Optimal monetary policy when asset markets are incomplete: an irrelevance result

R. Anton Braun[1] Tomoyuki Nakajima[2]

[1]Department of Econimics, University of Tokyo

[2]Institute of Economic Research, Kyoto University

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Outline

Optimal Monetary Policy in the New Keynesian model

- Previous results

  Suppose

  1. Cashless economy
  2. No other static distortions
  3. Price stickiness is the only dynamic distortion

  Optimal monetary policy: set inflation rate to zero.

  - With sticky prices, non-zero inflation distorts relative prices.
  - Such distortion can be eliminated by setting the inflation rate to zero in all periods.

- If assumptions 1 and 2 are relaxed optimal monetary policy involves some inflation/deflation, but ...

  a zero-inflation policy is still approximately optimal.

Standard New Keynesian model

- Representative agent model with complete markets
- Welfare cost of business cycles is negligible.

Uninsured idiosyncratic risk

- Idiosyncratic income shocks are very persistent and their variance fluctuates countercyclically.
  - Storesletten, Telmer and Yaron (2004), Meghir and Pistaferri (2004), etc.
- With incomplete asset markets, individuals cannot insure against idiosyncratic income shocks.
- When this risk is countercyclical welfare cost of business cycles is large.

How should monetary policy respond to countercyclical variation in idiosyncratic risk?

- We provide an answer to this question in a quantitatively relevant model.
  1. Over 80 % of variation in output over the business cycle is due to variation in labor input.

     We model labor supply
  2. Relative volatility of consumption is about 1/2. Relative volatility of investment is about 2.

     We model capital accumulation.
  3. The welfare cost of business cycles is large.

     In our model the welfare costs of business cycles is as large as 12 percent of consumption.

Our results

Optimal monetary policy:

1. A zero inflation rate is still optimal when there are no static distortions
2. The welfare costs of pursuing a zero inflation rate policy are still small when static distortions are present.

Some methodological issues

1. How to compute an equilibrium in incomplete market model with
   - Labor supply
   - Capital accumulation
   - Aggregate shocks (Technology)
   - Persistent idiosyncratic shocks with time varying risk.
2. How to find the optimal state-contingent (Ramsey) monetary policy?

Strategy 1: Numerical Methods

- Krusell, Mukoyama, Sahin and Smith (2009)
- Storesletten, Telmer and Yaron (2001)
- Chang and Kim (2007)

Disadvantages

- Hard to handle multiple shocks.
- Hard to compute optimal govt. policy (policies are indexed by each history).

Strategy 2: Extend Constantinides and Duffie (1996)

Bits and pieces

1. Labor supply: Heathcote, Storesletten and Violante (2008)
2. Capital accumulation, Krebs (2003)
3. Countercyclical risk, Krebs (2003) De Santis (2007)

We use strategy 2

- Extend Constantinides-Duffie (1996) to consider a model with all of the above features.
- The previous papers consider real economies.
- We introduce a New Keynesian nominal side to the economy.
  - monopolistic competition;
  - Calvo price setting;
- We can handle multiple shocks.
- We derive optimal monetary policy (Ramsey policy).

How do we get around the curse of dimensionality?

- Idiosyncratic shock hits labor and capital income in a symmetric way.
- Under this assumption we establish an aggregation result.
  - Labor supply of all individuals is identical
  - Consumption of all individuals is proportionate to aggregate consumption.
- All shareholders agree on value of firms.
- Objective of a benevolent Monetary Authority factors when using market clearing allocations.
- No opportunity for Monetary Authority to manipulate the price system to influence equity.

Outline

Composite good

- Y_t = aggregate output of a composite good:

$$Y_t = \left(\int_0^1 Y_{j,t}^{1-\frac{1}{\zeta}} \, dj \right)^{\frac{1}{1-\frac{1}{\zeta}}}$$

which can be consumed or invested:

$$Y_t = C_t + I_t$$

- P_t = price index:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\zeta} \, dj \right)^{\frac{1}{1-\zeta}}$$

Outline

Preferences of individuals

- A continuum of ex ante heterogeneous individuals.
- Preferences:

$$u_{i,0} = E_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left[c_{i,t}^{\theta} (1 - l_{i,t})^{1-\theta} \right]^{1-\gamma}$$

E_t^i includes history of i specific and aggregate shocks.

E_t includes history of aggregate shocks only.

