Asset Portfolio Choice of Banks and Inflation Dynamics^{*}

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March 14, 2012

Abstract

Since the middle of 1990s, the Japanese banks have drastically tilted their asset portfolio towards the government bonds, reducing their lending to firms. In this paper, we investigate the causes and consequences of the changes by introducing the banks' asset portfolio decision into an otherwise standard New Keynesian dynamic stochastic general equilibrium model. The banks in our model construct their portfolio under the so-called value at risk constraint that requires banks repay their debt regardless of the realization of the asset returns. Consequently, the banks' asset composition is affected by the maximum loss of the asset returns and the banks' net worth, in addition to the expected asset returns. We find that an increase in down-side risks, deterioration of the banks' net worth, and slow down of total factor productivity growth induce the banks to hold more government debt, dampening investment and reducing inflation. We estimate the model by Baynesian estimation and find that such banks' portfolio decision plays an important role in the accumulation of government bond and deflation since the latter 1990s.

Keywords: Value at Risk Constraint; Banks' Asset Allocation; Deflation; Lost Decade.

^{*}The authors would like to thank Takatoshi Ito, Toshiki Jinushi, Munehisa Kasuya, Koichiro Kamada, Takeshi Kimura, Takashi Kozu, Eiji Maeda, Koji Nakamura, Kenji Nishizaki, Masashi Saito, Takatoshi Sekine, Yosuke Takeda, Tsutomu Watanabe, participants at the fourth University of Tokyo – Bank of Japan conference in November 2011, and the staff of the Research and Statistics Department, the Bank of Japan, for their useful comments. The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

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1 Introduction

The long-lasting recession in Japanese economy since 1990s is often attributed to changes in the real side of the economy, such as the slow down of total factor productivity and reduction of mandatory working hour¹. Recent studies such as Hoshi and Kashyap (2004, 2010), Caballero, Hoshi, and Kashyap (2008) and Hirose and Kurozumi (2010), on the contrary, provide an alternative view, by emphasizing a channel through which the malfunction of financial intermediation adversely affects the economic activity. In fact, the banking sector has been under constant shifts in economic environment during the period. From the beginning of 1990s, the full-dress enforcement of Basel Committee agreement is called for, and the banks face the need to meet the capital requirement. In 1997, the banking crisis triggered by a collapse of Sanyo and Yamaichi Securities hit the economy, deteriorating the banks' profit structures and balance sheets².

In this paper, we explore the qualitative and quantitative role played by the banking sector during the lost decades. We shed lights on how the banks re-constructed their asset portfolio under the changing economic environments, and its consequences to the macroeconomy, paying special attention to inflation dynamics³⁴.

To do this, we focus on another peculiar change in the economic environment-secular and accelerating increase in government bond issuance-during the lost decade. Upper panel of Figure 1 displays the time path of government bonds outstanding relative to GDP⁵. The government bonds have a clear positive trend since the mid 1990s, partly reflecting the government's financial need for the successive implementation of economic stimulus packages and a growing spending associated with social security⁶. This expansion of government debt is, in fact, closely tied to the banks' financial intermediation activity. Figure 2 displays the banks' asset allocation as well as the ratio of the government bond outstanding held by the banks over the total government debt. Clearly,

¹Hayashi and Prescott (2002), based on a simple growth model, show that a slow down of growth in the total factor productivity can account for the economy downduring the 1990s. See also Otsu (2011) for the role played by a labor wedge.

 $^{^{2}}$ Kaihatsu and Kurozumi (2010) develop a model where frictions associated with the non-financial part of the economy and the financial sector are both incorporated and evaluate their relative importances quantitatively.

³Sugo and Ueda (2008), estimating a dynamic stochastic general equilibrium (DSGE) model a là Smets and Wouters (2003) based on the Japanese data, report that most of the variations in inflation in long horizon are accounted for by the variations in the target in monetary policy rule.

⁴Hayakawa and Maeda (1997) and Sudo (2011) argue that the banking crisis aggravates the financial intermediation activity, encourages the households' precautionary saving, and lowers velocity of circulation of money and price level.

⁵The government bond includes treasury discount bills, central government securities and FILP bonds, local government securities, and public corporation securities unless otherwise noted.

⁶There is a growing literature about the accumulation of government bond in Japanese economy from the perspective of government debt sustainability, including Doi et al. (2011) and Imrohologlu and Sudo (2011). See Enomoto and Iwamoto (2008) for the welfare implication of fiscal policy undertaken during the lost decade.

the banks' purchase of the government bonds starts to rise in the mid 1990s, while the banks' loan claim declined around the same period. As a consequence, a bulk of the government bond outstanding is held by the banks. This observation suggests that the banks' asset allocation has shifted from the loan claim to the government bond purchase, particularly since the latter half of 1997.

In order to explore the mechanism behind the banks' asset allocation and its macroeconomic implication, we incorporate an asset portfolio choice of banks into an otherwise standard New Keynesian DSGE model. The banks collect savings from the households and invest them into the two assets, the loan claim, which is equivalent to the investment in the productive capital, and the government bonds. The banks choose their portfolio so as not to violate the value-at-risk constraint (hereafter VaR constraint). Under the VaR constraint, the banks are required to repay their debt, regardless of the realization of the asset returns. While the returns from holding the loan claim and the government bond are uncertain and can be lower than the deposit rate, the banks construct their asset portfolio so that they do not go under even if the maximum losses are realized for both assets. The VaR constraint of this kind is analyzed by Adrian and Shin (2011) as a source of cyclical fluctuations of bank leverage. By incorporating two assets into the model, we depart from Adrian and Shin (2011) in that we investigate the banks' asset portfolio decision as well as their asset size and that we focus on the linkage between the banks' asset choice and the macroeconomic activity in a full-fledged DSGE model.

The central mechanism in our analysis is the banks' risk taking capacity⁷. When the VaR constraint is absent, the banks' optimal asset portfolio decision implies that the expected returns from the two assets are equalized in equilibrium. When the VaR constraint is present in the economy, the banks' asset portfolio is depends not only on the expected returns but also on the maximum loss from holding those assets and the banks' net worth. For instance, when the downside risk of holding the loan claim increases, the banks rearrange their asset portfolios so as to avert the bankruptcy in the state where the worst asset return realizes. The banks maintain their solvency even in the worst state, by investing more in the asset whose maximum loss is smaller. Changes in institutional environment, such as reinforcement of banks' capital requirement, may affect the economy in the similar manner to changes in the downside risks⁸. Such institutional initiatives encourage the rearrangement of the banks' asset portfolio by directly controlling the banks' risk taking capacity.

The banks' net worth also plays a significant role in the banks' asset portfolio decision.

⁷In the current paper, we focuse on the economy where banks risk taking is limited because of the VaR constraint. Consequently, the amount of capital accumulated in the economy is scarce compared with the economy where such constraint is absent. By contrast, recent studies including Korinek (2011) and Kato and Tsuruga (2011) investigate the economy where the fire-sale externality of assets leads to an ex-ante excessive investment by an individual bank.

⁸Gerali et al. (2009) show that the deterioration of the banks' net worth or reinforcement of capital requirement may increase the lending rate and dampen the output, based on the model where lending rate increases with the banks' net worth.

When the net worth deteriorates, the banks' repayment capacity in a worst state becomes smaller than otherwise. In such a case, the banks avert bankruptcy by reducing the leverage as is shown by Adrian and Shin (2011), and by shifting the asset portfolio from the asset with a large maximum loss to that with a smaller maximum loss.

We then draw some implications of the bank portfolio decisions under VaR constraint for the economic activity. Suppose that the uncertainty of capital return increases and hence the maximum loss of loan claim holding increases. The banks facing the VaR constraint reduce their investment in the loan claims, purchasing more government bond whose maximum loss is more limited. This implies that loan supply, reducing output (investment in particular) and inflation. The initial effect influences the expected asset returns and the banks' retained earnings in the subsequent periods, bringing about further effects to the economy.

Our model's implication is consistent with Japan's experience since 1990. Japan experienced the slow down in the growth of total factor productivity, the increased enforcement of banks' capital requirement, the burst of bubbles in the early half of the 1990s, and the banking crisis in the latter half of the 1990s followed by an increase in the bad loans and deterioration of the banks' net worth. In our model, all of these events may induce the banks' asset portfolio tilted toward the government bond purchase, generating downward pressure on the inflation.

