Deflation and debt:
A neoclassical framework for monetary policy analysis
(Very preliminary)

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Abstract

We construct a flexible-price model for monetary policy analysis. The key ingredient is the liquidity constraint in which both cash and credit are simultaneously used as media of exchange. The model reproduces the following plausible features similar to the New Keynesian models: Suboptimality of the Friedman rule due to the input-smoothing effect; The Phillips curve created by equilibrium response to financial shocks; The liquidity effect of monetary policy on output and money demand. This model may be able to provide a plausible framework for monetary policy analysis. In addition, this model can give a plausible interpretation of the persistent deflation in the last decade in Japan. The zero nominal interest rate policy (ZIRP) enhances efficiency by relaxing the liquidity constraint, whereas it generates the equilibrium deflation in the long run by the Fisher relation. With heterogeneity in the liquidity constraints for agents, the combination of a permanent negative financial shock and the long-term commitment by the central bank on ZIRP can cause permanent deflation under permanently low output.

Keywords: Commitment problem, deflation, flexible prices, the Fisher relation, the Friedman rule.

JEL Classification numbers: E32, E44, E50,

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1 Introduction

In this paper we construct a framework for the analysis of monetary policy, in which prices are flexible. A plausible framework of the monetary policy analysis should have the following three features.

First, the Friedman rule, i.e., the zero nominal interest rate policy (ZIRP), should be suboptimal. The starkest rupture between the neoclassical monetary theory and the reality is on the assessment of the Friedman rule. The Friedman rule is optimal in that it eliminates monetary distortion, while in the policy practices it is not adopted as an optimal policy in normal times. In practice, the ZIRP is considered an exceptional and irregular policy during a severe recession. In addition, as the standard form of the Friedman rule implies a decrease in money supply at a constant rate, it is never considered optimal by the central bankers. The neoclassical theory needs some twists to conform with the reality.

Second, the framework should be able to reproduce the Phillips curve, i.e., a positive correlation between inflation and output in the short-run.

Third, the framework should be able to reproduce a positive response of output to a decrease in the nominal interest rate, which may be called the "liquidity effect."[1] As the practitioners conduct monetary policy on the strong premise that reduction of the nominal rate induces an increase in output and employment, a plausible theory should reproduce this relationship.

This paper offers two potential contributions to the literature. First, our model offers a new account for the suboptimality of the Friedman rule, that is, the *input-smoothing effect*. Suppose that firms subject to loose constraints and tight constraints coexist. Distortionary tax on loose-constrained firms can be welfare enhancing because the tax can reallocate the input from loose-constrained firms to tight-constrained firms, since the marginal product of tightly constrained firms are higher than loosely constrained firms. The Friedman rule is thus suboptimal because the inflation tax can work as a device to materialize the input-smoothing effect. We show that our model provides a plausible framework for the monetary policy analysis because it reproduces the Phillips curve and the liquidity effect, in addition to the suboptimality of the Friedman rule. The Phillips curve is generated because tighter liquidity constraints reduce output and increase the value of money. The liquidity effect is generated because lower interest rates relax the liquidity constraints and increase the output.

The second contribution may be that our model provides a new account for a decade-long deflation in Japan. The decade-long deflation can be caused by a long-term com-

[1] Walsh (2010) calls the belief that faster money growth will initially cause nominal interest rates to fall the "liquidity effect." In this paper this term means slightly different notion that a decrease in nominal interest rates causes an increase in output.

mitment on the zero nominal interest rate policy (ZIRP) through the Fisher relation in equilibrium. The expectations on the future fiscal policy is Ricardian so that the transversality condition is satisfied under the persistent deflation. An increase in money supply can be compatible with persistent deflation if the prevailing expectations is that there will be a large-scaled tax increase in the far future.

Organization of the paper is as follows. In the next section the model is described and suboptimality of the Friedman rule in the steady-state equilibrium is derived. Section 3 analyzes the dynamics of the model and shows that the Phillips curve and the liquidity effect of monetary policy can be reproduced. Section 4 argues that the decade-long deflation in Japan could be caused by the zero nominal interest rate policy. Section 5 concludes.

2 The Model

We consider a closed economy inhabited with the representative consumer who owns heterogeneous firms. Firms are subject to heterogeneous financial constraints. We assume that there are two types of firms, firms 1 and firms 2. The owner of firm 1 (the representative consumer) can transfer cash to firm 1 as internal funds, while there is a severe asymmetry of information that makes the owner of firm 2 (the representative consumer) unable to transfer cash to firm 2 as an internal funds. This heterogeneity is interpreted as a technological difference: The consumer is the owner-manager of firm 1, while he is owner but not manager of firm 2; the manager of firm 2 can divert cash for private purposes without getting penalty from the firm owner.

The policy devices that can be chosen by the central bank are intra-period interest rate (nominal rate), j, and the amount of intra-period loan, Δ. The policy $(j,\ \Delta)$ decides the inflation rate π as an equilibrium outcome.

2.1 Setup

The model is a closed economy in which there lives a unit mass of identical consumers who consume, save, supply labor, and own firms. There are two types of firms: firm 1 and firm 2. Firms can produce y_t from labor L_t:

$$y_t = A_t L_t^{\alpha},$$

where A_t is the productivity and L_t is the labor demand by the firm. The wage payment $w_t L_t$, where w_t is the wage rate, must be financed by cash and/or credit subject to a liquidity constraint, described later. The measure of consumers is normalized to one. There are continuum of firms 1 with measure ϕ and continuum of firms 2 with measure $1-\phi$. In this paper we consider two exogenous shocks that may hit the economy: productivity shock A_t and financial shock θ_t. The financial shock θ_t is described shortly.

Consumer: A consumer maximizes the discounted present value of his utility from consumption c_t and leisure $1 - l_t$, where l_t is the labor supply, which is

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t,\ 1 - l_t),$$

where $U(\cdot)$ is the period utility and β (< 1) is the discount factor. The maximization is subject to the budget constraint that the consumption, cash holdings ϕm_t, intra-period borrowing $(1 + j_t)\phi\Delta_t$ and holdings of the government bond b_t must be financed by the period income, where j_t is the interest rate for intra-period borrowing and $\phi\Delta_t$ is the amount of the intra-period borrowing of cash from the central bank (CB). The period income consists of the wage income $w_t l_t$, the dividends from firms 1, $\phi\Pi(\frac{m_{t-1}}{\pi_t} + \Delta_t)$, the dividend from firms 2, $(1 - \phi)\Pi(0)$, and the bond repayment $(1 + r_t)b_{t-1}$, where π_t is the gross inflation rate from period $t - 1$ to t. We denote the dividend of a firm as a function of cash that is injected to the firm by the consumer. We assume that the period utility is logarithmic and therefore, the optimization problem for the consumer can be written as

$$\begin{aligned}
&\max E_0 \sum_{t=0}^{\infty} \beta^t [\ln c_t + \gamma \ln(1 - l_t)], \\
&\text{s.t. } c_t + \phi m_t + b_t \le w_t l_t + \phi\Pi\left(\frac{m_{t-1}}{\pi_t} + \Delta_t\right) + (1 - \phi)\Pi(0) - (1 + j_t)\phi\Delta_t + (1 + r_t)b_{t-1}.
\end{aligned}$$

Heterogeneous constraints for firms: Note that a consumer can invest his cash in firm 1 as internal funds, while he cannot invest cash in firm 2. The reason is a technological constraint that the manager of firm 2 can divert cash for private purposes without getting penalty from the consumer if the consumer invests cash in firm 2, while the consumer himself is the manager of firm 1 and there is no agency problem between the consumer and firm 1. As this heterogeneity implies that monetary injections by the central bank affect only firm 1 and not firm 2, our model can be understood as a limited participation model (Fuerst 1992, Christiano and Eichenbaum 1992, Christiano, Eichenbaum and Evans 1997, Williamson 2005).

