Debt-Ridden Borrowers and Productivity Slowdown

Keiichiro Kobayashi and Daichi Shirai

Institute of Economic Research, Hitotsubashi University
The Canon Institute for Global Studies

May, 2012

Motivation 1: Productivity slowdown after a financial crisis

- Decade after a financial crisis
  - Low growth (Reinhart and Rogoff 2009, Reinhart and Reinhart 2010)
  - Low productivity (Ohanian 2001, Hayashi and Prescott 2002, Kehoe and Prescott 2007, Kobayashi and Inaba 2006, Fukao and Miyagawa 2008)
- Why productivity slows down persistently after a financial crisis?

Decade after a crisis: the 1990s Japan

	HP	KI	JIP2011
1971–80	0.83		1.68
1981–90	1.93	2.06	1.39
1991–2000	0.36	0.35	0.04
2001–2007		0.48	1.13

Note: HP, KI, JIP2011 are from updated versions of Hayashi and Prescott(2002), Kobayashi and Inaba (2006), and Fukao and Miyagawa (2008).

Table: TFP growth rate in Japan

Motivation 2: Cause of financial shocks

- Great Recession
  - Financial shocks (Jermann and Quadrini 2012)
  - Risk shock (Christiano, Motto, and Rostagno 2010)
  - Shock to quality of capital (Gertler and Kiyotaki 2009)
  - These shocks tightened the financial constraints after a financial crisis.
- It is assumed that these financial shocks are *exogenous*.
- What causes these shocks?
  (Why are financial constraints tightened?)

Hypothesis

1. Exogenous shock (e.g., bubble collapse) makes a large fraction of firms default or debt-ridden.
2. Borrowing constraints for firms with excessive debt (or debt-ridden firms) are endogenously tightened.
   - We can analyze the borrowing constraints for working capital loans of debt-ridden firms by Modification of Jermann-Quadrini's bargaining
3. Tightened borrowing constraint lowers the aggregate productivity.
   - Tighter constraint on working capital loan lowers observed TFP (Chari, Kehoe, and McGrattan)

Jermann-Quadrini (JQ)'s bargaining

- Borrowing constraint derived from bargaining
  - Borrowing is bounded by limited enforceability of debt contract.
  - Firm can default on the debt.
  - If firm defaults, the firm and the lender renegotiate on repayment f.
    - If renegotiation breaks down the firm is liquidated and the lender confiscates the collateral, $\phi q_t k_t$.
    - If they agree the firm pays f and continues as a normal firm.
    - Nash bargaining

$$\max_f (V - f)^\sigma (f - \phi q_t k_t)^{1-\sigma}$$

    - In the limit of $\sigma \to 1$ the bargaining outcome is $f = \phi q_t k_t$
  - No-default condition: $m + b \leq f = \phi q_t k_t$.

What we do: Modification of JQ's bargaining to derive borrowing constraint of debt-ridden firm

- We define "debt-ridden firm" as an intermediate status between a normal firm and liquidation.
- Debt-ridden firm = defaulted already and not liquidated yet
- Institutional setting motivated by Japan's 1990s:
  - The status of debt-ridden firm is not derived as outcome of optimal contracting, but is an institutional setting.
  - Lender never releases firm if it defaults on the debt.
  - Defaulter must become either debt-ridden firm or liquidated. It cannot go back to a normal firm unless it repays all the original debt.

What we do: Definition of debt-ridden firm

- Debt-ridden firm
  - Lender can decide whether the firm continue next period or not.
  - Lender allows firm to continue if it promises to pay continuation fee d_{t+1}.
  - Continuation fee d_t determined by the bargaining between the debt-ridden firm and the lender.

Summary

Model Setup:

- An exogenous shock makes firms default.
  - Redistribution shock that changes the amount of debt

$$b = b' + \Delta.$$

- If firm defaults, it must become either debt-ridden firm or liquidated. It cannot return to normal firm unless repaying all original debt.

