding individuals younger than 16 (ER30736) and individuals with major assignments on their income and/or hours worked (ER30751, ER30755).

## **8.3 A3: U.S. Macroeconomic Data**

The data period is from 1947Q3 to 2009Q3. Output is “Real Gross Domestic Product (billions of chained 2005 dollars)” taken from Table 1.1.6 of the Bureau of Economic Analysis (BEA). Consumption is “Personal Consumption Expenditures (PCE)” less durable goods obtained from Table 2.3.5 of the BEA. Investment is the sum of durable goods consumption in Table 2.3.5 and private fixed investment (including residential investment) in Table 5.3.5. I compute the real values of consumption and investment using the price index for Gross Domestic Product in Table 1.1.4. The data on total labor hours are the data constructed by Cociuba, Prescott, and Ueberfeldt (2009).<sup>26</sup>

# **9 Appendix B: Solution Methods**

## **9.1 B1: Steady State**

The solution method for the steady state is similar to that of Chang and Kim (2007).

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<sup>26</sup>I am grateful to the authors for making the data available.

1. Discretize the idiosyncratic state  $(k, x)$ . Set 100 log-spaced points over  $[-2, 250]$  for  $k$ . For  $x$ , set 17 evenly spaced points over  $[-3\bar{\sigma}_{\varepsilon_x}/\sqrt{1-\rho_x^2}, 3\bar{\sigma}_{\varepsilon_x}/\sqrt{1-\rho_x^2}]$  and compute the transition matrix using the method of Tauchen (1986).
2. Set a guess for the discount factor  $\beta$ .
3. Solve the individual optimization problem and obtain the beginning-of-period value function  $V(k, x)$ . I omit the aggregate state  $(z, \mu, \sigma_{\varepsilon_x})$ , which is constant at the steady state.
  - (a) Compute the steady-state wage rate  $\bar{w} = \alpha\bar{z}((1-\alpha)\bar{z}/(\bar{r} + \delta))^{(1-\alpha)/\alpha}$  with the target steady-state rental rate of capital  $\bar{r} = 0.01$  and the steady-state aggregate TFP  $\bar{z} = 1.0$ .
  - (b) Set a guess for the beginning-of-period value function  $V_0(k, x)$ .
  - (c) Solve the consumption-saving problem for each employment choice:

$$V_1^E(k, x) = \max_{k' \geq \bar{k}} \{u(w\bar{h}x + (1+r)k - k', \bar{h}) + \beta \sum_{x'} \pi_x(x'|x) V_0(k', x')\}$$

and

$$V_1^N(k, x) = \max_{k' \geq \bar{k}} \{u((1+r)k - k', 0) + \beta \sum_{x'} \pi_x(x'|x) V_0(k', x')\},$$

where  $\pi_x(x'|x)$  is the transition probability from  $x$  to  $x'$ . Use cubic spline interpolation to approximate the conditional expectation at  $k'$  off the grid points. If  $V_1^E(k, x) \geq V_1^N(k, x)$ , then individuals with  $k$  and  $x$  choose to work. Otherwise, they do not work. Set  $V_1(k, x) = \max\{V_1^E(k, x), V_1^N(k, x)\}$ .

- (d) If  $V_1(k, x)$  becomes sufficiently close to  $V_0(k, x)$ , then set  $V(k, x) = V_1(k, x)$  and proceed to the next step. Otherwise, update the value function as  $V_0(k, x) = V_1(k, x)$  and return to (c).

