Dynamics of Firms and Trade in General Equilibrium

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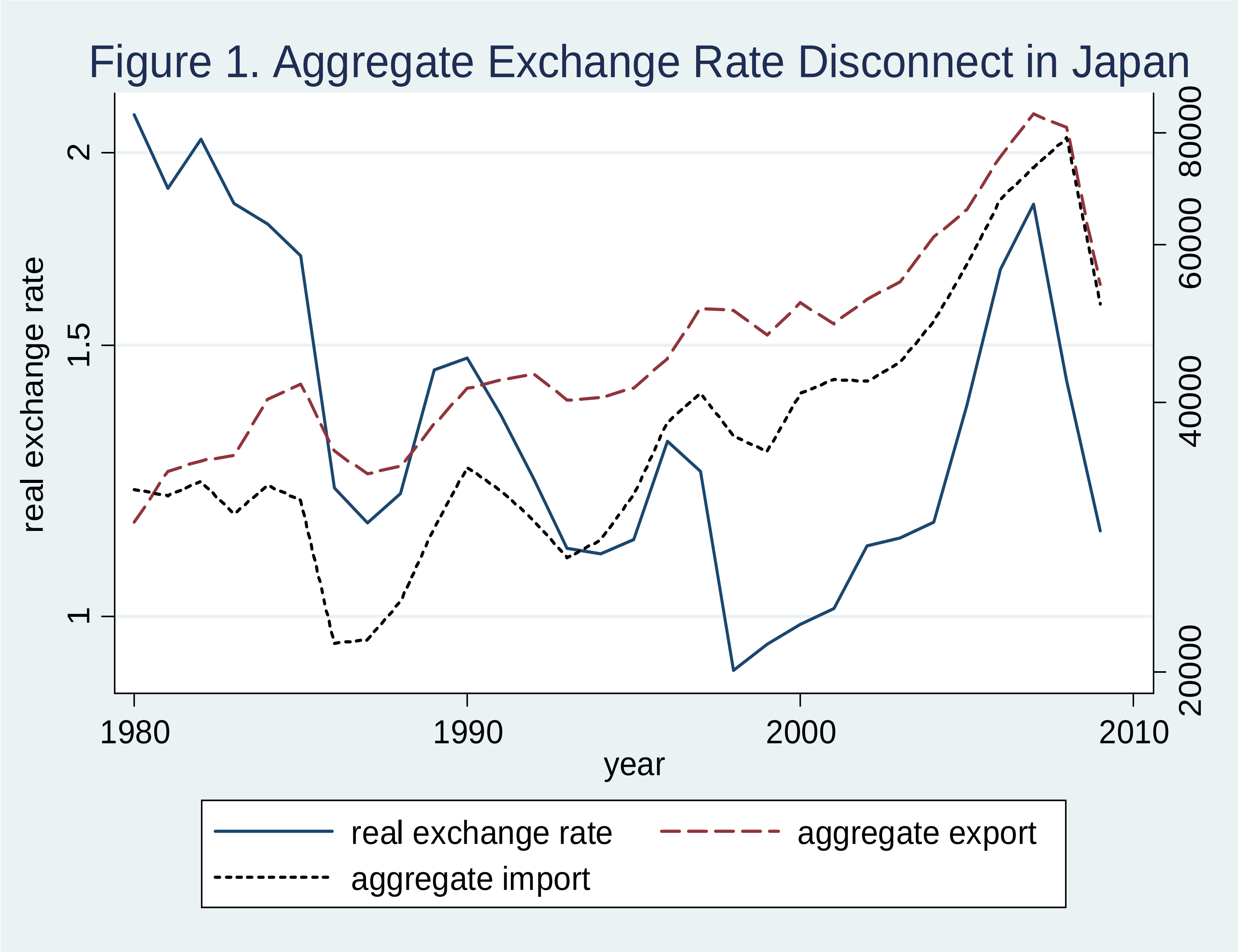


Figure 1. Aggregate Exchange Rate Disconnect in Japan
real exchange rate
2
1.5
1
80000
60000
40000
20000
1980
1990
2000
2010
year
real exchange rate
aggregate export
aggregate import

.

