Deflation and debt:
A neoclassical framework for monetary policy analysis

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Very Preliminary. Comments Welcome.

Can New Keynesian (NK) model account for Japan's Deflation?

- Deflation continued for more than a decade in Japan from 1998–present.
- New Keynesian models are not fully satisfactory in analyzing decade-long deflation.
    - Was the price stickiness a problem in Japan's 1990s and in the global crises?

What is necessary for a framework of monetary policy analysis

- Can we consider an alternative framework without price stickiness?
- Three features in the New Keynesian model
  1. Suboptimality of the Friedman rule:
     zero or moderate inflation maximizes social welfare.
  2. Phillips curve:
     a positive correlation between inflation and output.
  3. Liquidity effects:
     a negative correlation between nominal interest rate and output (or money supply).

What we do in this paper

- Construct a neoclassical model with flexible prices, which has the following features:
  1. Suboptimality of the Friedman rule: **input-smoothing effect**
     - Suppose that firms with loose constraints and tight constraints coexist. Distortionary tax on loosely-constrained firms can be welfare enhancing (input-smoothing effect).
     - Inflation works as a device for input-smoothing effect.
  2. Phillips curve
  3. Liquidity effect
- These features are generated from heterogeneous financial constraints.
- Our model may provide a new account for a decade-long deflation in Japan.

Overview of the model

- Closed economy, representative consumer, heterogeneous firms (firms 1, firms 2), central bank (CB).
- Heterogeneity in financial constraints
    - Consumer can transfer cash to firm 1 as internal funds.
        - Consumer is owner-manager of firm 1.
    - Consumer cannot transfer cash to firm 2.
        - Consumer is owner but not manager of firm 2. The manager of firm 2 can divert cash for private purposes without getting penalty.
- CB can choose intra-period interest rate (nominal rate), j, and the amount of intra-period loan, Δ.
- The policy $(j, \ \Delta)$ decides the inflation rate π as an equilibrium outcome.

Setup

- Time is discrete: $t = 0, 1, 2, \cdots$.
- Closed economy with representative consumer who owns two types of firms: firm 1 and firm 2.
- Firms can produce y_t from labor L_t:

$$y_t = A_t L_t^{\alpha},$$

  where the wage $w_t L_t$ must be financed by cash and/or credit subject to a liquidity constraint, described later.
- The measure of consumers is normalized to one.
- There are continuum of firms 1 with measure ϕ and continuum of firms 2 with measure $1 - \phi$.

Setup – Consumer

- A consumer can invest his cash in firm 1 as internal funds, while he cannot invest cash in firm 2.
  - The manager of firm 2 can divert cash for private purposes without getting penalty if the consumer invests cash in firm 2.
- Optimization of a consumer is

$$\max \sum \beta^t [\ln c_t + \gamma \ln(1 - l_t)],$$

$$\text{s.t. } c_t + \phi m_t + b_t \leq w_t l_t + \phi \Pi \left(\frac{m_{t-1}}{\pi_t} + \Delta_t \right) + (1 - \phi)\Pi(0) - (1 + j_t)\phi \Delta_t + (1 + r_t) b_{t-1},$$

  where $\Pi(m)$ is the profit from his own firm with internal funds m.
- Consumer can invest cash, $\phi \left\{ \frac{m_{t-1}}{\pi_t} + \Delta_t \right\}$, in his own firm 1.
  - m_{t-1} is the real money carried over from $t - 1$,
  - $\pi_t = P_t / P_{t-1}$ is the inflation from $t - 1$ to t,
  - Δ_t is intra-period loan from Central Bank, for which j_t is interest rate.

Setup – Firm 1

- Cash $\frac{m_{t-1}}{\pi_t} + \Delta_t$ is transferred from its owner (the consumer).
- Given $\frac{m_{t-1}}{\pi_t} + \Delta_t$, the firm solves

$$V_{t-1}^1 = \beta E_{t-1}\left[\frac{\lambda_t}{\lambda_{t-1}}\left\{\max_{L_{1t}}[AL_{1t}^{\alpha} - w_t L_{1t}] + \frac{m_{t-1}}{\pi_t} + \Delta_t + V_t^1\right\}\right],$$

$$\text{s.t. } w_t L_{1t} \leq \frac{m_{t-1}}{\pi_t} + \Delta_t + \theta V_t^1 \qquad (\mu_1)$$

where $\Pi\left(\frac{m_{t-1}}{\pi_t} + \Delta_t\right) = AL_{1t}^{\alpha} - w_t L_{1t} + \frac{m_{t-1}}{\pi_t} + \Delta_t$.

