

Knowledge, Diffusion and Reallocation

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1 Introduction

Knowledge is costly to replicate. It is often embodied, so there is limited capacity and opportunity costs involved in its transmission. Even in the case of learning by doing, it still takes time to learn and attempting to speed up this process may carry substantial costs. We identify in this paper knowledge transmission with the reproduction of its capacity. As a consequence the incentives and costs of knowledge reproduction play a prominent role in the determinants and gains from reallocation. This is a central point that has remained largely unnoticed in the literature of reallocation and the focus of this paper.

There is a growing literature emphasizing the gains from reallocation, both on the empirical and theoretical front. Bertola and Bentolila and Hopenhayn and Rogerson -among others- analyze the effect of firing costs, which tax reallocation of workers from less to more productive firms. The latter paper finds a sizable impact of these policies on productivity. Recent papers in international trade (e.g. Eaton-Kortum, Alvarez and Lucas) consider the impact of trade barriers on productivity and welfare. Tariffs are viewed as barriers to the reallocation of resources to most productive firms and constitute important wedges in the optimal allocation of production across countries. These papers also identify potentially large gains from reducing trade barriers. Finally, recent papers (Ramondo, McGrattan and Prescott and Burstein and Monge) have considered the gains from eliminating barriers to foreign direct investment, where the first two papers find very substantial gains.

All these papers make either explicit or implicit -mostly the latter- assumptions about the process of knowledge replication. In Eaton-Kortum and Alvarez-Lucas, firms face a constant marginal cost of production. The marginal cost is idiosyncratic to a firm -or a country in the latter paper- and continuously distributed, so in the absence of any barriers -including costs to trade- all world production of each differentiated good would be produced by a single producer (or country.) In contrast, in Hopenhayn-Rogerson firms also have idiosyncratic differences but face decreasing returns to scale, as in Lucas' (1977) classic paper on span of control. In absence of barriers to the reallocation of labor, marginal product is equalized across all productive units, but more than one producer will remain active. The gains to eliminating barriers to reallocation are mitigated by the strength of decreasing returns.

What lies behind assumptions about returns to scale? We believe that a key element is the process of transmission and replication of knowledge. With returns to scale -and no adjustment costs- knowledge can be costlessly and immediately replicated. But then, what prevents others from imitating the more advantaged firm? What is the source of this superior productivity that can be costlessly replicated inside but not outside of the limits of the firm? To the contrary, in most cases knowledge is embodied and as such costly to replicate. This gives an advantage and quasi-rents to the owner of this capital but at the same time places a limit on the ability to extend the associated knowledge. An extreme version of this appears in the Lucas span of control model, where the superior knowledge of a firm is embedded in its managerial ability, a non reproducible factor.

In this paper we attempt to go one step further in the microfoundations of returns to scale by providing a specific model of knowledge transmission and replication. The model puts together a cost of adjustment technology in a vintage capital structure. The basic component of the model is knowledge capital, a pair (z, k) where z represents the quality or productivity of the knowledge and k a measure of its capacity. This capacity can be accumulated according to a cost of adjustment technology. There is also a technology for developing new knowledge from scratch (e.g. R&D) that is drawn from a distribution with a support that shifts over time at a constant rate. There is a fixed endowment of labor that can be used either for production or as an input to R&D. The equilibrium determines -along a balanced growth path- the optimal degree of replication of existing knowledge capital and its life expectancy and the amount of resources devoted to outside R&D. Along this constant growth path, as new knowledge arrives and wages rise old knowledge becomes obsolete and unprofitable to use.