Summary

- Result 1: Borrowing constraint for debt-ridden firms
  - The borrowing constraint for working capital loan is tightened. It is tighter for debt-ridden firms than for normal firms.
    - Counterintuitive. Lenders should have more influence on debt-ridden firms than on normal firms.
    - Lenders could obtain more from debt-ridden firms than from normal firms in the bargaining.
    - Then the borrowing constraint for debt-ridden firms could be looser than for normal firms.
    - Our analysis shows the opposite.

Summary

- Result 2: Tight borrowing constraint lowers aggregate productivity
  - Mass default $\Rightarrow$ Emergence of debt-ridden firms
  - Tighter borrowing constraint on working capital
    - reduces the demand for inputs by debt-ridden firm, leading to inefficient production (Direct effect)
    - raises price of capital q_t because of higher demand for collateralizable asset by debt-ridden firms (Congestion effect)
      $\Rightarrow$ Higher cost for entry of new firms
      $\Rightarrow$ No entry of new firms
      $\Rightarrow$ Aggregate productivity becomes permanently low

Summary

- Result 2': Endogenous growth version of our model $\Rightarrow$ Zero Growth Path
  - We assume externality that enables endogenous growth.
  - Tight borrowing constraint raises the price of collateralizable capital q_t.
  - Higher q_t depresses the new entry of firms
  - No new entry leads to no growth in productivity
    $\Rightarrow$ Zero productivity growth
  - Externality induces multiple equilibria
    - Zero Growth Path (ZGP)
    - Balanced Growth Path (BGP)

Literature

- Debt overhang (Myers 1977, Lamont 1995, Philippon 2009)
  Debt holder is different from a lender of new money.
  - Debt holder *is* the lender of new money in our model.
- Zombie lending (Caballero, Hoshi, Kashyap 2008)
  Zombie firms are intrinsically unproductive. Zombie lending is inefficient subsidy from banks to unproductive firms.
  - Debt-ridden firms are not intrinsically unproductive in our model.
- Bubbles (Hirano and Yanagawa 2010, Aoki and Nikolov 2011)
  Collapse of bubble tightens borrowing constraints.
  - We consider default on debt and bargaining after default.

Model

- Expanding Variety Model (Rivera-Batiz, Romer 1991)
  - Intermediate good $i \in [0, N_t]$: Monopolistic competition.
  - R&D investment I_t expands N_t.
  - Capital $K = 1$ and labor $L = 1$ are fixed supply.
- Financial Friction: Jermann, Quadrini (2006, 2012)
- Household:

$$\max \quad E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln C_t \right],$$

$$\text{subject to} \quad C_t + \frac{b_{t+1}}{1+r_t} + I_t \leq w_t L + \int_0^{N_t} \pi_{it} di + b_t,$$

$$N_{t+1} = (1-\delta) N_t + \chi I_t,$$

$$I_t \geq 0.$$

Model

Household: R&D investment $I_t = 0$ if $\chi V_{nt} < 1$ and $I_t > 0$ if $\chi V_{nt} = 1$, where

- V_{nt} is the value of a new firm,
- χV_{nt} is the value of one unit of R&D investment.

Final good is produced in the competitive market:

$$\max \quad Y_t - \int_0^{N_t} p_{it} x_{it} di - w_t L_t,$$

$$\text{subject to} \quad Y_t = \frac{1}{\eta} \left(\int_0^{N_t} x_{it}^{\eta} di \right) L^{1-\eta}.$$

Model

- Intermediate good i produced by firm i:

$$x_{it} = A_{it} k_{it}^{\alpha} m_{it}^{1-\alpha}.$$

- Capital (land) is fixed supply: $\int_0^{N_t} k_{it} di = K$. Price is q_t.
- Material input m_{it} is the final good.

