Labor Market Heterogeneity and the Lucas Critique

Yongsung Chang University of Rochester and Yonsei University

Sun-Bin Kim

Yonsei University

Frank Schorfheide

University of Pennsylvania, CEPR, NBER

May 29, 2010

Prepared for Canon Institute for Global Studies

Motivation

- Representative Agent DSGE Models are commonly used to analyze the effects of monetary and fiscal policies.
- A key assumption in policy experiments is that taste and technology parameters are policy-invariant.

Motivation

- Representative Agent DSGE Models are commonly used to analyze the effects of monetary and fiscal policies.
- A key assumption in policy experiments is that taste and technology parameters are policy-invariant.
- Geweke (1985): Whenever econometric policy evaluation is undertaken using models estimated with aggregated data, it is implicitly presumed that the aggregator function is structural with respect to the policy intervention.

- Use Heterogeneous agent model economy as data generating process:
- Estimate a Representative Agent Model
- Aggregation error is captured by preference shocks in Rep Agent Model
 - Labor market "wedge" (Hall (1997), Chari, Kehoe, McGrattan (2005))
 - Often interpreted as market failure or inefficiency
- To what extent we can predict the effect of tax changes with the Rep Agent Model, assuming invariance of parameters and shocks?

Related Work

Heterogenous Agent Model and Aggregation

- Heterogeneous Agent Economy (e.g, Chang and Kim, 2006, 2007) features:
 - Individuals face stochastic idiosyncratic productivity
 - Incomplete capital markets
 - Face borrowing constraint
 - Supplies either zero or one unit of labor
- Aggregate labor supply curve depends on cross-sectional reservation wage distribution rather than individuals' willingness to substitute leisure over time.
- Aggregation is not perfect due to:
 - Incomplete capital markets
 - Indivisible nature of labor supply

Heterogeneous Agent Economy: Individual Worker's Problem

$$\max_{\{c_t, h_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - B \frac{h_t^{1+1/\gamma}}{1+1/\gamma} \right\} \right]$$

s.t. $c_t + a_{t+1} = a_t + (1 - \tau_H) W_t x_t h_t + (1 - \tau_K) R_t a_t + \bar{T}$
 $a_{t+1} \ge a$

- x_t: stochastic idiosyncratic productivity
- h_t : hours worked, either 0 or \bar{h}
- a_t: asset holdings
- τ_H , τ_K : tax rate
- \overline{T} : lump-sum transfers.

Het Agent Economy: Firm

• Each period t, The representative firm maximizes profits:

$$\max_{L_t, K_t} \lambda_t L_t^{\alpha} K_t^{1-\alpha} - W_t L_t - (R_t + \delta) K_t$$

• First-order conditions:

$$W_t = lpha Y_t / L_t$$
, and $(R_t + \delta) = (1 - lpha) Y_t / K_t$

- Capital accumulation: $K_{t+1} = (1 \delta)K_t + I_t$
- Exogenous technology: $\ln(\lambda_t/\bar{\lambda}) = \rho_\lambda \ln(\lambda_{t-1}/\bar{\lambda}) + \sigma_\lambda \epsilon_{\lambda,t}$

Het Agent Economy: Government

• Each period *t*, government spends tax revenues on lump-sum transfers and its own consumption:

$$\bar{T} + G_t = \tau_H W_t \int x_t h_t d\mu_t + \tau_K R_t \int a_t d\mu_t$$

- G is neutral to households' decisions
- Transfers are a fixed fraction of steady state tax revenues

$$ar{\mathcal{T}} = \chi \left(au_{\mathcal{H}} ar{\mathcal{W}} \int (x h) d\mu + au_{\mathcal{K}} ar{\mathcal{R}} \int a d\mu
ight)$$

Het Agent Economy: Equilibrium Conditions

• Capital Market Clearing:

$$K_t = \int a_t d\mu_t$$

• Labor Market Clearing:

$$L_t = \int x_t h_t d\mu_t.$$

• Goods Market Clearing:

$$Y_t = \lambda_t L_t^{\alpha} K_t^{1-\alpha} = \int c_t d\mu_t + I_t + G_t.$$

Equilibrium

Rep Agent Model: Household

• The representative household solves the following problem

$$\max_{k=1}^{\infty} \mathbb{E}_{t} \left[\sum_{s=0}^{\infty} \beta^{t+s} Z_{t+s} \left(\ln C_{t+s} - \frac{(H_{t+s}/B_{t+s})^{1+1/\nu}}{1+1/\nu} \right) \right]$$

s.t. $C_{t} + K_{t+1} = K_{t} + (1-\tau_{H}) W_{t} H_{t} + (1-\tau_{K}) R_{t} K_{t} + \bar{T}$