- Let $\gamma_c =$ (inverse of the) elasticity of intertemporal substitution of consumption (for a fixed level of leisure):

$$\gamma_c \equiv 1 - \theta(1 - \gamma)$$

Idiosyncratic shocks: Countercyclical variance

- $\eta_{i,t}$ = the idiosyncratic shock for individual i:

$$\ln \eta_{i,t} = \ln \eta_{i,t-1} + \sigma_{\eta,t}\epsilon_{\eta,i,t} - \frac{\sigma_{\eta,t}^2}{2}$$

  where

  - $\epsilon_{\eta,i,t}$ is i.i.d., and $N(0,1)$.
  - $\sigma_{\eta,t}$ = variance of innovations to idiosyncratic shocks.

- Assume that $\sigma_{\eta,t}$ fluctuates countercyclically.

Flow budget constraint

- The flow budget constraint of i is given by

$$c_{i,t} + k_{i,t} + s_{i,t} = \frac{\eta_{i,t}}{\eta_{i,t-1}} \left(R_{k,t} k_{i,t-1} + R_{s,t} s_{i,t-1} \right) + \eta_{i,t} w_t l_{i,t}$$

where $k_{i,t}$ = physical capital and $s_{i,t}$ = value of shares.

- Idiosyncratic shock $\eta_{i,t}$ affects i's income in two ways.
  - $\eta_{i,t}$ determines the productivity of individual i's labor.
  - $\eta_{i,t}$ also affects the return to savings of individual i.

Motivation for these assumptions

- In general, with uninsured idiosyncratic shocks, the wealth distribution, an infinite-dimensional object, must be included in the state variable.
- Under our assumptions distribution of wealth has a simple form.

Empirical Relevance

- Positive correlation between idiosyncratic unemployment and housing returns. Foote, Gerardi, Goette and Willen (2010).
- Positive correlation between idiosyncratic unemployment and stock return shocks. (Employee shareholding plans).
- private (proprietorship) capital, Angeletos (2007)
- Optimal (fiscal) policy in private information economies, Kocherlakota (2005).

Remarks

- This assumption produces large welfare costs of business cycles of as much as 12 % of consumption.
- This is about twice as large as e.g. Krebs (2003). (Only human capital is subject to this risk).
- Our principal finding is that the tradeoff faced by the monetary authority is little affected by the presence of idiosyncratic shocks.
- Dropping this assumption
  - Lowers the welfare cost of business cycles
  - Enhances an individual's ability to self-insure
  - Lowers the need for monetary policy to provide insurance via price manipulation.

Outline

Associated representative-agent problem

- Consider a representative-agent's utility maximization problem:

$$\max\ U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \nu_t \left[C_t^{\theta} (1 - L_t)^{1-\theta} \right]^{1-\gamma}$$

subject to

$$C_t + K_t + S_t = R_{k,t} K_{t-1} + R_{s,t} S_{t-1} + w_t L_t$$

- Here, ν_t is a preference shock defined by

$$\nu_t \equiv \exp \left[\frac{1}{2} \gamma_c (\gamma_c - 1) \sum_{s=0}^{t} \sigma_{\eta,s}^2 \right] \\ = E_t \left[\left(\frac{\eta_{i,t}}{\eta_{i,-1}} \right)^{1-\gamma_c} \right]$$

Aggregation result

Proposition

Suppose that $\{C_t^*, L_t^*, K_t^*, S_t^*\}_{t=0}^{\infty}$ *is a solution to the representative agent's problem. For each* $i \in [0,1]$, *let*

$$c_{i,t}^* = \eta_{i,t} C_t^*$$
$$l_{i,t}^* = L_t^*$$
$$k_{i,t}^* = \eta_{i,t} K_t^*$$
$$s_{i,t}^* = \eta_{i,t} S_t^*$$

Then $\{c_{i,t}^*, l_{i,t}^*, k_{i,t}^*, s_{i,t}^*\}_{t=0}^{\infty}$ *is a solution to the problem of individual* i.

Proof of the proposition

- Suppose that $\{C_t^*, L_t^*, K_t^*, S_t^*\}_{t=0}^{\infty}$ is a solution to the representative agent's problem.
- Then it satisfies

$$\theta(C_t^*)^{-\gamma_c}(1-L_t^*)^{(1-\theta)(1-\gamma)} = \lambda_t^*$$

$$\frac{1-\theta}{\theta}\frac{C_t^*}{1-L_t^*} = w_t$$

$$\lambda_t^* = E_t\beta\frac{\nu_{t+1}}{\nu_t}\lambda_{t+1}^* R_{k,t+1}$$

$$\lambda_t^* = E_t\beta\frac{\nu_{t+1}}{\nu_t}\lambda_{t+1}^* R_{s,t+1}$$

and the transversality conditions.