Our analysis is closely related to Braun and Nakajima (2011). They study the impact of accumulated government debt on the price level, focusing on the banks' asset allocation. The banks in their model hold the government bond as collateral to finance their asset purchase. So far as the they have optimistic view about the future bond price, they purchase the government bond by raising the fund from other agents, using the government bond as collateral. Consequently, pile-up of the government bond and deflation coexist in the economy. While our paper also stresses the implication of the banks' asset allocation, the economic mechanism behind the banks' government bond holding differs from the one discussed in Braun and Nakajima (2011). In our paper, the key determinant of the banks' asset portfolio is the severity of the VaR constraint. Whenever the constraint is tightened, the banks tilt toward less risky assets from the risky assets.

Another research in line with our work is Brunnermeier and Sanikov (2011). They construct a model economy where the market imperfection is present in the financial intermediation activity. Whenever the adverse shock hits the economy, the agents tilt their asset toward the safe asset, yielding deflation. This is because safe asset is in nominal term in their paper, and the higher demand for the safe asset results in the decrease in the price level. Although the same mechanism can be present in our model, the channel through which the banks' asset allocation affect the inflation is different. In our model, flight to safer asset prevents the capital accumulation, dampening the output and generating the inflation.

Regarding the role played by uncertainty, our analysis is also related to the work by

Fernandez-Villaderde et al. (2011). They empirically find, using the structural vector autoregression, that a higher volatility in the productivity lowers the price level and the output, and provide a theoretical framework to analyze the relationship between uncertainty and the households' asset allocation. In their model based on the inventory model of money demand, they show that facing a larger uncertainty, the households prefer safer and more liquid asset, money, to riskier goods.

The rest of the paper is organized as follows. Section 2 presents our model with banks that endogenously choose the asset portfolio under the VaR constraint. In addition, we explore the qualitative property of our model using a simplified setting. Section 3 demonstrates our economy's quantitative implications based on the model estimated using the Japanese data from 1980Q1 to 2007Q4. Section 4 concludes the analysis and discuss the future extension of our analysis.

2 The Model Economy

This section describes the structure of our model. The economy consists of seven types of agents: household, banks, intermediate goods producers, wholesale goods producers, final goods producers, government and central bank. See Figure 3 for model's brief outline.

The representative household supplies labor inputs to the intermediate goods producers, receive wage, make deposit to the banks, and receive repayment for the deposit in turn. She has no means to access to the financial market and cannot own the financial assets but the bank deposit. The banks collect the deposits from the household, and invest them on the two assets: the loan claim to the capital goods used by the intermediate goods producers, and the government bond. The banks construct their asset portfolio composition so as not to violate the VaR constraint. The intermediate goods producers hire the labor supply and capital goods from the household and the banks, respectively, to produce the final goods. The wholesale goods producers produce the differentiated final goods from the intermediate goods. They are monopolistic supplier of the final goods, and set their prices so as to maximize their profit. The final goods producers lump-sam tax from the household and issues the government bond to finance the government debt and the government expenditure. The central bank controls inflation by adjusting the nominal interest rate according to a Tayor rule.

2.1 Household

The infinitely-lived representative household makes decision for consumption and deposit holdings. She is barred from the financial market. She thus possess no real capital stock nor government bond, and hold all of her saving in the form of bank deposit. The household has preference over the consumption goods $c(s^t)$, and work effort $l(s^t)$, as described in the expected utility function, (1)

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c\left(s^t\right), l\left(s^t\right)) = E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c\left(s^t\right) + \eta \log\left(1 - l\left(s^t\right)\right)\right), \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor and η is the weighting assigned to leisure.

The budget constraint of the household is given by the equation below:

$$c\left(s^{t}\right) + d\left(s^{t}\right) = r_{d}\left(s^{t-1}\right)d\left(s^{t-1}\right) + \frac{W\left(s^{t}\right)}{P\left(s^{t}\right)}l\left(s^{t}\right) + \Pi\left(s^{t}\right) - \tau\left(s^{t}\right)$$
(2)

where $d(s^t)$ is the household's deposit, $r_d(s^{t-1})$ is the real deposit rate repaid by the banks for the deposit made in period t - 1, $W(h, s^t)$ is the nominal wage rate, $P(s^t)$ is the price index, $\Pi(s^t)$ is the sum of the real profits of the intermediate goods producers and the banks that are returned to the household as dividends. $\tau(s^t)$ is the lump-sum real tax collected by the government. We assume that the deposit is risk-free asset, and the real deposit rate is the real risk-free rate.

The first-order conditions associated with the household's intertemporal decision is given by

$$U_c(c(s^t), l(s^t)) = \beta r_d(s^t) \operatorname{E}_t U_c(c(s^{t+1}), l(s^{t+1})),$$

where U_c denotes the marginal utility with respect to the consumption. Because the household's only financial asset is banks' deposit, her consumption growth is dependent on the risk-free rate.

Since the labor market is competitive, we have

$$\frac{U_{l}(c\left(s^{t}\right),l\left(s^{t}\right))}{U_{c}(c\left(s^{t}\right),l\left(s^{t}\right))} = \frac{W\left(s^{t}\right)}{P\left(s^{t}\right)},$$

where U_l denotes the marginal utility with respect to the leisure.

2.2 Banks

The outline of banks' choice

There is a continuum of risk-neutral banks, indexed by $i \in (0, 1)$. Each bank *i* collects deposit $d(i, s^t)$ from the households, and purchases the loan claim, namely capital stock, $k(i, s^t)$, and the real government bond $b(i, s^t) \equiv \frac{B(i, s^t)}{P(s^t)}$, from the final goods producers and the government, respectively. The expenses are financed by the deposit $d(i, s^t)$ and the bank *i*'s own real net worth $n(i, s^t)$. The bank *i*'s balance sheet each period is therefore given by

$$k\left(i,s^{t}\right) + \frac{B\left(i,s^{t}\right)}{P\left(s^{t}\right)} = n\left(i,s^{t}\right) + d\left(i,s^{t}\right).$$

$$(3)$$

The bank i receives returns from the two assets invested in the previous period, repays the deposit to the households, and retains the rest of the earnings as the own net worth. Consequently, the bank's net worth evolves according to the following law of motion:

$$n(i, s^{t+1}) = r_k(s^{t+1})k(i, s^t) + r_b(s^{t+1})b(i, s^t) - r_d(s^t)d_t(i, s^t),$$
(4)

where $r_k(s^{t+1})$ and $r_b(s^{t+1})$ are the real return to the loan claim and the government bond, respectively. Note that the real return to the government bond is given by the policy rate $R_B(s^t)$ set by the central bank, divided by the inflation rate $\pi(s^{t+1})$ through the relationship below.

$$r_b\left(s^{t+1}\right) = \frac{R_B\left(s^t\right)}{\pi\left(s^{t+1}\right)}.$$

The bank *i* keeps the net worth accumulation up to the period when it exits from the economy.⁹ We assume that the bank's exiting probability each period is exogenously given by $1 - \gamma(s^t)$. The continuation value of the bank *i* is then given by

$$V\left(n\left(i,s^{t}\right)\right) = \beta \mathbf{E}_{t} \Lambda_{t,t+1}\left[\gamma\left(s^{t}\right) V\left(n\left(i,s^{t+1}\right)\right) + \left(1 - \gamma\left(s^{t}\right)\right) n\left(i,s^{t+1}\right)\right],\tag{5}$$

where $n(i, s^t)$ is the net worth held by the bank *i*, and $\Lambda_{t,t+1}$ denotes the households' stochastic discount factor from the period *t* to the period t + 1.

In choosing the asset portfolio composition between the two assets, the bank *i* considers a VaR constraint similar to the one discussed in Adrian and Shin (2011), together with the expected average returns of the two assets. Namely, the bank *i* constructs the asset portfolio in period *t* so that it is able to repay all of its debt to the household even if the two assets yield the maximum loss. Denoting the maximum loss from holding the two assets by $\underline{r}_k(s^{t+1}|s^t)$ and $\underline{r}_b(s^{t+1}|s^t)$, respectively, the value at risk constraint is expressed by

$$\underline{r}_{k}\left(s^{t+1}|s^{t}\right)k\left(i,s^{t}\right) + \underline{r}_{b}\left(s^{t+1}|s^{t}\right)b\left(i,s^{t}\right) - r_{d}\left(s^{t}\right)d\left(i,s^{t}\right) \ge 0.$$
(6)

Here, we assume that the loan claim holding has a larger risk compared with the government bond holding, so that $\underline{r}_k(s^{t+1}|s^t) < \underline{r}_b(s^{t+1}|s^t)^{10}$. There are two possible interpretation as to the time varying maximus loss. On the one hand, shocks to economic

 $^{^{9}}$ Following Gertler and Karadi (2011), we assume that the bank transfers all of the accumulated net worth to the household when it exits from the economy.

