Retirement in a Life Cycle Model With Home Production

Richard Rogerson
Johanna Wallenius

November 2009

Background/Motivation

This paper constitutes a first step toward understanding retirement in the context of optimal life cycle labor supply.

Two motivations:

1. Need a theory of retirement to assess changes in social security or medicare

2. May influence inference about important preference parameters

Retirement and Preference Parameters

- A large literature uses life cycle data to estimate the IES for labor supply
- Standard approach is to focus on labor supply during prime age years (prominent exception is French (2005))
- Common conclusion is that the IES is small
- If retirement is taken as exogenous then the retirement decision conveys no information about preference parameters.
- But if retirement is an endogenous decision then it would presumably also convey information about preference parameters.

Question: What does the retirement decision imply about the IES?

What We Do

We consider retirement in three models:

1. Standard life cycle model

2. Life cycle model with a nonconvexity

3. Life cycle model with a nonconvexity and home production

What We Find

In each case we find tensions in reconciling the model's predictions with various "consensus" estimates of key labor supply parameters.

- In standard model it is very hard to generate retirement
- In non-convex model it is hard to reconcile retirement with low IES and reasonable extent of nonconvexity
- In home production model it is hard to reconcile change in home production time at retirement with moderate substitution between time and goods

Characterizing Retirement in the Data

Question: What does the transition from full time work to retirement look like in the data?

We address this from two perspectives:

- cross-section data analysis using CPS
- panel data analysis using PSID

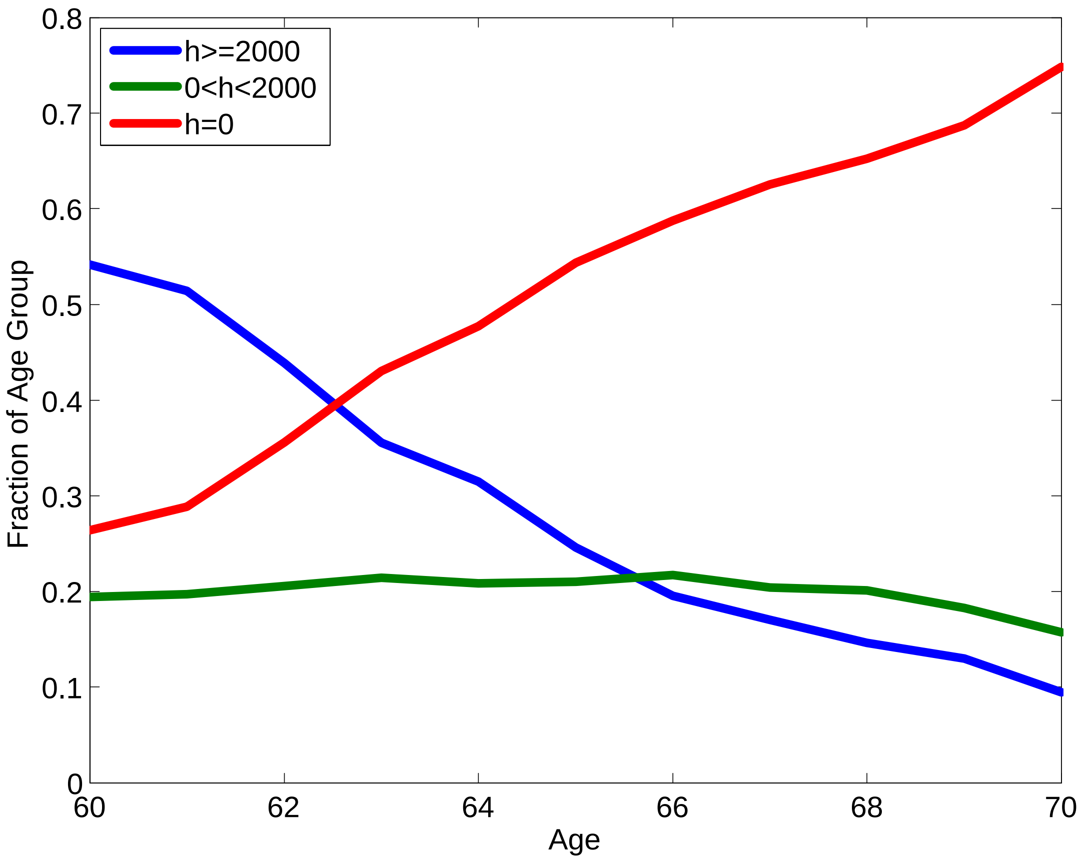
Analysis Using the CPS

We merge data from the 2002-2004 surveys about hours worked in the previous year.