Firm 1: In period t, the consumer invests cash in firm 1. The cash consists of $\phi\frac{m_{t-1}}{\pi_t}$, which is carried over from period $t - 1$, and $\phi\Delta_t$, which the consumer obtains in period t as the intra-period borrowing from the central bank. As the measure of the consumer is 1 and that of firm 1 is ϕ, the cash at hand of each firm 1 is $\frac{m_{t-1}}{\pi_t} + \Delta_t$. Given the cash at hand, $\frac{m_{t-1}}{\pi_t} + \Delta_t$, the firm 1 chooses the labor demand L_{1t} to solve

$$\begin{aligned}
&V_{t-1}^1 = \beta E_{t-1}\left[\frac{\lambda_t}{\lambda_{t-1}}\left\{\max_{L_{1t}}[A_t L_{1t}^{\alpha} - w_t L_{1t}] + \frac{m_{t-1}}{\pi_t} + \Delta_t + V_t^1\right\}\right], \\
&\text{s.t. } w_t L_{1t} \le \frac{m_{t-1}}{\pi_t} + \Delta_t + \theta_t V_t^1, \qquad (1)
\end{aligned}$$

where λ_t is the Lagrange multiplier for the consumer's problem associated with the budget constraint, θ_t is the pledgeability rate (the financial shock), which we describe shortly, and the constraint (1) is the liquidity constraint. The liquidity constraint is derived from the following commitment problem between the firm and the worker, similar to those in Kiyotaki and Moore (1997) and Jermann and Quadrini (2006, 2012): Before starting production, firm 1 pays cash at hand $\frac{m_{t-1}}{\pi_t} + \Delta_t$ to the worker and promises to pay the remaining wage after production; if the firm breaks the promise after production, the worker gets angry and destroys the firm with probability θ_t, in which case the firm cannot operate from $t+1$ on; As the firm loses the expected value $\theta_t V_t^1$ by breaking the promise, it can credibly pay at most $\theta_t V_t^1$ after production. The no-default condition for the firm implies that the total wage payment should be less than or equal to the sum of cash at hand and the pledgeable value as collateral. This condition leads to constraint (1). Note that $\Pi\left(\frac{m_{t-1}}{\pi_t} + \Delta_t\right)$ in the consumer's problem is defined as $\Pi\left(\frac{m_{t-1}}{\pi_t} + \Delta_t\right) = A_t L_{1t}^\alpha - w_t L_{1t} + \frac{m_{t-1}}{\pi_t} + \Delta_t$.

Firm 2: Given that the consumer does not invest his cash at hand into the firm 2, it solves the following optimization problem.

$$V_{t-1}^2 = \beta E_{t-1}\left[\frac{\lambda_t}{\lambda_{t-1}}\{\max_{L_{2t}}[A_t L_{2t}^\alpha - w_t L_{2t}] + V_t^2\}\right],$$
$$\text{s.t. } w_t L_{2t} \leq \theta_t V_t^2. \tag{2}$$

The liquidity constraint (2) emerges from the same commitment problem as that of firm 1. Note that $\Pi(0)$ in the consumer's problem is defined by $\Pi(0) = A_t L_{2t}^\alpha - w_t L_{2t}$.

Government: In this paper we do not explicitly deal with the optimization by the government, but we simply assume that the government follows an exogenous policy rules:

$$j_t = J(A_t, \theta_t; I_t),$$
$$\Delta_t = D(A_t, \theta_t; I_t),$$

where I_t is all information available for the government at t. The issuance of cash and bonds by the government is subject to the budget constraint:

$$(1 + r_t)B_{t-1} + \frac{M_{t-1}}{\pi_t} = B_t + M_t + j_t \phi \Delta_t,$$

where B_t and M_t are supplies of bonds and cash. The upper case variables do not represent nominal variables, but they are real variables. Note that we implicitly assume that the government has full capability to commit to the policy rule and it does not suffer from any time-inconsistency problem. The policy rule should have been endogenously determined such that they maximize the ex-ante social welfare if the government were benevolent. But in this paper we simply take these policy rules exogenous. Endogenizing the policy choice and finding the optimal policy should be a topic for future research.

Equilibrium conditions: The markets for the consumer goods, labor, cash, and bonds should clear in equilibrium, so that

$$
\begin{aligned}
&c_t = Y_t = \phi A_t L_{1t}^{\alpha} + (1-\phi) A_t L_{2t}^{\alpha}, \\
&l_t = \phi L_{1t} + (1-\phi) L_{2t}, \\
&\phi m_t = M_t, \\
&b_t = B_t.
\end{aligned}
$$

Nominal interest rate: The consumer's first order condition (FOC) with respect to Δ, and the envelope condition for $\Pi(m)$ in firm 1's problem imply that

$$j_t = \mu_{1t},$$

where μ_{1t} is the Lagrange multiplier for firm 1's optimization associated with the liquidity constraint (1). This means that the interest rate on the intra-period loans j_t equals the tightness of the liquidity constraint for firm 1.

2.2 Steady State equilibrium

In this subsection we analyze the steady state, where exogenous shocks are constant over time: $(A_t,\ \theta_t) = (A,\ \theta)$ and policy variables are also kept constant: $(j_t,\ \Delta_t) = (j,\ \Delta)$. Denote by μ_{2t} the Lagrange multiplier for firm 2's optimization associated with the liquidity constraint (2). The steady-state equilibrium is given as a solution to the following

14 equations for 14 unknowns $(c, w, l, L_1, L_2, \pi, \mu_2, m, V^1, V^2, r, b, M, B)$.