Demand $\quad p_{it} = p(x_{it}) = L^{1-\eta} x_{it}^{\eta-1}$,

Productivity parameter

$$A_{it} = A \quad \text{for all } i \text{ and } t.$$

Redistribution shock in this economy

- Firm i borrows inter-temporal debt $\frac{b'_{it+1}}{1+r_t}$ at the end of period t
- Redistribution shock changes the amount to be repaid from b' to b:

$$b_{it+1} = b'_{it+1} + \Delta_{it+1}.$$

- PDF of the redistribution shock Δ_{it} is known.

Bellman equation for normal firms

- Bellman eq: k and b' chosen at t, and m chosen at $t+1$

$$V_{nt} =$$

$$\max_{k,b'} \frac{b'}{1+r} - qk + E\left[\max_{m} \frac{\beta\lambda'}{\lambda}\left\{p(x)x - m - b + q'k + \tilde{V}\right\}\right], \tag{1}$$

$$\text{s. t.} \quad p(x) = L^{1-\eta}x^{\eta}, \tag{2}$$

$$x = A_{t+1}k^{\alpha}m^{1-\alpha}, \tag{3}$$

$$b = b' + \Delta, \tag{4}$$

$$m \leq \phi q_{t+1}k_{t+1} + V_{nt+1} - V_{zt+1} - b. \tag{5}$$

Derivation of Borrowing Constraint

- Borrowing constraint for normal firm:

$$m \leq \phi q_{t+1} k_{t+1} + V_{nt+1} - V_{zt+1} - b.$$

- Borrowing is bounded by limited enforceability of debt contract. $\Rightarrow$ Firm can default on their obligation.
- Lender cannot forgive debt of defaulter
  $\Rightarrow$ If defaults, either firm continues as debt-ridden or it is liquidated, unless it repays all the original debt.

What they do after default?

- Lender and firm have two stage bargaining after default:
  - First Bargaining in the middle of period on repayment of current debt, f $(< m + b)$.
    - If bargaining breaks down $\Rightarrow$ Liquidation. Lender obtains $(\phi + \psi)q_t k_t$
  - Second Bargaining at the end of period on continuation fee, d.
    - If bargaining breaks down $\Rightarrow$ Liquidation. Lender obtains $\psi q_t k_t$
  - A part of collateral $\phi q_t k_t$ is diverted by firm before the second stage bargaining.

Structure of Two stage bargaining after default

- First stage in the middle of period on repayment f.
  - If they do not agree, firm is liquidated.
    - Lender obtains $(\phi + \psi)q_t k_t$
    - Firm obtains $p(x)x + (1 - \phi - \psi)q_t k_t$
  - If they agree, firm continue as debt-ridden firm.
    - Lender obtains $f + D_t$
    - Firm obtains $p(x)x + q_t k_t - f + V_z$
- Second stage at the end of period on continuation fee d
  - If they do not agree, firm is liquidated.
    - Lender obtains $\psi q_t k_t$
    - Firm obtains $(1 - \psi)q_t k_t$
  - If they agree, firm continue as debt-ridden firm.
    - Lender obtains $D_t = \beta E\left[\frac{\lambda_{t+1}}{\lambda_t}\{d_{t+1} + D_{t+1}\}\right]$
    - Firm obtains $q_t k_t + V_z$

Bargaining outcome

- We assume that firm has all the bargaining power.
  $\Rightarrow$ Lender's payoff equals the liquidation value at each stage.

$$\text{First stage:} \quad f + D_t = (\phi + \psi) q_t k_t,$$

$$\text{Second stage:} \quad D_t = \psi q_t k_t$$

- First stage bargaining:

$$f = \phi q_t k_t.$$

- Second stage bargaining:

$$D_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \{ d_{t+1} + D_{t+1} \} \right] = \psi q_t k_t.$$