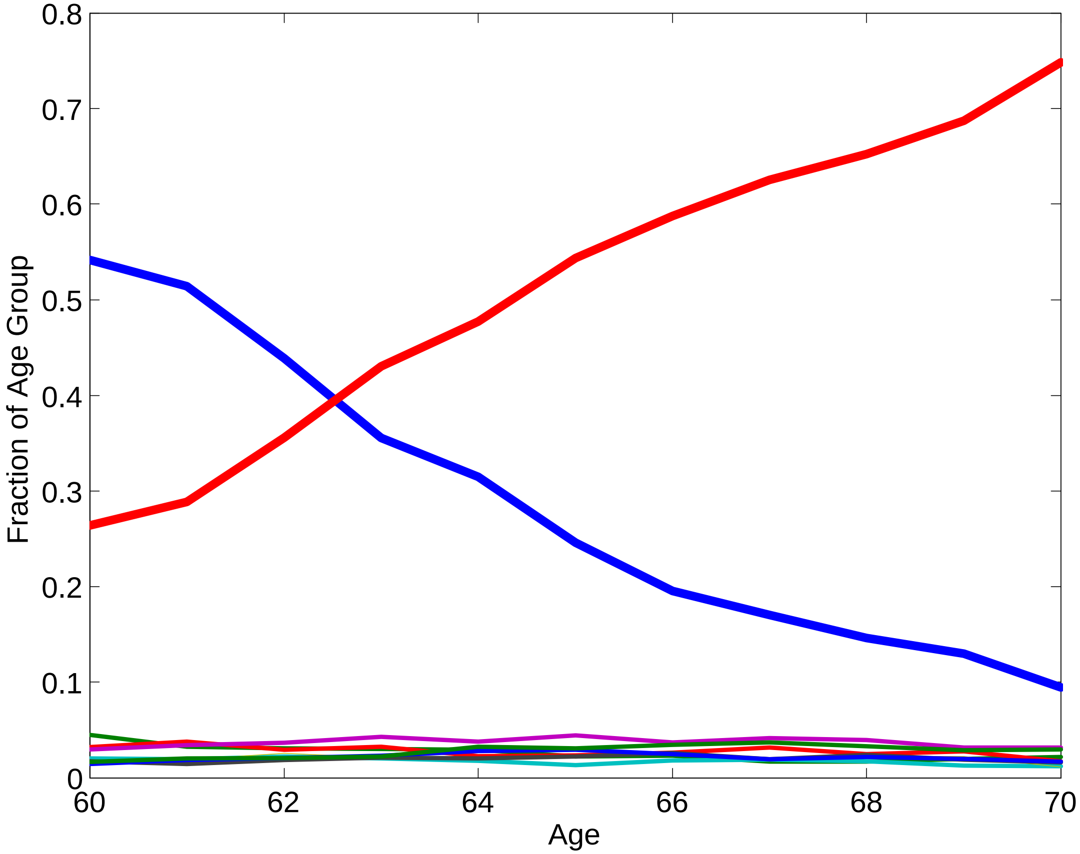
We group observations for each age from 60 to 70 into 10 bins with boundaries 2000, 1750, 1500, 1250,....0.

We do this for the total population as well as just males.