Setup – Firm 1 (cont'd)

- Liquidity constraint

$$wL_{1t} \leq \frac{m_{t-1}}{\pi_t} + \Delta_t + \theta V_t^1 \qquad (\mu_1)$$

is derived from the commitment problem (Kiyotaki and Moore 1997, Jermann and Quadrini 2012):

  - Before production, firm 1 pays cash to the worker and promises to pay the remaining wage after production.
  - After production, if the firm breaks the promise, the worker destroys the firm with probability θ, in which case the firm cannot operate from $t+1$ on.
  - As the firm loses the expected value θV_t^1 by breaking the promise, it can credibly pay the remaining wage as long as it is less than θV_t^1.

Setup – Firm 2

- The firm solves

$$V_{t-1}^2 = \beta E_{t-1}\left[\frac{\lambda_t}{\lambda_{t-1}}\{\max_{L_{2t}}[AL_{2t}^{\alpha} - w_t L_{2t}] + V_t^2\}\right],$$

$$\text{s.t. } w_t L_{2t} \leq \theta V_t^2 \qquad (\mu_2)$$

where $\Pi(0) = AL_{2t}^{\alpha} - w_t L_{2t}$.

Setup – Government or Central Bank

- This time, we do not specify the objective of the government. We assume the following assumption.
  - Government follows the exogenous policy rule:

$$j_t = J(A_t, \theta_t; I_t),$$
$$\Delta_t = D(A_t, \theta_t; I_t),$$

    where I_t is all information available at t.

- The government is subject to the budget constraint:

$$(1 + r_t)B_{t-1} + \frac{M_{t-1}}{\pi_t} = B_t + M_t + j_t\phi\Delta_t,$$

  where B_t and M_t are supplies of bonds and cash. The upper case variables do not represent nominal variables, but they are real variables.

Setup – Summary

Consumer: $\max \sum \beta^t [\ln c_t + \gamma \ln(1 - l_t)],$

s.t. $c_t + \phi m_t + b_t \leq w_t l_t + \phi \Pi \left(\frac{m_{t-1}}{\pi_t} + \Delta_t \right) + (1 - \phi)\Pi(0) - (1 + j_t)\phi\Delta_t + (1 + r_t) b_{1t-1}, \quad (\lambda_{1t})$

Firm 1: $V^1_{t-1} = \beta E_{t-1} \left[\frac{\lambda_t}{\lambda_{t-1}} \left\{ \max_{L_{1t}} [A L^{\alpha}_{1t} - w_t L_{1t}] + \frac{m_{t-1}}{\pi_t} + \Delta_t + V^1_t \right\} \right],$

s.t. $w_t L_{1t} \leq \frac{m_{t-1}}{\pi_t} + \Delta_t + \theta V^1_t \quad (\mu_1)$

Firm 2: $V^2_{t-1} = \beta E_{t-1} \left[\frac{\lambda_t}{\lambda_{t-1}} \{ \max_{L_{2t}} [A L^{\alpha}_{2t} - w_t L_{2t}] + V^2_t \} \right],$

s.t. $w_t L_{2t} \leq \theta V^2_t \quad (\mu_2)$

Government: $(1 + r_t) B_{t-1} + \frac{M_{t-1}}{\pi_t} = B_t + M_t + j_t \phi \Delta_t,$

Equilibrium conditions

- Equilibrium conditions

$$c = Y = \phi A L_1^{\alpha} + (1 - \phi) A L_2^{\alpha},$$

$$l = \phi L_1 + (1 - \phi) L_2,$$

$$\phi m = M,$$

$$b = B.$$

- Nominal interest rate: The consumer's FOCs wrt Δ and m_t, and the envelope condition for $\Pi(m)$ imply that

$$j_t = \mu_{1t}.$$

The short-term nominal interest rate j_t equals the tightness of liquidity constraint for firm 1.