No default condition $\Rightarrow$ Borrowing constraint

- If a normal firm defaults
  - Firm pays $f = \phi qk$.
  - Firm loses $V_{nt+1} - V_{zt+1}$, because it becomes debt-ridden firm inevitably.
- If the normal firm does not default
  - Firm pays $m + b$ and continues operation as a normal firm.
- No default condition for a normal firm:

$$m + b \leq f + V_{nt+1} - V_{zt+1} = \phi qk + V_{nt+1} - V_{zt+1}.$$

Appendix: Comparison with the Jermann-Quadrini model

- Jermann-Quadrini: Lender forgive debt if firm defaults.
  - Firm goes back to be a normal firm if it repays f.
- Our model: Lender cannot forgive debt once firm defaults.
  - Firm cannot be released even if it repays f if $f < m + b$.
  - Firm can be released only if it repays the original debt $m + b$.
  - Defaulter must become either debt-ridden or liquidated.
- Borrowing limit in our model:
  - Borrowing limit for normal firms is higher than JQ, because normal firm lose more by defaulting.
  - Borrowing limit is severely low for debt-ridden firm.

Appendix: Derivation of Borrowing Constraint

Assumption 1

- After receiving $p(x_t)x_t$ firm i can default on the debt $m_t + b_t$ and renegotiate on repayment f_t. (Bargaining 1 over f_t)
  1. Once a firm defaults, it cannot return to a normal firm unless it pays all original debt $m_t + b_t$.
  2. If the lender liquidates the firm she obtains $(\phi + \psi)q_t k_t$.
  3. If agreement is $f_t \geq m_t + b_t$, lender loses liquidation right.
  4. If agreement is $f_t < m_t + b_t$, lender retains liquidation right. At the end of t, they negotiate on d_{t+1}. (Bargaining 2 over d_{t+1})
     - If they agree on d_{t+1} firm continue in $t + 1$.
     - If they do not agree on d_{t+1}, firm is liquidated at end of t.
       - Lender confiscates only $\psi q_t k_t$.
       - $\phi q_t k_t$ is hidden by the firm.

Appendix: Derivation of Borrowing Constraint

- Assumption 1-1 $\Rightarrow$ Debt forgiveness is infeasible.
- Bargaining over f_t after default
  $\Rightarrow$ Debt-ridden firm or Liquidation?
  - If they agree on f_t, the firm continues as a debt-ridden firm.
    - Firm obtains $p(x_t)x_t + q_t k_t - f_t + V_{zt}$.
    - Lender obtains $f_t + D_t$.
    - V_{zt}: present value of dividend flow of debt-ridden firm
    - D_t: present value of repayment flow to the lender
    - V_{zt} and D_t are specified later.
  - If they do not agree on f_t, the firm is liquidated.
    - Firm obtains $p(x_t)x_t + (1 - \phi - \psi)q_t k_t$.
    - Lender obtains $(\phi + \psi)q_t k_t$.

Appendix: Derivation of Borrowing Constraint

- If the firm defaults,
  - Nash bargaining over f_t:

$$\max_{f_t}[(\phi+\psi)q_t k_t + V_{zt} - f_t]^{\sigma}[f_t + D_t - (\phi+\psi)q_t k_t]^{1-\sigma}.$$

  - With $\sigma = 1$, repayment is $f_t = (\phi+\psi)q_t k_t - D_t$
  - Firm becomes a debt-ridden firm.
  - We show later that $D_t = \psi q_t k_t$
    $\Rightarrow f_t = \phi q_t k_t$.

Appendix: Derivation of Borrowing Constraint

- No default condition for the firm:
  - If default, firm obtains
    $p(x_t)x_t - f_t + q_t k_t + V_{zt} = p(x_t)x_t + (1-\phi)q_t k_t + V_{zt}$.
  - If no default, firm obtains $p(x_t)x_t + q_t k_t - m_t - b_t + V_{nt}$.
  - No default condition $\Leftrightarrow$

$$p(x_t)x_t + (1-\phi)q_t k_t + V_{zt} \leq p(x_t)x_t + q_t k_t - m_t - b_t + V_{nt}.$$

- Borrowing constraint

$$m_t + b_t \leq \phi q_t k_t + V_{nt} - V_{zt}.$$

Default and debt-ridden firm

- Suppose that Δ_{it} is very large such that

$$\phi q_t k_{it} + V_{nt} - V_{zt} < b_{it}.$$

- In this case firm i cannot obtain working capital m_{it} and produce nothing.
- Firm i defaults on b_{it} and becomes a debt-ridden firm.