¹⁰In the current paper, we concentrate our analysis on the equilibrium where the banks hold both of the two risky assets, and the worst returns of the two risky assets are smaller than the risk-free rate, so that the two equations below hold.

environment, including an increase in downside risk about the asset return and an increase in uncertainty about macroeconomic outlook, lower the maximum loss of the assets. On the other hand, institutional changes, including reinforcement of capital requirement also bring the similar effect. While the size of these maximum losses may be endogenously affected by the economic surroundings, in the current paper, we treat them as given, concentrating our analysis on how these variations of losses affect the economy.

The banks' maximization problem

In Adrian and Shin (2011) where there is only one type of asset, the VaR constraint influences the size of bank's leverage. By contrast, in our model where there are two assets in the economy, the VaR constraint influences the asset portfolio allocation as well as the size of the leverage. The bank *i*'s optimization problem is formulated as the maximization of the value of the net worth at the last period it exists, which is shown by equation (5), subject to the bank *i*'s balance sheet equation (3), the law of motion for the bank *i*'s net wort accumulation (5), and the VaR constraint (3). Because the banks are risk-neutral, we first guess that the value function of the bank *i* is given by

$$V\left(n\left(i,s^{t}\right)\right) = \phi\left(s^{t}\right)n\left(i,s^{t}\right),$$

then the equation (5) is reduced to

$$\max V(n(i, s^{t})) = \beta E_{t} \Lambda_{t,t+1} \left[\gamma(s^{t}) \phi(s^{t+1}) \begin{pmatrix} q_{k}(s^{t+1}) k(i, s^{t}) \\ +q_{b}(s^{t+1}) b(i, s^{t}) \\ +r_{d}(s^{t}) n(i, s^{t}) \end{pmatrix} + (1 - \gamma(s^{t})) (q_{k}(s^{t+1}) k(i, s^{t}) + q_{b}(s^{t+1}) b(i, s^{t}) + r_{d}(s^{t}) n(i, s^{t})) \right].$$

The corresponding first order condition gives

$$\mathbf{E}_{t}\left[\frac{\left(\gamma\phi\left(s^{t+1}\right)+1-\gamma\left(s^{t}\right)\right)\Lambda_{t,t+1}q_{k}\left(s^{t+1}\right)}{\underline{q}_{k}\left(s^{t+1}\right)}\right] = \mathbf{E}_{t}\left[\frac{\left(\gamma\phi\left(s^{t+1}\right)+1-\gamma\left(s^{t}\right)\right)\Lambda_{t,t+1}q_{b}\left(s^{t+1}\right)}{\underline{q}_{b}\left(s^{t+1}\right)}\right].$$
(7)

Here $q_k(s^{t+1}) \equiv r_k(s^{t+1}) - r_d(s^t)$ and $q_b(s^{t+1}) \equiv r_b(s^{t+1}) - r_d(s^t)$ denote the excess return to the loam claim holding and that to the government bond holding relative to the deposit, respectively. Similarly, $\underline{q}_k(s^{t+1}) \equiv \underline{r}_k(s^{t+1}) - r_d(s^t)$ and $\underline{q}_b(s^{t+1}) \equiv \underline{r}_b(s^{t+1}) = \underline{r}_b(s^{t+1}) - r_d(s^t)$

$$\underline{r}_k \left(s^{t+1} | s^t \right) - r_d \left(s^t \right) < 0,$$

$$\underline{r}_b \left(s^{t+1} | s^t \right) - r_d \left(s^t \right) < 0.$$

 $\underline{r}_{b}(s^{t+1}) - r_{d}(s^{t})$ denote the excess return to the two risky assets when the worst return to the assets realize.

The equation (7) provides the bank's fundamental principle in allocating their assets into the loan claim and the government bond. When the VaR constraint is effective, there is no need that expected excess returns of the two assets are not equalized at the equilibrium. Instead, banks' asset portfolio is constructed so that the expected excess returns weighted by the maximum loss of each asset are equalized. Under the premise that the loan claim is riskier than the government bond, so that $\underline{r}_k(s^{t+1}) < \underline{r}_b(s^{t+1})$, the expected excess return of the loan claim needs to exceed that of the government bond, $E_t r_k(s^{t+1}) > E_t r_b(s^{t+1})$, for compensation.

From equations (6) and (7), we obtain the expression for $\phi(s^t)$.

$$\phi\left(s^{t}\right) = \beta \mathbf{E}_{t} \left[\Lambda_{t,t+1} \left\{ \gamma\left(s^{t}\right) \phi\left(s^{t+1}\right) + \left(1 - \gamma\left(s^{t}\right)\right) \right\} r_{d}\left(s^{t}\right) \left(1 - q_{k}\left(s^{t+1}\right) / \underline{q}_{k}\left(s^{t+1}\right) \right) \right].$$
(8)

Aggregation

The banks exit from the economy with probability $1 - \gamma(s^t)$ each period, and the aggregate banks' net worth evolves according to the following law of motion;

$$n\left(s^{t}\right) = \gamma\left(s^{t}\right)\left[r_{k}\left(s^{t}\right)k\left(s^{t-1}\right) + r_{b}\left(s^{t}\right)b\left(s^{t-1}\right) - r_{d}\left(s^{t-1}\right)d\left(s^{t-1}\right)\right],$$

where $n(s^t)$ is the aggregate banks' net worth. An increase in the exiting probability reduces the bank's net worth. As shown in the equation (6), the reduced net worth help tighten the banks' VaR constraint, affecting the banks' asset portfolio allocation in the subsequent period

 d^{1112} .

2.3 Intermediate Goods Producers

The intermediate goods producers produce intermediate goods $y(s^t)$, selling them to the wholesale goods producers with the price $P_y(s^t)$. They hire labor inputs $l(s^t)$ from the household and borrow the effective capital $v(s^t) K(s^{t-1})$ from the banks. Both the

¹¹Based on the financial accelerator model developed by Bernanke, Gertler, and Gilchrist (1999), Gilchrist and Leahy (2002) and Nolan and Thoenissen (2009) study the consequence of the exogenous deterioration of the entrepreneurial net worth to the economy. There the exogenous net worth change is considered as an irrational innovation in expected return to the entrepreneurial net worth or shock to the technology associated with the efficacy of the financial intermediation.

¹²There are alternative ways to incorporate the shocks to the banks' net worth into the model. In Gertler and Karadi (2011), the existing capital stock becomes out of date, deteriorating the value of the banks' loan claim and net worth. In Aoki and Nikolov (2011) where the banks' investment on the bubble is analyzed, the collapse of the bubble leads to a deterioration of the banks' net worth.

input and output market of the intermediate goods producers are competitive. The maximization problem of the intermediate goods producer is given by

$$\max_{y(s^{t}),v(s^{t})k(s^{t-1}),l(s^{t})}\frac{P_{y}\left(s^{t}\right)y\left(s^{t}\right)}{P\left(s^{t}\right)}-\tilde{r}\left(s^{t}\right)v\left(s^{t}\right)k\left(s^{t-1}\right)-W\left(s^{t}\right)l\left(s^{t}\right),$$

subject to

$$y(s^{t}) = \left(v(s^{t}) k(s^{t-1})\right)^{\alpha} \left(A(s^{t}) Z(s^{t}) l(s^{t})\right)^{1-\alpha},$$
(9)

where $v(s^t)$ is the capital utilization rate, $k(s^{t-1})$ is the capital stock, $\tilde{r}(s^t)$ is the real return to the use of effective capital, $A(s^t)$ is the stationary component of technology level, $Z(s^t)$ is the non-stationary component of technology level, and $\alpha \in [0, 1]$ is the capital share. The first order conditions of the intermediate goods producers yield the following equality.

$$\tilde{r}_{k}\left(s^{t}\right) = \alpha \frac{P_{y}\left(s^{t}\right)}{P\left(s^{t}\right)} \left(v\left(s^{t}\right)k\left(s^{t-1}\right)\right)^{\alpha-1} \left(A\left(s^{t}\right)Z\left(s^{t}\right)l\left(s^{t}\right)\right)^{1-\alpha}, \frac{W\left(s^{t}\right)}{P\left(s^{t}\right)} = \frac{P_{y}\left(s^{t}\right)}{P\left(s^{t}\right)} (1-\alpha) \left(v\left(s^{t}\right)k\left(s^{t-1}\right)\right)^{\alpha} \left(A\left(s^{t}\right)Z\left(s^{t}\right)\right)^{1-\alpha} \left(l\left(s^{t}\right)\right)^{-\alpha}.$$