Firm level Kaigin data (firm $h = 1, 2, ..., 352$, 52 four-digit level industries, $t = 1982 - 1997$) [Dekle-Jeong-Ryoo(2010)]

$$\begin{aligned}\ln\ Q^F_{ht} \ =\ & \beta_h + \underset{0.06}{\underset{0.77}{\beta_\epsilon}} \ln \epsilon_t + \underset{0.04}{\underset{-0.35}{\beta_r}} \ln r_t + \underset{0.07}{\underset{0.74}{\beta_f}} \ln p^F_t + \underset{0.16}{\underset{0.19}{\beta_{Y*}}} \ln Y^*_t \\ & + \underset{0.65}{\underset{0.19}{\beta_Z}} \ln Z_t + \underset{0.04}{\underset{0.68}{\beta_z}} \ln z_{ht} + \underset{0.03}{\underset{0.60}{\beta_s}} \ln \overline{s}^F_{ht} + u_{ht}\end{aligned}$$

ϵ_t: real exchange rate, r_t: real interest rate, p^F_t: foreign price, Y^*_t: world real income, Z_t: TFP, z_{ht}: industry productivity, $\overline{s}_{ht}$: firm $h's$ industry export share, excluding h. R^2 within 0.45, between 0.11, overall 0.13, # of observations 5445

Small Open Economy without Government

A continuum of home firm $h \in \mathcal{H}_t$. Firm h produces I_{ht} number of differentiated products for home and export market

$$q_{hit}^{H} = a_{hit} Z_t \left(\frac{l_{hit}^{H}}{\gamma_L}\right)^{\gamma_L} \left(\frac{m_{hit}^{*H}}{1-\gamma_L}\right)^{1-\gamma_L}, \text{ for } i = 1, 2, .., I_{ht}$$

$$q_{hit}^{F} = a_{hit} Z_t \left[\left(\frac{l_{hit}^{F}}{\gamma_L}\right)^{\gamma_L} \left(\frac{m_{hit}^{*F}}{1-\gamma_L}\right)^{1-\gamma_L} - \phi\right], \text{ for } i = 1, 2, ..I_{ht}$$

Home output for home and export markets are produced as

$$Q_t^H = \left[\int_{h\in\mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} {q_{hit}^{H}}^{\frac{\theta-1}{\theta}}\right) dh\right]^{\frac{\theta}{\theta-1}}$$

$$Q_t^F = \left[\int_{h\in\mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} {q_{hit}^{F}}^{\frac{\theta-1}{\theta}}\right) dh\right]^{\frac{\theta}{\theta-1}}$$

A new entrant who pays a sunk cost κ_E at date t draws an opportunity of producing $b \in \{1, 2\}$ number of new products from date t+1, where

$$b = \begin{cases} 2, \text{ with probability } (wp.) \ \iota \\ 1, \ wp. \ 1 - \iota \end{cases}$$

The productivity of each new product is independently distributed as

$$a_{hit} = \begin{cases} \in [1, a], \ wp. \ \lambda' F(a) = \lambda'(1 - a^{-\alpha}) \\ 0, \ wp. \ 1 - \lambda' \end{cases}$$

Assume $\lambda \equiv (1 + \iota)\lambda' < 1$, $\alpha > 1$ and $\alpha > \theta - 1$

A firm must pay the fixed maintenance cost κ for each product in order to produce and maintain its productivity. The maintained product has the same productivity in the next period ($a_{hit+1} = a_{hit}$) with probability $1 - \delta$, and receives a new productivity draw according to the same distribution as a new entrant with probability δ.

Each firm can produce many products. Each product multiplies and dies like "amoeba."

Home final goods market

$$Q_t^H = C_t + \kappa_E N_{Et} + \kappa N_t$$

N_{Et} is measure of entering firms, N_t is measure of differentiated products maintained. No government sector.

The representative household supplies labor L_t, consumes final goods C_t and holds home and foreign real bonds D_t^H and D_t^{*H} to maximize its expected utility

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \psi_0 \frac{L_t^{1+1/\psi}}{1+1/\psi} + \xi_t^{*H} \ln D_t^{*H} \right)$$

subject to the budget constraint

$$\begin{aligned} & C_t + \kappa_E N_{Et} + \kappa N_t + D_t^H + \epsilon_t D_t^{*H} \\ = \ & w_{Lt} L_t + \Pi_t + R_{t-1} D_{t-1}^H + \epsilon_t R_{t-1}^* D_{t-1}^{*H} \end{aligned}$$

ξ_t^{*H} : utility (liquidity) shock to foreign bond holding

Foreigners do not hold home bond $\longrightarrow$ $D_t^H = 0$ without government

Foreign bond holding of home household

$$D_t^{*H} = R_{t-1}^* D_{t-1}^{*H} + p_t^F Q_t^F - M_t^{*H}$$

where M_t^{*H} is total import

Foreign aggregate demand for home exports are given by

$$Q_t^F = (p_t^F)^{-\varphi} Y_t^*$$

where Y_t^* is an exogenous foreign demand parameter.