Rep Agent Model: Firms

• Each period *t*, The representative firm solves the following static profit maximization problem:

$$\max_{L_t,K_t} A_t H_t^{\alpha} K_t^{1-\alpha} - W_t H_t - (R_t + \delta) K_t$$

• Exogenous technology: $\ln(A_t/\bar{A}) = \rho_A \ln(A_{t-1}/\bar{A}) + \sigma_A \epsilon_{A,t}$

Rep Agent Model: Government

• Each period *t*, government spends tax revenues on lump-sum transfers and its own consumption:

$$\bar{T} + G_t = \tau_H W_t H_t + \tau_K R_t K_t$$

• Transfers are a fixed fraction of tax revenues

$$\bar{T} = \chi \left(\tau_H \bar{W} \bar{H} + \tau_K \bar{R} \bar{K} \right)$$

Het Agent Economy: Calibrate to Generate Data

- Labor share: $\alpha = 0.64$.
- Depreciation Rate: $\delta = 0.025$
- Aggregate productivity: $\ln \lambda_t = 0.95 \ln \lambda_{t-1} + 0.007 \epsilon_{\lambda,t}$.
- Substitution elasticity: $\gamma =$ 0.4, consistent with micro estimates.
- Work hour: $\bar{h} = 1/3$, from the Michigan Time-Use Survey.
- Borrowing constraint: $\underline{a} = -2$, two quarters of earnings.
- Disutility of working, B, and discount factor, β : target employment rate of 60% and quarterly interest rate 1% in steady state.
- Idiosyncratic Productivity: $\ln x_t = 0.94 \ln x_{t-1} + 0.287 \epsilon_{x,t}$, based on PSID data.
- Match wealth and earnings distribution.

Summary

Characteristics of Wealth Distribution

	Quintile of Wealth Distribution					
	1st	2nd	3rd	4th	5th	Total
PSID						
Share of wealth	52	.50	5.06	18.74	76.22	100
Group avg. / Pop avg.	02	.03	.25	.93	3.81	1
Share of earnings	7.51	11.31	18.72	24.21	38.23	100
Benchmark Model						
Share of wealth	-1.71	2.96	10.88	24.80	63.06	100
Group avg. / pop avg.	10	.15	.55	1.23	3.18	1
Share of earnings	9.60	15.60	19.61	23.91	31.27	100

Lorenz Curves

Business Cycle Statistics

	Model	U.S. Data
	3000 obs.	1964-2006
$\sigma(\ln Y)$.033	.041
$\sigma(\ln C)$.020	.021
$\sigma(\ln H)$.013	.042
$\sigma((\ln H)_{HP})$.007	.018
corr(ln Y, ln C)	0.84	0.83
corr(ln Y, ln H)	0.80	0.56
corr(ln C, ln H)	0.37	0.51

Quantitative Analysis

- Benchmark fiscal policy: $\tau_H = 0.29$, $\tau_K = 0.35$; $\chi = 0.36$. Remarks
 - Generate data from Het Agent Economy
 - Estimate Rep Agent model based on Het Agent Economy data
 - Estimate Rep Agent model based on U.S. data
 - Question 1: Does aggregation lead to sizeable preference shocks?
- Alternative fiscal policy:
 - Generate data from Het Agent Economy under alternative policy
 - *Question 2:* By how much do estimates of Rep Agent Model parameters/shocks change?
 - *Question 3:* How accurate are predictions based on the estimated benchmark Rep Agent Model?

Quantitative Analysis

- Benchmark: $\tau_H = 0.29$, $\tau_K = 0.35$; $\chi = 0.36$.
- Policy Changes we considered: Remarks

	$ au_{H}$	$ au_{K}$	χ
Labor Tax Cut	0.22		
Capital Tax Raise		0.47	
More Transfers			0.50
1960 Policy	0.23	0.44	0.22
2004 Policy	0.27	0.33	0.42

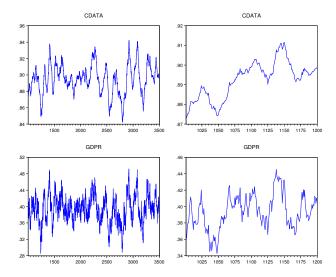
Benchmark Estimation – Setup & Priors

• Bayesian estimation of Rep Agent Model based on: *output, consumption, employment data.*