The capital utilization rate is determined by the banks. Assuming that choosing capital utilization $v(s^t)$, together with the capital stock $k(s^{t-1})$, incurs the real cost of

$$\frac{\kappa_v k \left(s^{t-1}\right) \left(v \left(s^t\right)\right)^{\phi+1} - 1}{\phi + 1},$$

to the banks, then the banks' optimal capital utilization rate is expressed by

$$\tilde{r}_k\left(s^t\right) = \left(\phi + 1\right)\kappa_v v_t^\phi,$$

where κ_v and ϕ are parameters that govern capital utilization rate. Consequently, the banks' net return to the investment on the productive capital $r_k(s^t)$ is given by

$$r_{k}\left(s^{t}\right)k\left(s^{t}\right) = \tilde{r}_{k}\left(s^{t}\right)k\left(s^{t}\right) - \kappa_{v}k\left(s^{t-1}\right)\left(v\left(s^{t}\right)\right)^{\phi+1} + (1-\delta)k\left(s^{t}\right),$$

where $\delta \in [0, 1]$ is the depreciation rate of the capital stock. Similarly, the real wage paid to the household is expressed by

$$\frac{W(s^{t})}{P(s^{t})} = s\left(s^{t}\right)\left(1-\alpha\right)\left(v\left(s^{t}\right)k\left(s^{t-1}\right)\right)^{\alpha}\left(A\left(s^{t}\right)\right)^{1-\alpha}\left(Z\left(s^{t}\right)\right)^{1-\alpha}\left(l\left(s^{t}\right)\right)^{-\alpha}.$$

2.4 Wholesale and Final Goods Producers

Optimization problem of wholesale and final goods producers

The wholesale goods sector contains a continuum of firms, each producing differentiated products, as indexed by $z \in [0,1]$, from the intermediate goods by the linear production technology

$$x(z,s^t) = y(z,s^t).$$

Here, $x(z, s^t)$ denotes the differentiated wholesale goods made by the wholesale goods producer z and $y(z, s^t)$ is the intermediate goods used as inputs by the producer z.

The final goods producer purchases these differentiated goods in a competitive market, producing the final goods from wholesale goods by the following CES aggregate technology

$$x\left(s^{t}\right) = \left[\int_{0}^{1} x(s^{t}, z)^{\frac{\varepsilon\left(s^{t}\right)-1}{\varepsilon\left(s^{t}\right)}} dz\right]^{\frac{\varepsilon\left(s^{t}\right)}{1-\varepsilon\left(s^{t}\right)}}, \quad \varepsilon\left(s^{t}\right) > 1$$

where $\varepsilon(s^t) \in (1, \infty)$ denotes the time-varying elasticity of substitution between the differentiated goods. Given this CES technology of the final goods, the demand for each differentiated goods $x(z, s^t)$ is given by a function of the price of its product $p(z, s^t)$, the aggregate price index $P(s^t)$, and the aggregate demand for the final goods $x(s^t)$, as below

$$x(z, s^{t}) = \left(\frac{p(z, s^{t})}{P(s^{t})}\right)^{-\varepsilon(s^{t})} x(s^{t}).$$

Each wholesale goods producer z maximizes its profit by choosing the product price optimally. The maximization problem of each differentiated producer is given by

$$\max_{p(z,s^{t+j})} \mathcal{E}_{t} \sum_{j=0}^{\infty} \beta^{j} \Lambda_{j-1,j} \begin{bmatrix} \left(\frac{p(z,s^{t+j})}{P(s^{t+j})}\right)^{1-\varepsilon(s^{t})} x\left(s^{t+j}\right) \\ -\left(\frac{P_{y}(s^{t+j})}{P(s^{t+j})}\right) \left(\frac{p(z,s^{t+j})}{P(s^{t+j})}\right)^{\varepsilon(s^{t})} x\left(s^{t+j}\right) \\ -\frac{\kappa}{2} \left(\frac{p(z,s^{t+j})}{p(z,s^{t+j-1})} - \frac{p(s^{t+j-1})}{p(s^{t+j-2})}\right)^{2} \left(\frac{p(z,s^{t+j})}{P(s^{t+j})}\right)^{-\varepsilon(s^{t})} x\left(s^{t+j}\right) \end{bmatrix},$$

where the third term denotes an adjustment cost that producer pays in changing its product price $p(z, s^t)$, and κ is the parameter that governs the size of adjustment cost.

Because all of the differentiated goods prices $p(z, s^t)$ set by the wholesale goods producers are identical at the symmetric equilibrium, we obtain the Phillips curve of the economy from the first order condition of this firm's maximization problem.

$$-\varepsilon \left(s^{t}\right) \left(1 - \frac{P_{y}\left(s^{t}\right)}{P\left(s^{t}\right)} - 0.5\left(\pi\left(s^{t}\right) - 1\right)^{2}\right) + 1 - \kappa\left(\pi\left(s^{t}\right) - 1\right)\pi\left(s^{t}\right) + \beta\kappa\left(\pi\left(s^{t+1}\right) - 1\right)\pi\left(s^{t+1}\right)\frac{x\left(s^{t+1}\right)}{x\left(s^{t}\right)} = 0.$$
(10)

The market clearing condition

The market clearing condition of the intermediate goods and the wholesale goods are given by

$$\int_0^1 x \left(s^t, z\right) dz = y \left(s^t\right),$$
$$x \left(s^t\right) = \int_0^1 x \left(s^t, z\right) dz$$

The final goods serves as the household's consumption, investment to productivity capital, and the government expenditure. The market clearing condition of the final goods is given by

$$c(s^{t})+k(s^{t})-(1-\delta)k(s^{t-1})+G(s^{t}) = x(s^{t})-\frac{\kappa}{2}(\pi(s^{t})-1)^{2}x(s^{t})-\kappa_{v}k(s^{t-1})v(s^{t})^{\phi+1}$$

2.5 Government and Central Bank

The government collects a lump-sum tax $P(s^t) \tau(s^t)$ from the household and issues a government bond $B(s^t)$ to finance the repayment $R_B(s^{t-1}) B(s^{t-1})$ to the banks and government expenditure $P(s^t) G(s^t)$. We assume that a balanced budget is maintained in each period t as follows:

$$R_B\left(s^{t-1}\right)B\left(s^{t-1}\right) + P\left(s^t\right)G\left(s^t\right) = P\left(s^t\right)\tau\left(s^t\right) + B\left(s^t\right).$$
(11)

The government tax policy is an increasing function of the outstanding government bond that is specified by the following equation:

$$\tau\left(s^{t}\right) = Tb\left(s^{t-1}\right)\left(\frac{b\left(s^{t-1}\right)}{x\left(s^{t}\right)}\right)^{\psi},\tag{12}$$

where $\psi \in (1, \infty]$ is an elasticity of lump-sum tax with respect to the government debt status, indicating that an increase in bond leads to an increase in tax, and T is a constant parameter.

The central bank sets the nominal interest rate according to a simple Taylor rule given by

$$\ln R_B\left(s^t\right) = (1 - \rho_M)\ln R + \rho_M \ln R_B\left(s^{t-1}\right) + (1 - \rho_M)\phi\ln\pi\left(s^t\right) + \epsilon_r\left(s^t\right), \quad (13)$$

where R is constant, $\rho_M \in [0, 1]$ is the autoregresive coefficient of the policy rate, and $\phi > 1$ is the policy weight attached to the inflation rate and $\epsilon_r(s^t)$ is an i.i.d. shock to the monetary policy rule¹³.