In order to provide a link to the gains from reallocation literature, we consider a very simple numerical exercise to evaluate the impact of a tax on knowledge accumulation. We consider three alternative scenarios: a benchmark case, a high cost of adjustment case and a high rate of technological improvement (and consequently high rate of obsolescence). In the last two scenarios the rate of reproduction of knowledge capital is lower and the differences in the rate across different z 's much less pronounced. In other words, compared to the benchmark these are scenarios where reallocation plays a much less prominent role. Our numerical results show that the negative impact of the tax on average productivity is considerably higher in the benchmark economy - the other two scenarios behave much like a case where

returns to scale decrease more strongly. This is in spite the fact that the technology we consider has actually constant returns to scale. The culprit lies in the interaction of adjustment costs and the limited durability and obsolescence of knowledge: when it is costly to invest in expanding the stock of existing knowledge and an alternative to invest in developing a new superior one, knowledge reproduction will be limited.

The paper models knowledge accumulation in the abstract. In practice, reproduction of knowledge takes places in multiple ways: learning by doing in a firm, transfer of knowledge through mobility in the labor market, spin-offs, chains stores opening new establishments, etc. In an attempt to connect better the theory to the data, the last section of the paper provides an aggregation procedure in the spirit of Klette and Kortum, where a firm is a collection of knowledge capital and can also draw stochastically new knowledge through R&D. The model generates patterns of entry, growth and exit of firms and productivity decompositions that match remarkably well the documented empirical evidence.

There are many papers that relate to our story. Vintage models have been widely used and our model draws heavily on that literature. In particular the paper by Chari and Hopenhayn considers explicitly the problem of reproduction and diffusion of new knowledge by the interaction of learning by doing and complementarities between skilled and unskilled workers. Franco and Filson develop a model of reproduction of knowledge and spin-offs. The importance of costly reproducible embodied knowledge has been used by Boldrin and Levine to suggest that intellectual property protection (patents) are not necessary in order for innovators to capture rents from their innovations and R&D. (references to be completed).

The paper is organized as follows. Section 2 develops the model and its comparative statics. Section 3 considers the policy experiment. Section 4 provides the link to firm level dynamics and finally Section 5 concludes.

2 A model of learning and diffusion

The basic unit of analysis is a knowledge capital pair (z, k) interpreted as an amount of capital k of knowledge type z . Production is CRS of the form $z f(k, n)$, where k is the capital of type z . The state of knowledge is a measure μ on (z, k) and given an aggregate amount of labor L efficient production

solves:

$$\begin{aligned} & \max \int z f(k, l(z, k)) d\mu \\ \text{subject to: } & \int l(z, k) d\mu \leq L. \end{aligned}$$

There is a technology for replication knowledge capital with adjustment costs. Letting $\dot{k}(z)$ denote the new flow of capital of type z , adjustment cost is $C\left(\frac{\dot{k}}{k}\right)zk$. In addition capital depreciates at an exogenous rate δ . Assume $C(0) = 0$ and C is strictly increasing and strictly convex. It follows that adjustment cost is homogenous of degree one in (k, \dot{k}) and linear in z , the latter reflecting that it is costlier to replicate better knowledge.

In the analysis below we focus on a balanced growth path where the economy grows at a constant rate g . In a general equilibrium framework and a closed economy, this requires preferences that are consistent with balanced growth. We assume there is a representative agent with CES separable preferences:

$$U = \int e^{-\rho t} u \frac{c(t)^{1-\theta}}{1-\theta} dt.$$

This implies that in a balanced growth path $r = \rho + \theta g$.

2.1 Optimal replication

Let $v(z, t)$ denote the value of one unit of $k(z)$ at time t and let $w(t)$ represent wages, taking output as a numeraire. Total output can be used for investment, as part of the adjustment cost or for consumption. It is standard to show that the value of k units of capital of type z is simply $v(z, t)k$. The Bellman equation for $v(z, t)$ is given by:

$$(r + \delta)v(z, t) = \max_n z f(1, n) - w(t)n + \left(\max_{\dot{k} \geq 0} v(z, t) \dot{k} - C\left(\frac{\dot{k}}{k}\right)z \right) + v_2(z, t)$$

where r is the interest rate. The interpretation is as follows: the net flow-value is the sum of the current flow of profits plus the value of expanding the knowledge capital and the third term captures changes in the value that arise from the time variation in w and r .