$$w = \frac{\gamma c}{1-l}, \tag{3}$$

$$1 + j = 1 + \mu_1 = \frac{\pi}{\beta}, \tag{4}$$

$$wL_1 = \frac{\alpha A L_1^\alpha}{1+\mu_1}, \tag{5}$$

$$wL_1 = \frac{m}{\pi} + \Delta + \theta V^1, \tag{6}$$

$$V^1 = \frac{\beta A L_1^\alpha}{1-(1-\theta)\beta}, \tag{7}$$

$$wL_2 = \theta V^2, \tag{8}$$

$$V^2 = \frac{\beta A L_2^\alpha}{1-(1-\theta)\beta}, \tag{9}$$

$$wL_2 = \frac{\alpha A L_2^\alpha}{1+\mu_2}, \tag{10}$$

$$Y = c = \phi A L_1^\alpha + (1-\phi) A L_2^\alpha, \tag{11}$$

$$l = \phi L_1 + (1-\phi) L_2, \tag{12}$$

$$1 + r = \beta^{-1}, \tag{13}$$

$$rB = \left(1 - \frac{1}{\pi}\right) M + \phi j \Delta, \tag{14}$$

$$b = B, \tag{15}$$

$$\phi m = M. \tag{16}$$

Characteristics of the steady-state equilibrium: It can be shown that the degree of inefficiency is pinned down by j and variables $(c, w, l, L_1, L_2, \pi, V^1, V^2)$ in the steady state are determined as j is given, but independent of Δ. Δ determines the value of (m, M, b, B).[2] This is shown as follows. The assumption that the utility is a log function implies that

$$c = [\gamma^{-1} - \phi L_1 - (1-\phi) L_2] w,$$

which then implies that the variables $(w, L_1, L_2, Y, V^1, V^2)$ do not depend on Δ. From (12), we have

$$\gamma^{-1} - \phi L_1 - (1-\phi) L_2 = \phi \frac{A L_1^\alpha}{w} + (1-\phi) \frac{A L_2^\alpha}{w}.$$

This equation can be solved for w:

$$w^{\frac{1}{1-\alpha}} = \gamma \phi A^{\frac{1}{1-\alpha}} \left\{ \left(\frac{\alpha\beta}{\pi}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\alpha\beta}{\pi}\right)^{\frac{\alpha}{1-\alpha}} \right\} + (1-\phi)\gamma A^{\frac{1}{1-\alpha}} \left\{ \left(\frac{\theta\beta}{1-(1-\theta)\beta}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\theta\beta}{1-(1-\theta)\beta}\right)^{\frac{\alpha}{1-\alpha}} \right\} \tag{17}$$

[2] The variables (r, μ_2) are independent of policy variables (j, μ_1), that is, $r = \beta^{-1} - 1$ and $1 + \mu_2 = \frac{\{1-(1-\theta)\beta\}\alpha}{\theta\beta}$.

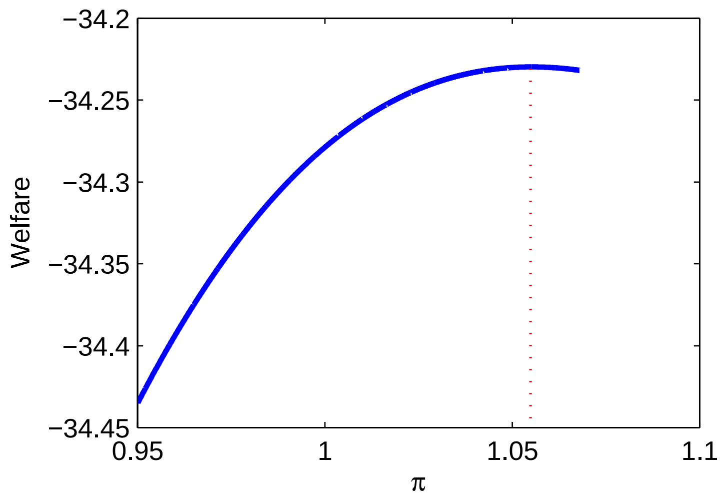

Figure 1: Welfare in the steady-state equilibrium

This result means that w is a decreasing function of π and w does not depend on the other policy variable (Δ). As $\pi = (1+j)\beta$ in the steady state, w is pinned down by j. Therefore, the values of $(c, w, L_1, L_2, V^1, V^2)$ are pinned down as j is fixed, and they do not depend on (Δ). The policy variable Δ is relevant only for deciding m, as long as $X(\pi) = \frac{m}{\pi} + \Delta = wL_1 - \theta V^1$ is pinned down by j. Given j, there are continuously infinite combinations of (m, Δ) that generate an identical level of social welfare.

2.3 Welfare analysis on the steady state

We focus on the deterministic steady-state equilibrium in which j and Δ are constant. Without loss of generality, we set $\Delta = 0$. We define the social welfare W by

$$\begin{aligned} W &= \frac{1}{1-\beta} U(c, l) \\ &= \frac{1}{1-\beta} \{\ln c + \gamma \ln(1-l)\}. \end{aligned}$$

We adopt the following parameter values:

α	β	ϕ	γ	A	θ
0.89	0.95	0.25	1.8	1	0.2

As shown in Figure 1, W is maximized by the policy: $j = 0.11$, which implies $\pi^* = 1.0552$, $Y^* = 0.349$. Therefore, the Friedman rule ($j = 0$) does not maximize the social welfare in this model. It should be underscored, however, that output Y is maximized at the Friedman rule as shown in Figure 2.

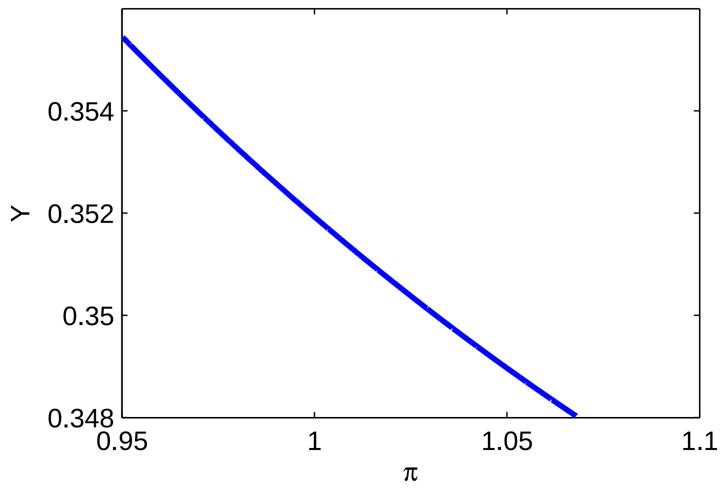

Figure 2: Output in the steady-state equilibrium

2.4 Why is the Friedman rule suboptimal in our model?

In the equilibrium, all monetary distortion, which is represented by μ_1, can be completely eliminated by setting $j = 0$ (or $\pi = \beta$) because $j = \mu_1$. Thus the Friedman rule seems to be the optimal policy, while it turns out it is not in numerical calculation. We derive the suboptimality of the Friedman rule rigorously and try to figure out the economic intuition behind this result.

