A Dynamic Model of Bank Runs

(Very incomplete and preliminary)

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Abstract

The global financial crisis in 2007-2009 was accompanied by sharp decreases in the short-term debt of the financial institutions. This phenomenon may be modeled as a collective action similar to the bank runs. In this paper we develop a simple dynamic model of bank runs. We introduce the banking sector which is subject to the refinance risk in an infinite horizon business cycle model. We show the existence and uniqueness of the equilibrium in which the banks can accumulate the debt over time and the depositors run on the banks if the amount of the banks' debt exceeds the debt capacity, which is determined endogenously. There is a trade-off concerning the debt restructuring after the bank run. If the depositors' bargaining power is high, the debt capacity is high, whereas once the bank run occurs it is likely to recur. If the depositors' bargaining power is low, the debt capacity is low, whereas once the bank run occurs, it is not likely to recur.

1 Introduction

We introduce the banking sector that is financed by the demand deposits, or the one-period debt, into a simplified real business cycle model. The demand deposit is a simplification of various short-term debt instruments in the financial markets in reality.

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The motivation is that we want to analyze the collective action problem (i.e., the bank runs) in a dynamic setting in which agents live forever.[1] To analyze the global financial crisis of 2007–2009, the business cycle models with financial frictions have been intensively studied (Christiano, Motto, Rostagno 2009, Gertler and Karadi 2009, Gertler and Kiyotaki 2010). These models consider borrowing constraints due to costly state verification à la Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999). While there are casual observations that the phenomenon like a systemic bank run occurred in the global financial markets in September 2008 when the Lehman Brothers collapsed (see Lucas 2009, Gorton and Metrick 2009, Adrian and Shin 2009, Uhlig 2009), the collective action problem is not analyzed in these models. In the present paper, we explicitly consider the collective action problem in a dynamic model. Although our motivation is quite close to He and Xiong (2010), their formulation of the model, which is in the literature of the global game, is quite different from ours and is difficult to reconcile with the standard business cycle model. Acharya, Gale, and Yorulmazer (2009) study an informational problem in the rollover of debts under a cost of liquidating the collateral.

This paper is also related to the models with default risks (Hopenhayn and Werning 2008, Arellano 2008, Chaterjee et. al. 2007). The models with defaults are used to study the sovereign debt crises and business cycle issues. The present paper, in which defaults occur due to the collective runs by the creditors on the borrowers, may be understood as a complement to these models, in which defaults occur as a result of optimal decisions by the borrowers. This is because in reality it seems that the collective action by creditors sometimes forces borrowers to choose to default in the case of sovereign debt crises and bankruptcies of financial or non-financial firms. Incidentally, the mathematical structure of our model is quite close to Arellano's (2008) model.

[1] In the existing literature, the bank runs are usually analyzed in a two-period or three-period models (see, for example, Diamond and Dybvig 1983, and Allen and Gale 1998). These models in the banking literature is not easily compatible with the standard dynamic models. There are the overlapping-generations models with infinite horizon, in which the banking sector plays the crucial role. See, for example, Smith (2002), Schreft and Smith (1996), Paal and Smith (2000), Cooper and Ejarque (1995), and Cooper and Corbae (2002).

2 The Model

2.1 The Environment

The economy is a closed economy, with discrete time that continues forever: $t = 0, 1, 2, \ldots$. There are the infinitely-lived consumers and banks inhabited in this economy, the measures of which are both a unit mass. The household maximizes the following utility:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \{u(c_t) - h_t\} \right], \tag{1}$$

where β is the intertemporal discount factor for the utility flow ($0 < \beta < 1$), c_t is consumption, h_t is the labor supply. We assume that the consumer can produce h_t units of the consumption goods from h_t units of labor, as in Lagos and Wright (2005). That the utility is linear in h_t and the output is linear in h_t makes the analysis of dynamics tractable by, as we see below, fixing the risk-free interest rate at β^{-1}. The budget constraint for the consumer is

$$c_t + b_t + d_t \leq h_t + (1 + r_{t-1})b_{t-1} + \tilde{R}_{t-1}d_{t-1}, \tag{2}$$

where b_t is the risk-free bond, d_t is the bank deposits, $(1 + r_t)$ is the gross rate of return on the risk-free bond, and $\tilde{R}_t$ is the gross rate of return on the bank deposits, the value of which varies depending on whether or not the bank run occurs. The value of $\tilde{R}_t$ is specified below.