For the rest of the paper we consider a balanced growth path where consumption and wages grow at a constant rate g and the interest rate is

constant. The above value function can be replaced by:

$$(r + \delta) v(z, w) = \max_n z f(1, n) - wn + \left(\max_{\dot{k} \geq 0} v(z, w) \dot{k} - C(\dot{k}) z \right) + v_2(z, w) wg \quad (1)$$

Proposition 1 *The value $v(z, w)$ is homogenous of degree one in (z, w) .*

Proof. Assume this property applies in the right hand side of the Bellman equation. It follows immediately that the left hand side value function is also homogeneous of degree one. ■

An immediate corollary to this proposition is that the choice variables n and \dot{k} are also homogenous of degree one in (z, w) and thus only depend -and are increasing- on (z/w) .

It is easy to characterize the time path of a knowledge capital (z, k) starting in a period with wage w . If z/w is sufficiently large so that initially $\dot{k} > \delta$, the stock of knowledge capital will grow. After some time, z/w reaches a value such that $\dot{k} = \delta$ so the knowledge capital will start to shrink. Finally, if $f_2(1, 0) < \infty$, when z/w reaches a critical value z^* , $v(z, w) = 0$ and that knowledge capital stops being used.

Homogeneity of degree one implies that:

$$v_2(z, w)wg = v(z, w)g - v_1(z, w)zg$$

so equation (1) may be rewritten as:

$$(r + \delta - g) v(z, w) = \max_n z f(1, n) - wn + \left(\max_{\dot{k}} v(z, w) \dot{k} - C(\dot{k}) z \right) - v_1(z, w) zg.$$

Together with the condition that $v(z, w) = 0$ whenever the corresponding profits are zero, the solution to this ODE gives the value function and the optimal accumulation rule.

2.2 Equilibrium with exogenous knowledge frontier and exogenous new arrivals

Suppose each period there is a constant flow m of units of knowledge $z = x\gamma(t)$, where $0 \leq x \leq 1$ is distributed according to a cdf $F(x)$. The frontier $\gamma(t) = e^{gt}$ grows at a constant rate g . These m units of knowledge constitute

a heterogenous cohort entering at time t . Conjecture that in the balanced growth path wages $w(t) = w_0 e^{gt}$, where w_0 is to be determined. The value of z measured relative to the frontier, *i.e.* $z/\gamma(t)$ is sufficient to determine the employment and growth rate of a unit of knowledge capital z at time t . In terms of this state variable, the stationary equilibrium involves a distribution μ over capital, with support in $[z^*, 1]$, where $v(z^*, 1) = 0$. This measure is characterized by equating the flow out of a given z to the flow into this z . Noting that

$$\mu_{t+\Delta}(z) \approx \mu_t(z e^{g\Delta}) + \Delta(\dot{k} - \delta)\mu(z) + \Delta f(z)m$$

it follows that

$$\begin{aligned} \lim_{\Delta \rightarrow 0} \frac{\mu_{t+\Delta}(z) - \mu_t(z)}{\Delta} &= \lim_{\Delta \rightarrow 0} \frac{\mu_t(z e^{g\Delta}) - \mu_t(z)}{\Delta} + (\dot{k} - \delta)\mu(z) + f(z)m \\ &= \mu'(z)gz + (\dot{k} - \delta)\mu(z) + f(z)m \end{aligned}$$

which must equal zero for a stationary distribution. This gives an ordinary differential equation on $[z^*, 1]$ with boundary condition $\mu(1) = mf(1)$.

To complete the equilibrium first note that the stationary measure μ will be decreasing in w_0 , since $\dot{k}(z)$ is also decreasing in w_0 . Moreover, for each (z, k) demand for labor must also be decreasing in w_0 . The two effects work in the same direction, so labor demand strictly and continuously decreases in w_0 . There is thus a unique equilibrium value w_0 that equates labor demand to L .

