# Screening and Adverse Selection in Frictional Markets 

Benjamin Lester<br>Philadelphia Fed

Venky Venkateswaran
NYU Stern

Ali Shourideh<br>Wharton

Ariel Zetlin-Jones
Carnegie Mellon University

May 2015

Introduction

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- Examples: insurance, loans, financial securities

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But many important questions

- Recent push to make these markets more competitive, transparent
- Is this a good idea?

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(1) Complete characterization of the unique equilibrium
(2) Explore positive predictions for distribution of contracts
(3) Policy experiments: changes in competition, transparency

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- Screening: Buyers offer general menus of non-linear contracts
- Price-quantity pairs: induce sellers to self-select
- Imperfect Comp: sellers receive either 1 or 2 offers (à la Burdett-Judd)
- Buyer competing with another with prob $\pi$, otherwise monopsonist.
- Contract offered before buyers know


## What We Know (Equilibrium)



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Perfect competition and "severe adverse selection" $\Rightarrow$ least-cost sep.

## What We Know (Equilibrium)



Perfect competition and "mild adverse selection" $\Rightarrow$ Mixed Strategy Eq.

## What We Know (Equilibrium)



Monopsony and "severe adverse selection" $\Rightarrow$ No Trade with High Type

## What We Know (Equilibrium)



Monopsony and "mild adverse selection" $\Rightarrow$ Full Trade

## Objective



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What are the implications of imperfect comp. for....

- Terms of trade
- Welfare
- Policy

Summary of Findings

## Methodology

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- Establish important (and general!) property of all equilibria:
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## Positive Implications

- Equilibrium can be pooling, separating, or mix
- Separation when adverse selection severe, trading frictions mild
- Pooling when adverse selection mild, trading frictions severe


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## Normative Implications

- Adverse selection severe: interior $\pi$ maximizes surplus from trade
- Adverse selection mild: welfare unambiguously decreasing in $\pi$
- Increasing transparency/relaxing info frictions can $\uparrow$ or $\downarrow$ welfare


## Related Literature

## Empirical

- Chiappori and Salanie (2000); Ivashina (2009); Einav et al. (2010); Einav et al. (2012)


## Adverse Selection and Screening

- Rothschild and Stiglitz (1976); Dasgupta and Maskin (1986); Rosenthal and Weiss (1984); Mirrlees (1971); Stiglitz (1977); Maskin and Riley (1984); Guerrieri, Shimer and Wright (2010); Many, many others

Imperfect Competition and Selection

- Search Frictions: Burdett and Judd (1983); Garrett, Gomes, and Maestri (2014)
- Specialization: Benabou and Tirole (2014), Mahoney and Weyl (2014), Veiga and Weyl (2015)


## Environment

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- Each Seller endowed with 1 divisible asset
- Seller values asset at rate $c_{i}$
- Two types of sellers $i \in\{I, h\}$ with prob. $\mu_{i}$
- Buyer values type $i$ asset at rate $v_{i}$


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- If $x$ units sold for transfer $t$, payoffs are
- Seller: $t+(1-x) c_{i}$
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- If $x$ units sold for transfer $t$, payoffs are
- Seller: $t+(1-x) c_{i}$
- Buyer: $x v_{i}-t$
- Assumptions:
- Gains to trade: $v_{i}>c_{i}$
- Lemons Assumption: $v_{l}<c_{h}$
- Adverse Selection: Only sellers know asset quality


## Model Environment

Screening

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- Refer to seller with 1 offer as Captive
- Refer to seller with 2 offers as non-Captive


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## Stylized Model of Trade

- best examples: corporate loans market; securitization (maybe)
- other examples: information-based trading; insurance


## Strategies

- Each buyer offers arbitrary menu of contracts $\left\{\left(x_{n}, t_{n}\right)_{n \in \mathcal{N}}\right\}$
- Captive seller's choice: best $\left(x_{n}, t_{n}\right)$ from one buyer
- Non-captive seller's choice: best $\left(x_{n}, t_{n}\right)$ among both buyers


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## Revelation Principle

sufficient to consider

- menus with two contracts $\mathbf{z} \equiv\left\{\left(x_{l}, t_{l}\right),\left(x_{h}, t_{h}\right)\right\}$

$$
\left(I C_{j}\right): \quad t_{j}+c_{j}\left(1-x_{j}\right) \geq t_{-j}+c_{j}\left(1-x_{-j}\right) \quad j \in\{h, I\}
$$

