Monetary Policy with Heterogeneous Agents*

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Abstract

We build a New Keynesian model in which heterogeneous workers differ with regard to their employment status due to search and matching frictions in the labor market, their potential labor income, and their amount of savings. We use this laboratory to quantitatively assess who stands to win or lose from unanticipated monetary accommodation and who benefits most from systematic monetary stabilization policy. We find substantial redistribution effects of monetary policy shocks; a contractionary monetary policy shock increases income and welfare of the wealthiest 5 percent, while the remaining 95 percent experience lower income and welfare. Consequently, the negative effect of a contractionary monetary policy shock to social welfare is larger if heterogeneity is taken into account.

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1 Introduction

Monetary policy, not unlike taxes or government expenditures, affects the level of aggregate activity and the distribution of income and consumption across households. Whereas the distributional consequences of monetary policy have long been recognized, once inflation had been tamed in the Volcker disinflation, they were widely considered of minor relevance.[1,2] This view has changed markedly upon the Federal Reserve's implementation of extraordinary monetary measures that have put the distributional concerns into sharp relief.[3] Indeed, there is some recent empirical evidence that even in an era of low inflation rates, monetary policy shocks have persistent effects on the distribution of income and consumption across households; Coibion et al. (2012).

Knowing how certain monetary policy measures, such as changes in the inflation target or a change in the relative weights on price stability and employment, affect different segments of the population can help policymakers communicate their decisions more effectively. For example, it can help address concerns that episodes of low nominal interest rates as witnessed currently induce a sizable redistribution of wealth. In addition, aggregate economic activity may be affected by the distributional effect of monetary policy decisions in ways that are overlooked in a representative-agent setting. In sum, distributional concerns may be an important input for judging the appropriateness of monetary actions and the stance of monetary policy.

The current paper provides a laboratory that allows us to study the distributional effects of monetary policy across socio-economic groups and the spillovers into aggregate activity in general equilibrium. At the aggregate level, our model builds on the New Keynesian sticky price framework. This class of models has been shown to be able to replicate salient features of the monetary transmission mechanism (Christiano et al., 2005) and of the business cycle more generally (Smets and Wouters, 2007; Altig et al., 2011).[4] Due to nominal rigidities, in our case in price setting, both the systematic and surprise component of monetary policy matter for real activity. As regards heterogeneity, we want to be able to finely trace out the effects of monetary policy on households with different characteristics. Toward that end, our model allows for household heterogeneity and imperfect consumption insurance. Households differ along three dimensions: their current potential productivity if employed, their current wealth, and their current employ-

[1] Pigou (1916) is an early reference discussing the distributional effects of different modes of financing government expenditures. In regard to the distributional consequences of monetary policy, Keynes states: "Thus a change in prices and rewards, as measured in money, generally affects different classes unequally, transfers wealth from one to another, bestows affluence here and embarrassment there, and redistributes Fortune's favours so as to frustrate design and disappoint expectation."citeKeynes:24 In the current paper, we are concerned with the distributional consequences of monetary policy across socio-economic groups at one moment in time, rather than with intergenerational redistribution.

[2] Three of the most prominent graduate textbooks on monetary policy, Galí (2008), Walsh (2010), and Woodford (2003), largely abstract from persistent heterogeneity on the household side.

[3] Compare, for example, Philadelphia Fed president Charles Plosser's op ed "When a Monetary Solution Is a Road to Perdition," Financial Times, May 17, 2012.

[4] Representative-agent versions of this model nowadays are the workhorse policy-models in most central banks. Examples are the Board of Governors' EDO model, Chung et al. (2010), and the European Central Bank's New Area-Wide model, Christoffel et al. (2008).

ment status. Labor markets are characterized by Mortensen and Pissarides (1994) search and matching frictions; Nakajima (2012) and Krusell et al. (2010).[5] Since there is no consumption insurance across different households, unemployment spells are costly to the individual worker. Accounting for the non-linearity of the labor market at the aggregate level is important; for example, Jung and Kuester (2011) document that with search and matching frictions business cycle fluctuations can have substantial effects on average unemployment, output, and consumption, Petrosky-Nadeau and Kuehn (2011) show that hiring decisions can be strongly non-linear. We document, therefore, how to solve the New Keynesian model with search and matching frictions and heterogeneity in a fully non-linear framework.

The model that we build can also be considered as extending existing incomplete-market general equilibrium models by introducing nominal frictions.[6] Traditionally, economists study the redistribution effects of fiscal policies and mostly ignore the effects of monetary policy on the distribution of income and wealth. As a consequence, models featuring heterogeneity across households are widely used to analyze the heterogeneous implications of fiscal policies. Heathcote (2005), for example, studies the effects of short-run tax changes in a model with incomplete markets. Chang et al. (2011) emphasize the pitfalls of basing inference regarding fiscal policy transmission on an estimated representative agent model when the actual economy, instead, is characterized by heterogeneity. Costain and Reiter (2004) investigate the role of fiscal stabilization policy in an incomplete-market model with labor market frictions. In their model, if fiscal policy stabilizes the economy, it helps to complete the market. In our model environment, too, there potentially is a positive role for monetary policy in ameliorating individual risk, and unemployment risk in particular. This role is absent in the standard New Keynesian complete-market representative agent framework. Last, our paper is also related to Glover et al. (2011), who emphasize the diverse effects of the Great Recession on heterogeneous households in an incomplete-market model that does not have nominal frictions.

In our model economy, we capture that monetary policy affects the distribution of income and consumption through a number of channels. First, different sources of income are affected differently by monetary policy, meaning that monetary policy does not affect the population evenly. This is important since, in the data, households' sources of income differ starkly: Wealthier households receive financial and business income, whereas other households rely primarily on labor income or transfers; Díaz-Giménez et al. (2011). Second, households' labor earnings may already be affected by monetary policy to a different extent. For example, it is well-known that unemployment risk in recessions rises disproportionately for the lower skill groups; Elsby et al. (2010). Similarly, along with different unemployment risks, and average savings, wages of differ-

[5] Our model is placed within an active literature that assesses how labor markets characterized by search and matching frictions affect the properties of New Keynesian sticky price models; early contributions in this literature are Trigari (2009), Walsh (2005) and Cheron and Langot (2000). Following the RBC models of Andolfatto (1996) and Merz (1995), these papers abstract from meaningful heterogeneity at the household level, however, and instead assume the fiction of a perfect insurance market or a representative household that insures individuals against labor income risk. Galí (2010) provides a recent overview.

[6] The first-generation steady-state incomplete market models of Aiyagari (1994) and Huggett (1996) were extended to allow for aggregate uncertainty by Krusell and Smith (1998) and Ríos-Rull (1996). Our model builds on the latter two references.

ent skill groups are likely to be affected differently by monetary policy measures; Heathcote et al. (2009).[7] Third, monetary policy will affect the value of different classes of assets and liabilities differently. As a result, to the extent that financial positions differ across households, monetary policy measures will redistribute wealth from one segment of the population to another. In our model, we currently abstract from liabilities on the household side, setting the borrowing limit to zero, and entertain zero government debt. There exist a variety of real assets, however, the return on which in equilibrium is affected by monetary policy: physical capital, shares in monopolistically competitive intermediate goods (sticky price) firms, and shares in one-worker labor firms. Monetary policy affects the value of each of these assets differently. For example, as markups rise in a contraction, sticky price firms' profits are affected less than, say, the returns on physical capital.[8]

An important result of our exercise is that monetary policy shocks have strikingly different implications for the welfare of different segments of the population. While households in the top 5 percent of the wealth distribution benefit slightly from a contractionary monetary policy shock, the bottom 5 percent would lose from this measure. For example, a monetary tightening of 1 percentage point (annualized) induces a loss equivalent to a permanent 0.1 percent cut in consumption for the lowest 5 percent of the wealth distribution. This heterogeneity in sign and size of welfare losses from monetary policy shocks stands in stark contrast to TFP shocks, which affect the population more uniformly.

Equilibrium models with heterogeneity due to incomplete markets have made considerable progress in recent years. However, while there is a vast amount of literature studying the distributional consequences of various fiscal policies, the literature on the distributional consequences of monetary policy is relatively limited. The work that exists mostly focuses on the long run (balanced-growth path or steady state). Erosa and Ventura (2002) and Albanesi (2007) analyze the distributional consequences of steady state inflation within incomplete markets models in which poorer households hold relatively more currency. Akyol (2004) studies how long run inflation hurts insurance in an incomplete-market model. There exists some previous literature, too, on the intersection of monetary policy and household heterogeneity. Williamson (2008) relies on segmented financial markets in which some agents are connected to the financial market and others are not. In the short run, a money injection therefore redistributes wealth from the connected to the unconnected, and the injection of liquidity disseminates only gradually through the trade in goods; similarly, Ledoit (2011). Both Doepke and Schneider (2006a) and Meh et al. (2010) focus on the effects that wealth redistribution of the size that would be associated with

[7] This effect is not captured in the current version of the model, due to an assumption that a) firms cannot differentiate their vacancy posting by a prospective worker's characteristics, b) wages in each skill group move in the same proportion with GDP, and c) separations are exogenous. We mention this effect here since future versions of the paper will relax a) and b).

[8] That said, for tractability, we do not currently allow households to trade directly in the different assets. Rather, households are constrained to save by acquiring shares in a mutual fund that in turn owns, and manages, all of these assets. Thus, we capture that households have a different exposure to the market portfolio due to the differing size of their savings but do not allow households to choose the asset class. We do not currently capture redistribution effects stemming from heterogeneity in the share of wealth that different segments of the population hold in nominal as opposed to real assets; see Doepke and Schneider (2006b).

a one-time unexpected change in the price level path would have on households' welfare and on aggregate economic activity. Our work differs from theirs in that we focus on the effects of monetary policy in a framework in which monetary policy would affect economic activity even absent any redistribution of wealth. In addition, our focus on labor market frictions allows us to study the distributional consequences of the interaction of monetary policy and unemployment.

The New Keynesian literature, too, has entertained some limited heterogeneity on the household side, typically in the form of splitting households into one of two exogenously specified types, and making assumptions such that all households of a certain type always make the same decisions. Galí et al. (2007), for example, study the transmission of government spending shocks when next to the typical representative Ricardian households, there is a set of "hand-to-mouth consumers" who can neither borrow nor save. Another common modeling device is to make assumptions such that households part into borrowers and savers, as done, for example, in the papers with credit market frictions in Iacoviello (2005) and Curdia and Woodford (2010). Lee (2010) assesses the implications for optimal monetary policy if labor is firm specific and if, with otherwise complete asset markets, it is costly for households to consume more or less than their current non-financial income. In a Calvo sticky price setup, he finds that real rigidities arise that make controlling inflation more costly in terms of output. Nevertheless, he finds that monetary policy should be firmly focused on price stability. The reason is that in his model income and consumption dispersion arise only if prices differ across firms. Price stability preempts such dispersion. In our model, instead, we allow asset markets to be incomplete, while labor markets are characterized by search and matching frictions. Heterogeneity in income and consumption then arises even under perfect price stability. In other words, we allow for an important role for monetary policy in insuring certain segments of the population against idiosyncratic risk.[9]

The current paper is organized as follows. Section 2 introduces the model. Section 3 highlights the steps involved in the computation. Section 4 discusses the calibration and business cycle statistics. Section 5 highlights the effects of monetary shocks and, for comparison, technology shocks on the aggregate economy, and on measures of inequality. The same section also shows that TFP shocks affect the population rather uniformly ("a rising tide lifts all boats"). An unexpected monetary tightening, instead, increases the welfare of the top 5 percent of the wealth distribution but has a sizable negative effect on the bottom 5 percent. A final section concludes. The appendix provides further details on the model and the algorithm used in solving it.

2 Model

Since the model straddles two strands of the literature, it may be useful to first provide a quite detailed overview of its structure. The model economy is characterized by incomplete markets and

[9] We have, recently, been made aware of parallel work by McKay and Reis (2012). They analyze the role that automatic fiscal stabilizers have in shaping the business cycle.

heterogeneity in income and wealth as in Krusell and Smith (1998).[10] In order to endogenously capture earnings heterogeneity over the business cycle, we introduce Mortensen and Pissarides (1994) labor market search and matching frictions into the heterogeneous agent environment, following Nakajima (2012) and Krusell et al. (2010). In the model, changes in the nominal interest rate affect aggregate activity because monopolistically competitive producers face quadratic price adjustment costs, following Rotemberg (1982). As a result, markups are endogenous, and in the short run, the economy becomes demand-driven.

As is standard in New Keynesian environments, we model a cashless-limit economy; see Woodford (1998). Monetary policy is characterized by a Taylor (1993)-type rule that governs the nominal interest rate at which financial entities (representative mutual funds that own all the firms in the economy) can borrow or lend to each other. The Taylor rule specifies how the nominal risk-free rate of interest fluctuates in response to inflation and output. The precise process of liquidity injection through which this comes to pass, in contrast, is not modeled explicitly.

We assume that the government balances its budget on a period-by-period basis. This and the cashless-limit assumption keep the number of state variables confined. These assumptions also imply that the government does not finance itself through an inflation tax. The absence of seigniorage may be tenable at some times more than others; compare Hall and Sargent (2011). We document that even with these assumptions, monetary interventions have persistent effects on the distribution of consumption, income, and wealth in the economy. We do not model the portfolio choice problem of each heterogeneous household. Instead, the model assumes that all assets are managed by a representative mutual fund that households can invest in. Next to the mutual funds, our model economy is characterized by four types of firms. First, capital producers invest in physical capital that they rent out in a competitive market. Second, there is a set of labor agencies that hire households in a frictional labor market. The labor agencies produce homogeneous labor services that they rent out in a competitive market. Third, intermediate-goods producers that are subject to price-adjustment costs rent labor and capital services and produce a differentiated good. Under monopolistic competition, they sell these goods to the fourth set of firms, representative competitive final goods firms; for the same structure linking the labor market and price-setting firms, see, for example Walsh (2005) or Trigari (2009).[11] We next turn to describing the model in more detail.

2.1 States

We define the model recursively. In order to keep the notation short, define X as the vector of aggregate state variables: $X = (K, N, Z, D, \mu)$. K is the aggregate capital stock. N is aggregate employment. Two shocks drive the cyclical fluctuations of the economy: the aggregate

[10] Gruber (1998) empirically finds that a higher replacement rate for unemployment insurance benefits is associated with a smaller drop in consumption expenditures upon unemployment, which suggests imperfect insurance against unemployment risks. The incomplete-market model is used to capture such imperfect insurance. We will calibrate the model such that the wealth distribution of the model matches the empirical distribution, as an (indirect) way to discipline the degree of insurance available to U.S. households.

[11] Linking labor markets and price-setting decisions through competitive markets makes the model more tractable; Kuester (2010), in a representative household setup with labor market frictions, makes the two decisions interdependent, showing that real rigidities arise.