Rigorous derivation of suboptimality of the Friedman rule: Note that $c = Y$ and $l = L = \phi L_1 + (1-\phi)L_2$. Given the gross inflation rate π, the utility of a consumer in this equilibrium is

$$U = \ln Y + \gamma \ln(1 - L),$$

where

$$\begin{aligned}
Y &= \phi A L_1^{\alpha} + (1-\phi) A L_2^{\alpha}, \\
L_1 &= \left(\frac{\alpha \beta A}{w(\pi)\pi} \right)^{\frac{1}{1-\alpha}}, \\
L_2 &= \left(\frac{\theta \beta A}{\{1-(1-\theta)\beta\} w(\pi)} \right)^{\frac{1}{1-\alpha}},
\end{aligned}$$

and $w(\pi)$ is defined by (17). We define the elasticity of wage rate with respect to inflation rate, $\varepsilon(\pi)$, by

$$\varepsilon(\pi) = -\frac{\pi}{w}\frac{dw}{d\pi}.$$

Note that (17) implies that

$$0 < \varepsilon(\pi) < 1 \qquad \forall \pi \geq \beta.$$

We can show

$$\frac{dL_1}{d\pi} = -\frac{(-\varepsilon(\pi)+1)L_1}{(1-\alpha)\pi},$$
$$\frac{dL_2}{d\pi} = \frac{L_2\varepsilon(\pi)}{(1-\alpha)\pi}.$$

Since $\frac{dU}{d\pi} = [(1-\phi)\mu_2\frac{dL_2}{d\pi} - \phi\mu_1\frac{dL_1}{d\pi}]\frac{w}{Y}$, we can show that

$$\frac{dU}{d\pi} = [(1-\phi)\mu_2\varepsilon(\pi)L_2 - (1-\varepsilon(\pi))\phi\mu_1 L_1]\frac{w}{(1-\alpha)\pi Y}$$

It is obvious that $\frac{dU}{d\pi} > 0$ at the Friedman rule because $\mu_1 = j = 0$ at the Friedman rule. This result means that the Friedman rule is suboptimal in this model.

Does this result depend on the functional form? As we used (17) this result seems to depend crucially on the fact that the utility is a logarithmic function. But in fact it is not. Suppose that the utility is a general function $U(c, 1-l)$. Then we have $w = U_2(Y, 1-L)/U_1(Y, 1-L)$, which implies that $Y = Y(w, L)$. Equation (12) implies that

$$Y(w, \phi L_1 + (1-\phi)L_2) = \phi A L_1^{\alpha} + (1-\phi)A L_2^{\alpha}.$$

We can solve this equation for w and get $w = w(\pi)$. Then we can define $\varepsilon(\pi)$ as the elasticity of w with respect to π. Thus in equilibrium

$$\frac{dU}{d\pi} = U_1\frac{dY}{d\pi} - U_2\frac{dL}{d\pi} = U_1\left[\frac{dY}{d\pi} - w\frac{dL}{d\pi}\right],$$
$$\frac{dY}{d\pi} = w\frac{dL}{d\pi} + \phi\mu_1 w\frac{dL_1}{d\pi} + (1-\phi)\mu_2 w\frac{dL_2}{d\pi}.$$

These two equations imply that at the Friedman rule where $\mu_1 = 0$, the welfare will be

$$\frac{dU}{d\pi} = (1-\phi)U_1\mu_2 w\frac{dL_2}{d\pi} = (1-\phi)U_1\mu_2 w\frac{L_2\varepsilon(\pi)}{(1-\alpha)\pi}.$$

Therefore, with a general utility function, the Friedman rule is suboptimal only if

$$\varepsilon(\pi) > 0, \quad \text{for } \pi \geq \beta.$$

Input-smoothing effect of inflation: The suboptimality of the Friedman rule can be understood as it is due to the *input-smoothing effect* of inflation tax. See the Appendix for an example of the input-smoothing effect of a general distortionary taxation. In our setting, the inflation tax is imposed selectively on firm 1, not on firm 2. This is because firm 1 can use cash to relax the liquidity constraint, while by assumption firm 2 cannot use cash to relax the liquidity constraint. Inflation reduces firm 1's demand for labor and decreases the wage rate w, which in turn increases firm 2's demand for labor. As the liquidity constraint for firm 2 is tighter than that for firm 1 because firm 1 can use cash and firm 2 cannot, the marginal product of labor (MPL) of firm 2 is strictly larger than MPL of firm 1. As some amount of labor is reallocated from firm 1 to firm 2, the decrease in total output due to the inflation is rather moderate compared to the decrease in total labor. Thus the inflation reduces more disutility from labor than utility from consumption of the output. Therefore, the overall effect of inflation can be welfare enhancing in the region of low or negative inflation. In other words, inflation is a second-best policy to reallocate labor from firm 1 to firm 2. If there were a policy that can reallocate labor from firm 1 to firm 2 without reducing total labor, it would have been better than inflation. But we ruled out the selective policy that explicitly reallocates labor from firm 1 to firm 2. There is a welfare reducing effect of inflation, which is that the inflation reduces total labor. Also, there is a welfare enhancing effect of inflation, which is that the inflation reallocates the labor from firm 1 to firm 2. In our setting, the second effect dominates the first and the inflation improves total welfare.

How to interpret the input-smoothing effect of inflation: We can interpret firm 1 represents old and traditional sectors, and firm 2 young and emerging industries. Old firms have easy access to funds, while young firms do not. Inflation reallocates resources from old firms to young firms by the input-smoothing effect. By doing so, inflation can enhance productivity of the economy in the low inflation region.

3 Dynamics

Now we examine the dynamics of this economy and show that our model can reproduce the Phillips curve relationship and the liquidity effect of monetary policy. We consider two stochastic shocks: the productivity shock A_t and the financial shock θ_t. When shock (A_t, θ_t) come, given the policy rule

$$
\begin{aligned}
j_t &= J(A_t, \theta_t; I_t), \\
\Delta_t &= D(A_t, \theta_t; I_t),
\end{aligned}
$$

the dynamic behavior of the economy is determined by the following equations.[3]

$$
\begin{aligned}
&w_t = \frac{\gamma c_t}{1-l_t},\\
&w_t L_{1t} = \frac{\alpha A_t L_{1t}^{\alpha}}{1+j_t},\\
&w_t L_{2t} = \frac{\alpha A_t L_{2t}^{\alpha}}{1+\mu_{2t}},\\
&w_t L_{1t} - \frac{m_{t-1}}{\pi_{t-1}} - \Delta_t = \theta_t V_t^1,\\
&w_t L_{2t} = \theta_t V_t^2,\\
&V_t^1 = \beta E_t \left[\frac{c_t}{\tilde{c}_{t+1}} \left\{ \tilde{A}_{t+1}\tilde{L}_{1t+1}^{\alpha} - \tilde{w}_{t+1}\tilde{L}_{1t+1} + \frac{m_t}{\tilde{\pi}_t} - \tilde{j}_{t+1}\tilde{\Delta}_{t+1} + \tilde{V}_{t+1}^1 \right\} \right],\\
&V^2 = \beta E_t \left[\frac{c_t}{\tilde{c}_{t+1}} \left\{ \tilde{A}_{t+1}\tilde{L}_{2t+1}^{\alpha} - \tilde{w}_{t+1}\tilde{L}_{2t+1} + \tilde{V}_{t+1}^2 \right\} \right],\\
&Y_t = c_t = \phi A_t L_{1t}^{\alpha} + (1-\phi) A_t L_{2t}^{\alpha},\\
&l_t = \phi L_{1t} + (1-\phi) L_{2t},\\
&(1+r_t)B_{t-1} + \frac{M_{t-1}}{\pi_{t-1}} = B_t + M_t + j_t\phi\Delta_t,\\
&1 = \beta E_t \left[\frac{c_t}{\tilde{c}_{t+1}} \frac{(1+\tilde{j}_{t+1})}{\tilde{\pi}_t} \right],\\
&1 = \beta E_t \left[\frac{c_t}{\tilde{c}_{t+1}} (1+\tilde{r}_{t+1}) \right],\\
&M_t = \phi m_t,\\
&B_t = b_t.
\end{aligned}
$$