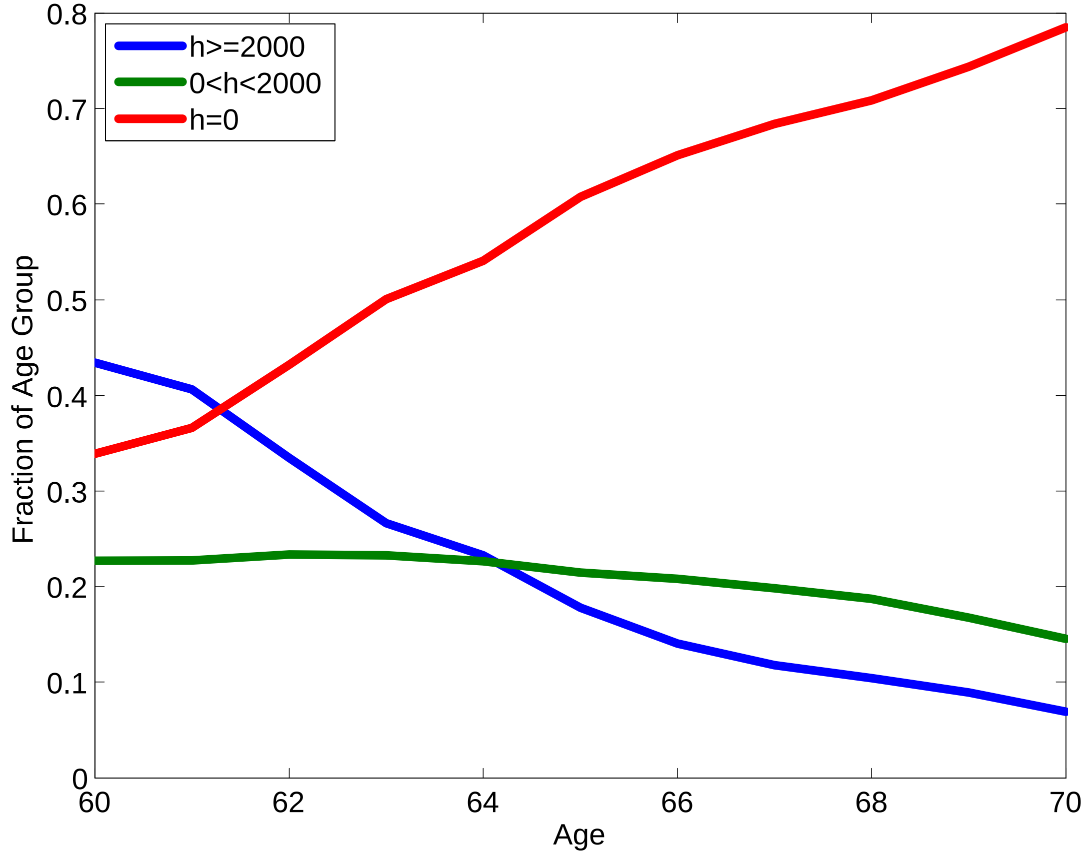
Results for Males



Results for Males, All Groups



Results for Males and Females:



Message from CPS Data:

As the population ages from 60 to 70 the key aggregate transition involves the the fractions of individuals doing full time work and no work.

Issue: This does not necessarily mean that individuals are moving directly from full time work to no work–they could be transitioning from full time work to no work over many years.

To assess this we next look at the PSID.

While the PSID does allow us to follow individuals, the sample size is quite small.

Analysis Using the PSID

We restrict attention to male head of households who have observations on hours worked for all ages from 60 to 70.

Sample size is only 307.

Aggregate statistics are similar to those in the CPS.

To focus on the transition from full time time to retirement we then focus on individuals with at least 1750 hours at age 60 and less than 250 at ages 69 and 70. Sample size is 151.

We then ask what fraction of these move from more than 1750 to less than 250 with at most one intermediate value during the transition.

Answer: 72.2%

Other statistics for the sample with 151 individuals:

age 60: mean hours $= 2152$ median hours $= 2048$

age 69: mean hours $= 5$ median hours $= 0$

age 70: mean hours $= 7$ median hours $= 0$

Message from analysis of the PSID:

The typical transition from full time work to retirement is for an individual to move from full time work to no work with at most one intervening year of intermediate work.

Retirement in a Standard Life Cycle Model

Individual solves:

$$Max \sum_{t=0}^{T} [\log(c_t) + \alpha_t v(1 - h_t)]$$

$$\text{s.t. } \sum_{t=0}^{T} c_t = \sum_{t=0}^{T} w_t h_t + Y$$

FOC for interior solution for h_t:

$$\alpha_t v'(1 - h_t) = \mu w_t$$

Generating Retirement

I will use the term "retirement" to describe a situation in which hours of work change from full time work to zero.