4. Compute the steady-state distribution  $\bar{\mu}(k, x)$ .
  - (a) Choose points used for approximating the distribution. Use 2,000 log-spaced points over  $[-2, 250]$  for  $k$  and the points chosen in Step 1 for  $x$ .
  - (b) Replace  $V_0(k, x)$  of the problems in Step 3 (c) with  $V(k, x)$  obtained in Step 3 (d). Solve the problems this time for  $2,000 \times 17$  pairs of  $(k, x)$  and find their optimal asset holding  $k'(k, x)$  and employment  $h(k, x)$ .
  - (c) Suppose  $k_m \leq k'(k, x) < k_{m+1}$ , where  $k_m$  and  $k_{m+1}$  are two sequential asset points. Starting from an initial guess, keep updating the distribution until the distribution converges as follows: Individuals with  $(k, x)$  move to  $(k_m, x')$  with probability  $\omega\pi_x(x'|x)$  and to  $(k_{m+1}, x')$  with probability  $(1 - \omega)\pi_x(x'|x)$ , where  $\omega = (k_{m+1} - k')/(k_{m+1} - k_m)$ . The result is the steady-state distribution  $\bar{\mu}(k, x)$ .
5. Compute the steady-state aggregate capital  $\bar{K} = \int k\bar{\mu}([dk \times dx])$  and aggregate efficiency-weighted labor  $\bar{L} = \int xh(k, x)\bar{\mu}([dk \times dx])$ . Calculate the implied steady-state rental rate of capital  $\bar{r} = (1 - \alpha)\bar{z}\bar{K}^{-\alpha}\bar{L}^\alpha - \delta$ . If  $\bar{r}$  becomes sufficiently close to the target rate (1 percent), then stop. Otherwise, set a different value for  $\beta$  and repeat Steps 3–5.

## 9.2 B2: Business Cycles

I analyze the model's business cycle generalizing the Krusell and Smith (1997, 1998) algorithm. The method is similar to that used in Takahashi (2014).

1. Discretize the aggregate state  $(z, \mu, \sigma_{\epsilon_x})$ . For aggregate TFP  $z$ , set nine evenly spaced points over  $[-3\bar{\sigma}_{\epsilon_z}/\sqrt{1 - \rho_z^2}, 3\bar{\sigma}_{\epsilon_z}/\sqrt{1 - \rho_z^2}]$ , and compute the transition matrix using the method of Tauchen (1986). Replace the individual distribution  $\mu$  with aggregate capital  $K$ . Use seven evenly spaced points over  $[0.80\bar{K}, 1.20\bar{K}]$ , where  $\bar{K}(= 11.57)$  is the steady-state aggregate capital. For  $\sigma_{\epsilon_x}$ , use the three risk states.

2. Discretize the individual state  $(k, x)$ . For  $k$ , use the 100 points chosen in the steady-state solution. For  $x$ , use 17 evenly spaced points over  $[-3\bar{\sigma}_{\epsilon_x}/\sqrt{1-\rho_x^2}, 3\bar{\sigma}_{\epsilon_x}/\sqrt{1-\rho_x^2}]$  for all of the risk states. The transition probabilities vary with the risk states. Compute these probabilities using the method of Tauchen (1986).
3. Individuals forecast  $K'$  and  $w$  using the following rules:

$$\ln \hat{K}' = a_{0,i} + a_{1,i} \ln K + a_{2,i} \ln z \quad (12)$$

and

$$\ln \hat{w} = b_{0,i} + b_{1,i} \ln K + b_{2,i} \ln z, \quad (13)$$

for each risk state ( $i = H, M, L$ ). Individuals compute  $\hat{r} = z(1-\alpha)(\hat{w}/(\alpha z))^{-\alpha/(1-\alpha)}$ .

4. Solve the individual optimization problem and obtain the beginning-of-period value function  $V(k, x; z, K, \sigma_{\epsilon_x})$ .
- (a) Set a guess for the beginning-of-period value function  $V_0(k, x; z, K, \sigma_{\epsilon_x})$ .
- (b) Solve the consumption-saving problem for each employment choice:

$$\begin{aligned} V_1^E(k, x; z, K, \sigma_{\epsilon_x}) = & \max_{k' \geq \bar{k}} \{u(\hat{w}\bar{h}x + (1+\hat{r})k - k', \bar{h}) \\ & + \beta \sum_{x'} \sum_{z'} \sum_{\sigma'_{\epsilon_x}} \pi_x(x'|x, \sigma_{\epsilon_x}) \pi_z(z'|z) \pi_{\sigma_{\epsilon_x}}(\sigma'_{\epsilon_x} | \sigma_{\epsilon_x}) V_0(k', x'; z', \hat{K}', \sigma'_{\epsilon_x}) \end{aligned}$$