2.3 Equilibrium with exogenous knowledge frontier change and endogenous arrivals

To endogenize the steady state arrival rate m , consider the following alternative technology for developing new knowledge capital. Suppose that a unit of labor generates a flow m_R of new technologies as described above. Then if L_R units of labor endowment are used in this research sector, $m = m_R L_R$. The cost of a unit flow of new knowledge capital is $\frac{1}{m_R} w_0 e^{gt}$ so in equilibrium this must also be the expected value created $v^e = e^{gt} \int v(z, w_0) dF(z)$. This free entry condition determines uniquely the equilibrium wages that solves:

$$\frac{1}{m_R} w_0 = \int v(z, w_0) dF(z) \quad (2)$$

or dividing by w_0

$$\frac{1}{m_R} = \int v\left(\frac{z}{w_0}, 1\right) dF(z)$$

Labor demand for production $L_P(w_0)$ is a strictly decreasing function of this wage and L_R is determined by

$$L_R = L - L_P$$

which we assume is strictly positive.¹ This will be the equilibrium considered in the numerical analysis of section ??.

2.3.1 Turnover

It is possible to trace any knowledge capital to a time when it was originated and started to be replicated. Starting from an initial (z, k_0) , we will call the corresponding sequence $\{z, k(t)\}$ a knowledge chain. A possible notion of turnover, is then the turnover of knowledge chains. This is the analogue of firm turnover in entry/exit models. Consequently, the rate of turnover (entry/exit) of chains along the balanced growth path is equal to the inverse of the expected lifetime of the average initial draw z and is thus monotonically decreasing in w_0 and the growth rate g .

The measure of turnover of chains is unweighted, i.e. it does not take into account that chains with different initial z 's will grow at different rates and thus will become overly represented in the economy's stock of knowledge. An alternative measure of turnover is obtained by measuring entry and exit (discontinuance) of knowledge capital. The flow of entry is proportional to L_R . The stock is $\int \mu(dz)$ which in general is of the form $L_R \phi(w_0)$, where ϕ is a decreasing function. For the special case where $f(k, n) = \min(k, n)$ the total stock of active capital equals employment in production so turnover of knowledge capital is $L_R/(1 - L_R)$. In our analysis below we report these two measures.

2.3.2 Comparative Statics

We consider here the effect of changes in parameter values. An increase in L is neutral, just changing proportionally entry and the density of the distribution $\mu(z)$. An increase in m_R reduces the cost of introducing new knowledge

¹In case this were negative, in equilibrium $m = 0$ and it can be shown that wages will increase (not at a constant rate) until ultimately all labor is employed in the highest z .

capital and leads to an increase in the equilibrium wage w_0 . This implies a reduction in $v(z)$, lower investment in replication and a shorter lifespan for knowledge capital. In turn, L_R increases and L_P decreases. Intuitively, as it becomes cheaper to introduce new knowledge capital relative to reproducing existing one, labor in research sector substitutes for reproduction of knowledge capital. Both measures of turnover discussed above increase.

An increase in g implies a faster wage growth and thus a faster depreciation (relative to the frontier) of existing knowledge capital. This implies a lower $v(z, w)$ and by equation (2) a lower value w_0 . The rise of g and fall in w_0 have opposite effects on the initial value $v(z, w_0)$ for new knowledge capital. From the free entry condition, it follows that as w_0 decreases the expected value of new knowledge capital must also decrease. However since w_0 decreases, for values of z close to the initial exit threshold value must increase and thus must decrease in a range corresponding to higher values. Though we have not proof, we conjecture the following relationship

itbpF3.045in1.7832in1.8957inFigure

so that the value increases up to some z (the intersection point) but is lower for higher z 's. The intuition for this results is as follows. In the initial life of a vintage of knowledge capital the wage is lower in the second scenario (higher g case.) Since wage grows faster, a point will be reached where some of these vintages will confront a higher relative wage and consequently a lower value. This higher wage will affect those initial knowledge stocks that grow faster and live longer, which are precisely the ones with higher initial z .