Competitive Equilibrium

Price of differentiated goods is a mark-up over the unit cost of composite input

$$p_{hit}^{H} = \frac{\theta}{\theta - 1} \frac{w_t}{a_{hit} Z_t}$$

$$p_{hit}^{F} = \frac{\theta}{\theta - 1} \frac{w_t / \epsilon_t}{a_{hit} Z_t}$$

$$w_t = (w_{Lt})^{\gamma_L} \epsilon_t^{1-\gamma_L}$$

Aggregate production of the composite input

$$X_t = \left(\frac{L_t}{\gamma_L} \right)^{\gamma_L} \left(\frac{M_t^{*H}}{1 - \gamma_L} \right)^{1-\gamma_L}$$

We conjecture that in equilibrium, all firms choose to pay the fixed maintenance cost with positive productivity

$$N_{t+1} = (1 - \delta + \delta\lambda)\, N_t + \lambda N_{Et} \tag{1}$$

Price index of home final output for home market

$$1 \;=\; p_t^H = \left[\int_{h\in\mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} p_{hit}^{H\;1-\theta}\right) dh\right]^{\frac{1}{1-\theta}} = \frac{\theta}{\theta\text{-}1}\frac{w_t}{\overline{a} N_t^{\frac{1}{\theta-1}} Z_t} \tag{2}$$

$$\overline{a} \;\equiv\; \left[\int_1^{\infty} a^{\theta-1} dF(a)\right]^{\frac{1}{\theta-1}} = \left(\frac{\alpha}{\alpha + 1 - \theta}\right)^{\frac{1}{\theta-1}}$$

Only products with higher than $\underline{a}_t$ productivity is exported.

$$\underline{a}_t = \left[\frac{\alpha\left(\theta-1\right)\phi}{\alpha+1-\theta}\frac{\overline{a}Z_t N_t^{\frac{\theta}{\theta-1}}}{\epsilon_t^{\varphi}Y_t^*}\right]^{\frac{\theta-1}{\alpha(\theta\text{-}1)+(\alpha+1\text{-}\theta)(1\text{-}\varphi)}} \tag{3}$$

$$\epsilon_t p_t^F Q_t^F = \left\{\left[\frac{\alpha\left(\theta\text{-}1\right)\phi}{\alpha+1\text{-}\theta}\overline{a}Z_t N_t^{\frac{\theta}{\theta-1}}\right]^{(\alpha+1\text{-}\theta)(1\text{-}\varphi)}\left(\epsilon_t^{\varphi}Y_t^*\right)^{\alpha(\theta\text{-}1)}\right\}^{\frac{1}{\alpha(\theta\text{-}1)+(\alpha+1\text{-}\theta)(1\text{-}\varphi)}} \tag{4}$$

φ is small relative to $\theta \longrightarrow$ aggregate export is not very sensitive to the real exchange rate

Low productive products drop like "flies" with an adverse shock, while export of high productive products is not very sensitive $\longrightarrow$ Disconnect between aggregate and firm-level responses

Equilibrium Dynamics

The input composite market equilibrium is

$$X_t = \frac{1}{\gamma_L(\psi_0 C_t)^{\psi}} \left(\frac{w_t^{1-\gamma_L+\psi}}{\epsilon_t^{(1-\gamma_L)(1+\psi)}} \right)^{\frac{1}{\gamma_L}} \tag{5}$$

$$= X_t^H + \phi \frac{\theta\alpha + 1 - \theta}{\alpha + 1 - \theta} \underline{a}_t{}^{-\alpha} N_t \tag{6}$$

The value function of the average product

$$\overline{V}_t = \overline{\pi}_t - \kappa + (1 - \delta + \delta\lambda) E_t \left(\Lambda_{t,t+1} \overline{V}_{t+1} \right) \tag{7}$$

where $\Lambda_{t,t+1} = \beta C_t / C_{t+1}$. The average profit per product

$$\overline{\pi}_t = w_t \left[\frac{X_t}{(\theta - 1) N_t} - \phi \underline{a}_t{}^{-\alpha} \right] \tag{8}$$

$$\kappa_{Et} = \lambda E_t \left(\Lambda_{t,t+1} \overline{V}_{t+1} \right) : \text{ free entry} \tag{9}$$