and

$$\begin{aligned}
V_1^N(k, x; z, K, \sigma_{\epsilon_x}) &= \max_{k' \geq \bar{k}} \{u((1 + \hat{r})k - k', 0) \\
&+ \beta \sum_{x'} \sum_{z'} \sum_{\sigma'_{\epsilon_x}} \pi_x(x' | x, \sigma_{\epsilon_x}) \pi_z(z' | z) \pi_{\sigma_{\epsilon_x}}(\sigma'_{\epsilon_x} | \sigma_{\epsilon_x}) V_0(k', x'; z', \hat{K}', \sigma'_{\epsilon_x}),
\end{aligned}$$

where  $\pi_x(x' | x, \sigma_{\epsilon_x})$  is the transition probability from  $x$  to  $x'$  under  $\sigma_{\epsilon_x}$ ,  $\pi_z(z' | z)$  is the transition probability from  $z$  to  $z'$ , and  $\pi_{\sigma_{\epsilon_x}}(\sigma'_{\epsilon_x} | \sigma_{\epsilon_x})$  is the transition probability from  $\sigma_{\epsilon_x}$  to  $\sigma'_{\epsilon_x}$ . Use bivariate cubic spline interpolation in  $(K, k)$  to approximate the conditional expectation at  $(\hat{K}', k')$  off their grid points. If  $V_1^E(k, x; z, K, \sigma_{\epsilon_x}) \geq V_1^N(k, x; z, K, \sigma_{\epsilon_x})$ , individuals with  $k$  and  $x$  work. Otherwise, they do not. Set  $V_1(k, x; z, K, \sigma_{\epsilon_x}) = \max\{V_1^E(k, x; z, K, \sigma_{\epsilon_x}), V_1^N(k, x; z, K, \sigma_{\epsilon_x})\}$ .

- (c) If  $V_1(k, x; z, K, \sigma_{\epsilon_x})$  becomes sufficiently close to  $V_0(k, x; z, K, \sigma_{\epsilon_x})$ , then proceed to the next step, setting  $V(k, x; z, K, \sigma_{\epsilon_x}) = V_1(k, x; z, K, \sigma_{\epsilon_x})$ . Otherwise, update the value function as  $V_0(k, x; z, K, \sigma_{\epsilon_x}) = V_1(k, x; z, K, \sigma_{\epsilon_x})$ , and return to (b).

5. Generate 3,500-period data using the beginning-of-period value function  $V(k, x; z, K, \sigma_{\epsilon_x})$ .

- (a) Set conditions for the initial period:  $z_1 = \bar{z}$ ,  $\sigma_{\epsilon_{x1}} = \sigma_{\epsilon_{x,M}}$ ,  $\mu_1(k, x) = \bar{\mu}(k, x)$ , and  $K_1 = \int k \mu_1([dk \times dx])$ .
- (b) Set  $\tilde{w}_1$ , as a guess for  $w_1$ . Then,  $\tilde{r}_1 = (1 - \alpha)z_1(\tilde{w}_1/\alpha z_1)^{-\alpha/(1-\alpha)} - \delta$ . The forecasting rule gives the individuals' forecast of the next period approximate aggregate state  $\hat{K}_2$ . Replacing  $V_0(k, x; z, K, \sigma_{\epsilon_x})$  with  $V(k, x; z, K, \sigma_{\epsilon_x})$ , solve the individual problems shown in Step 4 (b) under  $w = \tilde{w}_1$ ,  $r = \tilde{r}_1$ , and  $K' = \hat{K}_2$ , this time for  $2,000 \times 17$  pairs of  $(k, x)$ . Record the optimal asset holding  $k_2(k, x)$  and employment  $h_1(k, x)$ .
- (c) Check labor market clearing:  $\tilde{L}_1 \equiv (\alpha z_1/\tilde{w}_1)^{1/(1-\alpha)} K_1 = \int x h_1(k, x) \mu_1([dk \times dx])$ . If the labor market clears, proceed to the next step. Otherwise, reset  $\tilde{w}_1$  and



return to (b).<sup>27</sup>

(d) Compute aggregate variables:  $L_1 = \int x h_1(k, x) \mu_1([dk \times dx])$ ,  $K_2 = \int k_2(k, x) \mu_1([dk \times dx])$ ,  $H_1 = \int h_1(k, x) \mu_1([dk \times dx])$ ,  $Y_1 = z_1 K_1^{1-\alpha} L_1^\alpha$ ,

$I_1 = K_2 - (1 - \delta)K_1$ ,  $C_1 = Y_1 - I_1$ , and  $r_1 = (1 - \alpha)z_1 K_1^{-\alpha} L_1^\alpha - \delta$ .