• Fix
$$\alpha =$$
 0.64, $\delta =$ 0.025, τ_H , τ_K , and χ

Name	Domain	Density	Mean	Std Dev
R	\mathbb{R}^+	Gamma	1.00	0.50
ν	\mathbb{R}^+	Gamma	1.00	0.50
In Ā	\mathbb{R}	Normal	0.00	10.0
In $ar{B}$	\mathbb{R}	Normal	0.00	10.0
$ ho_A$	[0, 1)	Beta	0.50	0.25
ρ_B	[0, 1)	Beta	0.50	0.25
σ_A	\mathbb{R}^+	Inv. Gamma	.012	.007
σ_B	\mathbb{R}^+	Inv. Gamma	.012	.007
σ_Z	\mathbb{R}^+	Inv. Gamma	.012	.007

Consumption and Output Data



Benchmark Specification

1: Estimated steady states match

		7	T = 200		= 2,500
	"True"	Mean	90% Intv.	Mean	90% Intv.
K	15.2	14.7	[14.2, 15.1]	14.9	[14.7, 15.1]
H = E/3	0.20	.200	[.199, .201]	.200	[.200, .200]
С	0.89	0.89	[0.88, 0.90]	0.89	[0.89, 0.90]
Y	1.48	1.46	[1.44, 1.48]	1.47	[1.47, 1.48]
G	0.21	0.21	[.207, .211]	.211	[.210, .211]

• K is high in Heterogenous agent model due precautionary savings

Benchmark Specification

2: Estimates based on simulated versus actual data

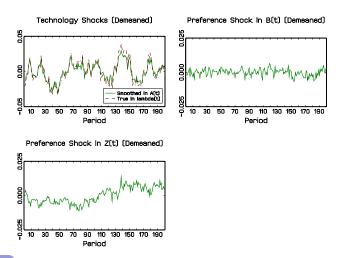
		T = 200	Т	= 2,500	l	J.S. Data
	Mean	90% Intv.	Mean	90% Intv	Mean	90% Intv
ν	1.72	[1.57, 1.86]	2.14	[2.01, 2.26]	0.34	[0.10, 0.60]
In Ā	-0.26	[-0.26, -0.26]	-0.26	[-0.26, -0.26]	-0.25	[-0.27, -0.22]
In $ar{B}$	-0.33	[-0.34, -0.32]	-0.32	[-0.32, -0.31]	-0.44	[-0.52, -0.37]
$ ho_A$	0.90	[0.89, 0.91]	0.91	[0.91, 0.92]	0.97	[0.96, 0.99]
$ ho_B$	0.76	[0.60, 0.92]	0.92	[0.92, 0.93]	0.98	[0.97, 1.00]
R	2.83	[2.68, 2.98]	2.77	[2.71, 2.83]	3.70	[3.25, 4.22]

- Aggregate elasticity ν is different from micro elasticity $\gamma = 0.4$.
 - Depends on the reservation wage dist
- Detects preference shocks.

Does aggregation lead to sizeable preference shocks?

		В		Z		
	Mean	90% Intv.	Mean	90% Intv.		
Benchmark Economy, $T=200$						
Output	5	[2, 8]	5	[4, 6]		
Consumption	3	[0, 7]	6	[4, 7]		
Hours	33	[18, 45]	5	[3, 7]		
Benchmark Economy, $T = 2,500$						
Output	9	[8, 10]	5	[4, 5]		
Consumption	9	[8, 10]	4	[4, 5]		
Hours	43	[41, 46]	4	[4, 4]		
U.S. Data						
Output	45	[21, 68]	5	[2, 9]		
Consumption	47	[21, 75]	6	[1, 10]		
Hours	98	[97, 99]	1	[0, 1]		

Benchmark Estimation – Smoothed Shock Processes



Remarks

Increase Transfers from $\chi = 0.36$ to 0.50

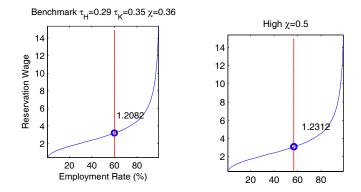
- Increased transfers generates positive income effect.
- Employment rate decreases from 60% to 57%.
- Less need for precautionary savings: K decreases.
- Output decreases.
- Average labor productivity increases (composition effect).
- Experiment:
 - Re-estimate Rep Agent Model
 - By how much do estimates of the Rep Agent Model parameters/shocks change?