- seller $j$ : chooses contract $j$ from available the set of menus available


## Equilibrium Price Dispersion

- Suppose $\pi \in(0,1)$ : no symmetric pure strategy equilibrium exists
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## Equilibrium Price Dispersion

- Suppose $\pi \in(0,1)$ : no symmetric pure strategy equilibrium exists
- buyers can guarantee positive profits: trade only with captive types
- in a pure strategy equilibrium: have to share non-captive types There is always an incentive to undercut
- Only mixed strategy equilibria possible $\Rightarrow$ equilibrium features price dispersion
$\Rightarrow$ equilibrium described by buyers' distribution over menus


## Equilibrium definition

A symmetric equilibrium is a distribution $\Phi(\mathbf{z})$ such that almost all $\mathbf{z}$ satisfy,
(1) Incentive compatibility:

$$
t_{j}+c_{j}\left(1-x_{j}\right) \geq t_{-j}+c_{j}\left(1-x_{-j}\right) \quad j \in\{h, /\}
$$

(2) Seller optimality:

$$
\chi_{i}\left(\mathbf{z}, \mathbf{z}^{\prime}\right) \text { maximizes her utility }
$$

(3) Buyer optimality: for each $z \in \operatorname{Supp}(\Phi)$

$$
\begin{equation*}
\mathbf{z} \in \arg \max _{\mathbf{z}} \sum_{i \in\{1, h\}} \mu_{i}\left(v_{i} x_{i}-t_{i}\right)\left[1-\pi+\pi \int_{\mathbf{z}^{\prime}} \chi_{i}\left(\mathbf{z}, \mathbf{z}^{\prime}\right) \Phi\left(d \mathbf{z}^{\prime}\right)\right] \tag{1}
\end{equation*}
$$

## Characterization

Equilibrium described by non-degenerate distribution in 4 dimensions

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Proceed in 4 steps

1. Show that menus can be summarized by a pair of utilities $\left(u_{h}, u_{l}\right)$

- Reduces dimensionality of problem to distribution in 2 dimensions

2. Show there is a 1-1 mapping between $u_{l}$ and $u_{h}$

- Reduces problem to distribution in 1 dimension + a monotonic function

3. Construct Equilibrium
4. Show that constructed equilibrium is unique

A utility representation

## Result (Dasgupta and Maskin (1986))

In all menus offered in equilibrium,

- the low types trades everything: $\quad x_{I}=1$
- $I C_{l}$ binds: $t_{l}=t_{h}+c_{l}\left(1-x_{h}\right)$


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## Result

Equilibrium menus can be represented by $\left(u_{h}, u_{l}\right)$ with corresponding allocations

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t_{l}=u_{l} \quad x_{h}=1-\frac{u_{h}-u_{l}}{c_{h}-c_{l}} \quad t_{h}=\frac{u_{l} c_{h}-u_{h} c_{l}}{c_{h}-c_{l}}
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Since we must have $0 \leq x_{h} \leq 1$,

$$
c_{h}-c_{l} \geq u_{h}-u_{l} \geq 0
$$

## A utility representation

Marginal distributions

$$
F_{j}\left(u_{j}\right)=\int_{\mathbf{z}^{\prime}} \mathbf{1}\left[t_{j}^{\prime}+c_{j}\left(1-x_{j}^{\prime}\right) \leq u_{j}\right] d \Phi\left(\mathbf{z}^{\prime}\right) \quad j \in\{h, /\}
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Then, each buyer solves

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\begin{aligned}
& \Pi\left(u_{h}, u_{l}\right)= \\
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Need to characterize the two linked distributions $F_{I}$ and $F_{h}$ !

Further Simplifying the Characterization

## Result

$F_{l}$ and $F_{h}$ have connected support and are continuous.

- Except for a knife-edge case (see paper)
- Proof more involved than standard case because of interdependencies

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## Result

The profit function $\Pi\left(u_{h}, u_{l}\right)$ is strictly supermodular.

- Intuition: $u_{l} \uparrow \Rightarrow \Pi_{h} \uparrow \Rightarrow$ stronger incentives to attract high types
- $\Rightarrow U_{h}\left(u_{l}\right) \equiv \operatorname{argmax}_{u_{h}} \Pi\left(u_{h}, u_{l}\right) \quad$ is weakly increasing


## Strict Rank Preserving

## Theorem <br> $U_{h}\left(u_{l}\right)$ is a strictly increasing function.

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Idea of Proof

- $U_{h}\left(u_{l}\right)$ increasing due to super-modularity of profit function
- $F_{l}$ and $F_{h}$ have no holes or mass points imply $U_{h}$ is strictly increasing and not a correspondence


## Strict Rank Preserving

## Theorem

$U_{h}\left(u_{l}\right)$ is a strictly increasing function.