2.6 Shock Process

The exogenous shocks in our economy, the shock to the markup-related elasticity $\varepsilon(s^t)$, the bank's net worth $\gamma(s^t)$, the maximum loss of the two assets $\underline{r}_k(s^t)$ and $\underline{r}_b(s^t)$, the stationary component of technology $A(s^t)$, the non-stationary component of technology $Z(s^t)$, and government expenditure $G(s^t)$, evolve according to the equation below:

$$\ln \varepsilon \left(s^t \right) = \ln \varepsilon + \epsilon_{\varepsilon} \left(s^t \right), \tag{14}$$

$$\ln \gamma \left(s^{t} \right) = \left(1 - \rho_{\gamma} \right) \ln \gamma + \rho_{\gamma} \ln \gamma \left(s^{t-1} \right) + \epsilon_{\gamma} \left(s^{t} \right), \tag{15}$$

$$\ln \underline{r}_{k}\left(s^{t}\right) = \left(1 - \rho_{\underline{r}_{k}}\right) \ln \underline{r}_{k} + \rho_{\underline{r}_{k}} \ln \underline{r}_{k}\left(s^{t-1}\right) + \epsilon_{\underline{r}_{k}}\left(s^{t}\right), \qquad (16)$$

$$\ln \underline{r}_{b}\left(s^{t}\right) = \left(1 - \rho_{\underline{r}_{b}}\right) \ln \underline{r}_{b} + \rho_{\underline{r}_{b}} \ln \underline{r}_{b}\left(s^{t-1}\right) + \epsilon_{\underline{r}_{b}}\left(s^{t}\right), \qquad (17)$$

$$\ln A\left(s^{t}\right) = (1-\rho_{A})\ln A + \rho_{A}\ln A\left(s^{t-1}\right) + \epsilon_{A}\left(s^{t}\right), \qquad (18)$$

$$\ln Z\left(s^{t}\right) = \ln Z\left(s^{t-1}\right) + u_{Z}\left(s^{t}\right), \qquad (19)$$

$$u_Z\left(s^t\right) = \rho_Z u_Z\left(s^{t-1}\right) + \epsilon_Z\left(s^t\right), \tag{20}$$

$$\ln G\left(s^{t}\right) = (1 - \rho_{G})\ln G + \rho_{G}\ln G\left(s^{t-1}\right) + \epsilon_{G}\left(s^{t}\right), \qquad (21)$$

where ρ_{ε} , ρ_{γ} , $\rho_{\underline{r}_{k}}$, $\rho_{\underline{r}_{b}}$, ρ_{A} , ρ_{Z} , and $\rho_{G} \in (0, 1)$ are the autoregressive root of the corresponding shocks, and $\epsilon_{\varepsilon}(s^{t})$, $\epsilon_{\gamma}(s^{t})$, $\epsilon_{\underline{r}_{k}}(s^{t})$, $\epsilon_{\underline{r}_{b}}(s^{t})$, $\epsilon_{A}(s^{t})$, $\epsilon_{Z}(s^{t})$ and $\epsilon_{G}(s^{t})$ are the exogenous i.i.d. shocks that are normally distributed with mean zero.

2.7 Equilibrium Condition

An equilibrium consists of a set of prices, $\{W(s^t), P(s^t), P_y(s^t), r_k(s^t), \tilde{r}_k(s^t), r_d(s^t), r_b(s^t), R_B(s^t)\}_{t=0}^{\infty}$, and the allocations $\{c(s^t), l(s^t), d(s^t), \Pi(s^t), k(s^t), v(s^t), x(s^t), y(s^t)\}_{t=0}^{\infty}$, for a given government policy $\{G(s^t), \tau(s^t)\}_{t=0}^{\infty}$, realization of exogenous

¹³Our parameterization of the policy parameters ψ and ϕ are both greater than unity implies that our economy is in the Ricardian regime for both fiscal and monetary policy. Relatedly, in the current paper, we do not consider the case of the government's default. In Non-Ricardian regime with government defaults, inflation rate is only uniquely pinned down when the central bank responds to inflation aggressively. See, for example, Kocherlakota (2012).

variables $\{\epsilon_{\varepsilon}(s^{t}), \epsilon_{\gamma}(s^{t}), \epsilon_{\underline{r}_{k}}(s^{t}), \epsilon_{\underline{r}_{k}}(s^{t}), \epsilon_{A}(s^{t}), \epsilon_{Z}(s^{t}), \epsilon_{G}(s^{t}), \epsilon_{r}(s^{t})\}_{t=0}^{\infty}$, the expected worst returns $\{\underline{r}_{k}(s^{t}), \underline{r}_{b}(s^{t})\}_{t=0}^{\infty}$, and initial conditions $\{B_{-1}\}, \{d_{-1}\}, \{k_{-1}\}$ such that for all t, i, and z:

(i) the household maximizes her utility given the prices;

(*ii*) the bank *i* maximizes its profits given the prices and the expected worst returns;

(*iii*) the intermediate goods producer maximizes its profits given the prices;

(iv) the wholesale goods producer z maximizes its profits given the prices;

(v) the final goods producer maximizes its profits given the prices;

(vi) the government budget constraint holds;

(vii) the central bank sets a policy rate following the Taylor rule; and

(viii) markets clear.

2.8 Steady State Analysis

Before investigating the model's dynamics, we explore the model's mechanism at the steady state to show the determinants of the banks' portfolio composition. In particular, we focus on how the returns from holding the two risky assets r_b and r_k are affected by the banks' VaR constraint, and how the banks' decision as to the portfolio allocation between the government bond b and the loan claim k is made¹⁴. For illustrative purpose, we made two simplifying assumptions in this subsection: (1) the households supply labor inelastically, l = 1, and (2) the banks' capital utilization cost is zero, $\phi = 0^{15}$.

Evaluating the portfolio choice equation, the VaR constraint equation, and the law of motion of the bank's net worth at the steady state values, we have,

$$\frac{r_k - r_d}{r_d - \underline{r}_k} = \frac{r_b - r_d}{r_d - \underline{r}_b},\tag{22}$$

$$(\underline{r}_k - r_d) k + (\underline{r}_b - r_d) b = -r_d n,$$

$$(23)$$

$$n = \frac{\gamma}{1 - \gamma r_d} \left[(r_k - r_d) \, k + (r_b - r_d) \, b \right]. \tag{24}$$

Notice that the household's Euler equation at the steady state implies that

$$r_d = \frac{1}{\beta}$$

¹⁴The definition of the steady state in our economy needs to be carefully stated. Suppose that we define the steady state as the economy where all of the exogenous shocks are absent and every endogenous variables grow at the constant rate. The banks' asset allocation then becomes indeterminate because the their portfolio choice is dependent on the riskiness of the assets. In the current paper, we define the steady state following the Devereux and Sutherland (2010, 2011) where the banks take the possibility that worst scenario of the asset return realize into the consideration. Consequently, the risks of holding the assets affect the banks' portfolio at the steady state.

¹⁵This assumption implies that the capacity utilization is unity.

The three equations above yield the excess return from holding the two risky assets, and the spread of the two risky assets;

$$r_b - r_d = \frac{1 - \gamma r_d}{\gamma r_d} \left(r_d - \underline{r}_b \right), \qquad (25)$$

$$r_k - r_d = \frac{1 - \gamma r_d}{\gamma r_d} \left(r_d - \underline{r}_k \right).$$
(26)

$$r_k - r_b = \frac{1 - \gamma r_d}{\gamma r_d} \left(\underline{r}_b - \underline{r}_k \right).$$
(27)

According to the equation (25) and (26), the excess return from holding the two risky assets and the spread between the two assets are expressed by the expected worst returns from holding the two risky assets \underline{r}_b and \underline{r}_k , together with the bank's survival probability γ .

When the risk of holding the loan claim \underline{r}_k increases, for instance, the bank's VaR constraint becomes tightened unless the bank reduces the loan claim. Consequently, the bank requires a higher spread in holding the loan claim than otherwise. The the government bond yield is unaffected by the change in \underline{r}_k . The similar mechanism works if the risk of holding the government bond \underline{r}_b increases.

By contrast, a reduction in the survival probability γ lead to a rise in the two excess returns. As indicated by the equation (24), the smaller probability prevents the banks from accumulating the net worth. Because the scarcity of the net worth tightens the VaR constraint by deteriorating the bank's balance sheet, it results in the reduction of the bank's purchase of both of the two risky assets. The excess returns for the two assets therefore increase to clear the demand.

Next, we discuss how the banks allocate their asset between the loan claim k and the government bond b. Because the return from holding the loan claim r_k equals to the return to the capital stock in the economy, we have

$$r_k = \alpha AZk^{\alpha - 1} + (1 - \delta).$$

Since the loan claim is equivalent to the capital stock in the equilibrium, we have

$$k = \left[\frac{r_k - (1 - \delta)}{\alpha AZ}\right]^{\frac{1}{\alpha - 1}}.$$
(28)

Taking that $\alpha - 1 < 0$ into the consideration, a higher return for the loan claim implies a smaller size of a loan claim and thus a smaller investment in the economy. Based on the discussions above, an increase in the risk of holding the loan claim or a decline in the banks' surviving probability reduce the loan claim through a rise in the return \underline{r}_k . The bank's decision as to the holding of the government bond is affected by the government policy regarding taxing and budget balance. From the equations (11) and (12), we have

$$r_{b}b = Tb^{\psi} + b,$$

$$b = \left[\frac{r_{b}-1}{T}\right]^{\frac{1}{\psi}}x = \left[\frac{r_{b}-1}{T}\right]^{\frac{1}{\psi}}AZk^{\alpha}.$$
(29)

Here we assume that inflation rate is unity as the steady state. These governmental equations suggest, for a value $\psi > 1$, that the banks tilt toward holding of the government bond as the corresponding return increases. Under the current tax policy, an increase in the government's interest rate payment is met by the comparable increase in government bond issuance, leading to a higher government bond holding by banks. Similarly to the working mechanism that determinants the loan claim k, the increasing risk of holding the government bond, given by a decline in \underline{r}_b , or the disruption of the bank's net worth, given by a decline in γ , leads to an increase in the bank's government bond holding, through a rise in the government bond yield.