Steady-state equilibrium

Given policy parameters $(j,\ \Delta)$, the steady-state equilibrium is given as a solution to the following 14 equations for 14 unknowns $(c, w, l, L_1, L_2, \pi, \mu_2, m, V^1, V^2, r, b, M, B)$.

$$w = \frac{\gamma c}{1-l}, \tag{3}$$

$$1 + j = 1 + \mu_1 = \frac{\pi}{\beta}, \tag{4}$$

$$wL_1 = \frac{\alpha A L_1^\alpha}{1+\mu_1}, \tag{5}$$

$$wL_1 = \frac{m}{\pi} + \Delta + \theta V^1, \tag{6}$$

$$V^1 = \frac{\beta A L_1^\alpha}{1-(1-\theta)\beta}, \tag{7}$$

$$wL_2 = \theta V^2, \tag{8}$$

Steady-state equilibrium

$$V^2 = \frac{\beta A L_2^\alpha}{1-(1-\theta)\beta}, \tag{9}$$

$$wL_2 = \frac{\alpha A L_2^\alpha}{1+\mu_2}, \tag{10}$$

$$Y = c = \phi A L_1^\alpha + (1-\phi) A L_2^\alpha, \tag{11}$$

$$l = \phi L_1 + (1-\phi) L_2, \tag{12}$$

$$1 + r = \beta^{-1}, \tag{13}$$

$$rB = \left(1 - \frac{1}{\pi}\right) M + \phi j \Delta, \tag{14}$$

$$b = B, \tag{15}$$

$$\phi m = M. \tag{16}$$

Characteristics of the steady-state equilibrium

- The degree of inefficiency is pinned down by π. The other policy variable (Δ) determines the value of m.
  - Variables $(w, L_1, L_2, Y, V^1, V^2)$ depend only on π, and independent of Δ. The value of w is given by

$$w^{\frac{1}{1-\alpha}} = \gamma\phi A^{\frac{1}{1-\alpha}} \left\{ \left(\frac{\alpha\beta}{\pi}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\alpha\beta}{\pi}\right)^{\frac{\alpha}{1-\alpha}} \right\} + (1-\phi)\gamma A^{\frac{1}{1-\alpha}} \left\{ \left(\frac{\theta\beta}{1-(1-\theta)\beta}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\theta\beta}{1-(1-\theta)\beta}\right)^{\frac{\alpha}{1-\alpha}} \right\}. \tag{17}$$

- Redundancy of Δ:
  - Δ does not affect the welfare.
  - There are continuously infinite combinations of (m, Δ) for given π.

Welfare analysis on the steady state

- We focus on the deterministic steady-state equilibrium in which j and Δ are constant.
- Inflation rate π is pinned down by

$$1 + j = 1 + \mu_1 = \frac{\pi}{\beta}.$$

- Tightness of constraint for firm 2 does not depend on monetary policy:

$$1 + \mu_2 = \frac{\{1 - (1 - \theta)\beta\}\alpha}{\theta\beta}.$$

- We define the social welfare W by

$$W = \frac{1}{1-\beta} U(c, l) = \frac{1}{1-\beta}\{\ln c + \gamma \ln(1 - l)\}.$$

Welfare analysis on the steady state (cont'd)

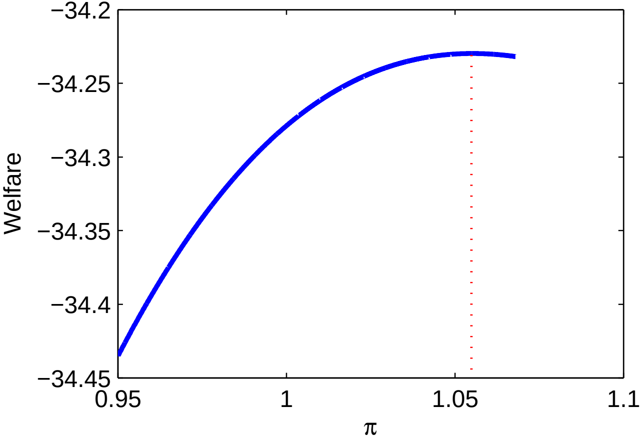
- Parameter values are

α	β	ϕ	γ	A	θ
0.89	0.95	0.25	1.8	1	0.2

- As shown in the next slide, W is maximized by the policy: $j = 0.11$, which implies $\pi = 1.0552$.
  $\Rightarrow$ A moderate inflation is optimal!