Implications for Characterization

- Rank ordering of equilibrium menus identical across types
- Menus attract same fraction of both types $F_{l}\left(u_{l}\right)=F_{h}\left(U_{h}\left(u_{l}\right)\right)$
- Greatly simplifies the analysis: only have to find $F_{l}\left(u_{l}\right)$ and $U_{h}\left(u_{l}\right)$


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Robust to any number of types

- Relies only on utility representation and ability to show distributions are well behaved

Constructing Equilibria

## Equilibria: The two limit cases

Monopsony: $\pi=0$

Bertrand: $\pi=1$

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- No Cross-subsidization
- $\mu_{h} \geq \bar{\mu}_{h} \Rightarrow$ Pooling with $x_{h}=x_{l}=1$ and $\Pi_{h}>0>\Pi_{l}$
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- Cross-subsidization

Intuition: Higher $\mu_{h} \Rightarrow$ Relaxing $I C^{\prime}$ more attractive

## Types of equilibria in the middle



## Types of equilibria in the middle



High $\mu_{h}$

- $\Pi_{h}>0>\Pi_{I}$
- All separating, all pooling or a mix

Low $\mu_{h}$

- $\Pi_{l}, \Pi_{h} \geq 0$
- All separating, $U_{h}\left(u_{l}\right) \neq u_{l}$

No cross-subsidization: Characterization

Focus on separating equilibrium in no-cross subsidization region

Recall problem of a buyer:

$$
\begin{aligned}
& \Pi\left(u_{h}, u_{l}\right)= \\
& \max _{u_{l} \geq c_{l}, u_{h} \geq c_{h}} \sum_{j \in\{l, h\}} \mu_{j}\left[1-\pi+\pi F_{j}\left(u_{j}\right)\right] \Pi_{j}\left(u_{h}, u_{l}\right) \\
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- In separating equilibrium we construct, $c_{h}-c_{l}>u_{h}-u_{l}>0$
- Sufficient to ensure local deviations unprofitable

No cross-subsidization: Characterization

Marginal benefits vs costs of increasing $u_{l}$

$$
\underbrace{\frac{\pi f_{l}\left(u_{l}\right) \Pi_{l}}{1-\pi+\pi F_{l}\left(u_{l}\right)}}_{\text {MB of more low types }}+\underbrace{\frac{\mu_{h}}{1-\mu_{h}} \frac{v_{h}-c_{h}}{c_{h}-c_{l}}}_{\mathrm{MB} \text { of relaxing } C_{l}}=\underbrace{1}_{M C}
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Boundary conditions

$$
F_{l}\left(c_{l}\right)=0 \quad F_{l}\left(\bar{u}_{l}\right)=1 \quad \rightarrow \quad F_{l}\left(u_{l}\right)
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Equal profit condition

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\left[1-\pi+\pi F_{l}\left(u_{l}\right)\right] \Pi\left(U_{h}, u_{l}\right)=\bar{\Pi} \quad \rightarrow \quad U_{h}\left(u_{l}\right)
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$$

Pursue similar construction in other regions of parameter space

## Equilibrium Regions in the Middle



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$\pi$

More Competition implies less pooling

- Gains to cream-skimming increase in $\pi$

Milder Adverse Selection (higher $\mu_{h}$ ) implies more pooling

- increased incentives to trade high volume
- increased cost of cream-skimming


## Equilibrium Regions in the Middle



More Competition implies less pooling

- Gains to cream-skimming increase in $\pi$

Milder Adverse Selection (higher $\mu_{h}$ ) implies more pooling

- increased incentives to trade high volume
- increased cost of cream-skimming


## Theorem

For every $\left(\pi, \mu_{h}\right)$ there is a unique equilibrium.

## Equilibrium Implications

Positive and Normative Implications

Is improving competition desirable for volume or welfare?

- For high $\mu_{h}$, monopsony dominates perfect competition
- For low $\mu_{h}$, perfect competition dominates monopsony
- Will show: for low $\mu_{h}$, welfare maximized at interior $\pi$


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Is increasing transparency desirable?