Proof of the proposition

- For each $i \in [0,1]$, let

$$c^*_{i,t} = \eta_{i,t} C^*_t, \quad k^*_{i,t} = \eta_{i,t} K^*_t, \quad s^*_{i,t} = \eta_{i,t} S^*_t,$$
$$l^*_{i,t} = L^*_t, \quad \lambda^*_{i,t} = \eta_{i,t}^{-\gamma_c} \lambda^*_t$$

Then it is straightforward to see that they satisfy

$$\theta (c^*_{i,t})^{-\gamma_c} (1 - l^*_{i,t})^{(1-\theta)(1-\gamma)} = \lambda^*_{i,t}$$
$$\frac{1-\theta}{\theta} \frac{c^*_{i,t}}{1 - l^*_{i,t}} = w_t \eta_{i,t}$$
$$\lambda^*_{i,t} = \beta E^i_t \lambda^*_{i,t+1} \frac{\eta_{i,t+1}}{\eta_{i,t}} R_{k,t+1}$$
$$\lambda^*_{i,t} = \beta E^i_t \lambda^*_{i,t+1} \frac{\eta_{i,t+1}}{\eta_{i,t}} R_{s,t+1}$$

and the transversality conditions.

Remarks

- Remark 1 Result applies when agents are ex ante heterogeneous: initial holdings of assets vary across individuals.

- Remark 2 The utility of the representative agent is indeed the cross-sectional average of individual utility:

$$U_0 = E_0[u_{i,0}]$$

Remark 3: Effective discount factor

- Idiosyncratic shocks affect the aggregate economy through the "effective discount factor":

$$\begin{aligned}\tilde{\beta}_{t,t+1} &\equiv \beta \frac{\nu_{t+1}}{\nu_t} \\ &= \beta \exp\left[\frac{1}{2}\gamma_c(\gamma_c - 1)\sigma^2_{\eta,t+1}\right]\end{aligned}$$

- It follows that

$$\uparrow \sigma^2_{\eta,t+1} \qquad \Longrightarrow \qquad \begin{cases} \uparrow \tilde{\beta}_{t,t+1} & \text{if } \gamma_c > 1 \\ \downarrow \tilde{\beta}_{t,t+1} & \text{if } \gamma_c < 1 \end{cases}$$

- Relate to Relative Prudence

Remark 4: Unanimity of stockholders' preferences

- The SDF used by individual i is independent of history of shocks

$$\beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} = \beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\eta_{i,t+1}}{\eta_{i,t}} \right)^{-\gamma_c}$$
$$= \beta \frac{\lambda_{t+1}}{\lambda_t} \exp \left(-\gamma_c \sigma_{\eta,t+1} \epsilon_{\eta,i,t+1} + \frac{\gamma_c}{2} \sigma^2_{\eta,t+1} \right)$$

- It follows that individuals agree on the present value of the profit stream of each firm.

- In particular, they agree with the representative agent, whose SDF is given by $\beta \frac{\lambda_{t+1} \nu_{t+1}}{\lambda_t \nu_t}$.

Outline

Firms

- Standard model with monopolistic competition and Calvo pricing.
- Production technology of firm j:

$$Y_{j,t} = z_t^{1-\alpha} K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} - \Phi_t$$

where z_t is aggregate productivity shock, and Φ_t is a fixed cost.

- Demand for variety j:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\zeta} Y_t$$

- $1 - \xi$ = rate of arrival of an opportunity to reset prices.

Outline

Aggregate shocks

- The productivity shock may either be permanent or temporary.
- The case of permanent productivity shock:

$$\ln z_t = \ln z_{t-1} + \mu + \sigma_z \epsilon_{z,t} - \frac{\sigma_z^2}{2}$$
$$\sigma_{\eta,t}^2 = \bar{\sigma}_\eta^2 + b\sigma_z \epsilon_{z,t}$$

- The case of temporary productivity shock:

$$\ln z_t = \rho_z \ln z_{t-1} + \sigma_z \epsilon_{z,t} - \frac{\sigma_z^2}{2(1+\rho_z)}$$
$$\sigma_{\eta,t}^2 = \bar{\sigma}_\eta^2 + b \ln z_t$$

Outline

Government

- Fiscal policy: no taxes, no debt, etc.
- Monetary policy sets $\{\pi_t\}$ (state-contingent path of inflation).
- Two monetary policy regimes:
  1. Ramsey regime:
     - Set $\{\pi_t\}$ so as to maximize the ex ante utility of individuals.
  2. Inflation-targeting regime:
     - Set $\pi_t = 1$ at all times.