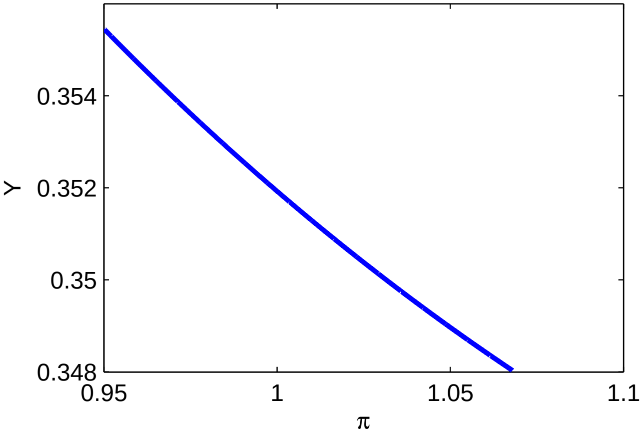
Suboptimality of the Friedman rule

- The welfare level in the steady state is maximized at $\Delta = 0,\ \pi^* = 1.0552,\ Y^* = 0.349$:



Suboptimality of the Friedman rule

- Output is maximized at the Friedman rule.



Why is the Friedman rule suboptimal?

- In the equilibrium,
  - all monetary distortion, which is represented by μ_1, can be completely eliminated by setting $j = 0$ (or $\pi = \beta$) because $j = \mu_1$ in equilibrium.
  - The Friedman rule seems to be optimal...
- ... But it turns out that the Friedman rule is suboptimal in our model.

Rigorous derivation of suboptimality of the Friedman rule

- Social welfare is $W = U/(1-\beta)$, where

$$U = \ln Y + \gamma \ln(1-L),$$

$$Y = \phi A L_1^{\alpha} + (1-\phi) A L_2^{\alpha},$$

$$L_1 = \left(\frac{\alpha\beta A}{w(\pi)\pi}\right)^{\frac{1}{1-\alpha}},$$

$$L_2 = \left(\frac{\theta\beta A}{\{1-(1-\theta)\beta\}w(\pi)}\right)^{\frac{1}{1-\alpha}},$$

- We can show for the elasticity of wage rate wrt inflation $\varepsilon(\pi)$, $0 < \varepsilon(\pi) < 1$, that

$$\frac{dU}{d\pi} = [(1-\phi)\varepsilon(\pi)\mu_2 L_2 - (1-\varepsilon(\pi))\phi\mu_1 L_1]\frac{w}{(1-\alpha)\pi Y}.$$

- Suboptimality of the Friedman rule: $\frac{dU}{d\pi} > 0$ at $\mu_1 = 0$.

Economic intuition: input-smoothing effect

- Inflation tax on firm 1 is welfare enhancing: Input-smoothing effect
  - Inflation tax is imposed selectively on firm 1, not on firm 2.
    - Firm 1 can use cash to relax the liquidity constraint.
    - Firm 2 cannot use cash to relax the liquidity constraint.
  - Input-smoothing effect:
    - Inflation tax on firm 1 reduces firm 1's demand for labor and decreases the wage rate w, which in turn increases firm 2's demand for labor.
    - MPL of firm 2 $>$ MPL of firm 1.
    - Decrease in total output is moderate compared to the decrease in labor.
    - Thus the inflation tax reduces disutility from labor more than utility from consumption of the goods.
    - Overall effect is welfare enhancing.

Economic intuition: input-smoothing effect (cont'd)

- Inflation: second-best policy to reallocate labor from firm 1 to firm 2.
  - If there exists a policy that can reallocate labor from firm 1 to firm 2 without reducing total labor, it should be better than inflation.
    - Inflation reduces total labor.
    - Inflation reallocate the labor from firm 1 to firm 2.
    - The second effect dominates the first and improve total welfare.
- Firm 1 represents old and traditional sectors, and firm 2 young and emerging industries.
  - Old firms have easy access to funds, while young firms do not.
  - Inflation reallocates resources from old firms to young firms.