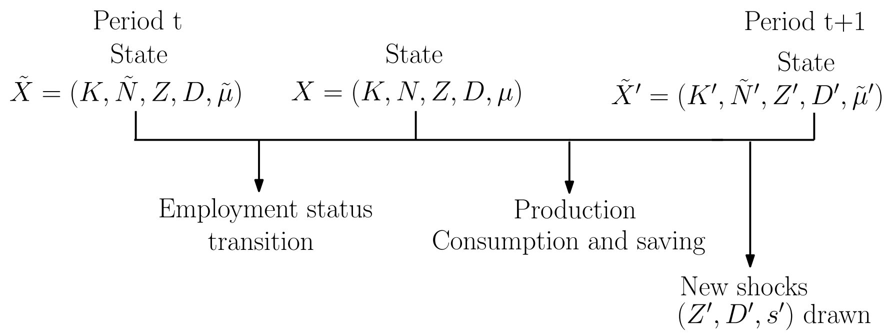


Figure 1: Timing of the model

productivity shock, Z, and the monetary policy shock, D.[12] Households are heterogeneous and characterized by a triplet (e, s, a). $e \in \{0, 1\}$ denotes the employment status: $e = 0$ indicates that a household is unemployed, while $e = 1$ indicates employment. A household works either full time or not at all. $s \in S$ represents the exogenously given discrete skill level of a household. $a \in A \subseteq \mathbb{R}$ denotes share holdings of a household. As described above, all households have access to a mutual fund through which they can save for the future. $\mu(e, s, a)$ is the type distribution of households, defined as an element of a canonical Borel σ-algebra $\mathcal{M}$ defined over $\{0, 1\} \times S \times A$.

2.2 Timing

Figure 1 summarizes the timing assumptions of the model. Households enter the period knowing their own employment status, skill type, and the state of the aggregate economy. For future reference, denote by $\widetilde{\mu}$ and $\widetilde{N}$ the type distribution and aggregate employment, respectively, at the beginning of the period, that is, before labor market transitions have occurred. Let $\widetilde{X} = (K, \widetilde{N}, Z, D, \widetilde{\mu})$ denote the corresponding state of the economy. Early in the period, previously employed households lose their jobs with exogenous probability λ. Unemployed households search for jobs and firms post vacancies. After matching has taken place, taking into account the number of newly employed households, the aggregate state becomes $X = (K, N, Z, D, \mu)$. Then, households make consumption and savings decisions. Intermediate goods firms set their prices. Capital producers make their investment decisions. Production takes place. At the beginning of the next period, shocks to the households' skill levels are drawn, as are new aggregate shocks.

2.3 Households

Preferences are time-separable with time-discount factor β and period utility function $u(c)$. The following describes the problem of a household that is employed at the production, consumption,

[12] That is, for reasons of computational tractability, we abstract from a number of other sources of business cycle shocks, particularly demand shocks or cost-push shocks that figure prominently in estimated New Keynesian models that adhere to the representative-agent paradigm; for example, Smets and Wouters (2007).

and saving stage ($e = 1$) and has skill level s and asset-holdings a:

$$W(X,1,s,a) = \max_{c,a' \geq 0} \left\{ u(c) + \beta \mathbb{E} \left[\left(1 - \lambda + \lambda\, f(\widetilde{X}') \right) W(X',1,s',a') + \lambda \left(1 - f(\widetilde{X}') \right) W(X',0,s',a') \right] \right\} \tag{1}$$

$$\text{s.t.} \qquad c + p_a(X)a' = \big(p_a(X) + d_a(X)\big)a + w(X)s\big(1 - \tau(X)\big), \tag{2}$$

Equation (1) is the household's Bellman equation. The household chooses consumption, c, and the number of shares of a mutual fund (explained further below) that it wants to carry into the next period, $a' \geq 0$, so as to maximize life time utility subject to its budget constraint, and (2) taking the job-finding rate, $f(\widetilde{X}')$, the price of shares, $p_a(X)$, dividends, $d_a(X)$, wage, $w(X)$, and the payroll tax rate, $\tau(X)$, as given. Notice that the household is subject to a short-sell constraint with respect to a'. The expectation operator $\mathbb{E}$ is taken with respect to the distribution of shocks going forward (Z', D', s', e'). In forming expectations, the household further takes into account the law of motion of the aggregate states, $\widetilde{X}' = \widetilde{G}(X)$ and $X' = G(X)$. A household that is currently employed will keep its job in the next period, too, with exogenous probability $1-\lambda$. Households without a job always search for employment. The search intensity is constant. Even if separated from a job, with probability $f(\widetilde{X}')$ the household will find a new job within the same period. The job-finding rate, $f(\widetilde{X}')$, depends on the aggregate state of the economy as described further below. If, in the next period, the household loses its job and does not find a new job immediately, its employment status changes to $e = 0$ and the household will go through a spell of unemployment. This happens with probability $\lambda(1 - f(\widetilde{X}'))$.

It may be useful, if only to fix the notation, to briefly describe the elements of the budget constraint: The right-hand side describes resources available to the household: the current value of the shares that the household owns, $p_a(X)a$, dividends associated with the shares, $d_a(X)a$, and after-tax labor income, $w(X)s(1 - \tau(X))$. $w(X)$ is the wage per efficiency unit and $\tau(X)$ is a constant proportional labor-income tax rate. The left-hand side of equation (2) describes how the household uses the resources for current consumption, c, and for purchasing shares that it carries into the next period, $p_a(X)a'$.

Similarly, the problem of a household of the same skill and wealth level that is unemployed during the production phase ($e = 0$) is given by:

$$W(X,0,s,a) = \max_{c,a' \geq 0} \left\{ u(c) + \beta \mathbb{E} \left[f(\widetilde{X}')W(X',1,s',a') + \left(1 - f(\widetilde{X}') \right) W(X',0,s',a') \right] \right\} \tag{3}$$

$$\text{s.t.} \qquad c + p_a(X)a' = \left(p_a(X) + d_a(X)\right)a + bs, \tag{4}$$

The Bellman equation (3) reflects that next period the unemployed household transitions into employment with state-dependent probability $f(\widetilde{X}')$ or otherwise remains unemployed. The budget constraint of the unemployed household mirrors that of the employed household. The only difference is that the unemployed household does not receive labor income but unemployment benefits. b is the unemployment insurance benefit per efficiency unit of labor.

For future reference, let the optimal decision rules for share holdings at the end of the period and consumption be denoted by $a' = g_a(X, e, s, a)$ and $c = g_c(X, e, s, a)$, respectively.

2.4 Aggregate Discount Factor

We assume that in pricing claims the mutual fund decisions are based on the average of the individual households' preferences, where the average is taken weighting their marginal utilities of consumption by their end-of-period share holdings. More precisely, the stochastic discount factor is given as follows:

$$Q(X, X') = \beta \int_{\mathcal{M}} a' \frac{u'(c')}{u'(c)} \, d\mu'. \tag{5}$$

This discount factor is arbitrage-free. Carceles-Poveda and Coen-Pirani (2009) have shown under somewhat more restrictive assumptions than entertained in the current paper that it can have the additional advantage of inducing unanimous agreement among share holders about firm investment decisions. Since the mutual funds own all non-financial firms in the economy, this is the discount factor that flows into the firms' respective decisions about investment, hiring, and pricing. The price of a share of the mutual fund, p_a, in turn, is competitively determined so as to equilibrate the demand for shares in the mutual fund.[13]

2.5 Final Good Producer

There is a representative competitive final goods sector in the economy. Final goods are used for consumption, investment and vacancy creation. The firm produces y units of the final good using a constant-elasticity-of-substitution production technology that requires differentiated intermediate goods as inputs. The types of intermediate goods are indexed by j and are uniformly distributed on the unit interval, so $j \in [0, 1]$. The final good producer takes each input price P_j of each type of intermediate good as given. It also acts as a price-taker in its own product market, taking its sell price, $P(X)$, as given. The problem of the representative final good producer thus is as follows:

$$\max_{y, y_{j \in [0,1]}} P(X) y - \int_0^1 P_j y_j dj \tag{6}$$

$$\text{s.t.} \quad y = \left(\int_0^1 y_j^{\frac{\epsilon - 1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon - 1}}. \tag{7}$$

Here y_j marks the quantity of intermediate good j demanded as input. The optimal decision of the final goods producer translates into the usual Dixit-Stiglitz demand functions for each intermediate good

$$y_j(X, P_j) = \left(\frac{P_j}{P(X)} \right)^{-\epsilon} y(X), \tag{8}$$

[13] Rather than using (5), the numerical results presented in the current version of the paper for now rely on the representative-agent discount factor $Q(X, X') = \beta u'(\int_{\mathcal{M}} c' d\mu')/(u'(\int_{\mathcal{M}} c \, d\mu))$.

where $y = y(X)$ is the total output of final goods. Parameter $\epsilon > 1$ marks the elasticity of demand for each intermediate good. From the zero-profit condition, the price of the final good is given by

$$P(X) = \left(\int_0^1 P_j^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \tag{9}$$

2.6 Intermediate Good Producer

An intermediate good producer j buys labor and capital services at the competitive rates $h(X)$ and $r(X)$, respectively, and sells its output to final goods firms under monopolistic competition at the nominal price P_j. This nominal price is subject to Rotemberg (1982)-type quadratic adjustment costs. The producer of intermediate good j solves the following problem:

$$J_I(X, P_{j,-1}) = \max_{P_j, \ell_j, k_j} y_j(X, P_j) \left(\frac{P_j}{P(X)} - \frac{\phi_\Pi}{2} \left(\frac{P_j}{P_{j,-1}} - \overline{\Pi} \right)^2 \right) \tag{10}$$
$$- r(X) k_j - h(X) \ell_j + \mathbb{E} \left[Q(X, X') J_I(X', P_j) \right]$$
$$\text{s.t.} \qquad y_j(X, P_j) = Z k_j^\theta \ell_j^{1-\theta}, \tag{11}$$

where $y_j(X, P_j)$ is the firm's demand function, given by equation (8). P_j is the price of intermediate good j. $P_{j,-1}$ is the price of the same good in the previous period. $\overline{\Pi}$ is the steady-state inflation rate. The adjustment costs are specified so that price adjustments are costly only to the extent that they deviate from this rate.[14] Parameter $\phi_\Pi > 0$ characterizes the size of the quadratic price adjustment cost (nominal rigidities). k_j is the amount of capital services that the firm chooses to rent and ℓ_j the amount of labor services. Equation (11) is the Cobb-Douglas production technology available to intermediate good producers. Z is total factor productivity (TFP). It follows a first-order autoregressive process:

$$\log(Z') = (1 - \rho_Z) \log(\overline{Z}) + \rho_Z \log(Z) + \epsilon_Z, \text{ where } \epsilon_Z \text{ is i.i.d. } N(0, \sigma_Z^2), \rho_Z \in [0, 1). \tag{12}$$

$\overline{Z}$ is the steady-state level of TFP.

In equilibrium, imposing symmetry, all intermediate good producers set the same price, which by equation (9) also coincides with the aggregate price level. While the firms' problem (10) suggests that past prices of intermediate goods, $P_{j,-1}$, are state variables, too, the equilibrium conditions show that keeping track of these is not necessary. Rather, due to symmetry, the equilibrium condition of each firm can be completely described by the current aggregate rate of inflation, $\Pi(X)$, and other contemporaneous aggregate variables or the expectations of each of these.[15] Also, in equilibrium, each intermediate firm therefore faces the same marginal costs and chooses the same amount of labor and capital inputs, so $k_j = k(X)$ and $l_j = l(X)$. Next, we turn to the production of these inputs.

[14] The current version of the paper accounts for price adjustment costs as affecting the behavior of firms but not causing actual resource costs.

[15] Note that if we, alternatively, were to model nominal rigidities using the Calvo sticky price setup, we would need to track a further state variable, namely, past price dispersion. Not having to do so is one advantage of the quadratic price adjustment cost framework that we are using here. Another is that the probability is virtually zero that any intermediate good firm incurs losses.

2.7 Capital-Producing Sector

There is a representative capital-producing firm, the value of which can be characterized recursively as follows:

$$J_K(X,K) = \max_{v,i,K'} \left\{ r(X)Kv - i + \mathbb{E}\left[Q(X,X')J_K(X',K')\right] \right\} \tag{13}$$

$$\text{s.t.} \qquad K' = \big(1-\delta(v)\big)K + \zeta\left(\frac{i}{K}\right)K \tag{14}$$

The capital-producing firm produces a homogeneous good, called "capital services," that it sells to the intermediate goods sector at the competitive rate $r(X)$. Capital services are the product of capital stock, K, and capacity utilization v. Capacity utilization is costly because the rate of depreciation of the existing capital stock, $\delta(v)$, depends positively on the chosen level of utilization; see equation (38) below for the functional form that we assume for this relationship. Next to capacity utilization, the capital-producing firm decides how much to invest in next period's capital stock, K'. Due to capital adjustment costs, capital investment, i, does not translate one-to-one into new capital. The functional form of these costs, $\zeta\left(\frac{i}{K}\right)$, will be introduced in equation (37) below.

The problem of the capital-producing firm characterizes aggregate investment $i(X)$, the aggregate utilization rate $v(X)$, and the aggregate capital stock in the next period $K'(X)$. In equilibrium, furthermore, $v(X)K = k(X)$ so that the amount of capital services produced in the capital-producing sector equals the demand of capital services by intermediate goods producers.

2.8 Labor Market

Labor agencies produce a homogeneous good called "labor services." They sell this to intermediate goods producers at the competitive rate of $h(X)$. Labor agencies may be matched with exactly one household or they are not matched. A labor agency that is already matched to a household produces an amount of labor services that is proportional to the skill level of the household it employs. The value of a labor agency that employs a household of skill level s can be described as:

$$J_L(X,s) = \big(h(X) - w(X)\big)s + \mathbb{E}\big[Q(X,X')(1-\lambda)J_L(X',s')\big], \tag{15}$$

where the first term reflects the fact that the labor agency pays the household a wage per efficiency unit of $w(X)$. Deviating from Nakajima (2012), instead of determining the wage as the outcome of a bargaining game between the household and its labor agency, we directly assume a functional form for the evolution of the wage per efficiency unit of labor over the business cycle. The functional form, $w(X)$, is presented in equation (39) in the calibration section. Note that this assumption is similar to assumptions found elsewhere in the New Keynesian literature; for example, Blanchard and Galí (2010). The continuation value in the second term of equation (15) reflects the fact that only with probability $(1-\lambda)$ the match between a labor agency and its household will be producing in the next period, too. With the remaining probability the match will be dissolved. The labor agency would need to hire a new household. The exposition here anticipates that due to free entry, the value of a labor agency that is not matched to a household is zero.