An implication of the flattening of this profile is that accumulation of knowledge capital will be less sensitive to differences in productivity z and as a consequence less reallocation will take place from lower to higher z 's. On the other hand, the higher drift in productivity caused by the higher exogenous growth g will increase the relative importance of higher z 's, with an obvious countervailing effect.

The net effect on turnover is not at all obvious and is likely to depend on the distribution F . In the standard vintage model where all new entrants get the highest z so F has all its mass at one, the expected lifetime of new knowledge capital must decrease so our first measure of turnover rises. Moreover, since in this case a lower w_0 implies -through entry equilibrium condition- a lower initial value $v(1, w_0)$ for the entrant, total knowledge accumulation of incumbents decreases pointwise and thus our second measure of turnover also increases. This has a natural interpretation as in the environment with higher

g , de novo creation of knowledge capital through entry has an advantage over replication of existing knowledge.²

Consider now a proportional increase in the knowledge accumulation cost $\gamma C(\dot{k})z$, where $\gamma > 1$. The equilibrium wage w_0 decreases and so does the expected initial value $\int v(z, w_0) F(dz)$. The effect is similar to an increase in g . On the one hand, lower wages increase the value of knowledge capital for all z 's. But the increase in g has an opposite effect, which is likely to impact higher z firms more because those are the ones that invest more. So a picture similar to the one obtained for our previous comparative statics exercise is likely to emerge, with a consequent flattening of profiles. The overall effect is more likely to be negative on total knowledge accumulation. As for the effect on turnover, the expected life of new knowledge increases, decreasing our first measure of turnover. On the other hand total turnover of knowledge capital is likely to rise as a consequence of reduced rates of replication.

Finally consider an improvement on the initial distribution F in first order stochastic dominance. This raises the expected value of de novo knowledge and thus raises w_0 , lowering the values $v(z, w_0)$ and knowledge accumulation for each z . At the same time it also increases the threshold at exit, lowering the life for a given initial z . The overall effect on expected lifetime is ambiguous as the shift in F puts more weight on higher z 's which are more durable and have higher accumulation rates. For example, if the original F has all its mass at some $z < 1$ and the new one on $z' > z$, the effect will be neutral as $z/w_0 = z'/w'_0$, implying that both durability and accumulation are unchanged. On the other hand, suppose that the initial distribution has two points of mass, x close to w_0 and 1 and the new distribution moves some of the mass resulting in $w'_0 > x$. Selecting out this lower x may lead to an increase in expected life and thus lower our first measure of turnover.

3 Policy experiment

In this section we evaluate the effect of a tax on knowledge replication as a barrier to reallocation. More specifically we consider a tax that increases the cost of reproduction of knowledge capital by $tC(\dot{k})z$. As seen above, this tax decreases investment $\dot{k}(z)$ and thus reallocation from less to more productive

²One might conjecture that if the distribution F has a large mass at lower levels of z that are very short lived and some mass at one, an increase in g might increase the expected lifetime and thus decrease our first measure of turnover.

capital. The impact of this tax depends intuitively on the importance of reallocation. We consider three scenarios, a baseline case, a high adjustment cost case and a high growth g in the knowledge frontier. Parameter values used in the simulations are as follows:

r	5%
δ	5%
m_R	0.5
g	3%

The value of m_R is chosen so that in equilibrium the lowest surviving z in the benchmark is approximately 1/2. This is consistent with results from Bartelsman and Domes where the ratio of the productivity of the top of top 5% vs. low 5% is 1.8. The rate of growth of 3% in the benchmark corresponds to the observed 3% growth in total factor productivity. In the high growth scenario the rate is twice as high.