Lastly, we discuss how the bank allocates its asset between the government bond holding and the loan claim to the firms. From the equations (28) and (29), the government bond holding relative to the loan claim is given by

$$\frac{b}{k} = \left[\frac{r_b - 1}{T}\right]^{\frac{1}{\psi}} \left[\frac{r_k - (1 - \delta)}{\alpha}\right].$$

According to the above equation, any changes in the economic environments that boost the return to the two risky assets r_b and r_k , including the increasing risk of lending to the firms, that of holding the government bond, or the disruption of the bank's net worth, cause the bank to purchase more of the government bond compared with the loan claim to the firms.

To summarize, the key determinants behind the banks' tilting toward the government bond are the relative increase in the risk of holding the real asset and the shortage of the banks' net worth. In the next subsection, we depart from the steady state analysis, and explore the implication of the bank's asset allocation to the inflation dynamics, by log-linearizing our model around the steady state.

2.9 VaR Constraint and Inflation Dynamics

By log-linearizing equations (7) and (13) around the steady state, we have

Here $\hat{\lambda}(s^t)$ denotes the log deviation of a variable $\lambda(s^t)$ from its steady state value. Taking the following relationship

$$\hat{r}_b\left(s^{t+1}\right) = \hat{R}_{b,t} - \hat{\pi}\left(s^{t+1}\right)$$

into our consideration, we have

$$\hat{\pi}(s^{t}) = \phi^{-1} \mathbf{E}_{t} \left[\hat{\pi}(s^{t+1}) + a_{1} \hat{r}_{k}(s^{t+1}) + a_{2} \underline{\hat{r}}_{k}(s^{t+1}) - a_{3} \underline{\hat{r}}_{b}(s^{t+1}) + a_{4} \hat{r}_{d}(s^{t}) \right].$$
(32)

Here, a_1 , a_2 , a_3 , and a_4 are all positive values that are denoted by

$$a_{1} = \frac{r_{b} - r_{d}}{r_{d}} \frac{r_{k}}{r_{k} - r_{d}}, \quad a_{2} = \frac{r_{b} - r_{d}}{r_{b}} \frac{\underline{r}_{k}}{r_{d} - \underline{r}_{k}}, \quad a_{3} = \frac{r_{b} - r_{d}}{r_{d}} \frac{\underline{r}_{b}}{r_{d} - \underline{r}_{l}}$$
$$a_{4} = \frac{r_{b} - r_{d}}{r_{b}} \left[\left(\frac{r_{d}}{r_{b} - r_{d}} - \frac{r_{d}}{r_{k} - r_{d}} \right) + \left(\frac{r_{d}}{r_{d} - \underline{r}_{b}} - \frac{r_{d}}{r_{d} - \underline{r}_{k}} \right) \right].$$

The equation (32) indicates the qualitative link between the banks' asset allocation and inflation rate in the economy. Other things, including the inflation expectation $E_t[\hat{\pi}(s^{t+1})]$, being equal, inflation is determined by the four variables, $\hat{r}_k(s^{t+1})$, $\underline{\hat{r}}_k(s^{t+1})$, $\underline{\hat{r}}_b(s^{t+1})$, and $\hat{r}_d(s^t)$. Suppose the return from holding the loan claim is high, an ample capital is accumulated, yielding a strength to the economy and generating inflation. The decrease in the risk of holding the loan claim $\underline{\hat{r}}_k(s^{t+1})$ or the increase in the risk of holding the government bond $\underline{\hat{r}}_b(s^{t+1})$ causes the inflation through the similar mechanism. The rise in the deposit rate $\hat{r}_d(s^t)$ generates inflationary pressures to the economy. While a higher deposit rate prevents the net worth accumulation of the banks, it encourages the banks' purchase of the loan claim relative to the government bond, increasing the capital goods in the economy. The mechanism behind is that under the premise that

$$\underline{r}_k < \underline{r}_b < r_d < r_b < r_k,$$

the excess return to the government bond is sensitive to a change in the deposit rate compared with that to the loan claim. Consequently, the banks' asset allocation tilt towards the real capital, leading to inflation in the economy.

3 Quantitative Analysis

In this section, we investigate the quantitative implication of our model, including the role played by the banks' VaR constraint. Based on the Japanese data, we first estimate the model's parameters and extract the eight structural shocks, the markup shock $\epsilon_{\varepsilon}(s^t)$, the bank's net worth shock $\epsilon_{\gamma}(s^t)$, the shock to the maximum loss of loan claim $\epsilon_{\underline{r}_k}(s^t)$, the shock to the maximum loss of government bond $\epsilon_{\underline{r}_k}(s^t)$, the temporary technology shock $\epsilon_A(s^t)$, the permanent technology shock $\epsilon_Z(s^t)$, and the government expenditure shock $\epsilon_r(s^t)$, using the Bayesian technique. We then explore the model's equilibrium response to these exogenous shocks. In particular, we discuss how the VaR constraint affects the model's dynamics after the shocks. Next, we explore the quantitative contribution of each shocks in explaining the variations of macroeconomic variables, including the banks' asset allocation, inflation, and GDP.

3.1 Data

Our benchmark dataset includes eight time series of the Japanese economy from 1980Q1 to 2007Q4: (1)the labor input¹⁶, corresponding to $l(s^t)$ in the model, (2)the real private investment, based on National Accounts of Japan, corresponding to $k(s^t)-(1-\delta)k(s^{t-1})$ in the model, (3)the sum of treasury discount bills, central government securities and FILP bonds, local government securities, and public corporation securities held by the domestically licensed banks, deflated by the GDP deflator, constructed from Flow of Funds, corresponding to $b(s^t)$ in the model, (4)stock price index of banks deflated by the GDP deflator, based on the data of Tokyo Stock Exchange, corresponding to $n(s^t)$ in the model, (5)the capacity utilization of manufacturing industry based on the indices of industrial production, corresponding to $v(s^t)$ in the model, (6)the GDP deflator, based on National Accounts, corresponding to $P(s^t)$ in the model, and (8)the real GDP based on National Accounts of Japan, corresponding to $x(s^t)$ in the model. All of the series, other than the series (5) and (7), are first differenced. The series (5) and (7) are used in level.

3.2 Prior and Posterior Distribution of the Parameters

The parameter values used for our quantitative analysis are reported in Table 1. The parameter values are quarterly unless otherwise noted. Since our model is a standard New Keynesian model except that our model incorporates the banks' asset portfolio choice, we set some of the parameters to the conventional values. These parameters are reported in Table 1(2).

 $^{^{16}}$ To construct the labor input series, we follow the methodology adopted in Hayashi and Prescott (2002).

Other parameters are estimated by the Bayesian technique, because they are specific in the current model. The third to the fifth columns of Table 1(1) report the prior distribution of the estimated parameters. The last three columns in Table 1(1) display the posterior mean and the confidence intervals of the model parameters.

3.3 Impulse Responses

In this subsection, we investigate the economy's dynamic response to structural shocks. Figure 4 to 11 display the economy's impulse response function to a negative shock to the technology growth rate $\epsilon_Z(s^t)$, a negative shock to the stationary component of technology $\epsilon_A(s^t)$, a negative shock to the banks' surviving probability (the deterioration of the banks' net worth) $\epsilon_{\gamma}(s^t)$, a negative shock to the maximum loss of loan claim $\epsilon_{\underline{r}_k}(s^t)$, a negative shock to the maximum loss of government bond purchase $\epsilon_{\underline{r}_b}(s^t)$, a positive shock to the monetary policy rule $\epsilon_r(s^t)$, a positive shock to the markup $\epsilon_{\varepsilon}(s^t)$, and a negative shock to the government expenditure $\epsilon_G(s^t)$, respectively. All of the equilibrium paths are approximated by the log-linearization around the steady state.

In order to underscore the role played by the VaR constraint in the economic variations, we plot the equilibrium response to a comparable shock under the alternative economy in which the VaR constraint is absent (labeled as "no VaR" and denoted by the line with black circles), together with the equilibrium response under the economy with the VaR constraint (labeled as "benchmark" and denoted by the line with white circles). The economy of "no VaR," is equivalent to our benchmark economy, except that the constraint equation (6) is not effective. Because the banks no longer take the assets' maximum loss or the net worth into consideration, the their investments are independent from these factors, and the expected return are equalized in the equilibrium.