3.1 The Phillips curve relationship

We show by numerical calculation that output and inflation move in the same direction in response to exogenous change in the financial shock θ_t. The parameter values are the same as in the previous section. We assume that the economy is initially at the steady state where the social welfare is maximized, i.e., $\pi = \pi^* = 1.05$. Then we change θ_t by temporary shocks and we assume that the policy $(j_t,\ \Delta_t)$ are kept at steady-state values: $j_t = \mu_{1t} = \bar{\mu}_1 = 0.11,\ \Delta_t = 0$. The shock on θ_t hits at period 0 and depreciates over time: $\theta_0 = \bar{\theta} + \varepsilon_0$ and $\theta_{t+1} = \rho\bar{\theta} + (1-\rho)\theta_t$ for $t \geq 0$. We calculate the dynamics backwardly. (We also confirmed that the dynamics can be calculated by the Dynare.) Figures 3 and 4 show the dynamic response of the economy to positive and negative shock on θ_t, respectively.

[3] In the deterministic case we can solve these equations backwardly for 14 unknowns $\mathbf{v}_t = (\pi_t, r_{t+1}, M_t, B_t, V_t^1, V_t^2, L_{1t}, L_{2t}, l_t, \mu_{2t}, w_t, c_t, x_t, z_t)$, where $x_t = \frac{m_{t-1}}{\pi_{t-1}}$ and $z_t = (1+r_t)b_{t-1}$, taking $\mathbf{v}_{t+1}$ as given.

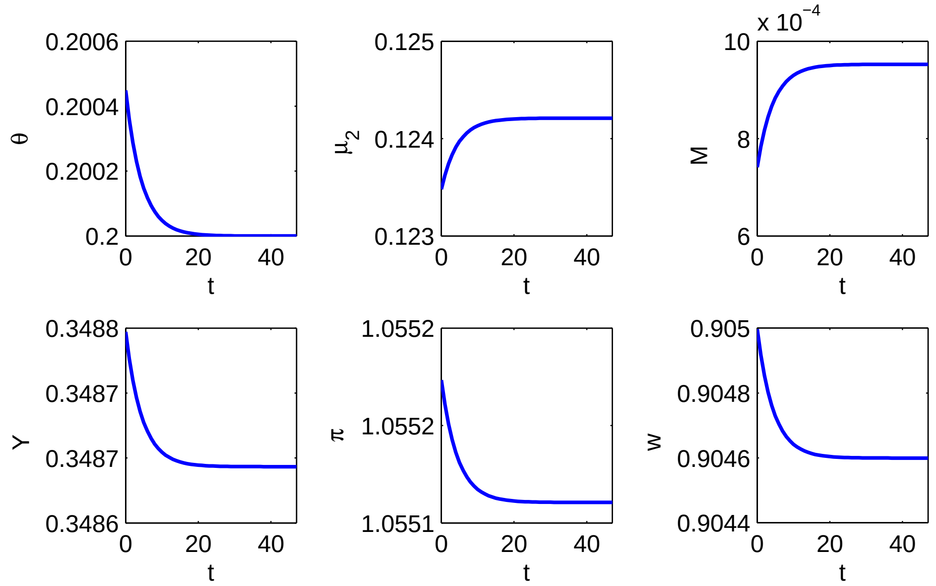

Figure 3: Dynamic response to positive shock on θ_t

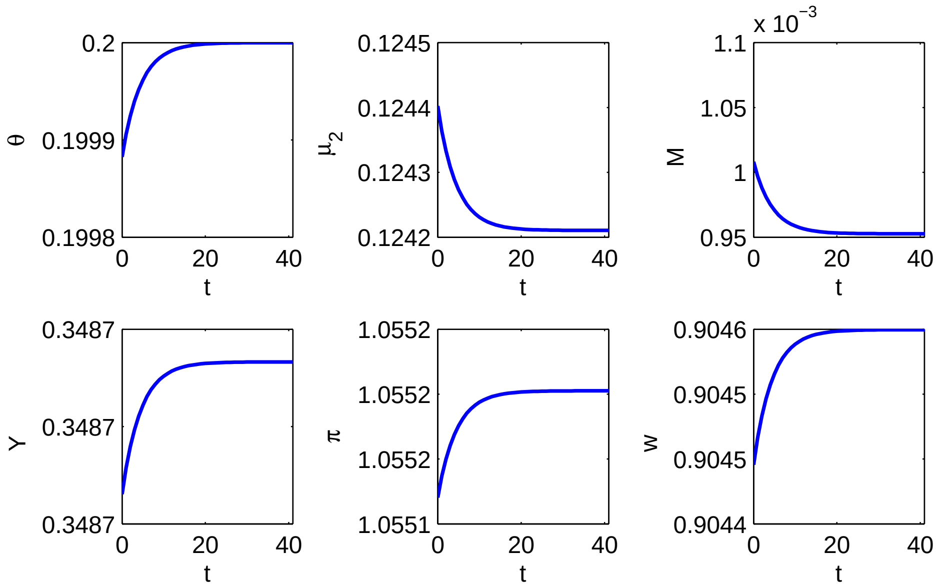

Figure 4: Dynamic response to negative shock on θ_t

Intuition for the Phillips curve: The intuition for this numerical result is straightforward. The Phillips curve relationship between output and inflation is an artifact that is generated by their responses to a common shock, which is the financial shock. Our experiment implies the following. As θ_t decreases, the liquidity constraints become tighter. On one hand, tightening of the liquidity constraint causes decrease in output. On the other hand, it also increase the value of cash as obtaining credit becomes more difficult, leading to a decline in the inflation rate. The straightforward implication of our model is that when θ_t decreases (increases), the output and the inflation rate both decreases (increases). Thus, variations in θ_t generates the short-run comovements of output and inflation, i.e., the Phillips curve.

3.2 Liquidity effect of monetary policy

Our model can easily reproduce the liquidity effect of monetary policy, that is, monetary easing (tightening) increases (decreases) output. We conducted numerical simulations of the dynamic response of our economy to a shock to the nominal interest rate j_t. In our experiment, we assume that θ_t and A_t are constant, that $j_t = \mu_{1t}$ is exogenously changed by monetary policy, and that Δ_t is kept at the steady-state value ($\Delta_t = 0$). The shock process is as follows: $\mu_{1,0} = \bar{\mu}_1 + \varepsilon_0$ and $\mu_{1,t+1} = \rho\bar{\mu}_1 + (1-\rho)\mu_{1t}$ for $t \geq 0$. Figures 5 and 6 show the dynamic response in the case of an increase and a decrease in μ_{1t} (j_t), respectively.