Stochastic shocks

- We consider two stochastic shocks in the economy.
  - A_t: the productivity shock.
  - θ_t: the financial shock (Jermann and Quadrini 2012).
- Given the policy rules:

$$j_t = \mu_{1t} = J(A_t, \theta_t; I_t),$$

$$\Delta_t = D(A_t, \theta_t; I_t),$$

we can calculate the dynamic response of the economy to these shocks (A_t, θ_t).

Dynamics

$$w_t = \frac{\gamma c_t}{1 - l_t}, \tag{18}$$

$$w_t L_{1t} = \frac{\alpha A_t L_{1t}^{\alpha}}{1 + j_t}, \tag{19}$$

$$w_t L_{2t} = \frac{\alpha A_t L_{2t}^{\alpha}}{1 + \mu_{2t}}, \tag{20}$$

$$w_t L_{1t} - \frac{m_{t-1}}{\pi_{t-1}} - \Delta_t = \theta_t V_t^1, \tag{21}$$

$$w_t L_{2t} = \theta_t V_t^2, \tag{22}$$

$$V_t^1 = \beta E_t \left[\frac{c_t}{\tilde{c}_{t+1}} \left\{ \tilde{A}_{t+1} \tilde{L}_{1t+1}^{\alpha} - \tilde{w}_{t+1} \tilde{L}_{1t+1} + \frac{m_t}{\tilde{\pi}_t} - \tilde{j}_{t+1} \tilde{\Delta}_{t+1} + \tilde{V}_{t+1}^1 \right\} \right], \tag{23}$$

$$V^2 = \beta E_t \left[\frac{c_t}{\tilde{c}_{t+1}} \left\{ \tilde{A}_{t+1} \tilde{L}_{2t+1}^{\alpha} - \tilde{w}_{t+1} \tilde{L}_{2t+1} + \tilde{V}_{t+1}^2 \right\} \right], \tag{24}$$

Dynamics (cont'd)

$$Y_t = c_t = \phi A_t L_{1t}^{\alpha} + (1-\phi) A_t L_{2t}^{\alpha}, \tag{25}$$

$$l_t = \phi L_{1t} + (1-\phi) L_{2t}, \tag{26}$$

$$(1+r_t)B_{t-1} + \frac{M_{t-1}}{\pi_{t-1}} = B_t + M_t + j_t \phi \Delta_t, \tag{27}$$

$$1 = \beta E_t \left[\frac{c_t}{\tilde{c}_{t+1}} \frac{(1+\tilde{j}_{t+1})}{\tilde{\pi}_t} \right], \tag{28}$$

$$1 = \beta E_t \left[\frac{c_t}{\tilde{c}_{t+1}} (1+\tilde{r}_{t+1}) \right], \tag{29}$$