Bank and deposits: At the initial period $t = 0$, each consumer owns one unit of land. While the consumers cannot produce anything from the land, the banks can produce the consumption goods from the land. For simplicity we assume that each bank can operate only one unit of land. This assumption on the banks is similar to that in Diamond and Rajan (2001): the banks have a specific human capital that can utilize the land in production of the consumption goods, while the depositors (consumers) do not have this specific human capital. The bank produces A_t (≥ 0) units of the consumption goods from one unit of land, where A_t is the aggregate productivity of the economy and is an exogenous random variable. We assume that A_t is independent from its past history such that the probability density function of A_t is $f(A_t)$, which does not depend on A_{t-i} $(i = 1, 2, \cdots)$. The support of A_t is $[0, +\infty]$ and $\int_0^\infty f(A)dA = 1$. For expositional convenience, we define that $f(A) = 0$ for $A < 0$. Thus $\int_{-\infty}^\infty f(A)dA = 1$. At $t = 0$, the consumers give the land to the banks in exchange for the bank deposit liabilities, which is X in terms of the consumption goods. X is determined as an equilibrium outcome. We assume that the financial market is incomplete and subject to the following constraint:

Assumption 1 *The financial contract cannot be contingent on the realization of A_t, which is observable for all agents but not verifiable in the court. Only the debt contract is available for the consumers (depositors) and the banks. The bank can voluntarily walk away without repaying the debt in the midst of production of the consumption goods, leaving z units of consumption goods and one unit of land to the depositors. If the depositors refuse to refinance the bank's debt and the banks are forced to repay a larger amount than A_t, the bank is forced to walk away, leaving z and the land to the depositors.*

We assume that if the bank walks away, voluntarily or involuntarily, the bank simply exit the economy and gets 0 utility thereafter, and that the new banks are born such that the total measure of the banks in the economy remain the same. The bank maximizes the following expected utility conditional on that the bank continues operation.

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(A_t - R_{t-1}D_{t-1} + D_t) \right], \tag{3}$$

where $U(C)$ is the flow utility, where $U'(C) > 0$, $U''(C) \leq 0$ and $U(0) = 0$; and D_t is the total amount of bank deposits at the end of period t. Since $U(C) > 0$ for $C > 0$ and the value of exiting for the bank is 0, the bank never walks away voluntarily. Therefore, unlike the optimal default models (e.g., Arellano 2008), the bank never choose to default unless all consumers refuse to refinance D_t, even though the bank has an option to default. In our model only the depositors choose whether or not to run on the bank.

Threshold of a bank run: The bank run is a situation that $R_{t-1}D_{t-1} > A_t$ and the depositors collectively refuse to refinance D_t. In this case the bank is forced to exit. The depositors decide whether or not to run on the bank. We construct an equilibrium where if D_t exceeds a threshold value, $\overline{D}$, all depositors refuse to deposit D_t with the bank. The threshold is determined as an equilibrium outcome. The bank run does not occur if $R_{t-1}D_{t-1} - A_t \leq D_t \leq \overline{D}$. If $R_{t-1}D_{t-1} - A_t > \overline{D}$, the bank run is inevitable because $D_t > \overline{D}$ even if the bank sets its consumption at 0.

Debt restructuring after a bank run: When the bank run occurs in period t, a depositor gets full amount of deposit, $R_{t-1}d_{t-1}$, with probability $z/(R_{t-1}D_{t-1})$. The $1 - \frac{z}{R_{t-1}D_{t-1}}$ depositors get equal fraction of the bank's land. The group of depositors need to sell the land to a new born bank at the price X. Therefore, the return for the $1 - \frac{z}{R_{t-1}D_{t-1}}$ depositors are $\frac{X}{R_{t-1}D_{t-1}-z}$. The maximum liabilities that a bank with one unit of land can sustain is $\overline{D}$. Because the banks have the specific human capital that can produce the consumption goods from the land, while the depositors do not have the ability to utilize the land, the price of land, X, is determined between 0 and $\overline{D}$ depending on the market structure or the bargaining between the new born banks and the groups of the depositors. We do not construct the market structure or the bargaining protocols, but simply assume the following as a reduced form:

$$X = \theta\overline{D}, \tag{4}$$

where θ $(0 \leq \theta \leq 1)$ is the parameter representing the bargaining power of the depositors.

2.2 Optimization Problems

The consumers maximize (1) subject to (2). The first-order conditions (FOCs) imply that in the equilibrium where the consumers hold bank deposits

$$1 + r_t = \beta^{-1}, \tag{5}$$

$$E_t[\tilde{R}_t] \geq 1 + r_t. \tag{6}$$

If (6) is not satisfied, the consumers do not purchase the bank deposits in period t and cause the bank run. Therefore, (6) is the participation constraint for the depositors that the bank needs to take into account. The bank's optimization is formulated as the following Bellman equation. We define $V(x_{t-1})$ as the value function of a bank at the beginning of period t when A_t is not revealed yet and $x_{t-1} = R_{t-1}D_{t-1}$ is the gross amount of debt liability of the bank at the beginning of period t. The Bellman equation is written as follows:

$$V(x_{t-1}) = \int_{x_{t-1}-\overline{D}}^{\infty} \max_{R_t(A_t),\ D_t(A_t)} \{U(A_t - x_{t-1} + D_t) + \beta V(R_t D_t)\} f(A_t) dA_t, \tag{7}$$