The distribution F on $z \in [0, 1]$ is given by:

$$F(z) = \frac{1 - \exp(-\lambda z)}{1 - \exp(-\lambda)}$$

with a value of $\lambda = 2$. The adjustment cost function has the following form:

$$\frac{c_0 k^2}{2}$$

where $c_0 = 15$ in the base line scenario and twice as high in the high adjustment cost case. Finally the production function used is $f(k, n) = \min(k, n)$.

The following figure give the value functions for each of three base scenarios and the corresponding replication rates. Note that as suggested by the theory, the value function of the benchmark case crosses from below the other two value functions and replication rates are flatter (and in average lower) under the high growth and high adjustment scenarios.

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In the benchmark scenario replication is high, particularly for the high z 's and as a consequence reallocation is also high. The following table gives the corresponding rates of reallocation to growing z 's in a unit time period as % of total capital/employment.

Reallocation to growing z 's

baseline	high g	high c
3.4%	0.9%	1.5%

The effect of a tax to the adjustment cost function of the form $tC(\dot{k})$ is considered, where $t \in \{0, 0.5, 1\}$. This tax affects productivity through several channels. In the first place, it is likely to reduce the rate of replication of high z 's and as a consequence lower the extent of reallocation from low to high z 's. This is similar to the effect of a hiring (or firing) tax that reduces reallocation from low to high marginal product firms. A second channel is that as the wage w_0 decreases, knowledge capital lasts longer and consequently there is less selection. As reallocation plays a lower effect on productivity in the high g and high c case, it is natural to expect that the effect of taxing investment should be lower in these two cases compared to the benchmark.

The results are given in the following table. Percentage changes of wages and productivity relative to the case of no taxation are given for the three scenarios. Note that the negative effect of taxation on productivity is twice as high in the benchmark scenario. A high cost of adjustment or obsolescence economy responds to this tax similarly to an economy with low returns to scale would respond to a tax on reallocation.

	Base case		high adj. cost		high g	
	w_0	prod	w_0	prod	w_0	prod
$t = 0$	100	100	93.7	93.7	83.9	80.7
$t = 0.5$	-6.5%	-2.1%	-3.5%	-0.9%	-3.9%	-0.8%
$t = 1.0$	-9.8%	-4.1%	-6.1%	-2%	-6.4%	-1.7%

4 Firms and Knowledge Capital

We have modelled knowledge accumulation in the abstract. In practice, reproduction of knowledge takes places in multiple ways: learning by doing in a firm, transfer of knowledge through mobility in the labor market, spin-offs, chains stores opening new establishments, etc. In an attempt to connect better the theory to the data, in this section we identify a firm as a portfolio of knowledge capital. The state of a firm can thus be described by a vectorframe $(z_1, k_1, z_2, k_2, \dots, z_n, k_n)$. At each instant a firm receives new arrivals at an exogenously given Poisson rate m , expands its knowledge capital for higher z 's (and reduces it for lower ones) and discontinues knowledge capital that becomes obsolete. The conjunction of these three sources leads to a process of growth and contraction of a firm. In the event that all the knowledge capital of a firm becomes obsolete (for the absence of new draws), we will say that the firm exits. A new potential entrants replaces the firm as

it gets a new draw. Notice that we are identifying firms through a somewhat arbitrary aggregation process -in spirit similar to the one used in Klette and Kortum- that is neutral to the equilibrium accumulation process discussed above. It just serves the purpose of providing a framework to compare model and data.