The final goods market clearing implies

$$C_t + \kappa_E N_{Et} + \kappa N_t = \overline{a} N_t^{\frac{1}{\theta-1}} Z_t X_t^H \tag{10}$$

Net foreign assets evolve as

$$D_t^{*H} = R_{t-1}^* D_{t-1}^{*H} + p_t^F Q_t^F - (1-\gamma_L)\frac{w_t X_t}{\epsilon_t} \tag{11}$$

Home demand for foreign bond is

$$D_t^{*H} = \frac{\xi_t^{*H} C_t}{\epsilon_t - R_t^* E_t \left(\Lambda_{t,t+1}\epsilon_{t+1}\right)} \tag{12}$$

$(1-12)$ determine $w_t, \underline{a}_t, X_t, X_t^H, C_t, \epsilon_t, \overline{V}_t, \overline{\pi}_t$,$N_{Et}$,$p_t^F Q_t^F$, N_{t+1} and D_t^{*H} as a function of the state variables N_t, D_{t-1}^{*H}, Z_t, ξ_t^{*H},Y_t^* and R_t^*.

Solving for the Model Equilibrium

A "shrunk" model: Short-run dynamics of real exchange rate is dominated by the liquidity shock $\longrightarrow$ Regard the real exchange rate as exogenous in the short-run.

From the free entry condition,

$$\kappa_E \left[1 - (1 - \delta + \delta\lambda) E_t \left(\frac{\beta C_t}{C_{t+1}}\right)\right] \tag{13}$$

$$= \lambda E_t \left\{ \frac{\beta C_t}{C_{t+1}} \left[\frac{\theta - 1}{\theta} \overline{a} N_{t+1}^{\frac{1}{\theta-1}} Z_{t+1} \left(\frac{X_{t+1}}{(\theta\text{-}1) N_{t+1}} \text{ - } \phi \underline{a}_{t+1}{}^{-\alpha} \right) \text{ - } \kappa \right] \right\}$$

From labor market clearing

$$X_t = \frac{1}{\gamma_L(\psi_0 C_t)^\psi}\left(\frac{\left(\frac{\theta-1}{\theta}\overline{a}N_t^{\frac{1}{\theta-1}}Z_t\right)^{1-\gamma_L+\psi}}{\epsilon_t^{(1-\gamma_L)(1+\psi)}}\right)^{\frac{1}{\gamma_L}} \tag{14}$$

