

# Optimal Timing of College Subsidies

## Enrollment, Graduation and the Skill Premium

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May 23, 2019

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- But almost half of the college enrollees in the US drop out.
- It is important to understand how policy can affect graduation.

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  - ▶ Front-loaded subsidies, vice versa
- Questions: What timing of subsidies will maximize the number of college graduates and social welfare?

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  - ▶ Focus on the relative sizes across years in college (slope).
  - ▶ I fix the total budget of college subsidies from now on.

# Outline

- 1 Introduction
- 2 Model**
- 3 Calibration
- 4 Results
- 5 Conclusion

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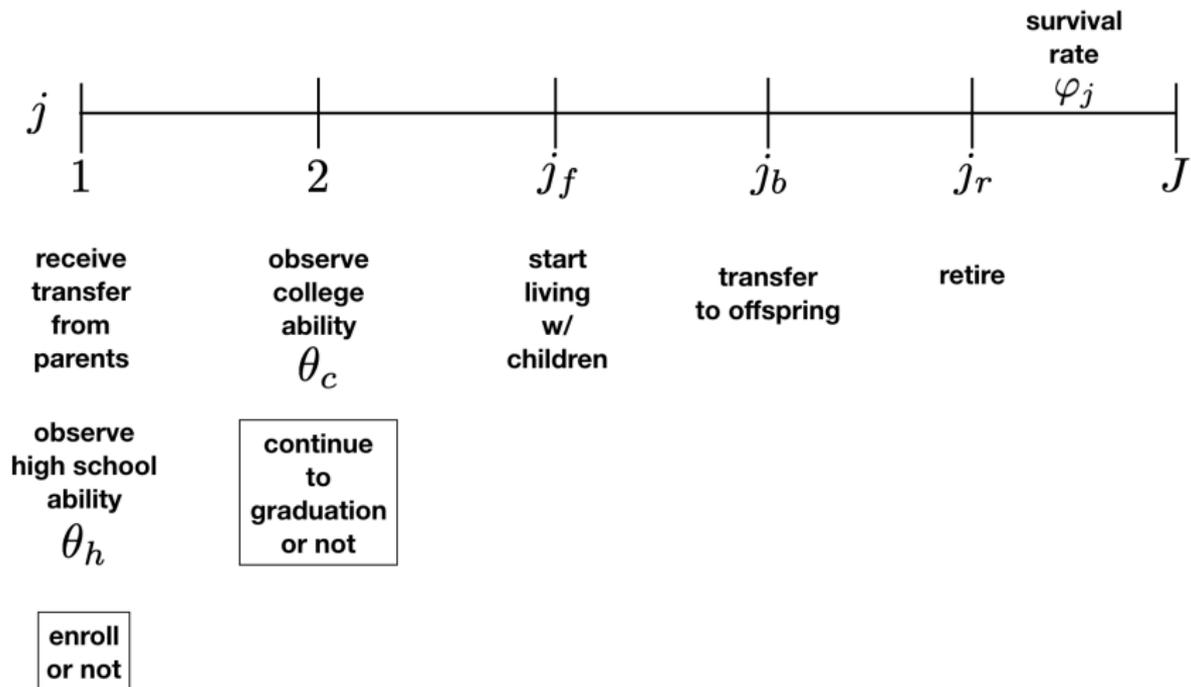
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- Once an individual finishes their schooling, they will be high school graduates ( $e = HS$ ), college dropouts ( $CD$ ), or college graduates ( $CG$ ).
- After that, they face a standard life cycle problem with income risk, incomplete markets for insurance, and borrowing constraints.

# Timeline



## Preferences: three parts

The lifetime utility is the sum of the following three parts:

- 1 The expected discounted sum

$$\mathbb{E}_1 \sum_{j=1}^J \tilde{\beta}_j u(c_j, \ell_j) \text{ where } u(c, \ell) = \frac{(c^\mu \ell^{1-\mu})^{1-\gamma}}{1-\gamma}$$

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- 2 Expected utility of college attendance:

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- 3 Parental altruism: They enjoy their children's lifetime utility with a weight  $\nu$ .

# Key Factors of Educational Decisions

## ④ Common factors

- ▶ Tuition  $p_e$ , **subsidies**  $s_1(q)$ ,  $s_2(q)$ , and credit limits  $\underline{A}_1^c$ ,  $\underline{A}_2^c$ .
- ▶ The price of effective labor  $w^{CG}$ ,  $w^{CD}$ ,  $w^{HS}$ .

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## ② Idiosyncratic factors: enrollment

- ▶ Initial asset  $a$ , family income  $q$ , and taste  $\phi \sim N(0, 1)$
- ▶ High school ability  $\theta_h$ : signal of  $\theta_c$  and labor productivity  $\varepsilon_j^{HS}(\theta_h, \eta)$ ,  $\varepsilon_j^{CD}(\theta_h, \eta)$ .
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## ③ Idiosyncratic factors: graduation

- ▶ Realized college ability  $\theta_c$  affects college utility  $\lambda_j(\theta_c, \phi)$  and  $\varepsilon_j^{CG}(\theta_c, \eta)$
- ▶ Idiosyncratic transitory productivity  $\eta \sim \Pi^{CD}(\cdot)$

## Education stage: Enrollment

$$V_0(a, \theta_h, \eta, q, \phi) = \max[\underbrace{V_1^c(a, \theta_h, \eta, q, \phi)}_{\text{enrolling}}, \underbrace{V_1(a, HS, \theta_h, \eta)}_{\text{not enrolling}}]$$