Bellman equation for debt-ridden firm

- Given $\{d_{t+j}\}_{j=1}^{\infty}$, firm chooses k in t and m in $t+1$.

$$V_{zt} =$$

$$\max_k -q_t k + E_t \left[\max_m \beta \frac{\lambda_{t+1}}{\lambda_t} \{p(x)x - m - d_{t+1} + q_{t+1}k + V_{zt+1}\} \right], \tag{6}$$

$$\text{subject to} \quad x = A_{t+1} k^{\alpha} m^{1-\alpha}, \tag{7}$$

$$m + d_{t+1} \leq \phi q_{t+1} k_{t+1}. \tag{8}$$

Derivation of Borrowing Constraint

- Debt-ridden firm can default on $m + d$.
- If debt-ridden firm defaults, the firm and the lender enter the two stage bargaining over repayment f_t and continuation fee d_{t+1}.
- The bargaining structure is identical to that for normal firms $\Rightarrow$ Bargaining outcome is
  - First stage bargaining:

$$f = \phi q_t k_t.$$

  - Second stage bargaining:

$$D_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \{ d_{t+1} + D_{t+1} \} \right] = \psi q_t k_t.$$

No default condition $\Rightarrow$ Borrowing constraint

- If debt-ridden firm defaults on $m + d$:
  - Firm pays $f = \phi qk$.
  - Firm continues operation as a debt-ridden firm.
- If firm does not default
  - Firm pays $m_t + d_t$.
  - Firm continues operation as debt-ridden firm.
- No default condition for a debt-ridden firm:

$$m_t + d_t \leq f = \phi qk.$$

Appendix: Second stage bargaining over d_{t+1}

- Bargaining over d_{t+1} at the end of t:
  - If agree, firm obtains $V_{zt}(d_{t+1})$ and lender obtains $D_t \equiv \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \{d_{t+1} + D_{t+1}\} \right]$.
  - If do not agree, firm obtains $(1-\psi)q_t k_t$ and lender obtains $\psi q_t k_t$ by liquidation.
  - Nash bargaining on d_{t+1} is

$$\max_{d_{t+1}} \{V_{zt}(d) - (1-\psi)q_t k_t\}^{\sigma} \left\{ \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \{d + D_{t+1}\} \right] - \psi q_t k_t \right\}^{1-\sigma}$$

  - With $\sigma = 1$, the outcome is

$$D_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \{d_{t+1} + D_{t+1}\} \right] = \psi q_t k_t.$$

Appendix: Second stage bargaining over d_{t+1}

- The values of d_t and D_t satisfy

$$D_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left\{ d_{t+1} + D_{t+1} \right\} \right] = \psi q_t k_t.$$

- Given $\overline{k}_{t+1}$,

$$d_{t+1} = \frac{\psi q_t k_t}{\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right]} - \frac{E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \psi q_{t+1} \overline{k}_{t+1} \right]}{E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right]}.$$

Appendix: First stage bargaining over m_t

- After d_{t+1} is agreed at the end of t, firm continues in $t+1$.
- Firm borrows m_{zt+1} and produces $x_{t+1} = A_{t+1}k_{t+1}^{\alpha}m_{zt+1}^{1-\alpha}$.
- After receiving $p(x_{t+1})x_{t+1}$, firm can default on $m_{zt+1} + d_{t+1}$.
- Renegotiation on repayment f_t:
  - If agree, firm obtains $p(x_{t+1})x_{t+1} + q_{t+1}k_{t+1} + V_{zt+1} - f$ and lender obtains $f + D_{t+1}$.
  - If do not agree, firm obtains $p(x_{t+1})x_{t+1} + (1-\phi-\psi)q_{t+1}k_{t+1}$ and lender obtains $(\phi+\psi)q_{t+1}k_{t+1}$.