Assuming $h_t > 0$, the optimal solution for $h_{t+1} > 0$ iff:

$$v'(1) < v'(1 - h_t)\frac{\alpha_t}{\alpha_{t+1}}\frac{w_{t+1}}{w_t}$$

Equivalently, $h_{t+1} = 0$ iff:

$$v'(1) \geq v'(1 - h_t)R_{t+1}$$

where

$$R_{t+1} = \frac{\alpha_t}{\alpha_{t+1}}\frac{w_{t+1}}{w_t}$$

Simple Quantitative Exercise

Functional Form:

$$v(1-h) = \frac{A}{1-\frac{1}{\gamma}}(1-h)^{1-\frac{1}{\gamma}}$$

Assume full time work corresponds to $h_t = .45$

Question: What value of R_{t+1} is required for $h_{t+1} = 0$ to be optimal.

Table 1

Value of R_{t+1} to Induce Retirement

$IES=2$	$IES=1$	$IES=.75$	$IES=.50$	$IES=.25$	$IES=.10$	$IES=.05$
.61	.48	.38	.23	.05	.001	.000

Table 2

Value of R_{t+1} to Induce Transition from Full-Time to Part-Time

IES=2	IES=1	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.80	.63	.54	.40	.16	.01	.000

Table 3

Value of R_{t+1} to Induce Retirement from Part-Time

IES=2	IES=1	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.88	.76	.70	.58	.34	.07	.01

Model With Fixed Costs of Working

Model of Prescott et al (2009)

Individual solves:

$$\max \int_0^1 [\log(c(t)) + \alpha(t)v(1 - h(t))]dt$$

$$\text{s.t. } \int_0^1 c(t)dt = \int_0^1 w(t)\max[0, h(t) - \bar{h}]dt + Y$$

Symmetry implies that we can rewrite the problem as:

$$\max_{e,h} \log[e(h - \bar{h})w + Y] + ev(1 - h) + (1 - e)v(1)$$

Assuming interior solutions for both e and h the FOCs are:

$$\frac{(h-\bar{h})w}{e(h-\bar{h})w+Y} = v(1) - v(1-h)$$

$$\frac{w}{e(h-\bar{h})w+Y} = v'(1-h)$$

Divide these two equations by each other to obtain:

$$h-\bar{h} = \frac{v(1) - v(1-h)}{v'(1-h)}$$

Assume as before:

$$v(1-h) = \frac{A}{1-\frac{1}{\gamma}}(1-h)^{1-\frac{1}{\gamma}}$$

Previous equation becomes:

$$h - \bar{h} = \frac{1}{1-\frac{1}{\gamma}}\left[1-(1-h)^{1-\frac{1}{\gamma}}\right](1-h)^{\frac{1}{\gamma}}$$

This equation must hold if the solution for e is interior.

Note that the value of e does not enter this equation.

Numerical Exercise

Similar to earlier exercise, we set $h = .45$

We now ask what value of $\bar{h}$ is required to induce an interior solution for e, i.e., retirement.

Note that one does not have to specify a value for e to compute the required value of $\bar{h}$

Table 4

Value of $\bar{h}$ Required for Retirement

IES=2	IES=1	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.08	.14	.18	.23	.32	.40	.43

Comparison with French (2005)

Alternative Form of Nonconvexity

Assume that wage rate is increasing in hours worked:

$$w(h) = w_0 h^{\theta}$$

Individual problem becomes:

$$\max_{e,h} \log(e w_0 h^{1+\theta} + Y) + e v(1-h) + (1-e) v(1)$$

Repeating the same steps as before, we arrive at the expression:

$$\frac{h}{1+\theta} = \frac{1}{1-\frac{1}{\gamma}}[1-(1-h)^{1-\frac{1}{\gamma}}](1-h)^{\frac{1}{\gamma}}$$

Numerical Results

Table 5

Value of θ Required for Retirement

$IES=2$	$IES=1$	$IES=.75$	$IES=.50$	$IES=.25$	$IES=.10$	$IES=.05$
.22	.46	.64	1.04	2.53	8.19	18.2

Combining the two nonconvexities:

$$\frac{h-\bar{h}}{1+\theta} = \frac{1}{1-\frac{1}{\gamma}}[1-(1-h)^{1-\frac{1}{\gamma}}](1-h)^{\frac{1}{\gamma}}$$