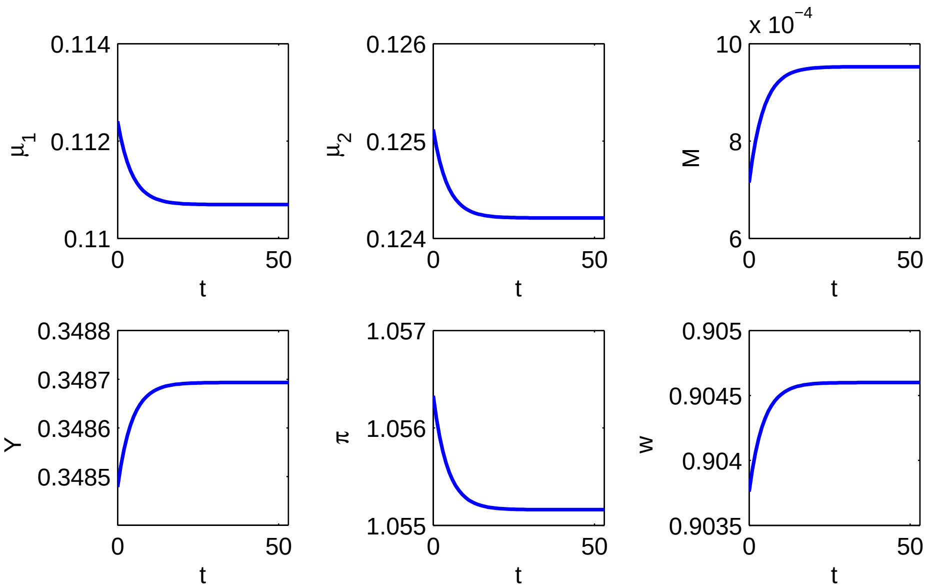

Figure 5: Dynamic response to positive shock on μ_{1t} (j_t)

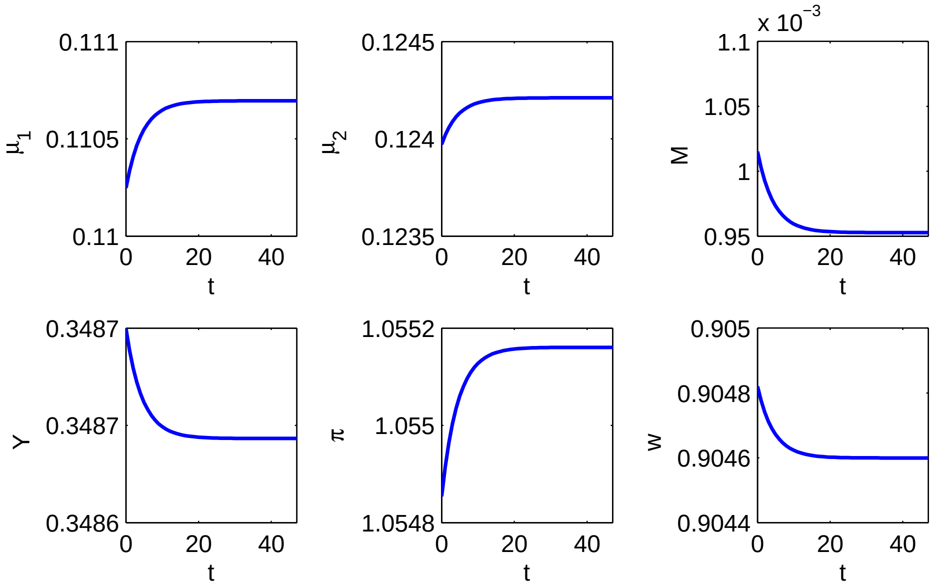

Figure 6: Dynamic response to negaitive shock on μ_{1t} (j_t)

Intuition for the liquidity effect: The intuition for the liquidity effect is also simple and straightforward. As the short-term nominal interest rate j_t represents the degree of tightness of the liquidity constraint for firm 1, μ_{1t}, a change in the short-term rate directly affects output. As $j_t = \mu_{1t}$ increases, liquidity constraints for firms 1 become tighter and decrease output. Thus, an increase (decrease) in the nominal interest rate j_t decreases (increases) output. This is what we call the liquidity effect of monetary policy.

3.3 Discussion: Comparison with the New Keynesian model

The New Keynesian model is very popular as a framework of monetary policy analysis because of its three plausible features: (1) The Friedman rule is suboptimal or the zero inflation is optimal in the steady state, (2) the Phillips curve is present, and (3) monetary easing (tightening) causes an increase (decrease) in output. The comprehensive descriptions of the New Keynesian model are given by e.g., Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005), Chrisiano, Trabandt and Walentin (2010).

What we show in this paper is that we can construct a neoclassical model without any nominal stickiness in prices or wages that reproduces these features. In our model, the Friedman rule is suboptimal because of input-smoothing effect of inflation in the economy where firms are subject to heterogeneous liquidity constraints; the Phillips curve is present because the financial shocks generate comovements of output and inflation as both cash and credit are simultaneously used as media of exchange; and monetary policy has real effect (the liquidity effect) because the nominal rate of interest represents the tightness of liquidity constraints. Our model indicates that workings of monetary policy could be understood from a financial perspective as interactions between money and credit, and that monetary policy could have nothing to do with nominal stickiness.

4 The Fisherian Deflation

The existing theories on the decade-long deflation in Japan are not fully satisfactory. One view is that deflation is an equilibrium outcome associated with decrease in base money and government debt (see Benhabib, Schmitt-Grohé, and Uribe 2002). This view is not compatible with the huge increase in base money and government debt in the past decade in Japan. Another view is that deflation is caused by a policy response to an unexpected large demand shock (Krugman 1998, Svensson 2001, Eggertson and Woodford 2003, Auerbach and Obstfeld 2005). This view does not satisfactorily explain decade-long persistence of deflation in Japan.

There should be a simpler explanation, which we call the "Fisherian deflation." In the steady-state equilibrium, if the nominal interest rate j is fixed at zero, then the inflation rate should be negative, i.e., $\pi < 1$, as the real interest rate is $\beta^{-1} - 1$ in equilibrium. The deflation is derived from the Fisher relation in the steady state: $1 + j = \beta\pi$. Thus the Fisherian deflation is an equilibrium outcome caused by the long-term commitment of ZIRP by the central bank. The theory of the Fisherian deflation should explain the following features in Japan consistently:

- Deflation was associated with low output.
- Deflation was associated with non-decreasing money supply, i.e., the transversality condition is satisfied in such a way that deflation continues with non-decreasing money supply.

We can consider a policy response to negative financial shocks that can demonstrate the above features in our model. The story goes as follows.