The labor market is characterized by search and matching frictions. Labor agencies that are not yet matched to a household can post a vacancy at cost κ. Labor firms cannot target their vacancies toward a certain skill level, nor can they condition on a prospective applicant's level of assets. The following free-entry condition governs the number of vacancies in equilibrium:

$$\kappa = \frac{M(\widetilde{X}, V)}{V} \int_{\mathcal{M}} J_L\left(\hat{G}(\widetilde{X}), s\right) d\,\mu. \tag{16}$$

where $X = \hat{G}(\widetilde{X})$ characterizes the law of motion for $\widetilde{X}$. Vacancies will be created up to the point where the cost of creating a vacancy (left-hand side) just balances the expected gain (right-hand side). The latter is determined by the product of the value of a match to the firm, compare equation (15), and the probability that an individual vacancy will be filled. This is given by the ratio of the aggregate number of new matches, M, to the aggregate number of vacancies, V.

In regard to match creation, we assume that the number of new matches that are created is governed by the following standard Cobb-Douglas matching function:[16]

$$M(\widetilde{X}, V) = \gamma \left(U(\widetilde{X}) + \lambda N(\widetilde{X})\right)^{\alpha} V^{1-\alpha}, \tag{17}$$

where $U(\widetilde{X}) + \lambda N(\widetilde{X})$ is the measure of households searching for a job (remember the timing assumption discussed in Section 2.2), and V is the number of vacancies posted.

With the number of vacancies $V(\widetilde{X})$ being characterized by equation (16), the probability that a household without employment finds a new job is given by:

$$f(\widetilde{X}) = \frac{M(\widetilde{X}, V(\widetilde{X}))}{U(\widetilde{X}) + \lambda N(\widetilde{X})}. \tag{18}$$

This probability is the same for each household, regardless of its skill level, wealth, or, indeed, unemployment duration. Next, we describe the functioning of the mutual fund, which is central to the way in which monetary policy decisions affect aggregate activity.

2.9 Mutual Fund

For the sake of tractability, we abstract from a portfolio choice by the individual households. Rather, we assume that they delegate financial management to a representative mutual fund. The households therefore only indirectly own the firms described above through their shareholdings in the mutual funds. These shares in mutual funds are the only assets households can hold. The shares of the mutual funds are traded in a competitive market at (ex-dividend) price p_a.[17]

[16] More precisely, $M(\widetilde{X}, V) = \min\left(U(\widetilde{X}) + \lambda N(\widetilde{X}), V, \gamma\big(U(\widetilde{X}) + \lambda N(\widetilde{X})\big)^{\alpha} V^{1-\alpha}\right)$. In practice, however, the model is calibrated such that the number of new matches always remains below both $U(\widetilde{X}) + \lambda N(\widetilde{X})$ and V.

[17] Due to adverse selection and enforcement costs, unemployment insurance typically is not provided by private markets. Thus, we quite realistically assume that insurance markets are incomplete in the sense that the mutual fund or individual households do not sell unemployment or salary insurance.

There are five types of assets in the economy that are owned by the mutual fund. Four of them are equity associated with final goods producers, intermediate goods producers, and producers of capital and labor services. In addition, the mutual funds can trade among each other nominal one-period bonds. As is a standard assumption in New Keynesian models, the central bank is assumed to control the rate of return on these. As is also common, but here mainly done for tractability, we abstract from an effect of central bank operations on the actual amount of outstanding private-sector debt. Since prices are sticky, by setting the nominal rate of return, the central bank influences the expected real rate of return on the nominal bonds and, in effect, the return on all other assets in the economy. In particular, equilibrium in the mutual funds market requires that all assets be priced according to the mutual fund sector's discount factor, (5), with the nominal bonds being in zero net supply.

To summarize, the mutual funds can lend and borrow using safe one-period pure-discount bonds that are traded in a centralized market. The equilibrium price, p_b, of these bonds is determined by the funds' discount factor as follows:

$$p_b(X) = \mathbb{E}\left[Q(X,X')\frac{1}{\Pi(X')}\right], \tag{19}$$

where $\Pi(X)$ is the gross rate of inflation. This, for the bond investment decision, yields a standard Euler equation (now for the mutual fund rather than the representative household)

$$1 = \mathbb{E}\left[Q(X,X')\frac{R(X)}{\Pi(X')}\right]. \tag{20}$$

The gross nominal interest rate $R(X) := 1/p_b(X)$ is the central bank's instrument. The values of the other assets that the mutual funds hold, namely J_I, J_K, and J_L, have been described in the previous subsections. The equilibrium value of final goods firms is equal to zero since they face both competitive product and factor markets.

We do not, currently, allow the mutual fund or the other firms in the economy to retain earnings. Rather, all the profits that are not reinvested are distributed to the share holders of the mutual funds in the form of dividends. Let $d_a(X)$ denote the dividends per share. These are given by the following:

$$\begin{aligned} d_a(X) \quad = \quad & \int_0^1 \left[y_j(X)\tfrac{P_j(X)}{P(X)} - r(X)k_j(X) - h(X)\ell_j(X)\right]\, dj \\ + \quad & r(X)K(X)v(X) - i(X) \\ + \quad & \int_{\mathcal{M}} (h(X) - w(X))s\; d\;\mu - \kappa V(\widetilde{X}). \end{aligned} \tag{21}$$

On the right-hand side, the first line shows the profits of the intermediate goods firms. The second line shows the profits in the capital services sector. The third, and final line, marks the profits in the labor services sector. The term in the integral refers to the profits of labor agencies that are matched to a household when production takes place. The second negative part is the costs that labor agencies without workers spend on posting vacancies.[18]

[18] Since, for each mutual fund, bond holdings are zero in equilibrium, income from risk-free bonds has been ignored in equation (21). Similarly, we omit profits in the final goods sector, which are equal to zero, too, in equilibrium.

2.10 Central Bank

The empirical literature finds that Taylor (1993)-type rules are a good representation of monetary policymaking in recent decades. The central bank adjusts the gross nominal interest rate, R, according to

$$\log\left(\frac{R(X)}{\overline{R}}\right) = \rho_\Pi \log\left(\frac{\Pi(X)}{\overline{\Pi}}\right) + \rho_y \log\left(\frac{y}{\bar{y}}\right) + D. \tag{22}$$

All else equal, the central bank thus raises the nominal rate above $\overline{R}$ whenever inflation exceeds the inflation target of $\overline{\Pi}$ (parameter $\rho_\Pi > 1$) and when output exceeds its target value $\bar{y}$, which is taken to be the value of output in the non-stochastic steady state (parameter $\rho_y \geq 0$). Often, estimated Taylor rules also allow for "intrinsic policy inertia," that is, a term involving last period's interest rate on the right-hand side of equation (22) in order to capture persistent deviations of the federal funds rate from the "desired" funds rate as specified above; compare, for example, Clarida et al. (1998). The extent to which such policy inertia reflects actual policymaking or to which it reflects missing information (and should thus be captured by correlated errors) is subject to debate; Rudebusch (2006) provides a critical appraisal. While we do not wish to take a specific stand on the discussion, here we assume that there is no intrinsic policy inertia. The nominal interest rate is hit by persistent monetary policy shocks that capture persistent deviations from typical behavior. We take these shocks, D, to follow a first-order autoregressive process:

$$\log(D') = \rho_D \log(D) + \epsilon_D, \text{ where } \epsilon_D \text{ is i.i.d. } N(0, \sigma_D^2)\ , \rho_D \in [0,1). \tag{23}$$

2.11 Fiscal Authority

The government runs a balanced-budget policy, its budget constraint being:

$$\int_{\mathcal{M}} \mathbb{1}_{e=0}\, bs\ d\mu = \tau(X) \int_{\mathcal{M}} \mathbb{1}_{e=1}\, w(X)s\ d\mu. \tag{24}$$

The government pays unemployment insurance benefits (left-hand side). The cost is b per efficiency unit of labor for each unemployed household ($\mathbb{1}$ marks the indicator function). Unemployment benefits are financed by a proportional tax $\tau(X)$ on the labor income of employed households (right-hand side).

2.12 Aggregate Laws of Motion

Next, we discuss how to construct the aggregate law of motion. For expositional purposes, we use two sets of aggregate state vectors, X and $\widetilde{X}$, that differ in time of measurement; compare Section 2.2. We therefore have three types of laws of motion, $X' = G(X)$, $\widetilde{X}' = \widetilde{G}(X)$, and $X = \hat{G}(\widetilde{X})$. Let us focus on one element of X at a time.

First, installed capital K does not change during a period. It therefore does not differ between X and $\widetilde{X}$, so we only need one law of motion for K; compare equation (14) :

$$K'(X) = \big[1 - \delta\big(v(X)\big)\big]\, K + \zeta\left(\frac{i(X)}{K}\right) K \tag{25}$$

where $v(X)$ and $i(X)$ are obtained from the optimization problem of the capital-producing sector. Next, the law of motion for employment during the production stage of this period is given by

$$N(X) = (1-\lambda)N(\widetilde{X}) + M(\widetilde{X}, V(\widetilde{X})). \tag{26}$$

Since there are no labor-market transitions at the end of the period, employment at the beginning of the next period coincides with employment at the end of the current period:

$$N'(\widetilde{X}') = N(X). \tag{27}$$

Last, we need to keep track of the type distribution of households. Remember that, at the beginning of a period, the type distribution is $\widetilde{\mu}(\widetilde{e}, s, a)$, where $\widetilde{e}$ is the employment status *before* the separations and hiring occur. The type distribution during the period (after the transitions in employment status) is $\mu(e, s, a)$. The laws of motion of $\widetilde{\mu}$ and μ are linked as follows:

$$\mu(1, s, a) = f(\widetilde{X})\widetilde{\mu}(0, s, a) + [1-\lambda+\lambda f(\widetilde{X})]\,\widetilde{\mu}(1, s, a), \tag{28}$$

$$\mu(0, s, a) = [1-f(\widetilde{X})]\widetilde{\mu}(0, s, a) + \lambda[1-f(\widetilde{X})]\,\widetilde{\mu}(1, s, a), \tag{29}$$

Notice that, between $\widetilde{\mu}$ and μ, only the employment status changes. The transition between the type distribution at the end of the period, μ, and the type distribution at the beginning of the next, $\widetilde{\mu}'$, is characterized by the following:

$$\widetilde{\mu}'(\overline{e}, \overline{s}, \overline{A}) = \sum_{s\in S} \pi_{s,\bar{s}} \int_{\mathcal{M}} \mathbb{1}_{e=\overline{e}}\, \mathbb{1}_s\, \mathbb{1}_{g_a(X,\overline{e},s,a)\in\bar{A}}\; d\,\mu(e, s, a), \tag{30}$$

with $\overline{A} \in A$ being a subset of the space of the share holdings and $\overline{e}$, $\overline{s}$ being individual states in $\{0, 1\}$ and S, respectively. $\pi_{s,\bar{s}}$ marks the probability to transition from skill state s to state $\overline{s}$ at the end of the period.

2.13 Market Clearing and Equilibrium

There are six markets operating in the model – final goods, intermediate goods, labor services, capital services, share of mutual funds, and inside bonds. Below are their market clearing conditions. The final goods market clears if

$$y(X) = \int_{\mathcal{M}} g_c(X, e, s, a)\; d\,\mu + i(X) + \kappa V(\widetilde{X}), \tag{31}$$

where the first two terms on the right-hand side are aggregate consumption and investment, respectively, and the other two terms refer to price adjustment costs and vacancy posting costs, respectively. By equation (8) and (11), the markets for all intermediate goods clear whenever

$$y(X) = Zk_j^{\theta} l_j^{1-\theta}, \forall\, j \in [0, 1]. \tag{32}$$

The market for labor services clears if

$$\int_{\mathcal{M}} s\mathbb{1}_{e=1}\; d\,\mu = \int_0^1 \ell_j\; dj. \tag{33}$$

The market for capital services clears if

$$v(X)K = \int_0^1 k_j \; dj. \tag{34}$$

The stock market (the market for shares of the mutual funds) clears if

$$\int_{\mathcal{M}} g_a(X, e, s, a) \; d \; \mu = 1. \tag{35}$$

Last, the (within mutual funds) bond market clears if inside bonds are in zero net supply. With this, we can define the recursive equilibrium in the model as follows.

Definition 1 (Recursive equilibrium) *A recursive equilibrium is a set of functions* $G(X)$, $\widetilde{G}(X)$, $\hat{G}(\widetilde{X})$, $W(X, e, s, a)$, $g_a(X, e, s, a)$, $g_c(X, e, s, a)$, $f(\widetilde{X})$, $p_a(X)$, $d_a(X)$, $w(X)$, $\tau(X)$, $h(X)$, $Q(X, X')$, $J_L(X, s)$, $V(\widetilde{X})$, $r(X)$, $J_K(X, k)$, $i(X)$, $v(X)$, $K'(X)$, $P(X)$, $y_j(X, P_j)$, $J_I(X, P_{j,-1})$, $k_j(X)$, $\ell_j(X)$, $P_j(X)$, $\Pi_j(X)$, $y(X)$, $R(X)$ *such that:*

1. *Given* $\widetilde{G}(X)$, $f(\widetilde{X})$, $w(X)$, $p_a(X)$, $d_a(X)$, $G(X)$, *and* $\tau(X)$, *value function* $W(X, e, s, a)$ *is a solution to the household's problem.* $g_a(X, e, s, a)$ *and* $g_c(X, e, s, a)$ *are the associated optimal decision rules.*

2. *Given* $h(X)$, $w(X)$, $Q(X, X')$, *and* $G(X)$, $J_L(X, s)$ *solves the problem of a labor agency.* $V(\widetilde{X})$ *satisfies the free-entry condition in the labor agency sector.* $f(\widetilde{X})$ *is consistent with* $V(\widetilde{X})$.

3. *Given* $r(X)$, $Q(X, X')$, *and* $G(X)$, $J_K(X, k)$ *solves the problem of a capital-producing firm.* $i(X)$, $v(X)$, *and* $K'(X)$ *are the associated optimal decision rules.*

4. *Given* $r(X)$, $v(X)$, $h(X)$, $P(X)$, $y_j(X, P_j)$, *and* $Q(X, X')$, *value function* $J_I(X, P_{j,-1})$ *solves the problem of an intermediate good producer.* $k_j(X)$, $\ell_j(X)$, $P_j(X)$, *and* $\Pi_j(X)$ *are the associated optimal decision rules.*

5. *Given* $P(X)$ *and* P_j, $y_j(X, P_j)$ *and* $y(X)$ *are the optimal decisions of final good producers.*

6. *The aggregate discount factor* $Q(X', X)$ *satisfies equation (5).*

7. $d_a(X)$ *satisfies the flow budget constraint of mutual funds (21).*

8. *The wage per efficiency unit of labor is given exogenously by* $w(X)$.