We simulate the process of birth-growth-death as follows. At birth, a firm gets m_R units of a randomly drawn $z(0)$ from distribution F . This capital grows over time according to the optimal replication rule $\dot{k}(z/w)$ where w starts initially at w_0 and grows exponentially at rate g . The following arrival is exponentially distributed with parameter m , the Poisson arrival rate. If the arrival occurs before $z(0)$ becomes obsolete, the firm survives and otherwise it exits and this arrival will be attributed to a new entrant. A surviving firm's state at any point in time can be conveniently expressed by a vector $\{(z_j, k_j)\}$ with maximal element \bar{z} . Letting $T(z)$ denote the life of an initial knowledge capital z , the life expectancy of this firm can be obtained as follows. The probability of exit before $T(\bar{z})$ is zero. Let $P(t, z)$ denote the probability that a firm gets an arrival before t periods that is greater than z . The probability of exit at $T(\bar{z})$ is then $1 - P(T, \bar{z})$. The probability of exit after $T(\bar{z})$ is difficult to characterize.

The process of birth and death depends critically on the arrival rate m : higher values of m make it more likely that a firm gets draws that extend its lifetime, thus decreasing the rate of entry and exit in the stationary equilibrium. There are very few free parameters left to match the data. These are basically: the parameter λ for the distribution of new entrants, the wage rate w_0 (determined by w_R) and the value of m . In our simulations we set $m = 1/3$, corresponding to an average time for a new draw of three years, $\lambda = 2$ and $w_0 = 0.5$ to match the range of total factor productivity as explained above. In spite of having few free parameters, the model fits remarkably well many features in the data.

The following table shows the results of a regression of firm growth rate on \ln size and age using the simulated data. The results are close to Gibrat's law of proportionate growth, though our estimates imply growth rates that are slightly increasing with firm size. This is somewhat mitigated by the negative effect of age.

The next table provides a series of statistics of firm dynamics for the model and US economy.

	model	US
size of entrants/incumbents	45%	35%
Rate of entry/exit (annual)	5%	7%
job creation/destruction rate	7%	10%
Share of entry/exit	30%	20%

Productivity growth in the economy results from the growth of productivity within a firm, the reallocation from less to more productive firms and through entry and exit. Bailey, Bartelsman and Haltiwanger proposed and analyzed the following decomposition. Let s_{it} denote the share of firm i in period t , let π_{it} be its productivity and Π_t aggregate productivity. Then

$$\begin{aligned}
\Delta\Pi_t &= \sum_{i \in C} s_{it-1} \Delta\pi_{it} \\
&+ \sum_{i \in C} (\pi_{it} - \Pi_{t-1}) \Delta s_{it} + \sum_{i \in C} \Delta\pi_{it} \Delta s_{it} \\
&+ \sum_{i \in N} s_{it} (\pi_{it} - \Pi_{t-1}) \\
&- \sum_{i \in X} s_{it-1} (\pi_{it-1} - \Pi_{it-1})
\end{aligned}$$

The first line corresponds to the within establishment productivity growth which in the model is the result of the change in the knowledge capital composition of a firm; the second term is the within firm change in productivity due to reallocation of shares from less to more productive firms; the third term measures the interaction of the first two and the two last components measure the effect of entry and exit. The following table gives this productivity decomposition as applied to the simulations from the model and the data.

Productivity decomposition	model	data
within establishments term	47.2	48
between establishments term	7.5	-8
interaction term	26.7	34
net entry	15.5	26

Except for the between establishments term, our numbers are in the ballpark. The model predicts a positive within establishment term because inside the firm better knowledge replicates faster. It predicts a positive between establishment term as firms that lose shares are to those that have lower z 's

while those that gain shares have higher $z's$.³ It predicts a positive interaction term because firms that have higher $z's$ will exhibit both reallocation of shares of its capital towards more productive knowledge and higher growth rates that increase their shares. The net entry effect is obviously positive, as firms that exit are always in the lower end of the distribution of productivity and new firms are the result of drawing from the frontier.

Exogenous technological change, in the form of new vintages of knowledge, spreads in this economy through two different channels: 1) as the result of draws taken by incumbent firms and draws made by new entrants. New plants in the model draw from the frontier technology and are thus likely to exhibit higher productivity. As firms age, two effects take place. On the one hand, existing knowledge capital loses its edge relative to the technological frontier. On the other hand, there is selection since survival is associated with success in getting draws from newer knowledge capital.