Appendix: First stage bargaining over m_t

- Nash bargaining

$$\max_f \; \{(\phi+\psi)q_{t+1}k_{t+1} + V_{zt+1} - f\}^{\sigma} \cdot \{f + D_{t+1} - (\phi+\psi)q_{t+1}k_{t+1}\}^{1-\sigma}$$

- With $\sigma = 1$, repayment is $f = (\phi+\psi)q_{t+1}k_{t+1} - D_{t+1}$.
- With $D_{t+1} = \psi q_{t+1}k_{t+1}$, this is written as $f = \phi q_{t+1}k_{t+1}$.

Appendix: First stage bargaining over m_t

- No renegotiation condition: $m_{zt+1} + d_{t+1} \leq f_t$.

$$m_{zt+1} + d_{t+1} \leq \phi q_{t+1} k_{t+1}.$$

Borrowing constraints for normal firm and debt-ridden firm

- Normal firm:

$$m_{t+1} + b_{t+1} \leq \phi q_{t+1} k_{t+1} + V_{nt+1} - V_{zt+1}.$$

Firm chooses b_{t+1} to maximize V_{nt}

- Debt-ridden firm:

$$m_{t+1} + d_{t+1} \leq \phi q_{t+1} k_{t+1}.$$

d_{t+1} is determined by the bargaining bw lender and firm.

- In our simulation:
  - Borrowing constraint is non-binding for normal firms.
  - Borrowing constraint is binding for debt-ridden firms.

Appendix: What if debt-ridden firms can make savings?

- Assumption 2: Debt-ridden firms cannot make savings s_t.
- Appendix C
  - We assume that firms can accumulate s_t.
  - Assumption: Lender can confiscate s_t if she liquidates debt-ridden firm at the end of t.
  - Focus on a deterministic equilibrium
  - It is shown that cost and gain of accumulating s_t cancel out with each other.
  - Debt-ridden firm has no incentive to accumulate s_t.
  - No savings in the deterministic equilibrium. (Corporate savings are neutral in equilibrium.)

Equilibrium with debt-ridden firms

- Initial steady state:
  $I_t = 0$, $N_t = N$, $k_t = \frac{K}{N}$. Parameter is $\chi = \chi_{SS}$.
  All firms are normal and there is no debt-ridden firms.
- Mass default due to one-time redistribution shock at $t = 0$:
  At $t = 0$, firm $i \in [0, Z]$ default on the debt, while firm $i \in [Z, N]$ do not default. $(0 < Z < 1)$
  - Firm $i \in [0, Z] \Rightarrow$ Debt-ridden firms
  - Firm $i \in [Z, N] \Rightarrow$ Normal firms
- No more shock $\Rightarrow$ Steady state equilibrium

Appendix: Equilibrium with debt-ridden firms

Parameters:

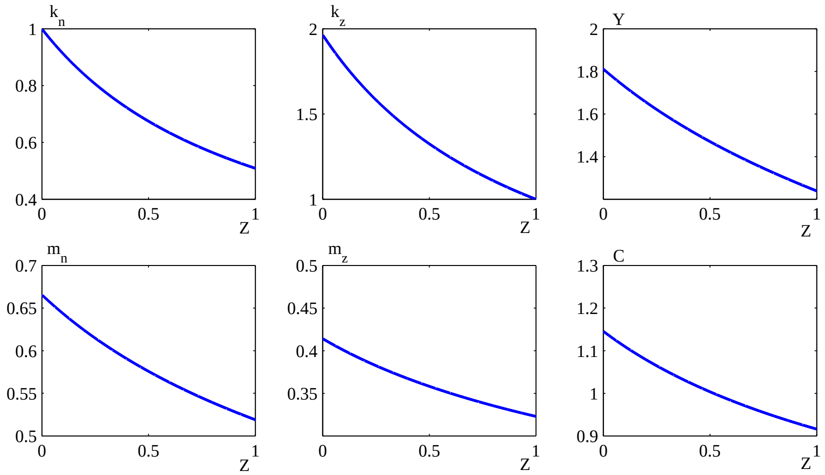
$$\alpha = 0.25, \quad \beta = 0.9, \quad \delta = 0, \quad \phi = 0.15, \quad \eta = 0.7, \quad A = 1.9048,$$
$$K = 1, \quad L = 1, \quad \psi = 0.4, \quad \chi_{SS}^{-1} = 3.4234, \quad V = \chi_{BGP}^{-1} = 2.9307.$$