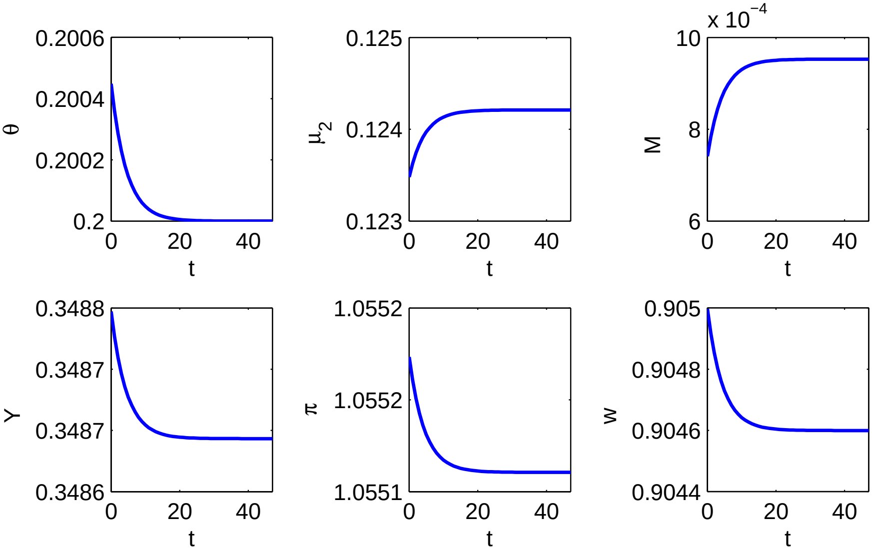
$$M_t = \phi m_t, \tag{30}$$

$$B_t = b_t. \tag{31}$$

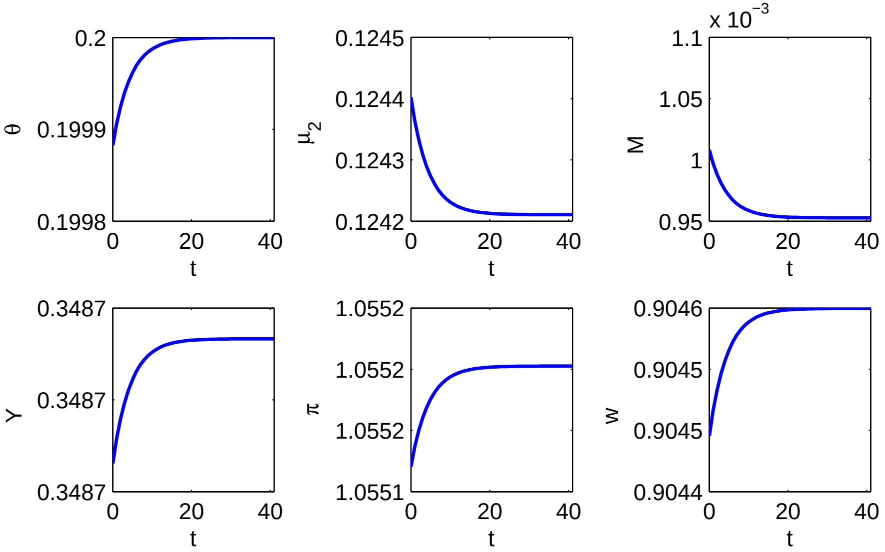
Phillips curve

- Consider the dynamic response of the model in which
  - θ_t changes exogenously,
  - (j_t, Δ_t) are kept at steady-state values ($j_t = \mu_{1t} = \bar{\mu}_1$, $\Delta = 0$).

Phillips curve



Phillips curve



Intuition for the Phillips curve

- Phillips curve relationship is an artifact generated by the responses of output and inflation to the financial shocks.

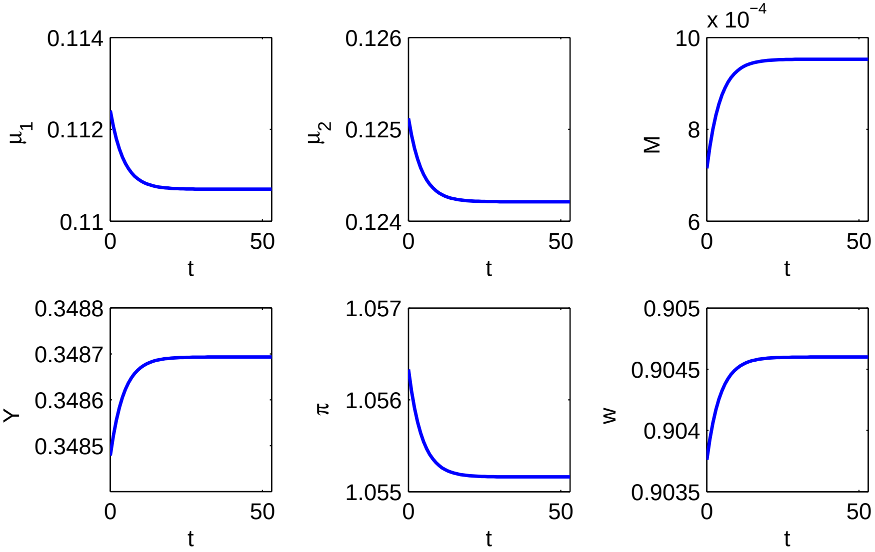
$$\theta \uparrow \Longrightarrow Y \uparrow \quad \text{and} \quad \theta \uparrow \Longrightarrow \pi \uparrow$$

$$\theta \downarrow \Longrightarrow Y \downarrow \quad \text{and} \quad \theta \downarrow \Longrightarrow \pi \downarrow$$