9. *The labor tax* $\tau(X)$ *satisfies the government budget constraint (24).*

10. *The nominal interest rate* $R(X)$ *satisfies the Taylor rule (22).*

11. *The aggregate laws of motion* $G(X)$, $\widetilde{G}(X)$, *and* $\hat{G}(\widetilde{X})$ *are consistent with the relevant optimal decision rules.*

12. *All market clearing conditions are satisfied.*

3 Computation

Once calibrated (see the next section), the model is solved numerically. A detailed description of the computation can be found in Appendix B. Labor-market models can exhibit substantial non-linearity; for example, Jung and Kuester (2011). In addition, the current model features incomplete markets with occasionally binding borrowing constraints. Therefore, the perturbation methods typically used for computing the equilibrium in New Keynesian DSGE models cannot be applied; instead, we use a version of the solution method developed by Krusell and Smith (1998).

4 Calibration

The purpose of the paper is to ascertain to what extent monetary policy in the U.S. may have distributional consequences through the channels that we model. Toward that end, we first seek to closely calibrate the model to the U.S. economy. In later sections, we then ask quantitatively by how much monetary policy affects inequality in this model economy. The calibration sample ranges from 1984Q1 to 2008Q3. One period in the model is a quarter. Tables 1 and 2 on the next pages summarize the calibrated parameters, the choice of which we will explain next.

4.1 Households

We start by calibrating the household sector. We choose the period utility function to be of the constant-relative-risk-aversion form:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{36}$$

where $\sigma = 1.5$, a standard value. The time-discount factor, β, is calibrated jointly with other parameters to obtain an annualized real return on saving of 6 percent. In the paper, we seek to investigate the implications of monetary policy for households with different levels of wealth. We, therefore, have to capture the distribution of wealth in the U.S. economy. Toward that aim, we calibrate the stochastic process for the idiosyncratic productivity shock. In particular, we use four discrete skill levels: $s \in S = \{s_1, s_2, s_3, s_4\}$. $\{s_1, s_2, s_3\}$ capture the productivity of "normal" households, with s_1 being the lowest skill level, s_2 a medium skill level, and s_3 a high skill level. The fourth skill level, s_4, is used to capture vastly more productive households, the "super-skilled." We need the latter group of households in order to capture the concentration of wealth in the U.S. economy; Díaz-Giménez et al. (2011). Conditional on being and staying within the "normal" (lower three) skill levels, we set the transition probabilities among the lower three states by discretizing an AR(1) process for the log of individual productivity with mean zero, persistence parameter ρ_s and a variance of the innovation of σ_s^2 using the algorithm by Tauchen (1986). We discuss the targets that determine the parameters of this AR(1) process below. We assume that the probability of becoming super-skilled is the same for each normal skill level. We also assume that a household that loses its super skills is equally likely to transition into each of the three normal skill levels. With these assumptions, there are three sets of parameters associated with the super-skilled state: the probability of staying super-skilled, π_{s_4,s_4}, the probability that a "normal" household becomes super-skilled, $\pi_{s_1,s_4} = \pi_{s_2,s_4} = \pi_{s_3,s_4}$, and the productivity of

Table 1: Calibrated Parameters

Parameter	Value	Description
Households		
σ	1.5	Relative risk aversion.
β	0.966	Time-discount factor.
$\pi_{s,s'}$	see Table 2	Skill transition probabilities.
s_1	0.123	Productivity low-skilled.
s_2	0.421	Productivity medium-skilled.
s_3	1.435	Productivity high-skilled.
s_4	34.65	Productivity super-skilled.
Capital services		
ζ_0	0.730	Parameter for capital adjustment cost.
ζ_1	0.100	Parameter for capital adjustment cost.
ζ_2	-.0017	Parameter for capital adjustment cost.
δ_0	0.015	Parameter for utilization cost function.
δ_1	0.030	Parameter for utilization cost function.
δ_2	0.040	Parameter for utilization cost function.
Intermediate goods		
ϵ	21.00	Elasticity of substitution across intermediate goods
θ	0.330	Exponent on capital in intermediate good production.
ϕ_Π	690.0	Slope of price adjustment cost.
ρ_Z	0.950	Persistence of total factor productivity shock.
σ_Z	0.006	Standard deviation of total factor productivity shock.
Labor services and labor market		
λ	0.100	Separation rate
α	0.600	Matching elasticity w.r.t. number of searchers.
γ	0.645	Matching efficiency
$\overline{w}$	0.637	Average wage.
ϵ_w	0.450	Wage elasticity w.r.t. output.
κ	0.240	Vacancy posting cost.
Monetary policy and fiscal policy		
$\overline{\Pi}$	1.005	Target gross inflation rate (qtrly.).
$\overline{R}$	1.020	Steady-state risk-free nominal interest rate.
ρ_Π	1.200	Responsiveness of policy rate to inflation.
ρ_D	0.700	Persistence of monetary policy shock.
σ_D	6.25e-4	Standard deviation of monetary policy shock
b	0.446	Unemployment insurance benefits per efficiency unit.

Notes: The table shows the calibrated parameters. See the main text for explanations and details regarding calibration targets.

Table 2: Skill Transition Matrix

		future			
		s_1	s_2	s_3	s_4
today	s_1	0.9719	0.0275	0.0000	0.0006
	s_2	0.0275	0.9444	0.0275	0.0006
	s_3	0.0000	0.0275	0.9719	0.0006
	s_4	0.0183	0.0183	0.0183	0.9450

Notes: Transition probabilities, π_{s_i,s_j}, across skill groups. s_1 : lowest skill group, s_4 : highest skill group.

the super-skilled, s_4. Table 2 reports the transition probabilities that we choose. Along with the level of productivity for the respective skill types, these are determined by the following five targets for the steady state: (i) The parameters for the skill transitions are calibrated such that, in the steady state, 1 percent of the households are super-skilled; (ii) The Gini index of wealth is 0.82; (iii) The proportion of households that are borrowing-constrained is 0.10; (iv) The Gini index of earnings is 0.64; and (v) The autocorrelation of annual earnings is 0.95. The first target is motivated by the fact that the dynamics of individual earnings in the Panel Study of Income Dynamics (PSID) is reasonably replicated by an AR(1) process, and the PSID is known to under-sample the highest-income or -wealth households. The next three targets are calculated using the 2007 Survey of Consumer Finances. The last target is motivated by existing empirical estimates of annual earnings persistence, most of which range between 0.9 and 1.

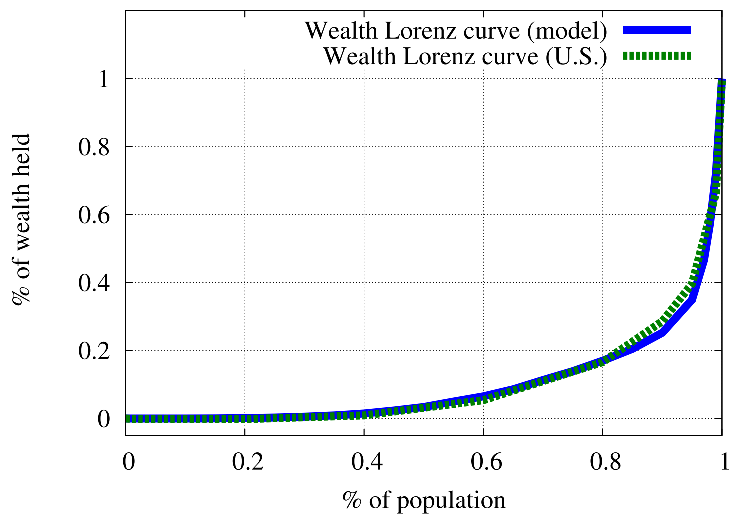
In order to allow the reader to judge the fit of the calibration, Table 3 reports how well the model matches the calibration targets discussed so far.

Table 3: Model vs. Calibration Targets

Parameter	Value	Target	Data	Model
β	0.966	Real interest rate (ann.)	0.060	0.060
ρ_s	0.998	Earnings autocorrelation	0.950	0.925
σ_s	0.071	Proportion of zero wealth	0.100	0.100
π_{s1}	0.945	Wealth Gini index	0.820	0.810
s_1	34.65	Earnings Gini index	0.640	0.645

Figure 2 compares the wealth Lorenz curve implied by the steady state of the model with the data. The Lorenz curve for the U.S. wealth distribution is taken from Díaz-Giménez et al. (2011), who use the 2007 Survey of Consumer Finances. The figure confirms that the model captures the essence of the distribution, especially the high concentration of wealth in the U.S. economy.

Figure 2: Wealth Inequality



Notes: Wealth Lorenz curve in the model (blue solid line) against Lorenz curve of wealth for the U.S. (green dashed line).

4.2 Capital-Producing Sector

The second set of parameters pertains to the capital-services sector. As to the functional form for the capital adjustment costs, we opt for the following specification:[19]

$$\zeta\left(\frac{i}{K}\right) = \zeta_0 \left(\frac{i}{K}\right)^{1-\zeta_1} + \zeta_2. \tag{37}$$

With regard to the parameters, we set the curvature parameter ζ_1 such that the volatility of investment relative to output is 3.5. This ratio is slightly lower than the observed ratio (see Table 4), but matching a higher ratio of the two volatilities implies that the response of investment to monetary policy shocks is too large compared with the strength of the response typically estimated in structural vector auto-regressions, such as Altig et al. (2011). In order to determine the other two parameters, we target a non-stochastic steady state in which, in equilibrium, the adjustment costs do not affect firms. The two parameters, ζ_0 and ζ_2, thereby follow from the steady state of the model and the first-order conditions. See the second set of rows in Table 1 for the corresponding values.

As regards the functional form for the depreciation rate of capital, we opt for the following

[19] The empirical New Keynesian literature often specifies investment adjustment costs so that what is costly to change is the current level of investment relative to the past level of investment; for example, Christiano et al. (2005). Such a specification results in a more drawn-out response of investment to shocks than a specification – as used here – with capital adjustment costs, where investment is costly to the extent that it is larger than a natural level in light of the existing capital stock. Relative to the former, the specification used here economizes on state variables.

specification that is quadratic in capacity utilization:

$$\delta(v) = \delta_0 + \delta_1(v-1) + \frac{\delta_2}{2}(v-1)^2. \tag{38}$$

As is common practice, we set δ_1 such that in the non-stochastic steady state of our model the utilization rate is equal to 1. We set the steady-state depreciation rate to $\delta_0 = 0.015$ (6 percent at annual rates). The depreciation rate was calculated using the average ratio of total private capital consumption over the total private capital stock in NIPA. As regards δ_2, we choose a value such that the utilization rate in log terms responds slightly more strongly than output to monetary policy shocks; compare Christiano et al. (2005). The resulting utilization rate in the model is somewhat less variable unconditionally than the standard deviation of capacity utilization measured in the U.S. data suggests (see Table 4). The resulting capital to quarterly output ratio in the steady state of the model turns out to be 10.59 (2.65 with annualized output), roughly in line with a ratio of the total private capital stock to GDP in NIPA of 10 (2.5 with annualized output).

4.3 Intermediate Good Producer

Turning to the intermediate goods producers, we set the elasticity of substitution across intermediate goods to $\epsilon = 21$. This is consistent with a steady-state markup of 5 percent, about in the middle of the range of choices made in the New Keynesian literature; Kuester (2010) has a discussion with a literature review. Our value is from Altig et al. (2011). We set parameter θ to a customary value of 0.33. Along with the markup just discussed, this implies that the capital share of income is 0.31. The parameter that governs the price adjustment cost is set to $\phi_\Pi = 690$. This value was chosen such that our model implies a reasonable slope of the New Keynesian Phillips curve (if the Phillips curve were linearized); compare Galí and Gertler (1999). The steady-state level of the TFP shock, $\overline{Z}$, is chosen so as to normalize steady-state output to unity. The persistence of the TFP shock $\rho_Z = 0.95$ is the standard estimate. The standard deviation of the TFP shock ($\sigma_Z = 0.0061$) was calibrated such that output in the model (once H-P-filtered) shows the same standard deviation as output in the data. The value for the standard deviation of the innovation to TFP thus obtained is quite standard.

4.4 Labor Market

With regard to the labor market, the separation rate is set to $\lambda = 0.10$. This 10 percent rate of separation per quarter is consistent with the JOLTS data. We set the elasticity of the matching function with respect to the number of searchers to $\alpha = 0.6$, so as to match the relative standard deviation of unemployment and vacancies in the model and the data. This number is in the middle of the "admissible" range identified by Petrongolo and Pissarides (2001). For comparison, Shimer (2005) estimates a value of 0.72. As for the wage function, we assume the following formula:

$$\log w - \log \overline{w} = \epsilon_w(\log y - \log \overline{y}), \tag{39}$$

where $\overline{w}$ is the steady-state wage level, and $\epsilon_w \in [0,1]$ represents the elasticity of the wage with respect to output. Values of $\epsilon_w < 1$ can be interpreted as reflecting "wage stickiness."[20] We set the wage per efficiency unit of labor in the steady state to $\overline{w} = 0.9425$. This choice, which implies that profits for labor agencies are small in the steady state, generates a reasonably large volatility of unemployment relative to output. Following Hagedorn and Manovskii (2008), we set $\epsilon_w = 0.45$. Two additional targets help determine the remaining two labor-market parameters. Following den Haan et al. (2000), in the model's steady state we assume a quarterly vacancy-filling rate of 0.71. Using the steady-state free-entry condition, this yields a vacancy posting cost of $\kappa = 0.24$. In addition, we target a steady-state unemployment rate of 6 percent. For comparison, using the steady-state conditions, this translates into a steady-state job-finding rate per quarter of $f = 0.61$ for the unemployed. Using the job-finding rate and vacancy-filling rate, we obtain the steady-state number of vacancies. The matching efficiency parameter γ is chosen so as to make vacancies, unemployment and the number of new matches (as determined by the rate of separation) internally consistent.

4.5 Central Bank and Government

Last, we have to parameterize the government sector. Looking at the central bank first, the inflation target ($\overline{\Pi}$) is set such that the model implies a steady-state inflation rate of 2 percent annualized, in line with the Federal Reserve System's inflation objective. Our model does not have mechanisms to explain the equity premium. The rate $\overline{R}$ used in the Taylor rule is chosen with a target for the real rate of return of 6 percent in mind, so as to approximate the return on capital in the data rather than that of risk-free bonds. The response of the policy rate to inflation in the Taylor rule is set at $\rho_\Pi = 1.2$, within the standard range of values used in the literature. The New Keynesian literature often allows for "policy inertia" by having a lagged federal funds rate on the right-hand side of the Taylor rule. A typical value of the coefficient on the lagged interest rate is 0.7; for example, Clarida et al. (2000). Such a response would considerably complicate the computation since the lagged policy rate would be an aggregate state variable. In the current paper, instead, we opt for inducing the interest rate persistence through autocorrelation of the monetary policy shock, the persistence of which we set at $\rho_D = 0.7$; Rudebusch (2006). Alternatively, this shock may be interpreted as a time-varying inflation target; for example, Coibion and Gorodnichenko (2011) and Cogley et al. (2010). The standard deviation of the monetary policy shock, σ_D, is such that a one-standard-deviation monetary shock has a size of 25 basis points annualized.