More Transfers: Parameter Estimates T = 200

	B	enchmark	Мо	re Transfers
	Mean	90% Intv.	Mean	90% Intv
R _A	2.83	[2.68, 2.98]	2.96	[2.81, 3.12]
ν	1.72	[1.57, 1.86]	2.68	[2.13, 3.34]
In Ā	-0.26	[-0.26, -0.26]	-0.24	[-0.24, -0.23]
In $ar{B}$	-0.33	[-0.34, -0.32]	-0.32	[-0.34, -0.31]
$ ho_A$	0.90	[0.89, 0.91]	0.92	[0.91, 0.92]
ρ_B	0.76	[0.60, 0.92]	0.90	[0.88, 0.92]

Smoothed Shocks

More Transfers: Why does $\hat{\nu}$ increase?



More Transfers: Why does \overline{A} rise?

- Lower level of employment
- ullet \longrightarrow Fewer low productivity workers are hired
- $\bullet \longrightarrow \mathsf{Aggregate}$ productivity $\bar{\mathsf{A}}$ in rep. agent model needs to rise.

More Transfers: Steady State Predictions [% Changes]

How accurate are predictions based on the estimated benchmark Rep Agent Model?

		Predi	Predicted $T = 200$		ted $T = 2,500$
	"True"	Mean	90% Intv.	Mean	90% Intv.
Hours	-5.25	-3.14	[-3.22, -3.04]	-3.38	[-3.43, -3.31]
Consumption	3.09	1.87	[1.79, 1.98]	1.62	[1.56, 1.68]
Output	-2.17	-3.14	[-3.22, -3.04]	-3.38	[-3.43, -3.31]

Message: Lack of invariance of the aggregator function is sufficiently strong to render predictions from representative agent model inaccurate (outside 90% prediction interval).

More Transfers: Explanation of Prediction Errors

- Policy predictions are based on
 - a $\hat{\nu}$ that is too low \longrightarrow under-predict the hours decline;
 - a composition effect \longrightarrow overpredict the output decline.
- Income effect is bigger in heterogeneous agent economy; transfers relax borrowing constraint for low wealth households; large effect on consumption and labor supply.

Labor Tax Cut

- low labor income tax encourages labor supply
- lowers the tax revenue, lump-sum transfer decreases by 18%,
- income effect on labor supply
- more need for precautionary savings
- $\ln \bar{A}$ falls to capture composition effect
- Key parameter estimates:

51			
	R	ν	In Ā
Benchmark	2.83	1.72	-0.26
Labor Tax Cut	2.64	1.12	-0.29

• Policy predictions:

	Hours	Consumption	Output
"True"	6.30	7.61	3.50
90 % Intv.	[2.96, 3.15]	[7.84, 8.03]	[2.96, 3.15]

Summary of other Policy Changes

- High capital tax has most impact on K, but has little effect on parameter estimates of DSGE model (due to choice of observables).
- "1960 Fiscal Policy" and "2004 Fiscal Policy" generates a combination of effects.
- Neither preference processes nor taste/technology parameters are invariant.
- "True" policy effects lie outside of predictive intervals generated from estimated Rep Agent Model.
- We chose a sample size of T = 200, because we wanted to compare the magnitude of aggregation biases to the posterior uncertainty based on a *realistic* sample size.

More Estimates Area More Predictions Area More Figures Area Estimates based on Efficiency Hours

Policy	"True"	Mean	90 % Interval
Labor Income Tax Cut:			
$ au_{H}=0.22$	0.0451	0.0664	[0.0660, 0.0668]
High Capital Tax:			
$ au_k = 0.47$	-0.0261	-0.0339	[-0.0352, -0.0325]
Higher Transfer:			
$\chi = 0.5$	0.0580	0.0313	[0.0310, 0.0318]
1960 Policy:			
$ au_H = 0.229, \ au_k = 0.443, \ \chi = 0.224$	-0.0309	0.0030	[0.0016, 0.0044]
2004 Policy:			
$ au_{H} = 0.27, \ au_{k} = 0.33, \ \chi = 0.42$	0.0407	0.0377	[0.0375, 0.0379]

Conclusion

- Incomplete markets and idiosyncratic productivity shocks can lead to time-varying parameters (preference shocks) in aggregate model.
- Neither labor supply elasticity nor preference shock process in the aggregate model are policy invariant.
- Prediction for policy effects obtained from Rep Agent Model are often inaccurate.