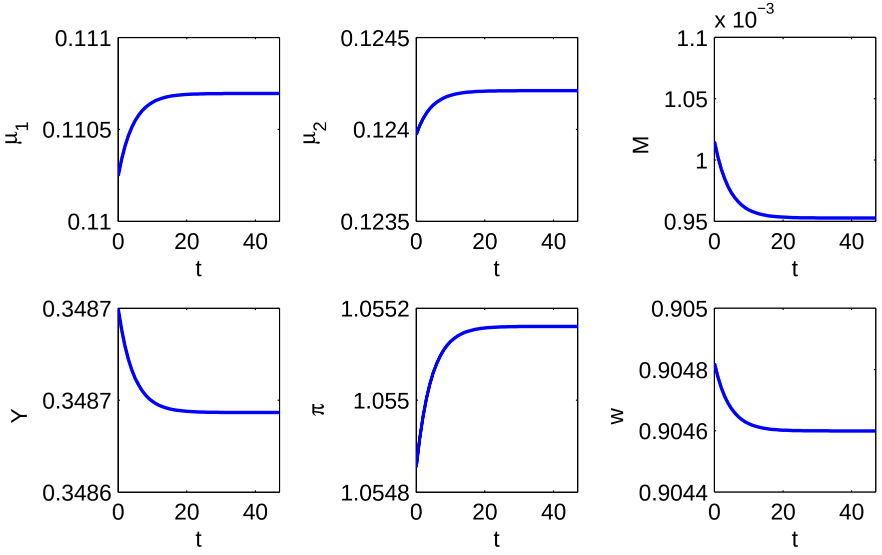
Liquidity effect

- Consider the dynamic response of the model in which
  - there is no exogenous shock: θ_t and A_t are constant,
  - $j_t = \mu_{1t}$ is changed by monetary policy,
  - Δ_t is kept at the steady-state value ($\Delta_t = 0$).

Liquidity effect



Liquidity effect



Intuition for the liquidity effect

- As $j_t = \mu_{1t}$ increases, liquidity constraints for firms 1 become tighter.
- Output decreases as liquidity constraints become tighter.
- Money demand decreases as the nominal short-term rate j_t increases.

$$j_t \uparrow \Longrightarrow Y \downarrow \quad \text{and} \quad j_t \uparrow \Longrightarrow M \downarrow$$

$$j_t \downarrow \Longrightarrow Y \uparrow \quad \text{and} \quad j_t \downarrow \Longrightarrow M \uparrow$$

Accounting for decade-long deflation

- A simple explanation: The Fisher relation in the steady state

$$1 + j = \pi(1 + r) = \frac{\pi}{\beta}.$$

  - If j is fixed at zero, then the inflation rate should be negative ($\pi < 1$), as the real rate of interest $r = \beta^{-1} - 1$ is positive.
  - If the following Ricardian expectations on fiscal policy prevail, then permanent deflation is compatible with increasing money supply.
    - Ricardian expectations:
      In the far futre, tax will be increased so that the government budget constraint is satisfied under permanent ZIRP.

Why deflation is associated with low output?

- In response to a negative financial shock the zero nominal interest rate policy (ZIRP, i.e., the Friedman rule) can maximize the output by relaxing the liquidity constraint.
- If the negative financial shock is **permanent**,
    - ZIRP can maximize the output of firm 1, while the level of total output is permanently lower than in the initial steady state,
    - ZIRP creates the equilibrium deflation by the Fisher relation,
    - ZIRP is not the long-run “optimal” policy. (ZIRP maximizes output, while it does not maximize welfare.)
- Permanent financial shock may represent structural changes in financial sector or in financial regulations.

Conclusion

- We construct a flexible-price model for monetary policy analysis.
- The model features
  1. suboptimality of the Friedman rule due to the input-smoothing effect,
  2. the Phillips curve created by equilibrium response to financial shocks,
  3. the liquidity effect.
- ZIRP (i.e., the Friedman rule) enhances efficiency by relaxing the liquidity constraint, whereas it generates the equilibrium deflation in the long run by the Fisher relation.