With regard to the unemployment insurance system, we target a replacement rate (in the steady state) of 70 percent of earnings. At the value of the average wage ($\overline{w}$) targeted above, this implies a value for benefits of $b = 0.446$ per efficiency unit of labor. The value for the replacement rate of 70 percent is higher than the value of around 40 percent that is often used; for example, Shimer (2005). Lower values for the replacement rate, however, would mean that unemployment would be so painful that more households move away from the borrowing constraint than the U.S. data suggest. At the current stage, we do not want to complicate our model any further, for

[20] In the current model, wage stickiness serves to amplify labor market fluctuations, as in Shimer (2004) and Hall (2005). Wage stickiness does not necessarily imply, however, that the marginal costs of price-setting firms are rigid; Krause and Lubik (2007). In particular, the price of labor services rather than wages feeds into marginal costs. The former is a mix of the wage and the value of having a household employed now and in the future. Therefore, the price of labor services and marginal costs tend to be less rigid than the wage.

example, by specifying different time-discount factors for different segments of the population. Instead, our calibration trades off a reasonable value for the replacement rate and a reasonable share of borrowing-constrained households.[21] The payroll tax rate is set so as to balance the budget on a period-by-period basis. To get a feel for its magnitude, the choices above imply a steady-state payroll tax rate of 4.46 percent.

4.6 Business Cycle Statistics

This section presents the usual business cycle facts and assesses how well the model is able to capture these. All data are quarterly. The cyclical properties are computed using the log of the data or model-implied series after filtering with the Hodrick-Prescott (H-P) filter. Following the standard practice in the business cycle literature for quarterly data, we use a smoothing parameter of 1,600.[22] We focus on the period of the Great Moderation, from 1984Q1 to 2008Q3, that is, right before the zero lower bound becomes binding for the federal funds rate.[23] We split the exposition into three parts, first showing output and its components (plus capacity utilization), then showing the labor market, and, last, showing prices and productivity.

All data are seasonally adjusted. Unless noted otherwise, the source of the data is the St. Louis Fed's FRED II database. Nominal variables are deflated by the GDP deflator. Personal consumption expenditures are from the national accounts. Consumption thus includes total durable and nondurable consumption expenditures as well as services. Investment is gross private domestic investment; it thus includes both residential and non-residential fixed investment and the change in private inventories. As the measure of output, we take the sum of consumption and investment. In order to measure the use of productive capacity over the business cycle, we use logs of the quarterly average of the Board of Governors' headline index of industrial capacity utilization.

The model succeeds in replicating the main cyclical properties of output and its components; Table 4. Consumption is less volatile than output and procyclical. Investment is considerably

[21] Alternatively, one may think of the replacement rate that we use currently as a composite of a constant disutility of labor and actual unemployment benefits, with both elements channeled through the government's budget constraint for convenience only. Costain and Reiter (2008) and Nakajima (2012), for example, find that a "gross" replacement rate of around 0.7 is consistent with the high volatility of the number of vacancies and unemployment in a business cycle model. In the current version of the model, unemployment benefits are constant over the business cycle and paid indefinitely. In practice, the duration of benefits is finite but often extended in recessions. However, since the model does not generate a large number of long-term unemployed, who would benefit from a countercyclical duration of unemployment benefits, the constant and permanent unemployment benefits are not problematic.

[22] Shimer (2005) uses a smoothing parameter of 10^5, instead. According to Hornstein et al. (2005) (see their Table 1), using the smoothing parameter of 10^5 instead of the standard value of 1,600 does not generate a substantial difference in terms of the volatility of the important labor market variables relative to the volatility of labor productivity.

[23] There is an ongoing discussion as to how the non-standard measures of monetary accommodation that the Federal Reserve implemented in the last recession have affected the economy and whether and how they can be mapped into Taylor-type rules with negative interest rates. The current paper is silent about non-standard monetary policy and the zero lower bound on nominal rates. We, therefore, want to steer clear of that episode and let our comparison sample end accordingly.

Table 4: Model vs. Calibration Targets: Output and Components

	SD%	SD/SD(y)	Corr with y	AR(1)
US: 1984Q1-2008Q3				
Output (y)	1.36	1.00	1.00	0.92
Consumption	0.77	0.56	0.84	0.82
Investment (i)	4.77	3.49	0.93	0.85
Capacity utilization (v)	1.87	1.36	0.75	0.91
Baseline model				
Output (y)	1.37	1.00	1.00	0.64
Consumption	0.58	0.43	0.96	0.73
Investment (i)	4.13	3.01	0.99	0.73
Capacity utilization (v)	1.00	0.73	0.79	0.28

Notes: The table compares moments of the data and from 200,000 simulations of the model, focusing on GDP and its components. The moments are taken after taking out an H-P-filtered trend with weight 1,600 from the log of the data, and multiplying the business cycle frequency series by 100. Consequently, all variables are reported in percent deviation from the H-P-trend. The first column reports the standard deviation of the variable, the second column its standard deviation relative to the standard deviation of output. The third and fourth columns, respectively, show the contemporaneous correlation with output and the first-order autoregression coefficient.

more volatile than output. Capacity utilization, too, is procyclical. It is less volatile in the model, however, than in the data.

Table 5: Model vs. Calibration Targets: Labor Market

	SD%	SD/SD(y)	Corr with y	AR(1)
US: 1984Q1-2008Q3				
Employment $N(X)$	0.50	0.36	0.81	0.94
Unemployment $U(X)$	8.48	6.20	-0.84	0.94
Vacancies (V)	10.05	7.34	0.89	0.91
Job finding rate (f)	5.84	4.27	0.75	0.78
Baseline model				
Employment $N(X)$	0.57	0.42	0.93	0.68
Unemployment $U(X)$	8.99	6.55	-0.92	0.67
Vacancies (V)	10.63	7.75	0.82	0.18
Job finding rate (f)	4.64	3.36	0.91	0.42

Notes: Same as Table 4 but focusing on the labor market.

Moving to the labor market, Table 5 compares business cycle properties of the labor market and the data. In order to measure unemployment, $U(X) := 1 - N(X)$, we use the civilian unemployment rate among those 16 years of age and older. Employment is one minus this measure. In regard to vacancies, V, we rely on Barnichon's (2010) composite help-wanted index.[24] We equate the job-finding rate, f, with the monthly transition probability from unemployment to employment in the Current Population Survey (CPS). The data are adjusted for time aggregation as in Shimer (2012). Table 5 shows that the model succeeds in replicating the key cyclical properties of the labor market data. Unemployment and vacancies are both highly volatile. Unemployment is strongly countercyclical both in the model and in the data, and vacancies are procyclical. The labor market in the model is showing somewhat less persistence than in the data, however, reflecting that output and its components, too, are a tad less persistent in the model than in the data; compare Table 4.

Table 6: Model vs. Calibration Targets: Productivity and Prices

	SD%	SD/SD(y)	Corr with y	AR(1)
<u>US: 1984Q1-2008Q3</u>				
Output per household $y/N(X)$	0.93	0.68	0.89	0.84
Wage per household	0.89	0.65	0.49	0.84
Inflation $\Pi^{[1]}$	0.68	0.48	0.22	0.16
Nominal interest rate $R^{[1]}$	1.16	0.84	0.60	0.92
<u>Baseline model</u>				
Output per household $y/N(X)$	0.86	0.63	0.97	0.61
Wage per household	0.62	0.45	1.00	0.64
Inflation $\Pi^{[1]}$	0.36	0.28	0.27	0.40
Nominal interest rate $R^{[1]}$	0.20	0.16	0.09	0.29

Notes: Same as Table 4 but focusing on productivity and prices. [1] : the nominal interest rate and the inflation rate are presented in annualized terms.

As regards productivity and prices, Table 6 provides a detailed comparison of the model and the data. We measure output per household in the data as GDP (defined, as before, as consumption plus investment) divided by employment. The model matches the properties of labor productivity well, apart from implying somewhat lower persistence. Total wages earned in the economy are computed as wage and salary accruals from the national accounts divided by the GDP deflator. The wage per household divides this measure by employment. As in the data, the wage per household is less volatile than output. Due to the functional form of the wage equation, however, the model – by design of the wage rule – implies a linear relation between wages and output in percentage terms, while the correlation of wages and output is only about one half that in the data. Next, the moments of the inflation rate reported in Table 6 refer to quarter-on-quarter inflation rates based on the GDP deflator. These rates are reported in annualized terms. The model manages to replicate about half of the standard deviation in inflation rates witnessed in the data and does reasonably well in terms of the persistence and serial correlation of inflation.

[24] Prior to 1995 Barnichon's series is the conventional newspaper help-wanted advertising series. Afterward, the index links the Conference Board's print and help-wanted advertising indexes and accounts for the changing shares of these modes of advertising over time.

5 The Transmission and Welfare Effects of Shocks

We split the presentation of the results into two blocks. First, we discuss the transmission of TFP and monetary policy shocks to the aggregate economy, and how they affect different segments of the population. Thereafter, we document the effect of these shocks on welfare. Anticipating the results, a positive TFP shock "lifts all boats," compressing inequality along the way. A contractionary monetary policy shock, in contrast, has strikingly different implications for different households: income and consumption rise for the wealthiest segment of the population, and fall for the rest. As a result, measures of inequality rise markedly – in stark contrast to the effect of a TFP shock. The welfare costs of the two types of shocks reflect this, too.

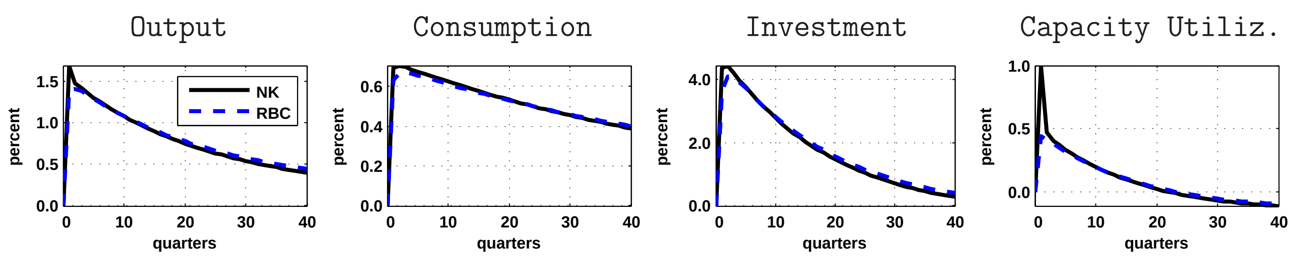
5.1 Transmission of a Technology Shock

For comparison with the literature we start by discussing the transmission of a technology shock in the calibrated model economy. First, we focus on the effect of TFP shocks on aggregate variables. Then, we discuss the effect of TFP shocks on measures of inequality.

5.1.1 Response of the Aggregate Economy to a Productivity Shock

Starting first with the effect of a 1 percent TFP shock on the aggregate economy, the figures below show the responses in the calibrated model as a blue solid line ("NK"). Also shown are the impulse responses in an economy that does not have nominal rigidities but otherwise is the same as the New Keynesian baseline ("RBC"). Figure 3 zooms in on output and its components. The TFP shock is realized in period 1. In the initial period, the responses are somewhat larger

Figure 3: Response to a TFP Shock: Output and Components

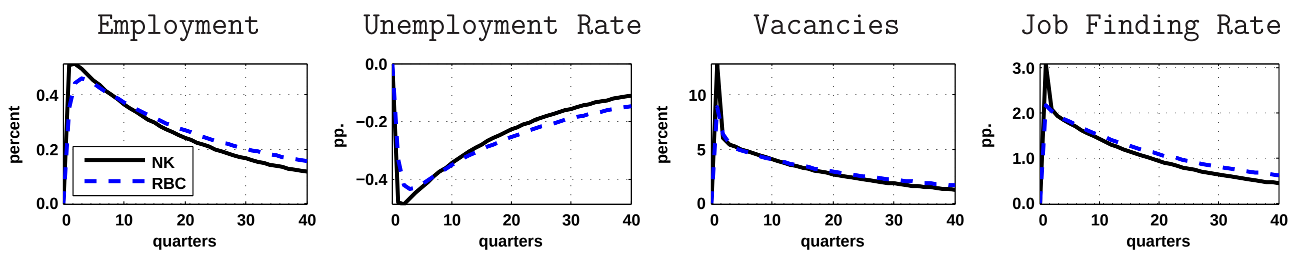


Notes: Impulse responses of output, aggregate consumption, investment, and capacity utilization in response to a 1 percent TFP shock. Black solid line: the New Keynesian economy. Blue dashed line: the same economy but without sticky prices.

in the New Keynesian economy than in the RBC counterpart. Afterward, however, the two economies move in remarkably similar ways. This is so despite the fact that we calibrated the model such that (if linearized) the slope of its New Keynesian Phillips curve would resemble the slopes estimated typically by the empirical literature. In other words, the Taylor rule in the baseline keeps the economy close to the flex-price equilibrium. Output, aggregate consumption and investment rise persistently in response to the shock. Capacity utilization rises, too, and particularly so in the New Keynesian model variant. This reflects the somewhat stronger demand effects in this variant.

The next set of charts, in Figure 4, focuses on the effect of the TFP shock on the labor market. In response to the TFP shock, demand for labor services increases and so does the price

Figure 4: Response to a TFP Shock: Labor Market



Notes: Impulse responses of unemployment rate, vacancies, employment, and job-finding rate to a 1 percent TFP shock. See also Figure 3.