Equilibrium with debt-ridden firms

- There is no R&D, i.e., $I_t = 0$, in equilibrium after default. (Verified later.)

  $\Rightarrow$ Equilibrium after default is a steady state equilibrium.

Equilibrium with debt-ridden firms



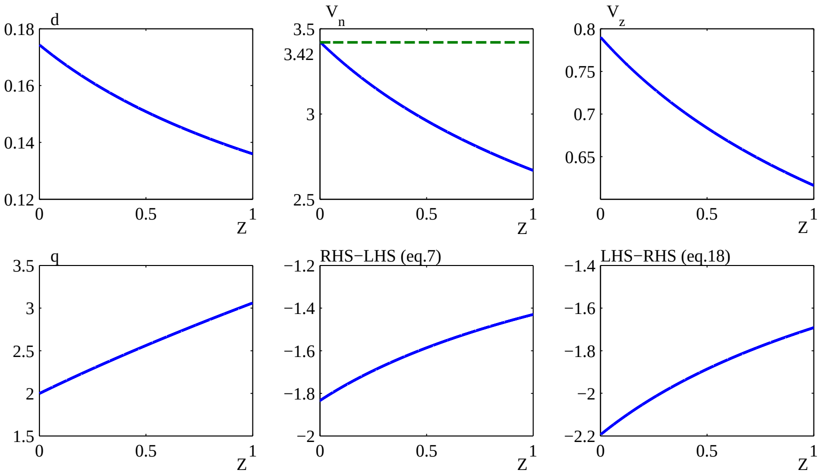
Equilibrium with debt-ridden firms

- Observed TFP, $\tilde{A}$, is decreasing in Z:

$$\tilde{A} = \frac{C(Z)}{K^{\theta} L^{1-\theta}},$$

where $C(Z)$ is consumption and $\theta = \frac{\alpha\eta}{1-(1-\alpha)\eta}$.

Equilibrium with debt-ridden firms



Equilibrium with debt-ridden firms

- V_{nt} decreases as Z increases, and $\chi_{SS} V_{nt} < 1$ for $Z > 0$.
  - Tighter borrowing constraint

    $\Rightarrow$ Debt-ridden firms buy k aggressively.
  - Price q rises. (Congestion effect)
  - Higher cost of k decreases the value of new entry V_{nt}.
- $\chi_{SS} V_{nt} < 1$

  $\Rightarrow$ $I_t = 0$ and N_t is constant over time.

  ($I_t > 0$ if $\chi_{SS} V_{nt} \geq 1$)

  $\Rightarrow$ Steady state equilibrium.

Policy implication

- If all defaulters are liquidated, the economy goes back to efficient steady state by new entry of firms.
- If debt forgiveness is allowed institutionally TFP goes back to normal immediately. (The equilibrium borrowing constraint for normal firms would be tighter. Appendix D.)

Endogenous growth and zero growth path (ZGP)

- Assumption 3
  Externality on productivity from variety: $A_t = \hat{A}N_t^{\alpha}$.
- We set parameter: $\chi = \chi_{BGP} \ \ (> \chi_{SS})$.