(e) Obtain the next period distribution  $\mu_2(k, x)$  as described in Step 4 (c) of the steady-state solution.

(f) Repeat (b)–(e) for 3,500 periods.

6. Using the simulated data (disregarding the first 500 periods), update the coefficients of the forecasting rules by ordinary least squares. If these coefficients converge, then proceed to the next step. Otherwise, repeat Steps 4 and 5 using the new forecasting rules.

7. Check whether the converged forecasting rules are sufficiently accurate. If not, assume different functional forms and repeat Steps 3–6. The forecasting rules of (11) and (12) are quite accurate, as reported in Appendix C.

## 10 Appendix C: Forecasting Rules

The tables below list the coefficients of the forecasting rules ( $\ln \hat{K}' = a_0 + a_1 \ln K + a_2 \ln z$  and  $\ln \hat{w} = b_0 + b_1 \ln K + b_2 \ln z$ ) and the accuracy of the rules. Two accuracy measures are the coefficient of determination  $R^2$  and the standard deviation of the forecasting error  $\hat{\sigma}$ . I use separate rules for each of the risk states.

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<sup>27</sup>Ensuring market clearing is an essential step of the Krusell and Smith (1998) algorithm and included for the bond market by Krusell and Smith (1997) and Pijoan-Mas (2007), for the goods market by Khan and Thomas (2003, 2007, 2008), and for the labor market by Takahashi (2014).

	Constant risk	Varying risk (H / M / L)	Psych risk (H / M / L)	
$\hat{K}'$	$a_0$	0.115	0.073 / 0.081 / 0.080	0.116 / 0.114 / 0.115
	$a_1$	0.953	0.971 / 0.967 / 0.967	0.953 / 0.954 / 0.953
	$a_2$	0.101	0.082 / 0.086 / 0.093	0.101 / 0.100 / 0.102
	$R^2$	1.000	1.000 / 1.000 / 1.000	1.000 / 1.000 / 1.000
	$\hat{\sigma}$	0.0079%	0.1122% / 0.0939% / 0.0969%	0.0101% / 0.0106% / 0.0107%
$\hat{w}$	$b_0$	-0.209	0.031 / -0.027 / -0.024	-0.209 / -0.209 / -0.209
	$b_1$	0.438	0.338 / 0.364 / 0.365	0.438 / 0.438 / 0.439
	$b_2$	0.818	0.928 / 0.900 / 0.859	0.818 / 0.818 / 0.815
	$R^2$	1.000	0.972 / 0.982 / 0.972	1.000 / 1.000 / 1.000
	$\hat{\sigma}$	0.0407%	0.6154% / 0.5032% / 0.5415%	0.0456% / 0.0503% / 0.0452%

	Countercyclical risk (H / M / L)	Recalibrated varying risk (H / M / L)	
$\hat{K}'$	$a_0$	0.245 / 0.204 / 0.229	0.096 / 0.099 / 0.091
	$a_1$	0.900 / 0.917 / 0.906	0.961 / 0.960 / 0.963
	$a_2$	0.093 / 0.100 / 0.106	0.091 / 0.096 / 0.093
	$R^2$	0.999 / 0.999 / 0.999	1.000 / 1.000 / 1.000
	$\hat{\sigma}$	0.0401% / 0.0418% / 0.0423%	0.0728% / 0.0592% / 0.0773%
$\hat{w}$	$b_0$	-0.988 / -0.722 / -0.855	-0.102 / -0.125 / -0.078
	$b_1$	0.756 / 0.648 / 0.703	0.393 / 0.404 / 0.387
	$b_2$	0.848 / 0.817 / 0.793	0.873 / 0.847 / 0.862
	$R^2$	0.974 / 0.983 / 0.981	0.989 / 0.990 / 0.986
	$\hat{\sigma}$	0.2518% / 0.2366% / 0.2291%	0.3958% / 0.3123% / 0.4054%