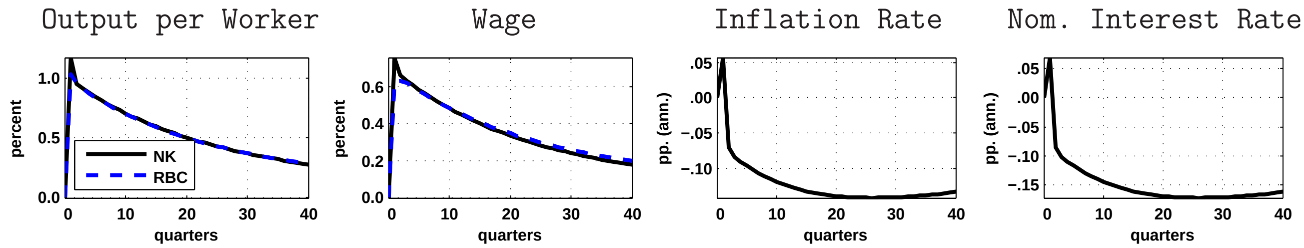
of labor services. Labor services firms, therefore, post more vacancies. The job-finding rate rises markedly, by about 3 percentage points on impact. Employment, therefore, rises in response to the TFP shock and the unemployment rate falls, namely, by about 0.4 percentage point. This response of unemployment and employment is consistent with the responses to *permanent* TFP shocks identified by Ravn and Simonelli (2008) and Altig et al. (2011) by means of long-run restrictions, and the responses identified by sign restrictions to persistent but possibly transitory TFP shocks in Dedola and Neri (2007).[25]

Figure 5 shows the responses of productivity and prices to an increase in TFP. On impact, the additional demand for production factors leads to a spike in the rental rates of labor and capital services (not shown). Despite the increase in productivity (and its negative effect on marginal costs), inflation therefore rises on impact. This increase is short-lived, however. Once the stock of employed households and productive capital expands, marginal costs fall. This gives rise to a period of inflation about one tenth of a percentage point below the target level. The aggregate wage rises less than output by design; equation (39). The resulting gap between the price of labor services and the wage means that the value of existing household-firm matches, J_L, increases (not shown), consistent with the increase in hiring activity.

Next, Figure 6 shows the response of asset-related variables to the TFP shock. Upon realization of the TFP shock the value of firms in the economy increases. The share price of the mutual fund, p_a, rises by a little over 0.6 percent. The share price remains elevated for an extended period of time, mirroring both the increased productivity and the increases in the stock of capital and employment. In the model, investment is funded by retained earnings only. Since the firms, which the mutual fund owns, initially increase their investment in physical assets and employment, for the first 10 or so quarters the mutual fund cuts back on dividends. Beyond that point, however, dividends rise persistently. This reflects the returns from the expanded capacity and that TFP remains well above the baseline. As regards the real return on the nominal bond,

[25] The sign of the response of hours worked and employment to technology shocks continues to be debated. Contrary to the references cited above, in an influential paper, Galí (1999), for example, finds that hours worked (as a measure of labor market activity) fall in response to a permanent technology shock identified in a VAR.

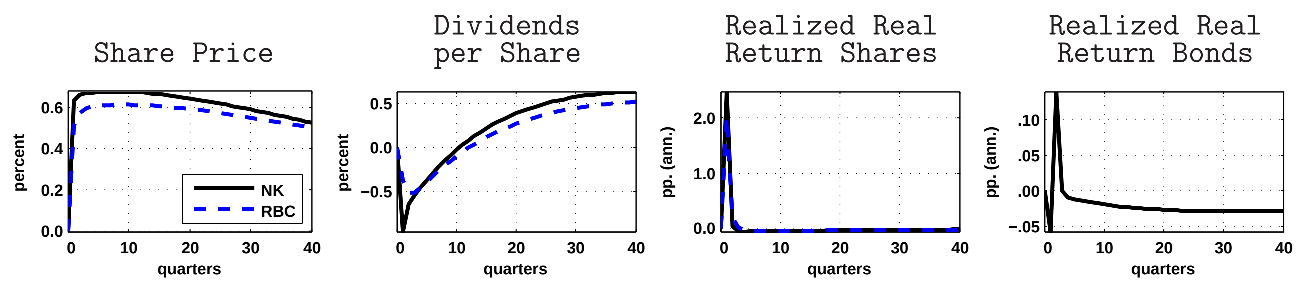
Figure 5: Response to a TFP Shock: Productivity and Prices



Notes: Impulse responses of output per household, wage, inflation, nominal rate, and rental cost of labor and capital services to a 1 percent TFP shock. See also Figure 3.

since inflation rises initially, the real return falls. Beyond that, however, monetary policy accommodates the TFP shock by reducing the nominal rate such that the realized real rate of return on nominal bonds falls from quarter three onward. The realized return on the shares deviates from this only in the short run, reflecting the sharp appreciation of the share price.

Figure 6: Response to a TFP Shock: Assets



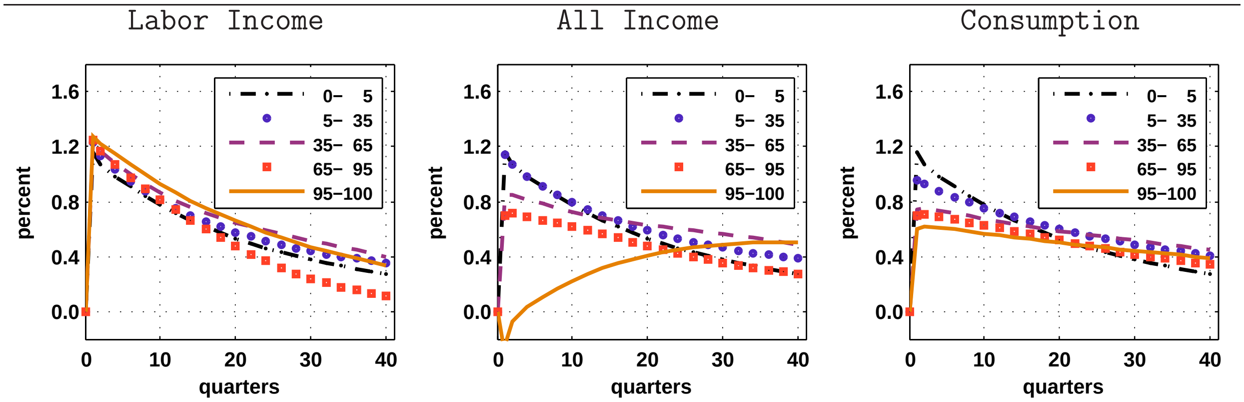
Notes: Impulse responses of the asset price, p_a, dividends per share, d_a, the realized (ex post) real return on stocks and the realized (ex post) real return on nominal bonds to a 1 percent TFP shock. See also Figure 3.

5.1.2 The Effect of a Productivity Shock on Inequality

An important feature of the model is that a positive TFP shock raises the standard of living of all households. For the New Keynesian model economy, Figure 7 shows the response of after-tax labor-related income (defined as earnings plus unemployment benefits), all income (labor-related income plus dividend income), and consumption across different percentiles of the population. For each of the figures, we cut the population according to the wealth of the household. Shown are the percentage responses when averaging along the respective dimension across the respective percentile of the wealth distribution. For example, the black dashed-dotted line in the first panel shows the percentage increase in the average labor income of the bottom 5 percent of the wealth distribution (relative to no TFP shock materializing).[26] In response to a TFP shock, wages rise and employment increases. As a result, labor-related income (left-most panel) rises for all

[26] The figures do not track the same households over time. Rather, households are assigned to the respective percentiles on the basis of their rank in the distribution of wealth at the end of each period.

Figure 7: TFP Shock: Individual Labor Income, Income, Consumption



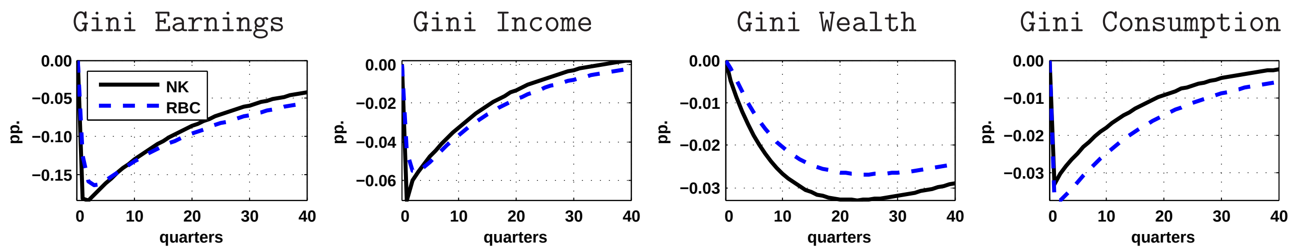
Notes: Impulse responses to a 1 percent TFP shock of after-tax labor-related income (earnings plus unemployment benefits), income (labor-related income plus dividends), and consumption by percentile of the wealth distribution in the New Keynesian economy.

segments of the population. The assumption of a common separation and job-finding rate, as well as a wage function that is common to all households, means that the labor earnings of all households are affected proportionally by the TFP shock.[27] In the left panel, labor-related income rises to a different degree for different segments of the wealth distribution because the figure shows repeated cross-sections: households who become reemployed after the TFP shock earn more labor income than unemployed households, and so are more likely to end up in a higher bin according to their end-of-period wealth. The response of all income (labor-related income plus capital income), instead, is more heterogeneous. The lowest percentiles of the wealth distribution show a markedly stronger response of income on impact than the higher percentiles. The former contain a larger proportion of the unemployed or formerly unemployed. Since both wages and employment rise after a positive TFP shock (compare Figure 4), the average income of these segments of the population rises strongly. Asset-rich households, instead, initially see a decline in dividends since firms choose to retain earnings so as to invest in physical capital. Once the capital is installed, however, the income of the highest segment of the wealth distribution recovers and eventually exceeds the increase in income (in percentage terms) that accrues to the less well-off households. Not surprisingly, the heterogeneity carries over to consumption (right panel). The heterogeneity in the consumption responses is more compressed, however, due to precautionary savings for the borrowing-constrained on the one hand, and the effect that for richer asset-holding households, current income is an incomplete guide to life time wealth.

Figure 8 shows the responses of the Gini indexes for earnings, income, wealth, and consumption. In sum, the responses of the Gini indexes to a TFP shock appear to be rather mild. In response to a 1 percent TFP shock, the earnings Gini falls by as much as 0.16 percentage point. The increase in TFP raises employment (and so reduces the share of households without labor earnings) while wages rise only rigidly. The income Gini falls by somewhat less than the earnings Gini (0.07 percentage point). The reason is that households that move into employment gain

[27] This, clearly, is unrealistic and will be changed in a future version of the current paper.

Figure 8: Response to a TFP Shock: Gini Indexes



Notes: Impulse responses of Gini indexes of earnings (not conditioning on being employed), income, wealth, and consumption to a 1 percent TFP shock. Black solid line: the New Keynesian economy. Blue dashed line: the same economy but without sticky prices. The figures show percentage point increases (a one percentage point increase in the Gini, being an increase from 0.7 to 0.71, for example).

wage income but also forfeit unemployment benefits. The wealth Gini declines, too. While, initially, a large fraction of households own few shares, and, therefore, do not benefit from the increase in asset prices, at the same time, these households strongly benefit from the increase in labor earnings. Overall, the poorer households improve their wealth position relative to wealthier households. The consumption Gini falls by about 0.03 percentage point, tracking the effect on the income Gini more closely than the wealth Gini, a reflection of the environment with idiosyncratic risk and borrowing constraints.

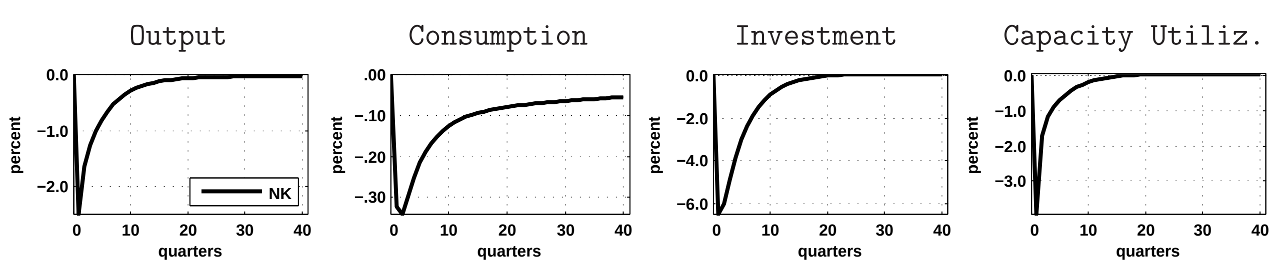
5.2 Transmission of a Monetary Policy Shock

We now turn to how monetary policy affects inequality, the main focus of the paper. In this section, we analyze the effect of a monetary surprise, as characterized by an innovation to the monetary shock, D; compare Taylor rule (22). We analyze how a contractionary monetary policy shock is transmitted to the aggregate economy and how it affects different segments of the population.

5.2.1 Response of the Aggregate Economy to a Monetary Policy Shock

On impact, the impulse of the monetary shock is chosen in such a way that, all else equal, it would raise the nominal rate by 1 percentage point (annualized). The monetary shock is realized

Figure 9: Monetary Policy Shock: Output and Components

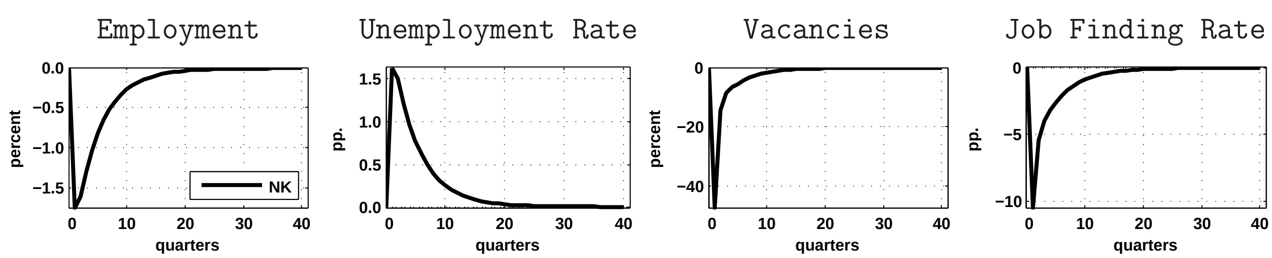


Notes: Impulse responses of output, aggregate consumption, investment, and capacity utilization in response to a 1 percentage point (annualized) monetary policy shock.

in period 1. By design, it persistently raises the expected long real rate of interest. Higher expected returns lead households to save more and cut back their spending for consumption (in the Figure 9, by 0.3 percent). Since prices are sticky, the ensuing fall in aggregate demand is met by an increase in intermediate goods firms' markups, validating the fall in demand. Firms invest less in light of the rising opportunity cost and falling demand. At the same time, capacity utilization falls. GDP overall falls by 2.5 percent. All the responses are front-loaded. The reason is that in order to maintain a tractable size of the state space, our model does not include additional features that generate more drawn-out, hump-shaped responses.