Outline

Factorization of social welfare function

Proposition

For all choices of χ_i that satisfy $\chi_i > 0, \forall i$ and $\int_i \chi_i di = 1$ the objective function for the Ramsey planner's problem is:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \nu_t \left[C_t^{\theta} (1 - L_t)^{1-\theta} \right]^{1-\gamma}$$

Proof

Proof.

Given that $c_{i,t} = \eta_{i,t} C_t$ and $l_{i,t} = L_t$ for all i in equilibrium, we obtain

$$\int_i \chi_i u_{i,0} di = \int_i \chi_i \left[E_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \eta_{i,t}^{1-\gamma_c} C_t^{1-\gamma_c} (1-L_t)^{(1-\theta)(1-\gamma)} \right] di \quad (1)$$

$$= \left(\int_i \chi_i \eta_{i,-1}^{1-\gamma_c} di \right) E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \nu_t C_t^{1-\gamma_c} (1-L_t)^{(1-\theta)(1-\gamma)}$$

$$= \left(\int_i \chi_i \eta_{i,-1}^{1-\gamma_c} di \right) U_0$$

Observe that the term in parenthesis in the final line is a constant that is independent of policy. □

Outline

Eliminating the monopoly distortions

- Let
  - τ = rate of subsidy to monopolists' revenue.
  - T_t = lump-sum taxes.
- Then after subsidy/tax profit of firm j is

$$(1+\tau)\frac{P_{j,t}}{P_t}Y_{j,t} - w_t L_{j,t} - r_t K_{j,t} - T_t$$

- Assume that

$$\tau = \frac{1}{\zeta - 1}$$

which eliminates the monopoly distortion at the zero-inflation steady state.

Optimality of inflation stabilization

Proposition

Assume that subsidies to the monopolists are given at the rate $\tau = \frac{1}{\zeta - 1}$*, which are financed by lump-sum taxes on the monopolists. Suppose also that the economy is initially at the zero-inflation steady state. Then the solution to the Ramsey problem is given by*

$$\pi_t = 1, \quad \textit{for all } t.$$

Outline

Motivation

- No subsidy (Static distortion)
- Welfare costs of business cycles is large.
- Strict zero inflation rule is nearly optimal.
- Explain intuition.

Effective Preference Discount rate

- Permanent technology shocks.

$$\ln \tilde{\beta}_{t,t+1} = \ln \beta + \frac{1}{2}\gamma_c(\gamma_c - 1)(\bar{\sigma}_\eta^2 + b\sigma_z\epsilon_{z,t+1})$$

- Temporary but persistent technology shocks

$$\ln \tilde{\beta}_{t,t+1} = \ln \beta + \frac{1}{2}\gamma_c(\gamma_c - 1)(\bar{\sigma}_\eta^2 + b \ln z_{t+1})$$

$$\ln z_t = \rho_z \ln z_{t-1} + \sigma_z\epsilon_{z,t} - \frac{\sigma_z^2}{2(1+\rho_z)}$$

Permanent productivity shock

Welfare costs of business cycles and the inflation-targeting regime

γ_c	0.7	0.7	2	2
b	0	-0.8	0	-0.8
Δ_{bc} (%)	-0.8191	-1.2983	2.0938	7.3301
Δ_{inf} (%)	0.0000	0.0000	0.0002	0.0006

- Even when welfare cost of business cycles is large, welfare costs of setting $\pi_t = 1$ are small.
- Welfare cost of business cycles negative when γ_c is low!

Temporary productivity shock

Welfare costs of business cycles and the inflation-targeting regime

γ_c	0.7	0.7	2	2
b	0	-0.8	0	-0.8
Δ_{bc} (%)	-0.0171	-0.6191	-0.0073	12.2258
Δ_{inf} (%)	0.0000	0.0001	0.0000	0.0024

- Welfare cost of business cycles is larger when technology shocks are temporary!
  - Expected preference discount rate increases for negative technology shock.
  - Individuals save more consume less.
- Welfare cost of price stabilization is still very small.

Countercyclical risk but state of technology held constant.

	i.i.d.	persistent
Δ_{bc} (%)	0.0061	11.0914
Δ_{inf} (%)	0.0000	0.0075

- If effective discount factor process is i.i.d. Welfare costs low.
- If effective discount factor process is persistent welfare costs are very large.

Outline

Conclusion

- We have developed a New Keynesian model with uninsurable idiosyncratic income shocks.
- The welfare cost of business cycles can be very large when the variance of idiosyncratic shocks fluctuates countercyclically.
- Nevertheless, the optimal monetary policy continues to call for stabilizing the price level.