Permanent financial shock: Suppose that the economy is initially in the steady state where $(\bar{\theta}, \bar{\pi}, \bar{j}, \bar{Y})$, and at $t = 0$ a permanent financial shock hits the economy such that θ_t changes from $\theta_t = \bar{\theta}$ for $t < 0$ to $\theta_t = \theta_L$ for $t \geq 0$, where $\theta_L \ll \bar{\theta}$. The permanent shock may represent institutional changes in the economy, e.g., tighter business practice in bank lending or more stringent bank regulations. We denote the nominal amount of money by $\aleph_t$ and the nominal price level by P_t.

$$
\begin{aligned}
P_t &= \pi_{t-1} P_{t-1} = (\prod_{i=1}^{t} \pi_{i-1}) P_0, \\
\aleph_t &= P_t M_t.
\end{aligned}
$$

Government policy: We assume that the government can set the lump-sum tax, τ_t, in addition to (j_t, Δ_t) subject to the budget constraint:

$$(1 + r_t)B_{t-1} + \frac{M_{t-1}}{\pi_{t-1}} = \tau_t + B_t + M_t + j_t \phi \Delta_t,$$

In response to the permanent financial shock, the government chooses the following policy.

- the government (mistakenly) considers that θ-shock would be temporary and adopt the monetary policy rule:

$$j_t = \mu_{1t} = \begin{cases} 0 & \text{if } \theta_t = \theta_L, \\ \bar{j} > 0 & \text{if } \theta_t = \bar{\theta}, \end{cases}$$
$$\Delta_t = \bar{\Delta} = 0.$$

- the government sets the nominal money supply constant: $\aleph_t = \aleph_0$, where $\aleph_0 = P_0 \bar{M}$. The real bond is set $B_t = \bar{B} = \frac{1-\bar{\pi}^{-1}}{\beta^{-1}-1}\bar{M}$.

Given these monetary policy responses, the macroeconomic expectations on fiscal policy is formed in such a way that there exists a far future period T such that $\tau_t = 0$ for $t < T$ and $\tau_T = \tau_H$ and $\tau_t = \bar{\tau}$ for $t \geq T$, where

$$\tau_H = \left(\frac{1}{\beta^{T+1}} - 1\right)\left(\frac{\aleph_0}{P_0} + \bar{B}\right) = \left(\frac{1}{\beta^{T+1}} - 1\right)(\bar{M} + \bar{B}),$$
$$\bar{\tau} = \frac{1-\beta}{\beta}(\bar{M} + \bar{B}).$$

Thus the prevailing expectations are that the future fiscal policy will be chosen in a way that the transversality condition is satisfied eventually, given the current and future monetary policy: $j_t = 0$ and $\Delta_t = 0$. In other words, the prevailing expectations are that the future fiscal policy is Ricardian.

Why deflation is associated with low output? In the steady state where $\theta_t = \theta_L$, total output Y is smaller than $\bar{Y}$ because the credit constraints are tighter. As $j = \mu_1 = 0$, the liquidity constraint for firm 1 is eliminated so that firm 1 can maximize its output and profit, whereas the liquidity constraint for firm 2 is tighter when $\theta = \theta_L$ than when $\theta = \bar{\theta}$, as $\mu_2 = \frac{\{1-(1-\theta)\beta\}\alpha}{\theta\beta} - 1$ is decreasing in θ. Thus the output and profit of firm 2 is smaller than in the original steady state. Thus the monetary easing ($j_t = 0$) cannot completely restore total output to $\bar{Y}$, i.e., $Y < \bar{Y}$.

Deflation is compatible with non-decreasing nominal money: In the steady state where $j = 0$, deflation is compatible with non-decreasing nominal money for $0 \leq t < T$: The consumer anticipates that tax will be increased at T from 0 to τ_H and they save more to prepare for the tax hike (Ricardian equivalence). The consumers are indifferent between holding cash and holding bonds as asset for savings because the real interest rate of cash is $\frac{1}{\pi} = \frac{1}{\beta}$, which is equal to the real rate of bonds. The consumers hold all cash $\aleph_t = \aleph_0$ either as media of exchange or as just a perfect substitute of the government bond. Therefore, non-decreasing cash $\aleph_t = \aleph_0$ is compatible with deflation for $0 \leq t < T$.

How to interpret the decade of deflation in Japan: This model implies the following concerning the decade-long deflation in Japan during 2000s. In response to a negative financial shock, the zero nominal interest rate policy (ZIRP) can maximize the output by relaxing the liquidity constraint. If the negative financial shock is permanent, however, ZIRP may result in unintended consequences, which is the equilibrium deflation. Under the heterogeneity of liquidity constraints among agents, ZIRP can mitigate the negative shock incompletely and the level of total output is permanently lower than in the initial steady state. The long-run "optimal" policy is not ZIRP under a permanent financial shock in this model because maximizing output is not optimal: As we saw in the previous section, the Friedman rule is suboptimal, while it maximizes total output. Given the expectations that fiscal policy in the future is Ricardian, an increasing or nondecreasing money and persistent deflation can be compatible. The permanent financial shock may represent structural changes in the business practices in financial sector or in financial regulations.

5 Conclusion

We construct a flexible-price model for monetary policy analysis. The key ingredient is the liquidity constraint in which both cash and credit are simultaneously used as media of exchange. The model shows the following plausible features similar to the New Keynesian models:

1. suboptimality of the Friedman rule due to the input-smoothing effect,
2. the Phillips curve created by equilibrium response to financial shocks,
3. the liquidity effect of monetary policy on output and money demand.

This model may be able to provide a plausible framework for monetary policy analysis. In addition, this model can give a plausible interpretation of the persistent deflation in the last decade in Japan. The zero nominal interest rate policy (ZIRP) enhances efficiency by relaxing the liquidity constraint, whereas it generates the equilibrium deflation in the long run by the Fisher relation. With heterogeneity in the liquidity constraints for agents, the combination of a permanent negative financial shock and the long-term commitment by the central bank on ZIRP can cause permanent deflation under permanently low output.

Appendix: Input-smoothing effect of distortionary taxation

In the text we show that inflation can enhance welfare due to the input-smoothing effect. In this appendix the input-smoothing effect is demonstrated in the case of a distortionary tax. We show this by constructing a simple example. The key ingredients for the input-smoothing effect are

- there exist two (or more) types of firms which have heterogeneous input constraints;
- the government can impose a distortionary tax selectively only on the less constrained firms;
- a decrease in the input price relaxes the input constraint for the heavily constrained firms.

Then social welfare in the steady state can be improved by increasing tax rate, in the neighborhood of zero tax. Although this result would hold in general settings, we leave the task of characterizing the input-smoothing effect in a general environment for future research. In what follows we just demonstrate the input-smoothing effect in a simple example.

Setup

We consider a one-period economy, in which a representative consumer owns two firms, Firm 1 and Firm 2. Production technology of Firm 1 is $y = A_1L_1$, while that for Firm 2 is $y = A_2L_2^{\alpha}$, where A_i is the productivity parameter and L_i is the labor input for $i = 1, 2$. Productivity parameters may or may not satisfy $A_1 = A_2$. Outputs of Firm 1 and Firm 2 are perfect substitutes. There are heterogeneity in input constraints for firms: Firm 1 has no constraint, whereas Firm 2 has the following input constraint, $wL_2 \leq D$, where w is the wage rate and D is an exogenous parameter. The government can impose a distortionary tax with tax rate t on the output of Firm 1 so that the after tax revenue for Firm 1 becomes $y' = (1-t)A_1L_1$. The government can choose the tax rate t.