A monetary policy tightening strongly affects the labor market (see Figure 10) for two reasons: On the one hand, the tightening reduces the demand for labor services and their price. On the other hand, such monetary policy raises the real rate of interest and therefore makes firms discount the future more. This further exacerbates the fall in hiring. Along with vacancy posting, the job-finding rate falls (by 10 percentage points). As employment falls, the unemployment rate rises by 1.5 percentage points (from a steady-state value of 6 percent). In the model with

Figure 10: Monetary Policy Shock: Labor Market

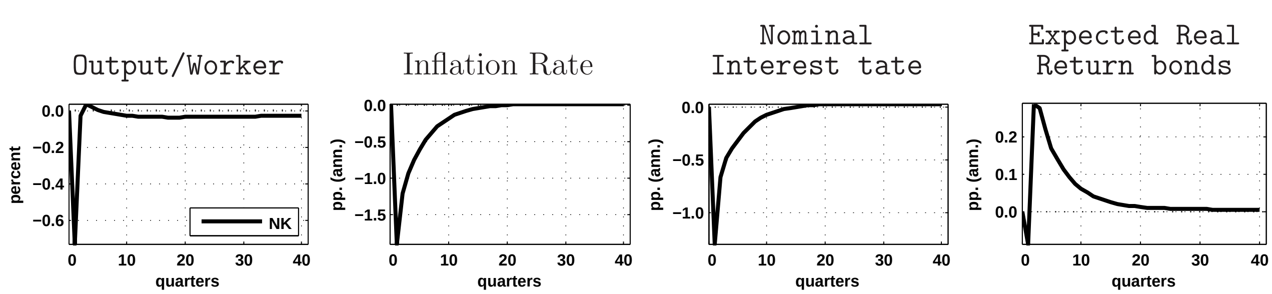


Notes: Impulse responses of unemployment rate, vacancies, employment, and job-finding rate to a 1 percentage point (annualized) monetary policy shock.

borrowing-constrained households, this increase in unemployment tends to further exacerbate the fall in aggregate demand, and the contractionary effects of monetary policy.

Figure 11 shows the response of productivity and wages to the monetary tightening. On

Figure 11: Monetary Policy Shock: Productivity and Prices

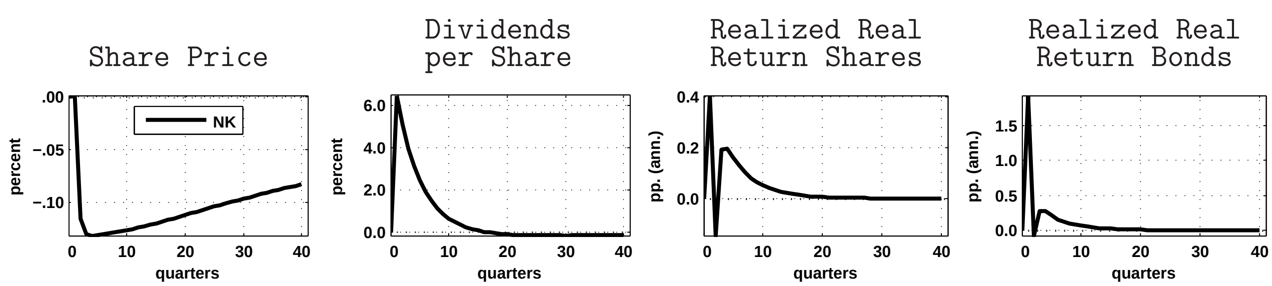


Notes: Impulse responses of output per employed household, wage, inflation rate, and expected one-period real return on bonds to a 1 percentage point (annualized) monetary policy shock.

impact, the reduced demand for production factors causes a reduction in capacity utilization and output per employed household. With production factors in lower demand, the rental rates for both labor and capital services fall (by 8 and 6 percent, respectively; not shown). Since marginal costs fall, inflation falls as well, namely, by about 2 percentage points (annualized). The response of the real rate of interest (right-most panel) deserves discussion. By the logic of the Taylor rule, equation (22), a positive monetary shock leads to a persistent increase in the *ex ante* long-term real rate of interest. In the simulations shown here, the increase in the long-term real rate of interest reduces inflation in a front-loaded manner. Initially, the short-term real rate, therefore, can fall, as is the case in the panels shown here. What matters for the contractionary effect of monetary policy is that the central bank commits to keeping the long-term real rate of interest higher than usual (right-most panel).

Figure 12 shows the response of asset-related variables to the monetary policy shock. As the discount rate rises and investment becomes less profitable, firms pay out "excess" cash-flow through dividends.[28] In the medium term, however, dividends fall as the below-average investment in both labor and capital drains the productive resources available to firms. Share prices decline somewhat on the back of the lower future dividends and heavier discounting of dividends in the near term. After the first few periods, the expected real return on the asset and the nominal bond align closely.

Figure 12: Monetary Policy Shock: Assets



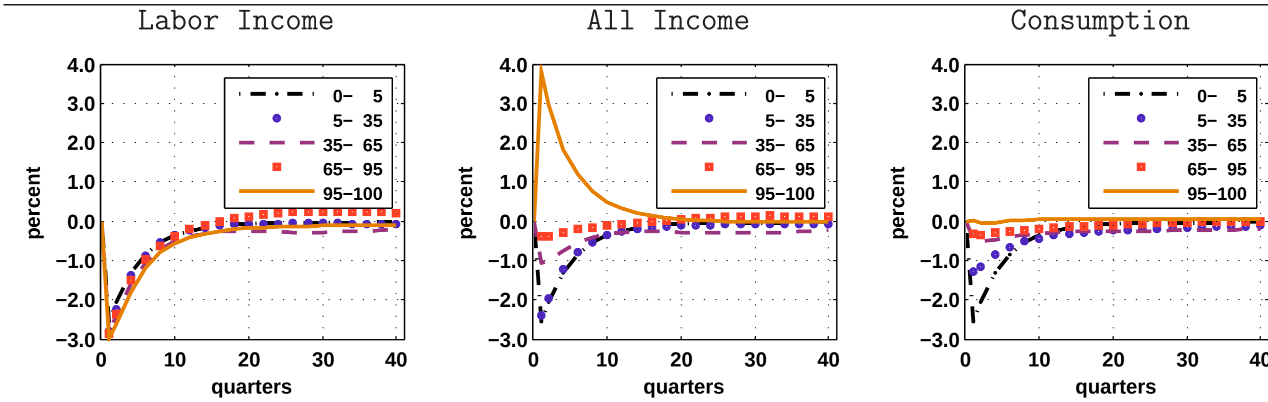
Notes: Impulse responses of the asset price, p_a, dividends per share, d_a, realized real return on stocks and realized real return on nominal bonds to a 1 percentage point (annualized) monetary policy shock.

5.2.2 The Effect of a Monetary Shock on Inequality

We are now in a position to examine how a contractionary monetary policy shock affects different segments of households. Figure 13 shows the response of labor income, all income, and consumption for different percentiles of the wealth distribution. The cut of percentiles is taken with respect to a household's position in the distribution of end-of-period wealth in each of the periods shown. Upon a monetary policy shock, the initial percentage drop in labor income is steepest for the higher wealth percentiles (left panel).[29] The picture is starkly different, however,

[28] The current paper does not allow firms to retain earnings. However, apart from accounting, this should not have a large effect on our results regarding consumption inequality and welfare. The timing of dividends matters primarily for households that are at the borrowing constraint. These households, however, do not hold shares in the first place. All other households can undo dividend payments that they consider ill-timed by adjusting the number of shares they hold.

Figure 13: Monetary Policy Shock: Individual Labor Income, Income, Consumption



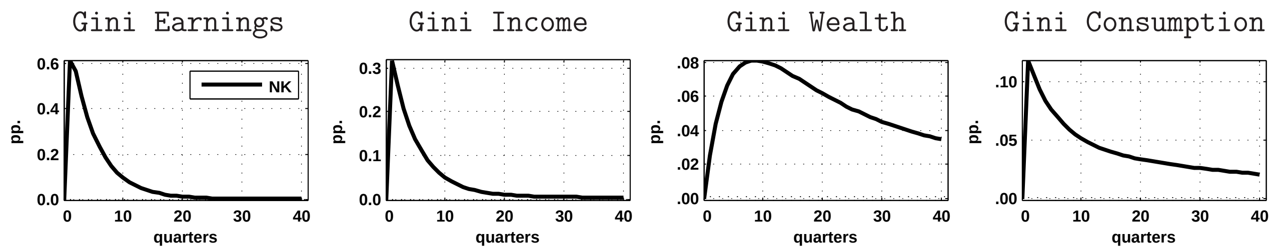
Notes: Impulse responses to a 1 percentage point (annualized) monetary policy shock of labor-related income, income, and consumption by percentile of the wealth distribution in the New Keynesian economy.

when considering the response of all income across different segments of the wealth distribution (middle panel). Monetary policy strongly affects the composition of income. In response to a contractionary monetary policy shock, the income of high-wealth households rises sharply on impact due to a spike in dividends. The income of lower-wealth households, in contrast, declines sharply on the back of lower earnings. The difference in the income responses of the top 5 percent of the households (by wealth) and the bottom 5 percent reaches an astonishing 7 percentage points. Our model broadly captures the effect on income heterogeneity emphasized in the empirical work of Coibion et al. (2012) but shows somewhat less persistence then the impulse responses reported by these authors. The consumption responses (right panel) are very heterogeneous, too. Consumption sharply declines for the lower-wealth households because of the weaker labor market combined with borrowing constraints. Consumption is stable on impact for the wealthiest, but then rises to levels higher than would have been observed in the absence of the monetary shock.

Figure 14 shows how this translates into the responses of the Gini indexes for earnings, income, wealth, and consumption. When monetary policy tightens unexpectedly, the earnings Gini rises sharply, by 0.6 percentage point. This is due to the incidence of higher unemployment, which means that fewer households earn labor income. At the same time, the earnings of those households who remain employed are somewhat rigid, exacerbating the increase in earnings inequality. The effect on the wealth Gini is very persistent, albeit smaller than for earnings (0.08 percentage point). Bear in mind, however, that wealth is already highly concentrated in the baseline. The income Gini spikes sharply on impact and rises somewhat more persistently than the earnings Gini. The consumption Gini rises considerably, too (by 0.12 percentage point). Indeed, the the increase is very persistent. While Figures 9 through 12 suggest that the effects of temporary monetary policy shocks dissipate within the first five years after the shock, the responses of the wealth and consumption Gini coefficients in Figure 14 speak a different language:

[29] Both employment and wages fall. This affects those households most who are employed before the shock. These households are thus more likely to have some wealth.

Figure 14: Response to a Monetary Policy Shock: Gini Indexes



Notes: Impulse responses of Gini indexes of earnings (not conditioning on being employed), income, wealth, and consumption to a 1 percentage point (annualized) monetary policy shock. The figures show percentage point increases (a one percentage point increase in the Gini, being an increase from 0.7 to 0.71, for example).

even short-lived monetary interventions have long-lasting effects on wealth inequality and, by implication, on consumption inequality as well. Next, we therefore explore the welfare effects of TFP shocks and monetary policy shocks.

5.3 Results: Welfare Effects of Productivity and Monetary Policy Shocks

Table 7 looks at the results discussed above from a somewhat different angle. In particular, it asks "What are the welfare costs or gains of TFP and monetary shocks of the size and sign examined above?" The welfare gains or costs are measured as consumption equivalents. For

Table 7: Welfare Effects of Monetary Policy and TFP Shocks

	TFP Shock	MP Shock
Social Welfare		
Representative Agent (RA)	0.109	–0.014
Average of all HHs (HA)	0.151	–0.050
By Wealth Holdings		
Top 5 percent	0.130	0.022
5–20 percent	0.185	–0.021
20-40 percent	0.142	–0.037
40-60 percent	0.122	–0.034
60-80 percent	0.149	–0.056
80-95 percent	0.166	–0.062
Bottom 5 percent	0.168	–0.104

Notes: The table shows the welfare gains (positive) and welfare costs (negative) of a positive 1 percent TFP shock and a negative 1 percent (ann.) monetary policy shock, respectively. The gains or losses are measured in consumption equivalents. The first two rows show welfare costs based on aggregate consumption and CRRA utility ("RA") and based on a utilitarian concept, giving all households the same weight ("HA"). Underneath "By Wealth Holdings," the table reports the consumption-equivalent welfare gains and losses for different households, when sorting these according to their wealth.

example, an entry of "-1" would mean that a household would be willing to forgo 1 percent of consumption in *every* period in the future in order not to live through the shock episode. Focus, first, on the column titled "TFP shock." The first row ("RA") shows the consumption equivalent when, counterfactually, computing welfare, and the consumption equivalent, using aggregate consumption and the CRRA utility function (36). By this measure, the 1 percent TFP shock (which is temporary) raises aggregate welfare by about 0.11 percent of life time consumption. The second row ("HA") averages over the consumption equivalents of each household in the heterogeneous agents world. Under this measure, society on average values the TFP shock at 0.15 percent of life time consumption. The difference between the RA and HA measures suggests tracing the welfare effects more finely according to the wealth distribution. In the rows below, we show the welfare effects for a range from the top 5 percent of the wealth distribution to the bottom 5 percent. The results show that all households benefit from the expansion generated by the TFP shock.

Matters are dramatically different for the monetary policy shock, however. Here, the 5 percent richest households gain from the monetary tightening (in consumption-equivalent terms by 0.02 percent) whereas all other households lose. Indeed, the losses are monotonic in wealth. The poorest 5 percent of households would be willing to pay 0.1 percent of their life time consumption in order not to be exposed to the one-time monetary tightening. The society-wide measures of welfare corroborate the importance of idiosyncratic risk: Under the representative-agent measure of welfare (RA) the monetary contraction barely affects aggregate welfare (the consumption equivalent is -0.014 percent). The average consumption equivalent of -0.05 when averaging the consumption equivalents of all households (HA) presents a more accurate picture but still masks much of the considerable heterogeneity.

6 Conclusions

Monetary policy affects different households differently. In this paper we have built a DSGE model featuring asset market incompleteness, a frictional labor market, as well as nominal frictions. Apart from the market incompleteness, our model deliberately stays close to existing formulations of the New Keynesian model with labor market search and matching frictions, as summarized, for example, in Galí (2008). We use the model setup to quantify how monetary policy affects different segments of the population.

We find that contractionary monetary policy shocks lead to a pronounced increase in earnings, income, wealth, and consumption heterogeneity. Particularly for wealth and consumption heterogeneity, the effects are very persistent and continue to be present even once the monetary impulse on the aggregate economy has largely died out. These findings are broadly consistent with the results in the empirical literature, for example, the recent evidence by Coibion et al. (2012).

An important result of our exercise is that monetary policy shocks have strikingly different implications for the welfare of different segments of the population. While households in the top 5 percent of the wealth distribution benefit slightly from a contractionary monetary policy shock, the bottom 5 percent lose. For example, a monetary tightening of 1 percentage point (annualized) induces a loss equivalent to a *permanent* 0.1 percent cut in consumption for the lowest 5 percent of the wealth distribution. This heterogeneity in sign and size of welfare losses from monetary policy shocks stands in stark contrast to TFP shocks, which affect the population more uniformly. In particular, with an expansionary TFP shock, in our model, a rising tide lifts all boats. A contractionary monetary policy, instead, lifts the boats of the wealthiest only.