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- I assume enrollees are overoptimistic on college abilities.

$$\theta_c = \underbrace{\mu_c(\theta_h)}_{\text{bias}} + \underbrace{\theta_h + \epsilon_c}_{\text{actual ability}} \text{ and } \epsilon_c \sim N(0, \sigma_c^2), \text{ (Perceived law of motion)}$$

where

$$\mu_c(\theta_h) = \mu_{c0} + \mu_{c1}\theta_h$$

## Education stage: First half of college

$$V_1^c(a, \theta_h, \eta, q, \phi) = \max_{c, h, a', y} u(c, 1 - h - \bar{h}) + \mathbb{E}_{\theta_c | \theta_h} \lambda_1(\theta_c, \phi) \\ + \beta \mathbb{E}_{\theta_c | \theta_h} \mathbb{E}_{\eta'} \max[\underbrace{V_2^c(a', \theta_c, \eta', q, \phi)}_{\text{continue}}, \underbrace{V_2(\tilde{a}(a'), CD, \theta_h, \eta')}_{\text{dropout}}]$$

subject to

$$c + a' + p_e = a + y + s_1(q) - T(c, a, y)$$

$$y = w^{HS} \varepsilon_1^{HS}(\theta_h, \eta) h, \quad a' \geq -\underline{A}_1^c \quad c \geq 0, \quad 0 \leq h \leq 1 - \bar{h}$$

$$\theta_c = \theta_h + \mu_c(\theta_h) + \epsilon_c, \quad \epsilon_c \sim N(0, \sigma_c^2), \quad \eta' \sim \Pi^{CD}$$

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- Going to college requires a fraction  $\bar{h}$  of time.
- At the beginning of  $j = 2$ , they observe  $\theta_c$  and  $\eta'$  and make a dropout decision.

## Education stage: Second half of college

$$V_2^c(a, \theta_c, \eta, q, \phi) = \max_{c, h, a', y} u(c, 1 - h - \bar{h}) + \lambda_2(\theta_c, \phi) + \beta \mathbb{E}_{\eta'} V_3(\tilde{a}(a'), CG, \theta_c, \eta)$$

subject to

$$c + a' + p_e - s_2(q) - y + T(c, a, y) = \begin{cases} (1+r)a & \text{if } a \geq 0 \\ (1+r^s)a & \text{if } a < 0 \end{cases}$$

$$y = w^{CD} \varepsilon_2^{CD}(\theta_c, \eta) h, \quad a' \geq -\underline{A}_2^c \quad c \geq 0, \quad 0 \leq h \leq 1 - \bar{h}, \quad \eta' \sim \Pi^{CG}$$

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- They can work as college dropouts.
- At the end of the period, one completes college and draws  $\eta'$  from  $\Pi^{CG}$ .
- [▶ Financial Market](#)

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- I assume retirees offer no labor, receive pension  $p(e, \theta)$ , and have no access to loans. ▶ Retirement Stage

## Goods Sector

- A representative firm produces final good from capital  $K$  and aggregate labor  $H$ :

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where  $\frac{1}{1-\rho}$  is the elasticity of substitution.

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where  $\frac{1}{1-\rho}$  is the elasticity of substitution.

- ▶ College graduates work as skilled labor:  $w^{CG} = w^S$
- ▶ High school graduates and college dropouts work as unskilled labor:  
 $w^{HS} = w^{CD} = w^U$  ▶ Share of skilled labor by college dropout

- A representative college requires  $\kappa$  units of skilled labor to provide education.

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- I assume colleges are competitive and there is free entry:  $p_e = w^S \kappa$

## Government

- The government collects tax  $T(c, a, y)$  and spend the revenues on
  - ▶ college subsidies

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- The tax function is assumed to be

$$T(c, a, y) = \tau_c c + \tau_k r a \mathbf{1}_{a \geq 0} + \tau_l y - d \frac{Y}{N}$$

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- The remaining parameters to match moments given the first set of parameter values.

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	HS	CD	CG
$\log$ AFQT	.61 (.32)	.74 (.32)	1.31 (.24)

## Transitory Labor Productivity Process

- I assume  $\pi_{\eta}^e(\eta'|\eta)$  is a two-state Markov chain approximating ▶ Markov Chain

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- Minimum Distance Estimator separately for each education level.

	HS	CD	CG
$\rho^e$	0.94	0.95	0.95
$\sigma_{\eta}^{e2}$	0.017	0.021	0.025

## Intergenerational Ability Transmission

- New independent individuals draw their high school abilities  $\theta'_h$ .

$$\theta'_h = m + m_\theta \theta + \epsilon_\theta, \quad \epsilon_\theta \sim N(0, \sigma_h^2)$$

- I regressed children's ability on parents' ability to get  $m_\theta = 0.46$ .

## Subsidies and Loans

$q$	family income	subsidies to students	subsidies to colleges	total $\bar{s}(q)$
1	- \$30,000	\$2,820	\$10,477	\$13,297
2	\$30,000 - \$80,000	\$668	\$10,477	\$11,145
3	\$80,000 -	\$143	\$10,477	\$10,620

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- Students' interest rate is the prime rate plus  $\iota^s = 2.3\%$ , annual.