- Allowing insurers, loan officers, dealers to discriminate on observables?
- Interpret increased transparency as increased spread in $\mu_{h}$
- Desirability depends on curvature of welfare function with respect to $\mu_{h}$
- Will show: Concavity/Convexity of welfare function depends on $\pi, \mu_{h}$

Equilibrium Implications: Competition

Competition with No Cross-Subsidization

Assume $\mu_{h}$ in no cross-subsidization region

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Equilibrium Distribution and $U_{h}\left(u_{l}\right)$ for $\pi=0.2$
Shaded Region indicates support of $F_{I}$

## Competition with No Cross-Subsidization

Assume $\mu_{h}$ in no cross-subsidization region


Equilibrium Distribution and $U_{h}\left(u_{l}\right)$ for $\pi=0.5$
Shaded Region indicates support of $F_{l}$

- Increase in $\pi$ increases $F_{l}$ in sense of FOSD


## Competition with No Cross-Subsidization

Assume $\mu_{h}$ in no cross-subsidization region


Equilibrium Distribution and $U_{h}\left(u_{l}\right)$ for $\pi=0.9$

$$
\text { Shaded Region indicates support of } F_{l}
$$

- Increase in $\pi$ increases $F_{l}$ in sense of FOSD
- Driven by increased competition for (abundant) low-quality sellers

Competition with No Cross-Subsidization

How is trade volume related to $U_{h}$ ?


$$
\begin{aligned}
& x_{h}\left(u_{l}\right)=1-\frac{U_{h}\left(u_{l}\right)-u_{l}}{c_{h}-c_{l}} \\
& x_{h}^{\prime}\left(u_{l}\right)>0 \Leftrightarrow U_{h}^{\prime}\left(u_{l}\right)>1
\end{aligned}
$$




Competition with No Cross-Subsidization


Equilibrium Objects for $\pi=0.2$

## Competition with No Cross-Subsidization



Equilibrium Objects for $\pi=0.5$

- From low $\pi$, increase in $\pi$ increases volume


## Competition with No Cross-Subsidization



Equilibrium Objects for $\pi=0.9$

- From moderate $\pi$, increase in $\pi$ decreases volume

Competition and Welfare

When no cross-subsidization

$$
W\left(\mu_{h}, \pi\right)=\left(1-\mu_{h}\right)\left(v_{l}-c_{l}\right)+\mu_{h}\left(v_{h}-c_{h}\right) \int x_{h}\left(u_{l}\right) d F\left(u_{l}\right)
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Why is welfare decreasing?

- $\mu_{h}$ low implies few high types
- Competition less fierce for high types
- Demand from high types relatively inelastic
- Equal profits $\Rightarrow$ greater dispersion in prices
- Implies $U_{h}^{\prime}\left(u_{l}\right)>1$

Welfare maximized for interior $\pi$

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Welfare maximized for interior $\pi$

With Cross-Subsidization, welfare (weakly) maximized in monopsony outcome

- Full trade $\Rightarrow$ all gains to trade exhausted

Equilibrium Implications: Transparency

## Desirability of Transparency

Do the following policies improve welfare ?

- Allowing insurance providers to discriminate based on observables
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- Each seller has individual $\mu_{h}^{\prime}$; Buyers know distribution over $\mu_{h}^{\prime}$
- Buyers restricted to offering contracts associated with $E\left[\mu_{h}^{\prime}\right]$
- Under transparency, buyers allowed to offer $\mu_{h}$-specific menus
- Need to compare $E\left[W\left(\mu_{h}^{\prime}, \pi\right)\right]$ to $W\left(E\left[\mu_{h}^{\prime}\right], \pi\right)$


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Is Transparency Desirable? Answer: Depends on $\pi$ !

- $W$ is linear when $\pi=0$ and $\pi=1 \Rightarrow$ no effect on welfare
- $W$ is concave when $\pi$ is high $\Rightarrow$ bad for welfare

Desirability of Transparency: The two limit cases

Monopsony: $\pi=0$

Bertrand: $\pi=1$

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In these cases, welfare is linear in $\mu_{h}$ so that mean-preserving spread (locally) has no impact on welfare

## Desirability of Transparency: The general cases




- With cross-subsidization, welfare is concave
$\Rightarrow$ increases in transparency harm welfare
- Without cross-subsidization, welfare is concave only for high $\pi$
$\Rightarrow$ increases in transparency harm welfare when markets competitive


## Conclusion

Methodological contribution

- Imperfect competition and adverse selection with optimal contracts
- Rich predictions for the distribution of observed trades

Substantive insights

- Depending on parameters, pooling and/or separating menus in equilibrium
- Competition, transparency can be bad for welfare

Work in progress

- Generalize to $N$ types, curved utility
- Non-exclusive trading

No cross-subsidization: Price vs quantity (conditional)

$$
\pi=0.2
$$



Correlation $<0$ for suff. high $\pi$

A strategy to infer competitiveness ?