The above results suggest that there might be a positive role of monetary policy in mitigating market incompleteness through business cycle stabilization. In on going work, we seek to investigate to what extent the aforementioned results argue for a monetary policy strategy that is based on both inflation and a measure of employment. At the same time, the results presented here have kept the heterogeneity of households to a bear minimum. In particular, we have not allowed for substantial heterogeneity with the respect to labor-market prospects. In future work, we plan to introduce more substantial earnings heterogeneity. We plan to do so in two ways: first, by allowing for heterogeneity in average unemployment and job-finding rates across different skill groups; second, by incorporating a state of long-term unemployment. Last, we have not allowed for positive government debt or money holdings or heterogeneous portfolio allocations. Looking forward, we would like to bring those back into the picture.

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A Model Appendix

For better accessability, Table 8 presents a list of the variables used in the paper. In the following, equilibrium conditions of some sectors of the economy are derived. At the end of this section, steady-state conditions are derived and listed.

A.1 Capital-Producing Sector

Using q as the Lagrange multiplier for the law of motion of capital, we obtain the following first-order conditions with respect to i, v, and k':

$$r(X) = q\delta'(v)$$
$$q\zeta'\left(\frac{i}{k}\right) = 1$$
$$\mathbb{E}Q(X, X')J'_K(X', k') = q$$

The envelope condition with respect to k yields:

$$J'_K(X, k) = r(X)v + q\left(1 - \delta(v) + \zeta\left(\frac{i}{k}\right) - \zeta'\left(\frac{i}{k}\right)\frac{i}{k}\right).$$

Combining them and imposing an equilibrium condition $k = K$, we obtain the following two conditions that characterize:

$$r(X)\zeta'\left(\frac{i}{K}\right) = \delta'(v)$$
$$\frac{1}{\zeta'(i/K)} = \mathbb{E}Q(X, X')\left[r'(X')v' + \frac{1}{\zeta'(i'/K')}\left(1 - \delta(v') + \zeta\left(\frac{i'}{K'}\right)\right) - \frac{i'}{K'}\right]$$

These conditions characterize investment $i(X)$ and utilization $v(X)$ and indirectly the law of motion for capital stock $K'(X)$.

A.2 Final Good Producer

The first-order condition with respect to y_j for the problem of the representative final good producer implies the following for $\forall j$:

$$y_j = \left(\frac{P_j}{P(X)}\right)^{-\epsilon} y$$

Notice that the zero profit condition for final good producers implies:

$$P(X)y = \int_0^1 P_j y_j dj$$

Moreover, since we will focus on a symmetric equilibrium in which (y_j, P_j) are the same for all j, we have $P(X) = P_j$ and $y = y_j$ for $\forall j$ in equilibrium. Going back, the following first-order

Table 8: List of Variables

Variable	Description
$X = (K, N, Z, D, \mu)$	Vector of aggregate state variables
K	Aggregate capital stock
N	Total employment
Z	Total factor productivity (TFP) shock
D	Monetary policy shock
$\mu = (e, s, a) \in \mathcal{M}$	Type distribution of households
G	Law of motion of X
w	Wage per efficiency unit
h	Rental rate of labor (per efficiency unit)
r	Rental rate of capital
p_b	Price of a risk-free one-period discount bond
p_a	Price of a share of mutual funds
d_a	Dividends per share of the mutual funds
P	Price of the final good
P_j	Price of the intermediate good j
Π	Inflation rate
$e \in \{0, 1\}$	Employment status (0: unemployed, 1: employed)
$s \in S$	Skill level
$a \in A \subseteq \mathbb{R}^+$	Share holdings of the mutual funds
$W(X, e, s, a)$	Households' value function
c	Consumption
$a' = g_a(X, e, s, a)$	Optimal decision rule of households with respect to share holdings
$c = g_a(X, e, s, a)$	Optimal decision rule of households with respect to consumption
$Q(X, X')$	Aggregate discount factor
$J_L(X, s)$	Value function for labor agencies
f	Job-finding rate
M	Number of new matches created
$U = 1 - N$	Number of unemployed households
V	Number of vacancy postings
$J_K(X, k)$	Value of capital-producing firm
$J_I(X, P_{j,-1})$	Value function for intermediate good producer
y	Output of a final good producer
y_j	Output of an intermediate good j
v	Utilization rate of capital
i	Investment
ℓ_j	Labor inputs used by an intermediate good producer
k_j	Capital inputs used by an intermediate good producer
τ	Labor tax rate
b	Unemployment insurance benefits per efficiency unit
R	Risk-free nominal rate

condition for good j characterizes the demand function used for the problem of the monopolistic intermediate good producer's problem:

$$y_j(P_j, X) = \left(\frac{P_j}{P(X)}\right)^{-\epsilon} y(X) \tag{40}$$

where, in equilibrium, $y_j(P_j, X) = y_j(P(X), X) = y(X)$ for $\forall j$.

A.3 Intermediate Good Producer

Notice that the problem of an intermediate good producer j can be split into two problems: (i) the cost minimization problem given output $\overline{y}_j$, and (ii) the choice of the price P_j given the demand function of the final good producer and the solution to the cost minimization problem given output level y_j. Let's look at the problems one by one.

The cost-minimization problem of an intermediate good producer can be characterized as follows:

$$\min_{k_j,\ell_j} \quad r(X)v(X)k_j + h(X)\ell \tag{41}$$

$$\text{s.t.} \qquad \overline{y}_j = Zk_j^{\theta}\ell_j^{1-\theta} \tag{42}$$

Arranging the first-order conditions, we can obtain the following two equations:

$$\frac{r(X)v(X)}{h(X)} = \frac{\theta\ell_j}{(1-\theta)k_j} \tag{43}$$

$$\overline{y}_j = Zk_j^{\theta}\ell_j^{1-\theta} \tag{44}$$

Plugging them back into the total cost yields:

$$\text{total cost} = \left(\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{1-\theta}\right)^{1-\theta}\frac{(r(X)v(X))^{\theta}h(X)^{1-\theta}}{Z}\overline{y}_j \tag{45}$$

This implies:

$$\text{marginal cost} = m(X) = \left(\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{1-\theta}\right)^{1-\theta}\frac{(r(X)v(X))^{\theta}h(X)^{1-\theta}}{Z} \tag{46}$$

Second, an intermediate good producer j sets the price of its product P_j, taking the demand function of the final good producer (40) and prices as given. The profit maximization problem of an intermediate good producer j can be recursively characterized as follows:

$$J_I(X, P_{j,-1}) = \max_{P_j} \left\{\left(\frac{P_j}{P(X)}\right)^{-\epsilon} y(X)\left(\frac{P_j}{P(X)} - m(X) - \frac{\phi_P}{2}\left(\frac{P_j}{P_{j,-1}} - \overline{\Pi}\right)^2\right) + \mathbb{E}Q(X,X')J_I(X', P_j)\right\} \tag{47}$$

subject to $X' = G(X)$. $m(X)$ is the marginal cost defined in (46). It is necessary because the price adjustment cost depends on the change in the price from the previous period $(\Pi_{j,-1})$ to the current period (P_j). However, as shown below, we will not need to keep track of $P_{j,-1}$ in our computations. Also notice that the demand function by the final good firm (40) is already taken into account. The first-order condition with respect to P_j and the envelope condition are the following:

$$
\begin{aligned}
y(X)\left(\frac{P_j}{P(X)}\right)^{-\epsilon-1}\Bigg[\frac{(1-\epsilon)}{P(X)}\left(\frac{P_j}{P(X)}\right)+\frac{m(X)\epsilon}{P(X)}+\frac{\phi_P\epsilon}{2P(X)}\left(\frac{P_j}{P_{j,-1}}-\overline{\Pi}\right)^2& \\
-\frac{\phi_P}{P_{j,-1}}\left(\frac{P_j}{P(X)}\right)\left(\frac{P_j}{P_{j,-1}}-\overline{\Pi}\right)\Bigg]+\mathbb{E}Q(X,X')J'(X',P_j)&=0
\end{aligned}
$$

$$
J_I'(X,P_{j,-1}) = y(X)\phi_P\left(\frac{P_j}{P_{j,-1}^2}\right)\left(\frac{P_j}{P_{j,-1}}-\overline{\Pi}\right)\left(\frac{P_j}{P(X)}\right)^{-\epsilon}
$$

Combining the two, applying $\Pi(X) = P(X)/P_{-1}(X)$, and imposing the symmetric equilibrium condition $P = P_j\ \forall j$:

$$
\begin{aligned}
1-\epsilon+m(X)\epsilon+\frac{\phi_P\epsilon}{2}(\Pi(X)-\overline{\Pi})^2-\phi_P\Pi(X)(\Pi(X)-\overline{\Pi})& \\
+\mathbb{E}Q(X,X')\frac{y(X')}{y(X)}\phi_P\Pi(X')(\Pi(X')-\overline{\Pi})&=0 \quad (48)
\end{aligned}
$$

Notice that, as mentioned, it is no longer necessary to keep track of past prices. Instead, Euler equation (48) implicitly characterizes the inflation rate of intermediate and final goods $\Pi(X)$.

A.4 Steady-State Conditions

The following equations characterize the labor market in the steady state:

$$
\begin{aligned}
&J_L^*(s) = (h^*-w^*)s + Q^*\sum_{s'}\pi_{s,s'}J_L^*(s') \\
&M^* = \gamma((1-N^*)+\lambda N^*)^{\alpha}(V^*)^{1-\alpha} \\
&\kappa = \frac{M^*}{V^*}\int_{\mathcal{M}} J_L^*(s)\ d\ \mu \\
&\lambda N^* = M^*
\end{aligned}
$$

In the steady state, the optimal decisions of capital-producing firms can be characterized as follows. Notice that we assume $\zeta(i^*/K^*) = i^*/K^*$, $\zeta'(i^*/K^*) = 1$, $\delta(v^*) = \delta^*$

$$
\begin{aligned}
&\delta^*K^* = i^* \\
&1 = Q^*[r^*+1-\delta^*]
\end{aligned}
$$

The optimal decisions of the intermediate good producers in the stead-state equilibrium are characterized by the following:

$$
\begin{aligned}
&\frac{r^* v^*}{h^*} = \frac{\theta L^*}{(1-\theta)K^*} \\
&m^* = \left(\frac{1}{\theta}\right)^{\theta} \left(\frac{1}{1-\theta}\right)^{1-\theta} \frac{(r^*)^{\theta}(h^*)^{1-\theta}}{Z^*} \\
&1 - \epsilon + m^* \epsilon = 0
\end{aligned}
$$

B Computation Appendix

This appendix outlines the solution method of an equilibrium with aggregate uncertainty. The method is a version of the method developed by Krusell and Smith (1998) and Krusell and Smith (1997) and is closely related to the solution method of Nakajima (2012).

1. Following Krusell and Smith (1998), we approximate the type distribution of households μ by aggregate capital stock K and aggregate employment N. Consequently, the aggregate state variable becomes $X = (K, N, Z, D)$.

2. Parameterize the forecasting functions for the price of shares $p_a(X)$ and the aggregate discount factor $Q(X, X')$. Denote the set of parameters for the forecasting functions as Φ.

3. Set a guess Φ^0.

4. Given the forecasting functions with Φ^0, iterate on $N'(X)$, $K'(X)$, $\Pi(X)$, and $v(X)$.

    (a) Set a guess for $K'(X)$, $N'(X)$, $\Pi(X)$, and $v(X)$.
    (b) Using the Taylor rule and the Euler equation associated with the mutual funds, an updated $\hat{\Pi}$(X) can be obtained.
    (c) Using the equilibrium conditions of the intermediate-good producers, $h(X)$ can be obtained.
    (d) Using the zero profit condition for labor agencies and the Bellman equation for labor agencies, $V(X)$ can be obtained. Once $V(X)$ is obtained, an updated $\hat{N}'(X)$ can be obtained as well.
    (e) Using the equilibrium conditions associated with capital-producing firms, $i(X)$ and thus $\hat{K}'(X)$ and $\hat{v}(X)$ can be obtained.
    (f) If the updated values are close to the guessed values, this step is done. Otherwise, go back with an updated guess.

5. Given the forecasting functions with Φ^0, and $K'(X)$, $N'(X)$, $\Pi(X)$, $v(X)$ that were obtained in the previous step, iterate on the value function of households.

    (a) Set a guess for the value function $W(X, e, s, a)$.

(b) Use the Bellman equation to update the value function.

(c) If the updated value function is close to the guess, this step is done. The optimal decision rules $a' = g_a(X, e, s, a)$ and $c = g_c(X, e, s, a)$ are obtained. Otherwise, go back with an updated value function.

6. Simulate the model. Notice that, for each period a market-clearing p_a has to be found, in the same manner as in Krusell and Smith (1997).

   (a) Set the initial state and the initial type distribution. Use the steady-state values as the initial guess.

   (b) At the beginning of a period t, draw a new set of shocks. We have the aggregate state in period t, (K_t, N_t, Z_t, D_t).

   (c) Using the forecasting functions, compute $\hat{Q}_t(Z', D')$.

   (d) Set a guess for the share price $\hat{p}_a$, using the forecasting function with Φ^0.

   (e) Conditional on $\hat{p}_a$, and the aggregate state variables in period t, solve the problem of households.

   (f) Check market-clearing. Compute the excess demand for the shares. If it is zero, a market-clearing price in period t, $p_{a,t}$, is obtained for period t. K_{t+1} and N_{t+1} can be computed. Go to the next step. Otherwise, update $\hat{p}_a$ and go back to the previous step.

   (g) Using the market-clearing p_a, and the optimal decision rules, the aggregate stochastic discount factor implied by the current type distribution of households in period t, $Q_t(Z', D')$ can be computed.

   (h) Update the type distribution and aggregate state variables using $p_{a,t}$ and go to period $t + 1$.

   (i) Keep simulating until period $T = T_0 + T_1 = 500 + 5000$ periods.

7. The previous step generates a time series of $\{K_t, N_t, Z_t, D_t, K_{t+1}, N_{t+1}, Q_t(Z', D'), p_{a,t}\}_{t=0}^{T}$. Drop the first T_0 periods. Using the time series for $t = T_0{+}1, ..., T$, regress $\{K_{t+1}, N_{t+1}, Q_t(Z', D'), p_{a,t}\}$ on $\{K_t, N_t, Z_t, D_t\}$, which gives an updated of Φ, here denoted by Φ^1.

8. Compare Φ^0 and Φ^1. If they are close, an equilibrium is obtained. Stop. Otherwise, update Φ^0 and go back to step 4.