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- In the current system, college subsidies are constant and  $s_1(q) = s_2(q) = \bar{s}(q)$ .
- Students' interest rate is the prime rate plus  $\iota^s = 2.3\%$ , annual.
- The loan limit for the first half  $\underline{A}_1^c$  is \$6,125 (= \$2,625 + \$3,500) from Stafford loan.
- The loan limit for the second half  $\underline{A}_2^c$  is \$23,000.

## The Remaining Parameters

Parameter	Description	Value
$\mu_c^0$	college ability bias intercept	0.190
$\mu_c^1$	college ability bias slope	-0.409
$\lambda$	college utility intercept	-23.2
$\lambda^\theta$	college utility slope	241
$\lambda_1^\phi$	first period college taste	64.1
$\lambda_2^\phi$	second half college taste	41.3
$a^S$	productivity of skilled labor	0.457
$\epsilon^{CD}$	productivity of CD	1.02
$\sigma_c$	s.d. of college ability	0.340
$\kappa$	education cost	0.226
$\mu$	consumption share of preference	0.418
$\beta$	time discount rate	0.938
$\nu$	altruism	0.0948
$d$	lump-sum transfer ratio	0.125
$\iota$	borrowing wedge ( $r^- = r + \iota$ )	18.0%
$m$	intergenerational ability transmission intercept	-0.0471
$\sigma_h$	intergenerational ability transmission s.d.	0.171

## Matched Moments

Moment	Model	Target
Enrollment rate of ability quartile	(figure)	(figure)
Graduation rate of ability quartile	(figure)	(figure)
Enrollment rate of family income quartile	(figure)	(figure)
Graduation rate of family income quartile	(figure)	(figure)
Skill premium for CG	90.8%	90.2%
Skill premium for CD	19.6%	19.9%
Expected/Actual graduation rate -1	0.431	0.433
Education cost/mean income at 48	0.320	0.33
Hours of work	33.8%	33.3%
$K/Y$	1.298	1.325
Transfer/mean income at 48	67.0%	66%
Log pre-tax/post-tax income	61.2%	61%
Borrowers	6.59%	6.3%
Mean of AFQT	-0.0135	0
Standard deviation of AFQT	0.217	0.213

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- To match this fact, the calibrated  $\mu_c^0$  is positive and
  - ▶ the bias for the mean ability is 48% of the standard deviation of college ability.
- Low ability students are more optimistic ( $\mu_c^1 < 0$ ), which is consistent with data.

# Model Fit

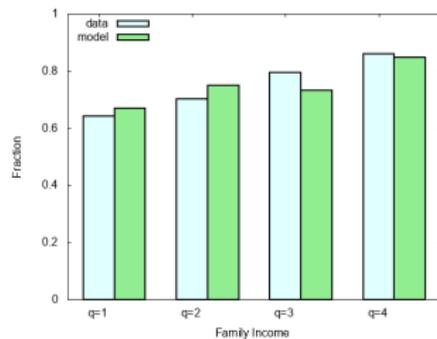
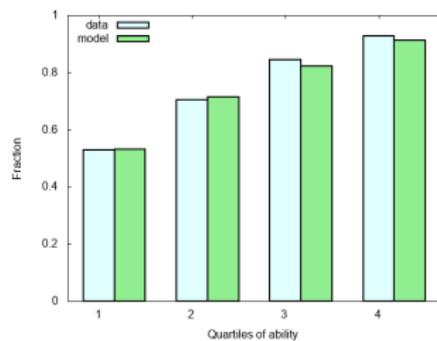


Figure: Enrollment rates

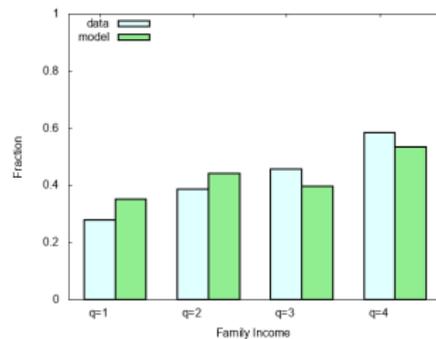
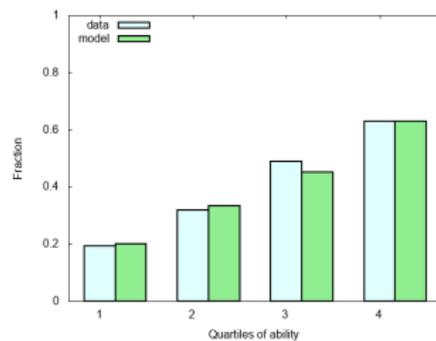


Figure: Graduation rates

## Validation 1: Partial Equilibrium Effect of Year-Invariant subsidies

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- The aggregate enrollment rate of the affected generation increases by 1.05 percentage points in the simulation, which is broadly in the range.
  - ▶ The fraction of college graduates increases by 0.45 percentage points.
  - ▶ The fraction of college dropouts increases by 0.60 percentage points.

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## Validation 2: Sluggish increase in college graduates

- In the US, the number of college graduates increased sluggishly despite the increase in the skill premium.
- Derive the two steady states'  $a^S$  and  $\epsilon^{CD}$  imitating 1980 and 2000 skill premiums.
- Compare the changes of the numbers of college graduates and dropouts with data.

	1980	2000	change (model)	change (data)
college graduate premium	46.2%	90.9%	44.7pp	43.2pp
college dropout premium	12.1%	19.6%	7.5pp	7.4pp
share of college graduates	28.0%	32.9%	4.9pp	4.98pp
share of college dropouts	42.8%	41.3%	-1.5pp	2.41pp

# Outline

- 1 Introduction
- 2 Model
- 3 Calibration
- 4 Results**
- 5 Conclusion

## Main Exercises

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- Exercise 2: Keep total spending fixed but choose subsidies by year (**year-dependent subsidies**) to maximize the number of college graduates in steady state.
- Exercise 3: Keep total spending fixed and choose subsidies to maximize welfare in steady state.

## Exercise 1: Year Invariant Subsidies

$G_e$	$0.75 \bar{G}_e$	$\bar{G}_e$	$1.5 \bar{G}_e$	$2 \bar{G}_e$
enrollment rate	72.7%	74.2%	77.2%	77.8%
share of college graduates	32.1%	32.9%	34.2%	35.0%
skill premium	95.0%	90.9%	82.8%	78.3%

## Exercise 2: Year Dependent Subsidies That Maximize College Graduates

$$\max_{g_1, g_2, \tau_\ell} \int_{S_2^{CG}} d\mu_2^{CG}$$

subject to

$$g_1 \int_{S_1^c} \bar{s}(q) d\mu_1^c + g_2 \int_{S_2^c} \bar{s}(q) d\mu_2^c = G_e$$

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$s_j(q)$	year-invariant $\bar{G}_e$	year-dependent $\bar{G}_e$
$s_1(1)$	\$13,599	\$4
$s_1(2)$	\$11,447	\$4
$s_1(3)$	\$10,922	\$3
$s_2(1)$	\$13,599	\$42,436
$s_2(2)$	\$11,447	\$35,720
$s_2(3)$	\$10,922	\$34,082

- Back-loaded

## Exercise 2: Year Dependent Subsidies That Maximize College Graduates

year-invariant/dependent	invariant $\bar{G}_e$	dependent $\bar{G}_e$	invariant $1.5\bar{G}_e$
enrollment rate	74.2%	68.7%	77.2%
share of college graduates	32.9%	34.5%	34.2%
skill premium	90.9%	82.6%	82.8%

- Share of college graduates increases more than increasing the total budget by 50%.
- Skill premium decreases more than increasing the total budget by 50%.
- Enrollment decreases.

## Mechanism

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- Decreasing subsidies for the first period serves mainly to discourage people who are unlikely to graduate from enrolling.
- The higher subsidies for the second period encourages marginal dropouts to finish.
- In addition, we can shift subsidies away from college dropouts to college graduates.

### Exercise 3: Year Dependent Subsidies That Maximize Welfare of Newborns

$$\sum_j N_j \left( \int V_j(\mathbf{s}_j) d\bar{\mu}_j(\mathbf{s}_j) + \int V_j^c(\mathbf{s}_j^c) d\bar{\mu}_j(\mathbf{s}_j^c) \right)$$

subject to

$$g_1 \int_{S_1^c} \bar{s}(q) d\mu_1^c + g_2 \int_{S_2^c} \bar{s}(q) d\mu_2^c = G_e$$

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	Current state	Optimal
$s_1(1)$	\$13,599	\$10,721
$s_1(2)$	\$11,447	\$9,025
$s_1(3)$	\$10,922	\$8,611
$s_2(1)$	\$13,599	\$19,858
$s_2(2)$	\$11,447	\$16,716
$s_2(3)$	\$10,922	\$15,949

- Optimal subsidies are back-loaded.

## Aggregates

	Current state	Optimal
share of college enrollees	74.2%	73.8%
share of college graduates	32.9%	33.6%
skill premium	90.9%	87.3%
welfare gain		+0.15%

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Optimal	+0.07%	+0.15%	+0.04%	-0.09%

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- Back-loaded subsidies improve welfare.
- The level effect is positive while inequality at the initial state increases.

## Welfare

	Current state	Optimal
$Y$	0.318	0.318
$K$	0.413	0.413
$C$	0.211	0.211
$w^S$	0.355	0.352
$w^U$	0.405	0.408
std $c$	0.129	0.129
std $a$	0.478	0.475
std $h$	0.0834	0.0833
std wage	0.544	0.540

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std wage	0.544	0.540

	$q = 1$	$q = 2$	$q = 3$
$\theta = 1$	+0.6%	+0.1%	+0.5%
$\theta = 2$	+0.2%	-0.4%	+0.5%
$\theta = 3$	-0.8%	-0.3%	+0.5%
$\theta = 4$	-0.9%	-0.0%	+0.4%

- High-ability poor-family enrollees lose welfare.

## Conclusion

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## Conclusion

- Back-loaded subsidies maximize the number of college graduates and social welfare.
- The number of college graduates increases and the skill premium decreases as much as the case with increasing the total budget by 50%.
- Enrollment decreases despite an increase in college graduates. Policies increasing enrollment might be misguided.

## Student Loan Transformation

- The fixed payment to repay full debt for 20 years (10 periods)  $d$  is given by

$$a' = \sum_{t=0}^9 \frac{d}{(1+r^s)^t} = \frac{d}{1+r^s} \frac{1-(1+r^s)^{-10}}{1-(1+r^s)^{-1}} = d \frac{1-(1+r^s)^{-10}}{r^s}$$

- To have the same payment schedule  $d$  with interest  $r^-$ , the initial balance has to be

$$\tilde{a}(a') = \sum_{t=0}^9 \frac{d}{(1+r^-)^t} = \frac{d}{1+r^-} \frac{1-(1+r^-)^{-10}}{1-(1+r^-)^{-1}} = d \frac{1-(1+r^-)^{-10}}{r^-}$$

- As a result,

$$\tilde{a}(a') = a' \times \frac{r^s}{1-(1+r^s)^{-10}} \times \frac{1-(1+r^-)^{-10}}{r^-}$$

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$$V_j(a, e, \theta, \eta) = \max_{c, h, a', y} u\left(\frac{c}{1 + \mathbf{1}_{\mathcal{J}_f} \zeta}, 1 - h\right) + \beta \mathbb{E}_{\eta' | \eta} V_{j+1}(a', e, \theta, \eta')$$

subject to

$$c + a' - y + T(c, a, y) = \begin{cases} (1 + r)a & \text{if } a \geq 0 \\ (1 + r^-)a & \text{if } a < 0 \end{cases}$$

$$y = w^e \varepsilon_j^e(\theta, \eta) h, \quad a' \geq -\underline{A}^e \quad c \geq 0, \quad 0 \leq h \leq 1, \quad \eta' \sim \pi^e(\cdot | \eta)$$

where  $\mathbf{1}_{\mathcal{J}_f}$  is an indicator function which is one when the individual lives with its children ( $j \in [j_f, j_b - 1]$ ). [▶ Back](#)

## Transfer

$$V_j(a, e, \theta, \eta) = \max_{c(\theta'_h), h(\theta'_h), a'(\theta'_h), y(\theta'_h)} \mathbb{E}_{\theta'_h | e, \theta} \{u(c(\theta'_h), 1 - h(\theta'_h)) + \tilde{V}_{j_b+1}(a', \theta, \theta'_h, e, \eta)\}$$

subject to

$$c(\theta'_h) + a'(\theta'_h) - y(\theta'_h) + T(c(\theta'_h), a(\theta'_h), y(\theta'_h)) = \begin{cases} (1+r)a & \text{if } a \geq 0 \\ (1+r^-)a & \text{if } a < 0 \end{cases}$$

$$y(\theta'_h) = w^e \varepsilon_j^e(\theta, \eta) h(\theta'_h), \quad a' \geq -\underline{A}^e \quad c(\theta'_h) \geq 0, \quad 0 \leq h(\theta'_h) \leq 1, \quad \eta' \sim \pi^e(\cdot | \eta)$$

where

$$\tilde{V}_{j_b+1}(a, \theta, \theta'_h, e, \eta) = \max_{b \in [0, a]} \beta \mathbb{E}_{\eta' | \eta} V_{j_b+1}(a-b, e, \theta, \eta') + \nu \mathbb{E}_{\eta'' | \eta, \phi} V_0(b, \theta'_h, \eta'', \tilde{q}(w^e \varepsilon_j^e(\theta, \eta)), \phi)$$

for all  $\theta'_h$ .

- Individuals can make parental transfers  $b$  to their children only at this age.
- Before making any decisions, individuals observe only their children's high school ability  $\theta'_h$  from  $\pi_\theta(\theta'_h | e, \theta)$ .

## Family income level

- Family income level

$$\tilde{q}(w^e \varepsilon_j^e(\theta, \eta)) = \begin{cases} 1 & \text{if } w^e \varepsilon_j^e(\theta, \eta) \times 0.35 \in [0, q_1] \\ 2 & \text{if } w^e \varepsilon_j^e(\theta, \eta) \times 0.35 \in [q_1, q_2] \\ 3 & \text{else} \end{cases}$$

where  $q_1$  and  $q_2$  correspond to \$30,000 and \$80,000.

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## Retirement Stage

$$V_j(a, e, \theta) = \max_{c, a'} u(c, 1) + \beta \varphi_{j+1} V_{j+1}(a', e, \theta)$$

subject to

$$c + a' = (1 + r)\varphi_j^{-1}a + p(e, \theta) - T(c, \varphi_j^{-1}a, 0)$$

$$a' \geq 0 \quad c \geq 0$$

- The sources of income is asset earnings and retirement benefits  $p(e, \theta)$ .
- The asset inflated by  $\varphi_j^{-1}$  reflects that assets of expiring households are distributed within cohorts (perfect annuity market).

- [▶ Back](#) [▶ Social Security](#)

- The average life time income is

$$\hat{y}(e, \theta) = \frac{\sum_{j=j_a+2}^{j_r-1} w^e \varepsilon_j^e(\theta, 1) \bar{h}}{j_r - 2}$$

- The pension formula is given by

$$p(e, \theta) = \begin{cases} s_1 \hat{y}(e, \theta) & \text{for } \hat{y}(e, \theta) \in [0, b_1) \\ s_1 b_1 + s_2 (\hat{y}(e, \theta) - b_1) & \text{for } \hat{y}(e, \theta) \in [b_1, b_2) \\ s_1 b_1 + s_2 (b_2 - b_1) + s_3 (\hat{y}(e, \theta) - b_2) & \text{for } \hat{y}(e, \theta) \in [b_2, b_3) \\ s_1 b_1 + s_2 (b_2 - b_1) + s_3 (b_3 - b_2) & \text{for } \hat{y}(e, \theta) \in [b_3, \infty) \end{cases}$$

where  $s_1 = 0.9$ ,  $s_2 = 0.32$ ,  $s_3 = 0.15$ ,  $b_1 = 0.22\bar{y}$ ,  $b_2 = 1.33\bar{y}$ ,  $b_3 = 1.99\bar{y}$ ,  
 $\bar{y} = \$28,793$  annually.

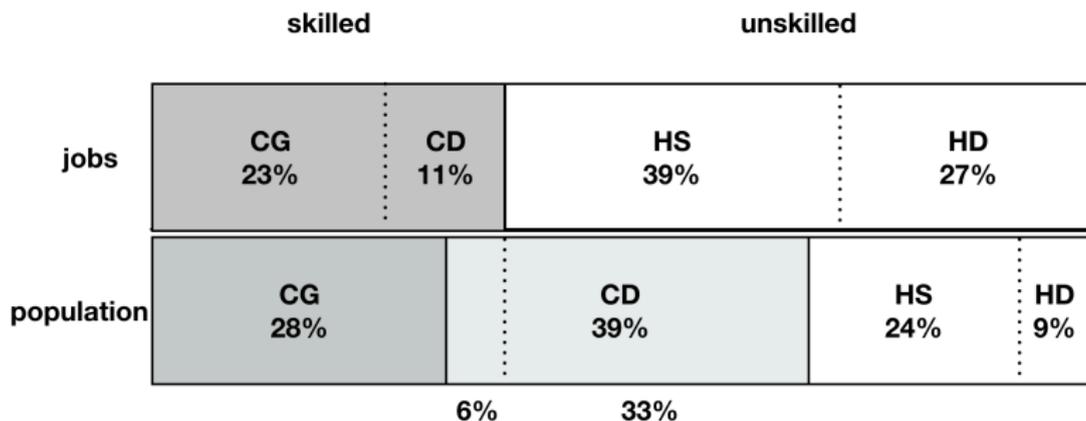
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# Financial Market

- There is no insurance market and individuals can self-insure using only risk-free assets.
- Borrowing wedge:
  - ▶ Overseeing cost  $\iota$  for workers:  $r^- = r + \iota$
  - ▶ Overseeing cost  $\iota + \iota^s$  for enrollees:  $r^s = r^- + \iota^s$
- Borrowing limit:
  - ▶  $\underline{A}^e$  for workers with education  $e$
  - ▶  $\underline{A}_j^c$  for enrollees at age  $j$
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## Share of Skilled Labor from College Dropouts

- Two separate data:
  - ▶ the fraction of jobs requiring each education level
  - ▶ the fraction of workers acquiring each education level
- Interpreting jobs for college dropouts and more as skilled labor. [▶ Back](#)



# Government Budget

- Government Budget Constraint

$$G_c + G_e + \sum_{j=j_r}^J \int_{S_j} p(e, \theta) d\mu_j = \sum_{j=1,2} \int_{S_j^c} T(c_j^c(\mathbf{s}_j^c), a_j^c(\mathbf{s}_j^c), y_j^c(\mathbf{s}_j^c)) d\mu_j^c \\ + \sum_j \int_{S_j} T(c_j(\mathbf{s}_j), a_j(\mathbf{s}_j^s), y_j(\mathbf{s}_j^s)) d\mu_j^s$$

where

$$G_c = gF(K, H) \\ G_e = \sum_{j=1,2} \int_{S_j^c} s_j(q, \theta) d\mu_j^c$$

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## Market clearing

- Aggregate labor

$$H^S + \kappa E = H^{CG}$$
$$H^U = H^{HS} + H^{CD}$$

where

$$H^{CG} = \sum_{j=3}^{j_r-1} \int_{S_j^{CG}} \epsilon_j^{CG}(\theta, \eta) h_j(\mathbf{s}_j) d\mu_j^{CG}$$

$$H^{CD} = \sum_{j=2}^{j_r-1} \int_{S_j^{CD}} \epsilon_j^{CD}(\theta, \eta) h_j(\mathbf{s}_j) d\mu_j^{CD} + \int_{S_2^c} \epsilon_2^{CD}(\theta, \eta) h_2^c(\mathbf{s}_2^c) d\mu_2^c$$

$$H^{HS} = \sum_{j=1}^{j_r-1} \int_{S_j^{HS}} \epsilon_j^{HS}(\theta, \eta) h_j(\mathbf{s}_j) d\mu_j^{HS} + \int_{S_1^c} \epsilon_1^{HS}(\theta, \eta) h_1^c(\mathbf{s}_1^c) d\mu_1^c$$

- Capital

$$K = \sum_{j=1}^{j_r-1} \int_{S_j} a_j'(\mathbf{s}_j) d\mu_j + \sum_{j=1,2} \int_{S_j^c} a_j'^c(\mathbf{s}_j^c) d\mu_j^c$$

- Education

$$E = \sum_{j=1,2} \int_{S_j^c} d\mu_j^c$$

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# Equilibrium

## Definition

A stationary equilibrium is a list of value functions of workers and college enrollees  $\{V_j(\mathbf{s}_j), V_j^c(\mathbf{s}_j^c)\}$ , decision rules of enrollment  $d_0(\mathbf{s}_0)$  and graduation  $d_1(\mathbf{s}_1^c)$ , decision rules of consumption, asset holdings, labor, output, parental transfers of workers  $\{c_j(\mathbf{s}_j), a'_j(\mathbf{s}_j), h_j(\mathbf{s}_j), y_j(\mathbf{s}_j), b(\mathbf{s}_j)\}$ , decision rules of college enrollees  $\{c_j^c(\mathbf{s}_j^c), a_j^c(\mathbf{s}_j^c), h_j^c(\mathbf{s}_j^c), y_j^c(\mathbf{s}_j^c)\}$ , aggregate enrollees, capital, and labor inputs  $\{E, K, H^S, H^U\}$ , prices  $\{r, w^S, w^U, p_e\}$ , policies  $\tau_\ell$ , measures  $\mu = \{\mu_j^c(\mathbf{s}_j^c), \mu_j(\mathbf{s}_j), \mu_j^e(\mathbf{s}_j^e)\}$  such that

- 1 Taking prices and policies as given, value functions  $\{V_j^c(\mathbf{s}_j^c), V_j(\mathbf{s}_j)\}$  solve the household Bellman equation\*s and  $d_0(\mathbf{s}_0), d_1(\mathbf{s}_1^c), \{c_j(\mathbf{s}_j), a'_j(\mathbf{s}_j), h_j(\mathbf{s}_j), y_j(\mathbf{s}_j), b(\mathbf{s}_j)\}, \{c_j^c(\mathbf{s}_j^c), a_j^c(\mathbf{s}_j^c), h_j^c(\mathbf{s}_j^c), y_j^c(\mathbf{s}_j^c)\}$  are associated decision rules.
- 2 Taking prices and policies as given,  $K, H^{HS}, H^{CG}$  solve the optimization problem of the good sector and  $E$  solves the optimization problem of the education sector.
- 3 The government budget is balanced.
- 4 Human capital, asset, and education markets clear.
- 5 Measures  $\mu$  are reproduced for each period.



## Labor Productivity Process Estimation

- PSID: SRC sample, only people with 8 or more individual-year observations
- keep only positive hours of labor aged 25-63
- eliminate extreme changes in earnings
- quadratic ages are separately estimated by education group with year dummies

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	HS	CD	CG
<i>Age</i>	.0530181 (.0030501)	.0684129 (.0040353)	.0955783 (.0036997)
<i>Age</i> <sup>2</sup>	-.0005314 (.0000356)	-.0006872 (.0000474)	-.0009521 (.0000429)

## Labor Productivity

- For high school graduates,  $\theta = \theta_h$  which is approximated by  $\ln \text{AFQT80}$ .
- For college dropouts and college graduates, I use high school ability ( $\theta_c = \theta_h + \epsilon_c$ ).

$$\ln \epsilon^e + \ln \psi_j^e + \epsilon_\theta^e \theta_c + \ln \eta = \ln \epsilon^e + \ln \psi_j^e + \epsilon_\theta^e \theta_h + (\ln \eta + \epsilon_\theta^e \epsilon_c)$$

because  $\theta_h$  is uncorrelated with  $\ln \eta + \epsilon_\theta^e \epsilon_c$ .

# Markov Chain Approximation

- Two state Markov chain with education-specific states for  $\{-\sigma_e, \sigma_e\}$  and transition matrix

$$\Pi = \begin{bmatrix} \pi_e & 1 - \pi_e \\ 1 - \pi_e & \pi_e \end{bmatrix}$$

where

$$\rho^{e2} = 2\pi_e - 1$$

$$\sigma_e = \frac{\sigma_\eta^e}{\sqrt{1 - \rho^{e2}}}$$

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## Parameters Determined outside the Model

Parameters	Interpretation	Value
$\gamma$	Coef of relative risk aversion	4
$\bar{h}$	Study time	0.25
$\zeta$	Adult equivalence scale	0.3
$\alpha$	Capital share	33.3%
$\delta$	Depreciation (annual)	7.55%
$\rho$	Elasticity of substitution in production 1.41	0.2908
$\iota^s$	Stafford interest premium (annual)	2.3%
$\underline{A}_1^c$	Borrowing constraint for 1st half (Stafford loan)	\$6,125
$\underline{A}_2^c$	Borrowing constraint for 2nd half (Stafford loan)	\$23,000
$\underline{A}^{HS}$	Borrowing constraint, HS (SCF)	\$17,000
$\underline{A}^{CD}$	Borrowing constraint, CD (SCF)	\$20,000
$\underline{A}^{CG}$	Borrowing constraint, CG (SCF)	\$34,000
$\tau_c$	Consumption tax rate	7%
$\tau_k$	Capital income tax rate	27%
$g$	Gov cons to GDP ratio	17.1%

## Minimum Distance Estimator

- The residual process is assumed to be

$$y_{ia} = \alpha_i + z_{ia} + u_{ia}$$

where

$$z_{ia} = \rho z_{ia-1} + \epsilon_{\eta ia}, \quad \epsilon_{\eta ia} \sim N(0, \sigma_\eta^2)$$

- Then

$$\text{cov}(y_{ia}, y_{ia-d}) = \sigma_\alpha^2 + \rho^d \frac{1 - \rho^{2a}}{1 - \rho^2} \sigma_\eta^2 + \mathbb{1}_{d=0} \sigma_u^2$$

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## Responding to the consumption loss at the first period

	% of subsidy loss
Subsidies	-100%
Labor income	+24%
(Price of an hour of working)	+13%
(Leisure)	(-0.061)
Transfer from parents	+0.03%
Reducing savings	+65%
Less tuition	+4%
Consumption	-7%

- Consumption at the first period does not decrease much because:
  - ▶ The wage of college enrollees increases due to a smaller skill premium.

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  - ▶ Parents increase transfer.

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- **Front-loaded subsidies** are optimal when correcting bias.

## Correcting Bias

	Total	Level	Uncertainty	Inequality
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Correct bias (Optimal)	+2.05%	-2.31%	+3.51%	-1.37%

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	Current state	Correcting bias	Optimal
share of college enrollees	74.2%	45.5%	45.8%
share of college graduates	32.9%	26.2%	26.0%
skill premium	90.9%	124%	125%
welfare gain		-9.28%	-9.25%

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- Correcting bias reduces welfare significantly.
- Enrollment is excessively low due to no insurance on college ability.

## No Optimism

- In this paper, optimism is a key factor for college dropouts.
- A different approach to explain college dropouts: High option value due to high uncertainty of college ability.
- I assume that the standard deviations of college ability can vary across high school ability.

$$\sigma_c(\theta_h) = \sigma_c \exp(\sigma_c^\theta \theta_h)$$

## No Optimism: The Remaining Parameters

Parameter	Description	Value
$\lambda$	college utility intercept	-16.6
$\lambda^\theta$	college utility slope	287
$\lambda_1^\phi$	first period college taste	68.8
$\lambda_2^\phi$	second half college taste	40.0
$a^S$	productivity of skilled labor	0.435
$\epsilon^{CD}$	productivity of CD	0.985
$\sigma_c$	s.d. of college ability intercept	0.721
$\sigma_c^\theta$	s.d. of college ability slope	0.158
$\kappa$	education cost	0.422
$\mu$	consumption share of preference	0.422
$\beta$	time discount rate	0.931
$\nu$	altruism	0.0630
$d$	lump-sum transfer ratio	0.131
$\iota$	borrowing wedge ( $r^- = r + \iota$ )	18.7%
$m$	intergenerational ability transmission intercept	-0.0384
$\sigma_h$	intergenerational ability transmission s.d.	0.0764

## No Optimism: Matched Moments

Moment	Model	Target
Enrollment rate of ability quartile	(figure)	(figure)
Graduation rate of ability quartile	(figure)	(figure)
Enrollment rate of family income quartile	(figure)	(figure)
Graduation rate of family income quartile	(figure)	(figure)
Skill premium for CG	90.7%	90.2%
Skill premium for CD	20.1%	19.9%
Education cost/mean income at 48	0.308	0.33
Hours of work	33.3%	33.3%
$K/Y$	1.241	1.325
Transfer/mean income at 48	67.2%	66%
Log pre-tax/post-tax income	60.5%	61%
Borrowers	6.07%	6.3%
Mean of AFQT	0.0880	0
Standard deviation of AFQT	0.204	0.213

# No Optimism: Model Fit

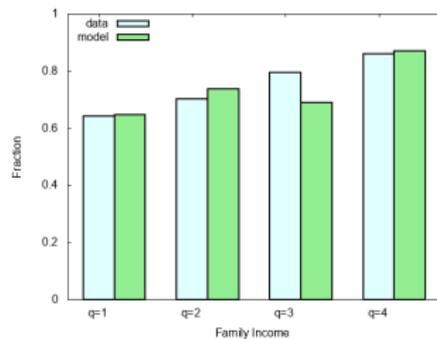
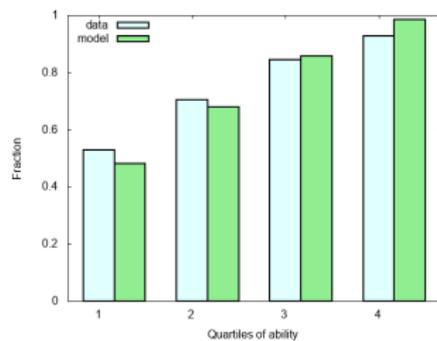


Figure: Enrollment rates

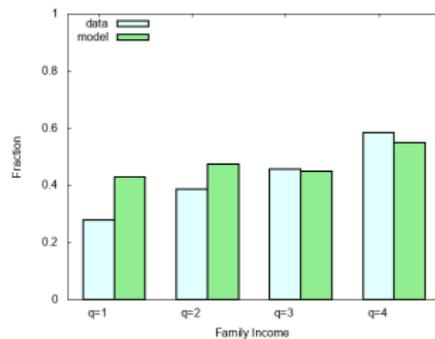
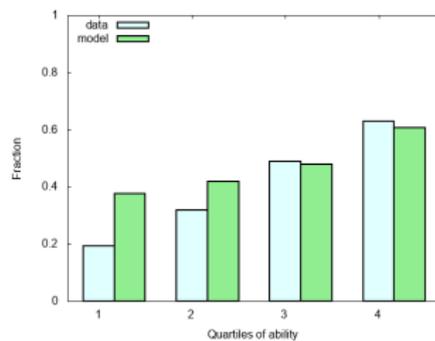


Figure: Graduation rates

## Mo Optimism: Optimal Policy

	Current state	Optimal
$s_1(1)$	\$13,600	\$14,153
$s_1(2)$	\$11,448	\$11,913
$s_1(3)$	\$10,923	\$11,367
$s_2(1)$	\$13,600	\$12,478
$s_2(2)$	\$11,448	\$10,503
$s_2(3)$	\$10,923	\$10,021

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