Table 6

Value of θ Required for Retirement When $\bar{h} = .1h$

IES=2	IES=1	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.09	.31	.48	.84	2.17	7.27	16.3

Model With Home Production

Preferences:

$$\int_0^1 [\log c(t) + v(1 - h_m(t) - h_n(t))]dt$$

where:

$$c(t) = [ag(t)^{\varepsilon} + (1 - a)h_n(t)^{\varepsilon}]^{1/\varepsilon}.$$

Budget equation:

$$\int_0^1 g(t)dt = \int_0^1 w_0[h_m(t) - \bar{h}]^{1+\theta}dt$$

Solution to the problem can be summarized by the following values:

- fraction of life in market employment, e
- hours of market work when working, h
- consumption of goods when working and retired, g_w and g_r
- home production time when working and retired, h_w and h_r

Numerical Results

Take elasticity parameters γ and ε as given

Set $\bar{h} =.045$

Normalize $w_0 = 1$

Choose values of a, A and θ so that $h =.45$, $h_w =.10$, $e = 2/3$

Results

Table 8

Values of θ for Home Production Model

	$\varepsilon = 0$	$\varepsilon = .20$	$\varepsilon = .40$	w/o HP
$IES = 1.00$	$.18$	$.17$	$.16$	$.31$
$IES = .50$	$.41$	$.38$	$.34$	$.84$
$IES = .25$	$.70$	$.63$	$.52$	2.17
$IES = .10$	1.03	$.86$	$.70$	7.27

Table 9

Values of h_r and g_r/g_w in the Home Production Model

	h_r			g_r/g_w		
	$\varepsilon = 0$	$\varepsilon = .2$	$\varepsilon = .4$	$\varepsilon = 0$	$\varepsilon = .2$	$\varepsilon = .4$
$IES = 1.00$	$.23$	$.23$	$.24$	1.00	$.96$	$.90$
$IES = .50$	$.24$	$.26$	$.29$	1.00	$.93$	$.83$
$IES = .25$	$.35$	$.37$	$.39$	1.00	$.89$	$.76$
$IES = .10$	$.46$	$.47$	$.48$	1.00	$.86$	$.70$

ATUS Data

Time Use By Age: Men and Women

Age	MW	HP	SH	LE	ED	PC
56	23.4	16.7	5.9	35.5	9.1	65.5
58	23.6	15.5	5.6	37.3	8.5	65.2
60	19.6	16.0	6.0	36.6	10.2	66.3
62	16.3	16.6	5.9	39.5	9.8	66.3
64	13.3	18.4	6.2	43.9	9.4	64.9
66	7.55	18.2	6.2	44.0	10.1	68.2
68	8.25	17.4	5.4	46.8	9.9	66.6
70	3.78	17.4	6.3	49.7	10.5	67.5
72	5.67	18.5	6.0	47.6	10.4	67.1

Table 11

Estimated Time Use Effects–Total

	MW	HP	SH	ED	LE	PC
a	-1.5(.1)	.16(.04)	.02(.02)	.09(.02)	1.0(.06)	.17(.04)
h	-	-.12(.02)	-.01(.01)	-.06(.01)	-.65(.04)	-.12(.02)

HP: home production

SH: shopping

ED: eating and drinking

LE: leisure time

PC: personal care

Table 12

Estimated Time Use Effects–Men

	MW	HP	SH	ED	LE	PC
a	-1.7(.2)	.01(.01)	.08(.03)	.12(.03)	1.2(.1)	.17(.04)
h	-	-.03(.03)	-.05(.02)	-.07(.02)	-.7(.04)	-.11(.02)

Conclusion

We have considered models in which utility from leisure is strictly concave, implying that all else equal, individuals prefer smooth leisure over time.

Retirement generates a very dramatic change in hours of market work

In a model in which leisure and work are mirror images of each other, this is hard to reconcile with low values of the IES.

In a model with home production, it is hard to reconcile the small increase in home production time with moderate elasticity of substitution between time and goods and low values of the IES