The consumer's utility is logarithmic and he solves the following optimization problem.

$$\max_{c,\ l} \ \ln c + \gamma \ln(1-l),$$
$$\text{subject to} \quad c = wl + \pi_1 + \pi_2 + T,$$

where c is the consumption, l is the labor supply, π_1 is the dividend from Firm 1, π_2 is the dividend from Firm 2, and T is the transfer from the government. The optimization by Firm 1 is simply as follows.

$$\pi_1 = \max_{L_1} \ (1-t)A_1L_1 - wL_1.$$

The FOC implies

$$w = (1-t)A_1.$$

The optimization by Firm 2 is as follows.

$$\pi_2 = \max_{L_2} \; A_2 L_2^{\alpha} - wL_2.$$
$$\text{s.t.} \quad wL_2 \leq D.$$

The following government budget must be satisfied:

$$tA_1 L_1 = T.$$

In equilibrium the markets for goods and labor must clear. Thus the following resource constraint should be satisfied.

$$\begin{aligned} c &= A_1 L_1 + A_2 L_2^{\alpha}, \\ l &= L_1 + L_2. \end{aligned}$$

Equilibrium

Given the tax rate t, the equilibrium values of macroeconomic variables are determined as follows.

$$\begin{aligned} w &= (1-t)A_1, \\ L_2 &= \frac{(D/A_1)}{1-t}, \\ L_1 &= \frac{(1-t)A_1 - D - \gamma A_2 (D/A_1)^{\alpha}(1-t)^{-\alpha}}{(1-t+\gamma)A_1}, \\ c &= Y = A_1 L_1 + A_2 L_2^{\alpha}, \\ l &= L = L_1 + L_2. \end{aligned}$$

The welfare $W(t) = \ln c + \gamma \ln(1-l)$ can be expressed as

$$W(t) = \ln(1-t) + (1+\gamma)\ln(1 - L(t)) + const.$$

As the total differentials of labor with respect to the tax rate are given by

$$\begin{aligned} \frac{dL_2}{dt} &= \frac{(D/A_1)}{(1-t)^2}, \\ \frac{dL_1}{dt} &= \frac{-\gamma A_1 - D - (1+\alpha)\gamma A_2 (D/A_1)^{\alpha}(1-t)^{-\alpha} - \alpha\gamma^2 A_2 (D/A_1)^{\alpha}(1-t)^{-\alpha-1}}{(1-t+\gamma)^2 A_1}, \end{aligned}$$

it is easily shown that the total differential of W with respect to t, evaluated at $t=0$, is:

$$\left.\frac{dW}{dt}\right|_{t=0} = (1+\gamma)\left(\frac{\alpha A_2 (D/A_1)^{\alpha} - D}{A_1 - D + A_2 (D/A_1)^{\alpha}}\right).$$

If $A_1 = A_2 = 1$, then

$$\left.\frac{dW}{dt}\right|_{t=0} = (1+\gamma)\left(\frac{\alpha D^{\alpha} - D}{1 - D + D^{\alpha}}\right) > 0 \quad \text{if} \quad D \leq \alpha^{\frac{1}{1-\alpha}}.$$

This result means that the distorting tax that smooths inputs of heterogeneous firms can enhance the representative agent's welfare. We can regard this welfare enhancing effect of distortionary taxation a version of the input-smoothing effect. The intuition behind this result is the same as in the case of the inflation in the text.

Case where the tax revenue is not transferred back to the consumer

When we evaluate the optimality of the distortionary taxation, a rigorous method should be to compare the distortionary tax and a lump-sum tax, subject to the constraint that the government needs to raise the same amount of tax revenue.[4]

In this subsection we consider the case where the government just throw the tax revenue away and the consumption c is not equal to the output Y:

$$c = (1-t)A_1 L_1 + A_2 L_2^{\alpha}.$$

If $\left.\frac{dW}{dt}\right|_{t=0} > 0$ then it means that the distortionary tax is welfare improving than the lump-sum tax subject to the constraint that the government must raise a (very small) fixed revenue.

Given the tax rate t, the equilibrium is determined as follows.

$$\begin{aligned}
w &= (1-t)A_1, \\
L_2 &= \frac{(D/A_1)}{1-t}, \\
L_1 &= \frac{(1-t)A_1 - D - \gamma A_2 (D/A_1)^{\alpha}(1-t)^{-\alpha}}{(1-t)(1+\gamma)A_1}, \\
c &= (1-t)A_1 L_1 + A_2 L_2^{\alpha}, \\
l &= L = L_1 + L_2.
\end{aligned}$$

As in the previous case, the welfare can be expressed as

$$W(t) = \ln(1-t) + (1+\gamma)\ln(1 - L(t)) + const.$$

As the total differentials of labor are given by

$$\begin{aligned}
\frac{dL_2}{dt} &= \frac{(D/A_1)}{(1-t)^2}, \\
\frac{dL_1}{dt} &= -\frac{(D/A)}{(1+\gamma)(1-t)^2} - \frac{(1+\alpha)(A_2/A)(D/A)^{\alpha}\gamma}{(1+\gamma)(1-t)^{2+\alpha}},
\end{aligned}$$

[4] I thank Selahattin Imrohoroglu for pointing out this argument.

it is easily shown that the total differential of W with respect to t is

$$\frac{dW}{dt}=-\frac{1}{1-t}+\frac{(1+\gamma)^2\left\{-\frac{\gamma(D/A)}{(1+\gamma)(1-t)^2}+\frac{(1+\alpha)\gamma(A_2/A)(D/A)^\alpha}{(1+\gamma)(1-t)^{2+\alpha}}\right\}}{\left\{1-\frac{(D/A)}{1-t}+\frac{(A_2/A)(D/A)^\alpha}{(1-t)^{1+\alpha}}\right\}\gamma},$$

which is evaluated at $t=0$ as follows:

$$\left.\frac{dW}{dt}\right|_{t=0}=\frac{-1-\gamma(D/A_1)+(\alpha+(1+\alpha)\gamma)(A_2/A_1)(D/A_1)^\alpha}{1-(D/A_1)+(A_2/A_1)(D/A_1)^\alpha}.$$

If $A_1=A_2=1$, then

$$\left.\frac{dW}{dt}\right|_{t=0}=\frac{-1-\gamma D+(\alpha+(1+\alpha)\gamma)D^\alpha}{1-D+D^\alpha}.$$

This value can be positive for an appropriate value of D if, for example, $\gamma>1$ and $\alpha=0.9$. In this case the distortionary tax is welfare improving, whereas the lump-sum tax on the consumer always reduces the welfare. Therefore, the distortionary tax on firm 1 can be a better policy than the lump-sum tax on the consumer to raise a (very small) fixed amount of the government revenue.

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