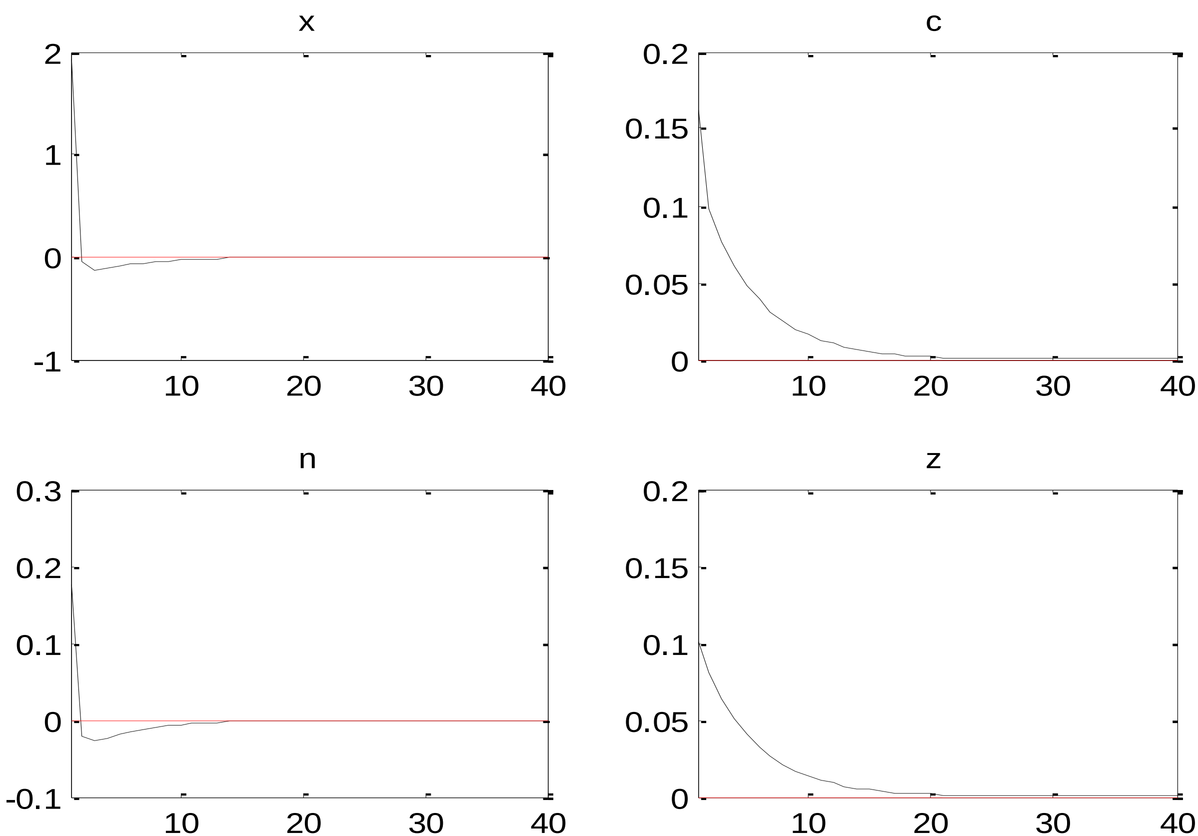
The goods market clearing condition is

$$\begin{aligned} & C_t + \kappa N_t + \frac{\kappa_E}{\lambda}[N_{t+1} - (1-\delta+\delta\lambda)N_t] \qquad (15) \\ = \ & \overline{a}N_t^{\frac{1}{\theta-1}}Z_t\left\{X_t - \phi\frac{\alpha\theta+1-\theta}{\alpha+1-\theta}\underline{a}_t{}^{-\alpha}N_t\right\} \end{aligned}$$

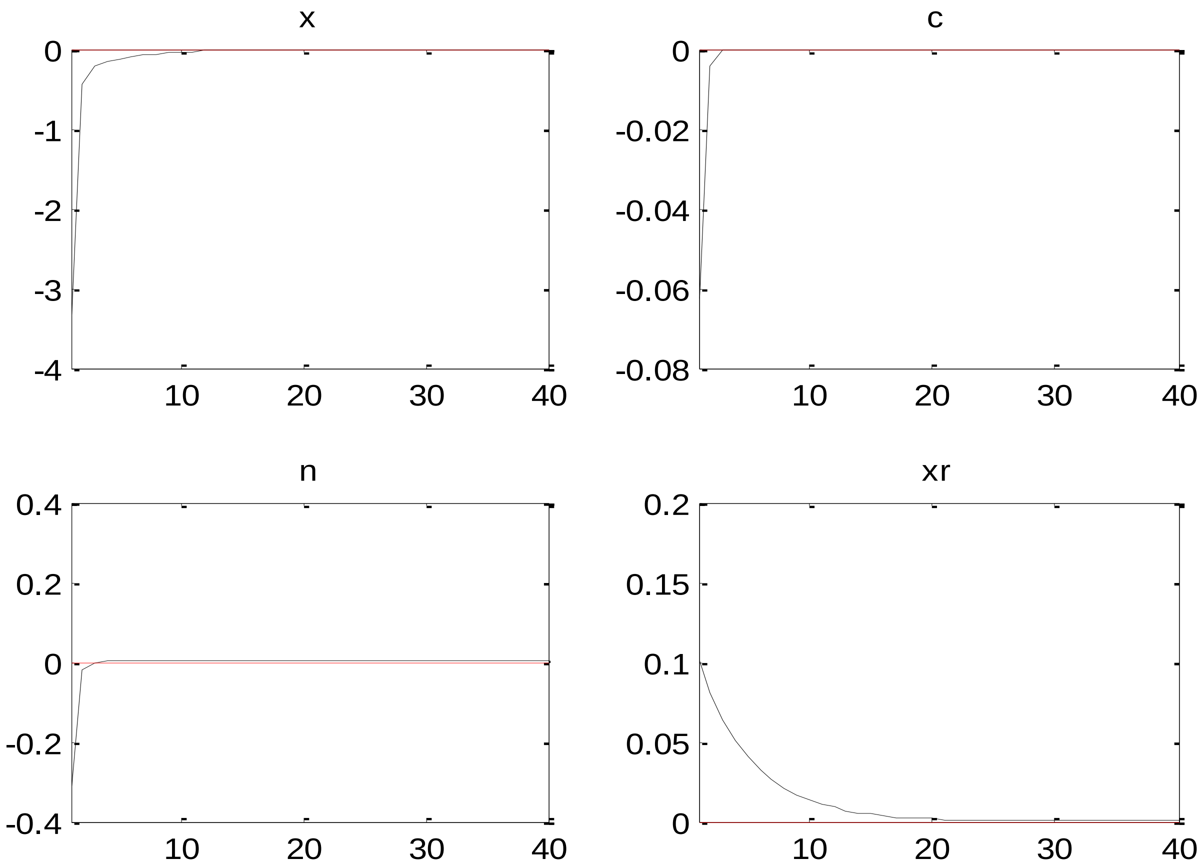
$(\underline{a}_t, X_t, C_t, N_{t+1})$ solve (3,13,14,15) as a function of N_t, Z_t, Y_t^* and ϵ_t.