Related Work

- Chang and Kim (IER, 2006): Calibrate heterogeneous agent model; simulate date; estimate aggregate Frisch elasticity (slope of reservation wage distribution), value is about 1.
- Chang and Kim (AER, 2007): Same model economy; calculate "wedge" between marginal rate of substitution and labor productivity; investigate cyclical properties of this "wedge"
- An, Chang, and Kim (AEJ Macro, 2009): Same model economy; focus on GMM based estimates of equilibrium conditions; apparent failure of equilibrium conditions due to aggregation rather than market failure.
- Scheinkman and Weiss (1987), Krüger and Lustig (2007); Liu, Waggoner, and Zha (2008)

Back

- There are two assets: (i) claims to physical capital; (ii) IOU's. The returns on the claims to capital are taxed; the returns on the IOU's are not. IOU's are in zero net supply. After tax returns on both assets are identical.
- Lump-sum transfers are independent of asset holdings and productivity. They are constant over time \longrightarrow no additional state variable.

Back

Recursive Representation of Worker's Problem

• Value of working:

$$V^{E}(a, x, A, \mu) = \max_{a' \in A} \left\{ \ln c - B \frac{\bar{h}^{1+1/\nu}}{1+1/\nu} + \beta E \left[V(a', x', A', \mu') | x, A \right] \right\}$$

s.t. $c + a' = a + (1 - \tau_{H}) W x \bar{h} + (1 - \tau_{K}) R a + \bar{T}$

Value of Not-working:

$$V^{N}(a, x, A, \mu) = \max_{a' \in \mathcal{A}} \left\{ \ln c + \beta E \left[V(a', x', A', \mu') | x, A \right] \right\}$$

s.t. $c + a' = a + (1 - \tau_{\mathcal{K}})Ra + \overline{T}$

• Labor supply decision:

$$V(a, x, A, \mu) = \max \{V^{E}(a, x, A, \mu), V^{N}(a, x, A, \mu)\}.$$

Back

Equilibrium

- Value functions: $V^{E}(a, x, A, \mu)$, $V^{N}(a, x, A, \mu)$ and $V(a, x, A, \mu)$
- Decision rules: $a'(a, x, A, \mu)$, $c(a, x, A, \mu)$ and $h(a, x, A, \mu)$
- Aggregate factor inputs: $K(A, \mu)$ and $L(A, \mu)$
- Factor prices: $W(A,\mu)$ and $R(A,\mu)$
- Government consumption: $G(A, \mu)$
- Law of motion for distribution: $\mu' = \mathbf{T}(A, \mu)$

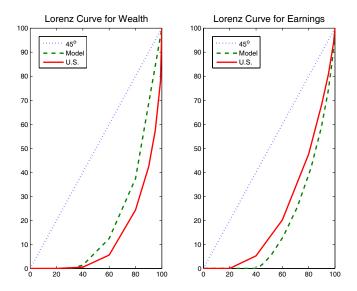
such that

- Individual workers optimize.
- The representative maximizes profits.
- Markets clears.
- Government balances budget.
- Individual and aggregate behaviors are consistent.

Parameters of the Benchmark Economy: Summary

Parameter	Description			
$\alpha = 0.64$	Labor share in production function			
eta= 0.98332	Discount factor			
$\delta = 0.025$	Capital depreciation rate			
$\gamma = 0.4$	Individual labor-supply elasticity with divisible labor			
B = 101.0	Utility parameter			
$\overline{h} = 1/3$	Labor supply if working			
<u>a</u> = -2.0	Borrowing constraint			
$ ho_x = 0.939$	Persistence of idiosyncratic productivity shock			
$\sigma_{x} = 0.287$	St. dev. of innovation to individual productivity			
$ ho_A = 0.95$	Persistence of aggregate productivity shock			
$\sigma_A = 0.007$	St. dev. of innovation to aggregate productivity			

Lorenz Curves of Wealth and Earnings



Remarks

- Capital and labor tax rates correspond to 1984 values as reported in Chen, Imrohoruglu, and Imrohorogul (2007)
- To choose a value for $\chi = T/(T + G)$ we used data on Government Consumption (G) and Net Government Social Benefits (T):

Year	T/(T+G)	G/Y
1960	0.22	0.16
1984	0.36	0.16
2004	0.41	0.15

▶ Back

U.S. Tax Rates

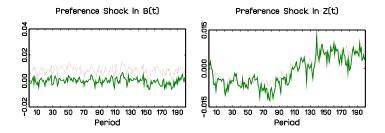


▶ Back

Remarks

- The measured technology shock from the Rep Agent Model is less volatile than the true technology shock.
- In booms, low-efficiency workers enter the labor force, which dampens measured productivity.
- Correlation between technology and intratemporal (intertemporal) preference shock is 0.3 (0.2).