As shown in Figure 4, the permanent downward shift in the technology permanently dampens economic activity, investment, and banks' net worth, yielding the downward pressure to the inflation. Although this shock also generates the inflationary pressure by directly lowering the productivity of the wholesale goods production, this effect is offset by the weakened household's demand caused by the output decline. Compared with the case where the VaR is absent, the shock under the benchmark economy generates quantitatively larger macroeconomic effects. This is because of the endogenous development of the banks' net worth after the adverse shock. Since the technology shock reduces the banks' net worth, the banks under the VaR constraint shrink their leverage and avert the defaults. Consequently, less capital is accumulated in the economy, leading to a further decline in output. Figure 5 shows the economic response when the negative temporary shock strikes the economy. In this case, the response of inflation becomes positive, as the marginal cost increase of the wholesale goods producer, stemming from the lowered productivity, becomes dominant mechanism. The output falls only temporarily, but the magnitude of the fall is greater under the VaR model than the no VaR model.

Figure 6 displays the macroeconomic consequence of the banks' net worth disruption.

Note that in the economy where the VaR constraint is absent, the banks' net worth cannot be a source of the economic fluctuations. The deterioration brings about deflation and recession to the economy. There are two channels in which the net worth shock influences the banks' investment decisions. First, as pointed out by Adrian and Shin (2011), the banks shrinks their balance sheet, reducing the purchase of both the loan claim and government bond. When their net worth is scarce, the banks find it difficult to repay their debt in the case that the maximum loss realize. Second, the banks reduce the purchase of the loan claim disproportionately compared with the government bond. Since the maximum loss of the loan claim is large, the banks tilt their assets toward relatively safer asset, as the VaR constraint is tightened. The both channels lead to a reduction in the aggregate investment on the real capital, dampen the output and suppress the inflation.

Figure 7 displays the equilibrium response to the increase in the maximum loss in holding the loan claim. As indicated by the equation (7), since the loan claim becomes relatively riskier than otherwise, the banks tilt the asset portfolio toward the government bond whose expected return weighted by the maximum loss is now higher. The reduced loan claim implies a decline in the productive capital, leading to a depressionary pressure to the economy.

Figure 8 displays the equilibrium response to the increase in the maximum loss in holding the government bond. In this case, since the government bond holding becomes relatively risky, the banks shift the asset portfolio towards loan claim. They reduce the government bond, increasing the capital goods purchase, following the equation (7), resulting in the output expansion and inflation¹⁷.

The responses of our model economy to the positive markup shock, contractionary monetary policy and government expenditure shock, are, in general, not qualitatively different from those under the standard New Keynesian model, as shown in Figure 9, 10, and 11, respectively. The rise in markup dampens output and raises inflation, and the contractionary policy shocks lower both output and inflation. Similarly to the outcome of the shocks discussed above, because of the endogenous evolvement of the banks' net worth, the markup shocks and government expenditure shocks yield larger quantitative response of the economy under the benchmark model than the no VaR model. After the monetary policy shock, the rise in the policy rate increases the government debt repayment to the banks, help accumulate the banks' net worth. Consequently, adverse effect of the shock is offset.

¹⁷The mechanism behind the expansionary effect of the shocks to the maximum loss of the government bond depends on our setting that the economy is closed economy. Suppose that there is a room for the domestic banks to invest overseas, the capital flow may go overseas, reducing domestic capital accumulation.

3.4 The Role of VaR Constraint

To summarize the effect of incorporating the VaR constraint into the model, we report the steady state values and the theoretical moments of the macroeconomic variables around the steady state, under the benchmark model and the no VaR model, in Table 2. The values of shared parameters in the two models are set to equal values. Because three of the structural shocks, shocks to the banks' net worth $\varepsilon_{\gamma}(s^t)$, shocks to the maximum loss of the loan claim $\varepsilon_{\underline{r}_k}(s^t)$, and shocks to the maximum loss of the government bond $\varepsilon_{\underline{r}_b}(s^t)$, are absent in the no VaR model, we report the standard deviations for the two settings, a case where all of the eight shocks are present and a case where five out of the eight shocks are present in the economy, for comparison.

According to Table 2(1), the output and capital accumulation are smaller, and government bond accumulation is larger at the steady state under the VaR. Because the VaR constraint suppresses the banks' risk taking capacity, the banks tilt toward government bond holding compared with the economy where such constraint is absent. Consequently, capital and output are reduced.

Table 2(2) reports the standard deviation of the growth rate of macroeconomic variables. It is seen that the VaR amplifies the macroeconomic effect of exogenous shocks hitting the economy. Under the VaR model, in addition to the direct effect of shocks, the endogenous development of the banks' net worth leads to a further variations in the banks' leverage and the asset portfolio allocation, giving a higher volatility to the growth rates.

3.5 Contribution of Structural Shocks during Lost Decades

Using the estimated model parameters and extracted shocks, we investigate the role of each structural shock in explaining macroeconomic variations during the lost decades. Table 3 displays the decomposition of the variations in GDP, inflation, and government bond purchase into the eight shocks for the two sub-sample periods: period before the bubble burst (from 1981Q1 to 1990Q4), which we call period I, and the period during and after the banking crisis (from 1997Q1 to 2007Q4), which we call period II.

In explaining GDP variations, temporal shocks to the technology play the key role. During the period II where the average output growth rate is significantly slower than that during the period I, the technology shock remains as the key source of the GDP fluctuations. The contribution of other shocks are quantitatively limited, and most of them contribute negatively to the GDP growth.

In explaining inflation variations, the quantitative role played by the permanent technology shock rather than the temporal technology shock is significant. One reason behind this observation stems from that the inflation rate is forward-looking variable that is less affected by the temporal change in the technology. Comparing the two sub-sample periods, it is seen that the monetary policy shock ranks the second important shock during the period I and the banks' net worth shocks contributes the most in the period II. Both the permanent technology shock and the bank's net worth shock contribute the reduction in inflation from period I to period II. As the impulse response exercise suggests, a slow down of the technology growth, a deterioration of the banks' net worth, and an increase in the maximum loss of the government bond reduce the supply of capital to the economy, by shrinking the banks' leverage and changing the banks' asset portfolio allocation toward government bond, leading to lowered output and inflation.

While the growth rate of government bond purchase increases from period I to period II, the bulk of the variations is accounted for by the shocks to the permanent technology shock. Comparison between period I and period II suggests that a slow down of the technology growth in period II is responsible for the accumulation of the government bond and reduction of the capital. Contribution of the monetary policy shock is large during period I while it is negligible during period II. During period I, the policy rate is well above zero, leaving a room for the government bond purchase to response to a monetary policy shock. During period II, the variation of the policy rate is limited and the government bond variation is affected less by the shock. During period II, the shocks to the banks' net worth and the maximum loss of the loan claim play the important The deterioration of the banks' net worth lowers government bond growth by role. reducing the banks' leverage. The shocks to the maximum loss contribute negatively to the government bond growth by shifting the banks' asset portfolio from the loan claim. Contribution of the shocks to the maximum loss of the government is negligible. Possible interpretation behind this result is that the default risk of the Japanese government is not actualized, and the government bond is regarded as safe asset even after the enforcement of capital requirement.

Our analysis above suggest that shocks to the banks' net worth and the maximum loss of the assets are not the dominant shocks in explaining the output and inflation. This results, however, does not indicate that VaR is not important in explaining the economic fluctuations As discussed in subsection above, the VaR plays the important role in amplifying the macroeconomic effect of exogenous shocks through the endogenous development of the banks' net worth.

4 Conclusion

During the lost decades, the Japanese economy has experienced the long-lasting deflation and the unprecedented government debt. Focusing on that the banks' asset allocation increasingly shift to the government bond holding from the lending to the firms during the period, we propose a theoretical explanation that stresses the implication of the banks' asset allocation to the macroeconomy.

To this end, we introduce the banks' portfolio allocation between the government bond holding and the loan claim to the firm into an otherwise standard New Keynesian model. The banks in the model choose their asset allocations so that the value at risk constraint is not violated. Our model implies that a slow down of the total factor productivity growth rate, the disruption of the banks' net worth after the bubble burst, and an introducing of the capital requirement, encourage the banks to accumulate the government bond holding, reducing the loan claim to the firms. As the real capital investment is reduced, the output growth slows down, leading to deflation.

Our results have policy implications when the economic recession is associated with the reduction of banks' risk taking behaviors. In the economy where the uncertainty about capital investment plays as the key obstacle, the policy aiming to reduce the minimum loss of investment may be effective.