Jensen, McGuckin and Stiroh have proposed the following decomposition for US manufactures.

Average Labor Productivity (Value added / hours worked in 1987 dollars)							
	1963	1967	1972	1977	1982	1987	1992
Industry	25.2	28.3	33.1	35.2	38.3	48.8	54.0
New Plants	25.2	23.7	28.3	28.3	30.5	41.4	45.0
				Surviving Cohorts			
a=1		33.7	34.5	34.6	36.7	44.6	51.4
a=2			38.9	36.6	38.4	45.5	53.6
a=3				41.0	39.4	48.4	49.3
a=4					44.3	50.2	52.7
a=5						55.6	53.3
a=6							60.8

The data is calculated as follows. The cohort age 1 is the cohort that entered between the first and second census years (63-67) *and survived* up to the last period. Cohort age 2 entered between the next two census dates (67-72) and survived up to the end of the period...and so on. The calculation is

³It is actually very difficult to understand why it is negative in the data, suggesting that in average expanding establishments are less productive than those contracting.

not the ideal one, for it introduces an important selectivity bias which is more pronounced for older vintages. The last column is perhaps the most interesting one. It indicates a little difference in productivity between new entrants and the cohorts of past entrants, where the latter actually show higher levels of productivity. Again, this is not very surprising if one considers the effect of selection. On the other hand, the differences in productivity between cohorts of existing firms are not large and do not follow any predictable pattern.

The following table reproduces numbers obtained from our simulations. It should be noted that numbers are a bit higher, since the above data shows a rate of growth of 2.5%, while our numbers were generated under a 3% steady growth rate of total factor productivity (which is also labor productivity in our calibration.) The patterns obtained are quite similar.

	Surviving Cohorts					
a=1	29.6	33.8	40.2	49.5	60.9	62.6
a=2		34.4	39.3	46.7	57.6	70.7
a=3			39.9	45.6	54.2	66.9
a=4				46.4	53.0	63.0
a=5					53.9	61.6
a=6						62.6

5 Final Remarks

We developed a model that explicitly considers the process of replication of knowledge. In particular we use the model to analyze the impact of policies that tax replication and thus the reallocation of resources from less to more productive knowledge. Our numerical analysis suggests that the impact of these policies depends critically on the incentives for replication: if the rate of obsolescence is high (due to high exogenous vintage specific progress) or the cost of replication of knowledge high, such a tax is likely to have much less impact on the economy's productivity.

In an attempt to match the model with the data, we considered a simple aggregation procedure identifying firms as collections of knowledge capital. Preliminary calculations with the model shows a remarkably good fit with firm dynamics data and productivity decompositions.

Several extensions come immediately to mind. Knowledge is transmitted also through different locations. The experience of some retail chains such

as Walmart (see Holmes and Jia) and results from gravity equations for foreign direct investment suggest that proximity plays an important role in the spatial transfer of knowledge. Ramondo's (2007) estimates countries that are twice as far have 45% higher cost to FDI. We propose here a simple extension that may capture the role of distance in replication..This can be done by allowing the use of capital at a location i to build capital in location j with a cost of adjustment that increases with the distance of a location. There are good reasons to justify this assumption.. In the first place, knowledge might be to a large extent location specific and hard to replicate in distant or dissimilar locations. Such is the case of knowledge and links to highly productive local service suppliers. In addition, if knowledge is embodied in people that need to travel to transfer it, long distances may impose a large penalty on replication. These may be reasons why many retail chains tend to develop locally prior to extending to other regions.

In a similar vein, knowledge that is used to produce certain products may be costly transferrable for the production of other products. Such replication can be linked to firm product diversification, where the analogue to geographic distance is some measure of technological or marketing distance between products.

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