Benchmark versus High Transfers – Smoothed Shock Processes



Posterior Means Based on Het Agent Economy Data

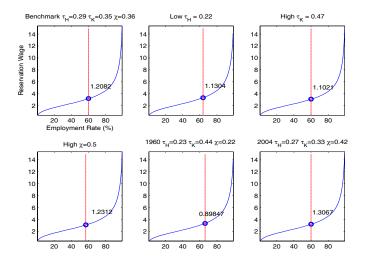
	Bench-	Lab. Tax	Cap. Tax	More	1960	2004
	mark	Cut	Raise	Transfers	Policy	Policy
$ au_H$	0.29	0.22			.229	.269
$ au_{K}$	0.35		0.47		.443	.327
χ	0.36			0.50	.224	.417
R	2.83	2.64	2.84	2.96	2.61	2.80
u	1.72	1.12	1.67	2.68	1.07	1.70
$\ln A_0$	-0.26	-0.29	-0.26	-0.24	-0.30	-0.26
$\ln B_0$	-0.33	-0.33	-0.33	-0.32	-0.32	-0.33
$ ho_{\mathcal{A}}$	0.90	0.94	0.92	0.92	0.95	0.94
$ ho_B$	0.76	0.90	0.87	0.90	0.91	0.92
σ_A	.005	.006	.006	.005	.006	.006
σ_B	.003	.003	.003	.003	.003	.003
σ_{ζ}	.003	.003	.003	.002	.002	.003



Predictions Based on Estimated Benchmark Rep Agent Model

		Hours	Consumption	Output
Lab.Tax Cut	"True"	6.30	7.61	3.50
$\tau_H = 0.22$	90 % Intv.	[2.96, 3.15]	[7.84, 8.03]	[2.96, 3.15]
Cap.Tax Raise $ au_{\mathcal{K}}=$ 0.47	"True"	-0.15	-2.69	-2.85
	90 % Intv.	[-0.31, -0.28]	[-3.63, -3.37]	[-4.07, -3.84]
More Transf. $\chi = 0.5$	"True"	-5.25	3.09	-2.17
	90 % Intv.	[-3.22, -3.04]	[1.79, 1.98]	[-3.22, -3.04]
1960 Policy	"True"	9.95	1.75	2.60
	90 % Intv.	[5.18, 5.51]	[2.25, 2.65]	[2.28, 2.63]
2004 Policy	"True"	-0.15	3.93	0.82
	90 % Intv.	[-0.21, -0.20]	[3.66, 3.71]	[0.36, 0.41]

Employment Rate Based on the Reservation Wage Distribution



Estimates based on Efficiency Unit of Hours

	Bench-	Lab. Tax	Cap. Tax	More	1960	2004
	mark	Cut	Raise	Transfers	Policy	Policy
$ au_{H}$	0.29	0.22			.229	.269
$ au_{K}$	0.35		0.47		.443	.327
χ	0.36			0.50	.224	.417
r _A	2.75	2.54	2.71	2.82	2.51	2.73
ν	0.64	0.54	0.67	0.80	0.47	0.64
In Ā	0.01	0.00	0.01	0.01	0.00	0.01
In $ar{B}$	-0.81	-0.82	-0.81	-0.79	-0.83	-0.81
ρ_A	0.91	0.94	0.93	0.92	0.94	0.92
ρ_B	0.91	0.91	0.93	0.91	0.89	0.90
σ_{A}	.007	.007	.007	.007	.007	.007
σ_B	.002	.002	.002	.002	.002	.002
σ_{ζ}	.003	.003	.003	.002	.002	.002



Auxiliary Model Economies

		Labor Market	
		Divisible Indivisib	
Capital Market	Complete	1	2
	Incomplete	3	4

Comparisons: Chang and Kim (2007)

Incomplete Market (& Divisible Labor) economy:

- Similar to representative agent economy
- Households labor supply respond similarly to aggregate shocks
- Aggregation holds approximately (Krusell & Smith, 1998)

Indivisible Labor (& Complete Markets) economy:

- Aggregation theorem holds
- Aggregate FOC holds in efficiency units
- Aggregate elasticity is not the same as individual elasticity

Labor Market Wedges