Impulse Response Functions

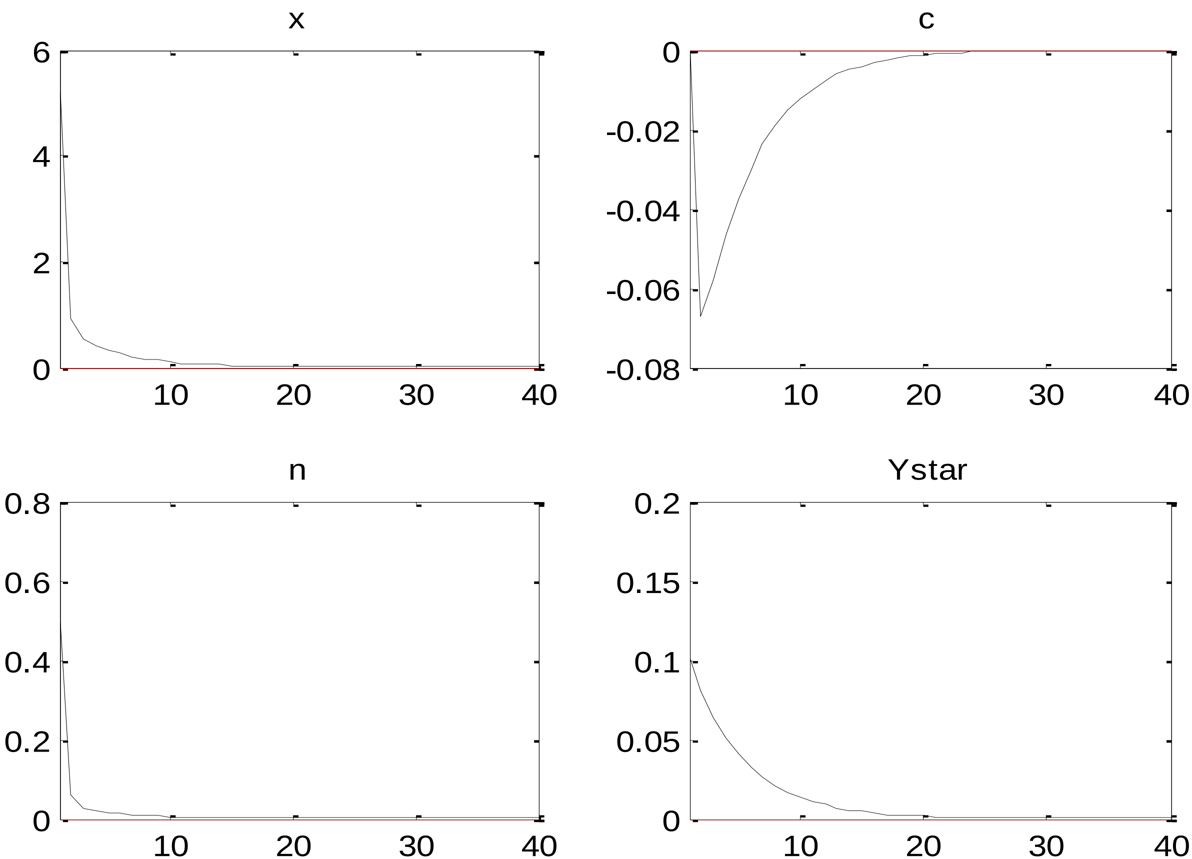
1. Impulse response functions from one unit of productivity innovation:



2. Impulse response functions from one unit of real exchange rate innovation:



3. Impulse response functions from one unit of foreign demand innovation:



4. Impulse response functions from one unit of government purchases innovation: