Discouraging Deviant Behavior in Monetary Economics

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Northwestern and Hitotsubashi

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- Message from models: Taylor rule not sufficient to stabilize inflation globally.

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  - The policy delivering uniqueness is of no economic interest.
- Our finding is that Cochrane's conclusion is *not* correct in a production economy.
  - While correct in his endowment economy.

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  - Tiny trembles activate escape clause,
    - Negative consequences for welfare if there are money demand shocks.

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- We use a refinement of *rationalizability* to answer the big question.
  - Rationalizable implementation is more desirable for policy design.
    - Bergemann, Morris, and Tercieux(2011).

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- Conclusion

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where $\bar{\pi}^* = \bar{\mu}^* \geq 1$ and $\bar{R}^*$ are *desired* inflation and interest rate.

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- Household first order conditions:

$$\frac{W_t}{P_t} = c_t^{\gamma} l_t^{\psi}, \quad c_t^{-\gamma} = \beta c_{t+1}^{-\gamma} \frac{\bar{R}_t}{\bar{\pi}_{t+1}}, \quad \text{‘Euler equation’}$$

plus transversality and cash in advance conditions.

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- In equilibrium, the Euler equation is the Fisher equation:

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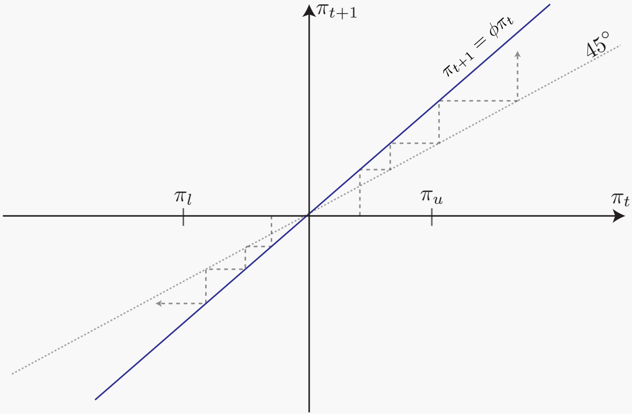
Properties of Taylor Rule Equilibrium

Multiplicity and Local Uniqueness of Desired Equilibrium

- Multiple equilibria, $\{\pi_t\}$, each indexed by π_0.
- Desired equilibrium is unique equilibrium that never violates *monitoring range*, $[\pi_l, \pi_u]$.
  - If $\pi_0 \neq 0$, then $|\pi_t| \to \infty$.



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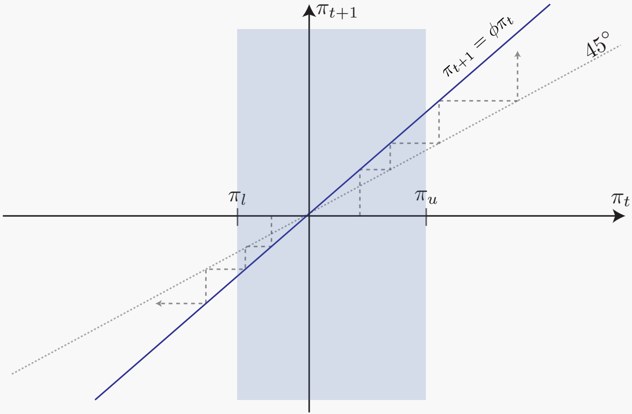
- Result: under Taylor rule with escape clause, desired equilibrium is the globally unique equilibrium.

Uniqueness of Equilibrium Under Escape Clause

✓ If $\pi_0 \neq 0$, then $|\pi_t| \to \infty$.

- Activation of escape clause is not consistent with the equilibrium conditions.
- Unique equilibrium associated with $\pi_0 = 0$.



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$$\text{Taylor rule} \quad : \quad R_T = \phi\pi_T > \pi_u, \text{ because } \phi > 1$$

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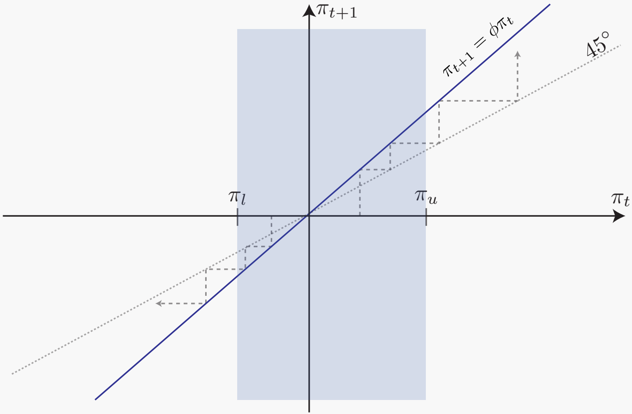
- So,

$$R_T > \pi_u \text{ and } R_T \leq \pi_u, \qquad \text{contradiction!}$$

Uniqueness of Equilibrium Under Escape Clause

- ✓ If $\pi_0 \neq 0$, then $|\pi_t| \to \infty$.
- ✓ Activation of escape clause is not consistent with the equilibrium conditions.
- Unique equilibrium associated with $\pi_0 = 0$.



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  - In the monetary model, no one would believe such policy, and hyperinflation is not excluded!

Cochrane's Critique in Our Production Economy

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- Euler equation in Cochrane's endowment economy:

$$R_T = \pi_{T+1}$$

  - in *and* out of equilibrium because $c_t = Y$ for all $t \geq 0$ (Cochrane (2010,p.574).

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- What is it about the escape clause that implies $\pi_T > \pi_u$ cannot occur in equilibrium?
- We need an equilibrium concept which allows for out-of-equilibrium.

Exit Ramp Off Equilibrium

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- Best response analysis goes back at least to Diamond and Dybvig (1983)
  - Describe what would happen, off-equilibrium paths, and discourage undesirable actions.

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  - In the afternoon, the rest happens so W_t/P_t^c is determined as a function of "history," (h_{t-1}, P_t^c).

$$p_{i,t} = P_t^c \times \frac{W_t}{P_t^c} = P_t^c \left(c_t^b\right)^{\gamma+\psi} = P_t^c \left(c_t^b \left(h_{t-1}, P_t^c\right)\right)^{\gamma+\psi}.$$

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Continuation Equilibrium

- Let

$$a_t = \left(I_t, \pi_t, c_t, R_t, W_t, \mu, \bar{M}_t\right)$$
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Definition

A *continuation equilibrium* conditional on (h_{t-1}, π_t^c) is a sequence, a_{t+s}, for $s \geq 0$, with two properties:

(a) a_{t+s}, $s > 0$ satisfies all $t+s$ equilibrium conditions.

(b) a_t satisfies all time t equilibrium conditions except intermediate good firm optimality.

Strategy Equilibrium

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A strategy equilibrium is a competitive equilibrium with the property that for each possible history h_{t-1}: (i) there is a well-defined continuation equilibrium corresponding to any value of π_t^c and (ii) there exists a π_t^c that is a fixed point:

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- Part (i) provides an exit-ramp from the competitive equilibrium in each t.
  - Allows us to think coherently about why people privately choose not to take the exit ramp.
  - Can ask 'why does the escape strategy' trim non-desired equilibria?

Why is $\pi_T^c > \pi_u$ not an Equilibrium Under Escape Clause?

- Easy to show that actual inflation would be:

$$F\left(h_{T-1}, \pi_T^c\right) = \pi_T^c + \underbrace{(\gamma + \psi) \overbrace{\left[\frac{\phi}{1-\gamma}\pi_T^c\right]}^{\ln c_T}}_{\text{real wage}}.$$

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- So, intermediate firms post lower prices, and actual inflation is low,

$$\pi_T^c > F\left(\pi_T^c\right)$$

- No fixed points.

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Proposition

If $\gamma > 1$ and $1 < \phi \leq 2\frac{\gamma-1}{\gamma+\psi}$, then for any large compact set Π, $F^{\infty}(\Pi) = \{0\}$.

- Rational firms convince themselves that desired equilibrium occurs!
  - A desired property for policy design.

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- Logic by which it works looks like an ‘Out-of-equilibrium Taylor Principle’.

- Common knowledge of rationality is enough to ensure that firms spontaneously come up with the rational expectation.

Concluding Observations

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  - Need to revisit New Keynesian canon that thinking about money demand is unnecessary.