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(Figure1)



Accumulation of Government Debt and Deflation (1)Government bond outstanding over GDP

(2)Inflation dynamics



(note) Statistics Released from the Ministry of Finance and Cabinet Office.



Banks' Asset Allocation





(3) Portion of government bond outstanding held by domestically licensed banks



(note1) Statistics Released from the Ministry of Finance and the Bank of Japan. (note2) Dotted line in (3) displays the portion of government bond held by the banks including postal savings.





Economic Response to Permanent Technology Shock

(note) Vertical axis denotes the deviation from the steady state and horizontal axis denotes the number of quarter after the shock.



Economic Response to Temporal Technology Shock

(note) Vertical axis denotes the deviation from the steady state and horizontal axis denotes the number of quarter after the shock.





(note) Vertical axis denotes the deviation from the steady state and horizontal axis denotes the number of quarter after the shock.





(note) Vertical axis denotes the deviation from the steady state and horizontal axis denotes the number of quarter after the shock.





(note) Vertical axis denotes the deviation from the steady state and horizontal axis denotes the number of quarter after the shock.

Economic Response to Markup Shock



(note) Vertical axis denotes the deviation from the steady state and horizontal axis denotes the number of quarter after the shock.



Economic Response to Monetary Policy Shock

(note) Vertical axis denotes the deviation from the steady state and horizontal axis denotes the number of quarter after the shock.



Economic Response to Fiscal Expenditure Shock

(note) Vertical axis denotes the deviation from the steady state and horizontal axis denotes the number of quarter after the shock.

Parameter of Model

(1) Estimated Parameters

Daromatar		Prior	Prior	Duion Std	Posterior		
	Parameter	Dist.	Mean	Prior Sta.	mean	5%	95%
κ_p	Adjustment Cost of Price	norm	4	1.5	6.09	6.08	6.11
ϕ	Monetary Policy Rule (coefficient for inflation)	norm	1.5	0.125	1.19	1.19	1.19
$ ho_{\scriptscriptstyle A}$	Monetary Policy Rule AR	norm	0.75	0.125	0.93	0.93	0.93
γ	Survival Probability of Banks	norm	0.9	0.01	0.90	0.90	0.90
r_k	Maximum Loss of Loan Claim	norm	0.7	0.01	0.70	0.70	0.70
r_b	Maximum Loss of Bond Purchase	norm	0.95	0.01	0.95	0.95	0.95
Т	Fiscal Policy Rule	norm	0.01	0.01	0.003	0.002	0.004
<u>Ψ</u>	Fiscal Policy Rule (coefficient for debt)	norm	2	0.125	1.97	1.96	1.97
G / X	Government Expenditure Share	norm	0.17	0.1	0.17	0.16	0.17
ε	Preference about Goods Variety	norm	7	0.125	6.97	6.97	7.00
ρ_7	Permanent Technology Shock AR	beta	0.75	0.22	0.98	0.98	0.98
ρ_{A}	Temporal Technology Shock AR	beta	0.129	0.22	0.64	0.63	0.64
ρ_{r}	Banks' Net Worth Shock AR	beta	0.75	0.22	0.98	0.98	0.98
ρ_r	Shock to Maximum Loss of Loan Claim AR	beta	0.75	0.22	0.83	0.83	0.83
$ ho_{r_b}$	Shock to Maximum Loss of Bond Purchase AR	beta	0.129	0.22	0.74	0.74	0.75
$ ho_{_G}$	Government Expenditure Shock AR	beta	0.129	0.22	0.99	0.99	0.99
$\sigma_{_r}$	Monetary Policy Shock SD	invg	0.009	Inf	0.005	0.002	0.009
$\sigma_{ m z}$	Permanent Technology Shock SD	invg	0.035	Inf	0.005	0.005	0.005
$\sigma_{\scriptscriptstyle A}$	Temporary Technology Shock SD	invg	0.009	Inf	0.083	0.079	0.086
σ_{ϵ}	Price Markup Shock SD	invg	0.009	Inf	2.646	2.612	2.617
$\sigma_{_{\gamma}}$	Banks' Net Worth Shock SD	invg	0.035	Inf	0.039	0.034	0.042
σ_{r_k}	Shock to Maximum Loss of Loan Claim SD	invg	0.009	Inf	0.154	0.134	0.174
σ_{r}	Shock to Maximum Loss of Bond Purchase SD	invg	0.035	Inf	0.005	0.002	0.009
$\sigma_{G}^{"}$	Government Expenditure Shock SD	invg	0.035	Inf	0.280	0.260	0.320

(2)Calibrated Parameter

α	Capital Share in Final Goods Production	0.35		
β	Households' Discount Factor	0.99		
η	Households' Preference over Leisure	0.5		
δ	Depreciation Rate	0.025		

Effect of VaR constraint on the economy

(1) Comparison of the steady state value of macroeconomic variables

		Economy with VaR	Economy without VaR
У	output at steady state	1.5	2.3
k	capital at steady state	6.6	19.3
b	government bond at steady state	1.5	0.9
<i>b / k</i>	relative size of government bond w.r.t. capital	0.2	0.1

(2) Variation of macroeconomic variables around the steady state

		Economy	with VaR	Economy without VaR
$std(\Delta y)$	standard deviation of output growth	0.30	[0.26]	0.20
$std(\Delta k)$	standard deviation of capital growth	0.07	[0.04]	0.02
$std(\Delta b)$	standard deviation of growth of government bond	0.15	[0.12]	0.11
$std(\Delta(b+k))$	standard deviation of leverage	0.07	[0.05]	0.03
$std(\pi)$	standard deviation of inflation rate	0.15	[0.08]	0.07

(note) [] denotes standard deviation of the variable when shocks to the banks' net worth and maximum loss of assets are absent from the econom

Contribution of Structural Shocks in Macroeconomic Variations

	(a) before 1990 (1981Q1-1990Q4)	(b) after banking crisis (1997Q4-2007Q4)	(b)-(a)	full sample (1981Q1-2007Q4)
average growth rate (%)	0.4	0.1		
Permanent Technology Shock	5.6	2.1	(▼)	6.7
Temporary Technology Shock	68.8	82.0	(▼)	72.0
Banks' Net Worth Shock	3.8	3.8	(▼)	4.3
Shock to Maximum Loss of Capital	2.7	2.5	(▼)	2.7
Shock to Maximum Loss of Bond	0.0	0.0	(▼)	0.0
Other Shock	14.8	6.7	(▼)	10.5
Monetary Policy Shock	1.3	0.0	(▼)	0.8
Government Expenditure Shock	3.1	2.9	(Δ)	3.0

(1) Output Growth (Ouarterly)

(2) Inflation (Quarterly)

	(a) before 1990	(b)after banking crisis	(b)-(a)	full sample
	(1981Q1-1990Q4)	(1997Q4-2007Q4)		(1981Q1-2007Q4)
average growth rate $(\%)$	0.3	-0.3		
Permanent Technology Shock	49.7	30.0	(▼)	41.9
Temporary Technology Shock	5.0	3.2	(Δ)	6.2
Banks' Net Worth Shock	2.3	42.1	(▼)	13.0
Shock to Maximum Loss of Capital	4.1	14.3	(Δ)	6.0
Shock to Maximum Loss of Bond	0.0	0.0	(▼)	0.0
Other Shock	11.4	5.8	(▼)	9.3
Monetary Policy Shock	26.6	3.6	(Δ)	22.8
Government Expenditure Shock	0.9	1.0	(Δ)	0.9

(3) Bond Purchase Growth (Quarterly)

	(a) before 1990	(b)after banking crisis	(b)-(a)	full sample
	(1981Q1-1990Q4)	(1997Q4-2007Q4)		(1981Q1-2007Q4)
average growth rate $(\%)$	-0.2	0.4		
Permanent Technology Shock	23.1	32.5	(Δ)	35.0
Temporary Technology Shock	9.7	16.8	(▼)	13.2
Banks' Net Worth Shock	3.4	20.4	(▼)	5.9
Shock to Maximum Loss of Capital	5.7	19.9	(Δ)	5.8
Shock to Maximum Loss of Bond	0.0	0.0	(▼)	0.0
Other Shock	17.2	4.2	(▼)	9.5
Monetary Policy Shock	31.6	1.9	(▼)	23.0
Government Expenditure Shock	9.2	4.2	(Δ)	7.5

(note1) \triangle ($\mathbf{\nabla}$) indicates that the corresponding shock contributes positively (negatively) during period (b) relative to period (note2) Other Shock includes contribution of initial values as well as contribution of markup shocks.

The contribution of the former component is, however, negligible compared with the latter component.