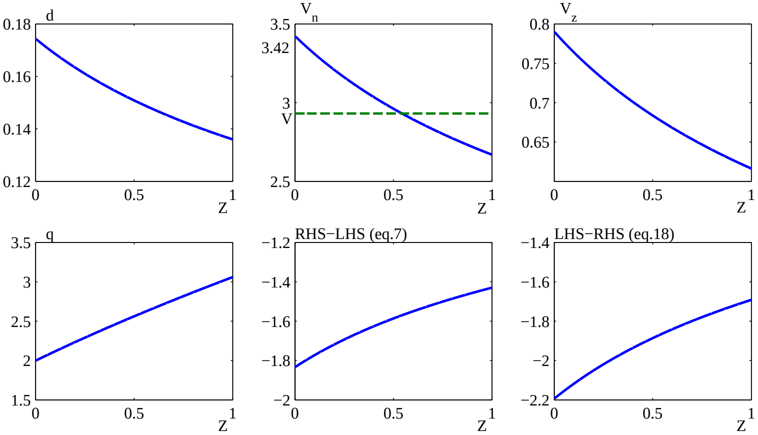
- Under this assumption, the equilibrium without debt-ridden firms is the balanced growth path (BGP) with $\frac{N_{t+1}}{N_t} = 1 + g$.
- Same parameters as before, except for χ; The value of χ is chosen such that $g = 0.01$ in the BGP.

Endogenous growth and zero growth path (ZGP)

- Emergence of debt-ridden firm decreases V_{nt}.
- Condition for N_t to grow is $\chi V_{nt} = 1$.
- If Z is large than $\underline{Z}$, then the equilibrium is ZGP
  If $Z > \underline{Z}$ then V_{nt} become so small that $\chi_{BGP} V_{nt} < 1$.
  - If $\chi V_{nt} < 1$, no R&D investment takes place.
  - N_t does not grow
  - No productivity growth in equilibrium $\Rightarrow$ ZGP

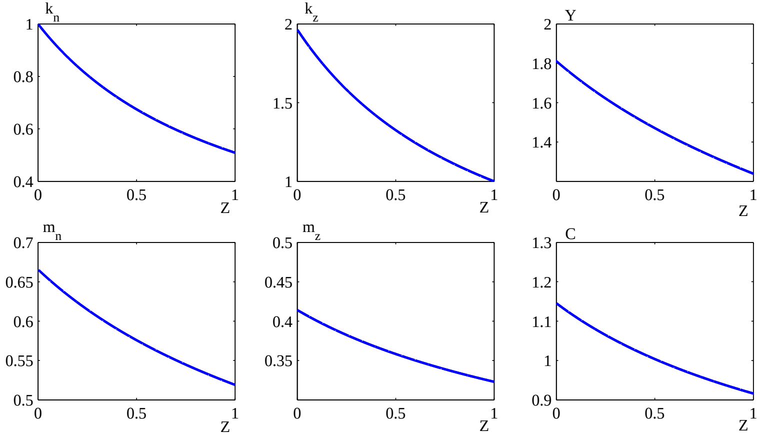
Endogenous growth and zero growth path (ZGP)

- If $Z > 0.5439$, then $V_{nt} < V$ and equilibrium is ZGP.
- Variables in Figure are those in ZGP.



Endogenous growth and zero growth path (ZGP)

- If $Z > 0.5439$, then $V_{nt} < V$ and equilibrium is ZGP.
- Variables in Figure are those in ZGP.



Conclusion

- We consider a model in which
  - exogenous shock makes a large proportion of firms default, and
  - defaulted firms cannot go back to normal firms unless they repay all original debt. Defaulted firms continue as debt-ridden firms,
    (c.f. defaulted firms can go back to normal after renegotiation in Jermann-Quadrini model.)
- In this economy,
  - borrowing constraints are tighter for debt-ridden firms,
  - TFP decreases as debt-ridden firms increase,
  - a sufficient increase of debt-ridden firms lowers TFP growth to zero in the endogenous growth version of our model.

Future research

- Embed this model into a DSGE model with variable capital and labor.
- Money
- Various policy assessments