subject to

$$E[\tilde{R}_t] \geq 1 + r_t, \tag{8}$$

$$\max\{0, x_{t-1} - A_t\} \leq D_t, \tag{9}$$

$$D_t \leq \overline{D}. \tag{10}$$

Note that to set $D_t > \overline{D}$ is infeasible for the bank because the depositors do not finance D_t if $D_t > \overline{D}$ and the bank is forced to exit due to the bank run. To set $D_t \leq \overline{D}$ is feasible if and only if $x_{t-1} - A_t \leq \overline{D}$. If $D_t \leq \overline{D}$ is feasible, the bank optimally chooses $D_t \leq \overline{D}$ and avoid the bank run. Therefore, the bank run occurs if and only if $A_t < x_{t-1} - \overline{D}$. Given this condition, $E_t[\tilde{R}_t]$ can be written as follows:

$$\begin{aligned} E[\tilde{R}_t] &= R_t \int_{R_t D_t - \overline{D}}^{\infty} f(A_{t+1}) dA_{t+1} + \left(\frac{z}{R_t D_t} R_t + \frac{R_t D_t - z}{R_t D_t} \frac{X}{R_t D_t - z} \right) \int_0^{R_t D_t - \overline{D}} f(A_{t+1}) dA_{t+1} \\ &= R_t \int_{R_t D_t - \overline{D}}^{\infty} f(A_{t+1}) dA_{t+1} + \left(\frac{z}{D_t} + \frac{X}{R_t D_t} \right) \int_0^{R_t D_t - \overline{D}} f(A_{t+1}) dA_{t+1}. \end{aligned} \tag{11}$$

We can show the following proposition concerning this Bellman equation.

Proposition 1 *The Bellman equation (7), subject to (8)–(11), has a unique solution.*

See Appendix A for proof. It is easily shown that (8) is always binding:

Lemma 1 *Given D_t, the value of R_t is determined as a solution to (8) with equality. If (8) has two or more solutions, the bank chooses the smallest one.*

See Appendix B for proof.

If $A_t < R_{t-1}D_{t-1} - \overline{D}$, the bank run occurs in period t and the existing banks exit and new born banks enter into the economy. In this case, output becomes z in period t and the new banks issue bank deposits X to the consumers. Therefore, $\{D_{t+j}, R_{t+j}\}_{j=0}^{\infty}$ evolves from $D_t = X$, where R_t is determined as a solution to (8), given $D_t = X$.

2.3 Equilibrium

The equilibrium is characterized by the sequence $\{D_t, R_t, A_t\}_{t=0}^{\infty}$, where given A_t, D_t and R_t are the solution to the bank's optimization (7) subject to (8)–(11). When $A_t < R_{t-1}D_{t-1} - \overline{D}$, D_t jumps to X as a result of the bank run. The upper bound for the sustainable debt, $\overline{D}$, is uniquely determined by

$$\overline{D} = \max\left\{ D \left| \max_R R \int_{(R-1)D}^{\infty} f(A)dA + \left(\frac{z}{D} + \frac{X}{RD}\right) \int_0^{(R-1)D} f(A)dA \geq \frac{1}{\beta} \right.\right\}. \quad (12)$$

We can show the following lemma for $\overline{D}$:

Lemma 2 *Suppose that $\overline{D}$ is given by (12). This $\overline{D}$ is the unique value that satisfies the following claim: Given that the depositors in period $t+1$ refuse to refinance D_{t+1} if and only if $D_{t+1} > \overline{D}$, it is optimal for the depositors in period t to refuse to refinance D_t if and only if $D_t > \overline{D}$.*

(Proof) We consider the pair of debt capacities $\{\overline{D}_t, \overline{D}_{t+1}\}$ such that,given that the depositors in period $t+1$ refuse to refinance D_{t+1} if and only if $D_{t+1} > \overline{D}_{t+1}$, it is optimal for the depositors in period t to refuse to refinance D_t if and only if $D_t > \overline{D}_t$. Taking $\overline{D}_{t+1}$ as given, $\overline{D}_t$ must be determined by

$$\overline{D}_t(\overline{D}_{t+1}) = \max\left\{ D \left| \max_R G(R, D; \overline{D}_{t+1}) \geq \frac{1}{\beta} \right.\right\}, \quad (13)$$

where

$$G(R, D; \overline{D}_{t+1}) = R \int_{RD-\overline{D}_{t+1}}^{\infty} f(A)dA + \left(\frac{z}{D} + \frac{X}{RD}\right) \int_0^{RD-\overline{D}_{t+1}} f(A)dA.$$

The function $\overline{D}_t(\overline{D}_{t+1})$ is the best response of the depositors in period t to those in period $t+1$. We consider $G(R, D; \overline{D}_{t+1})$ only for D that satisfies

$$D > \beta z + \beta^2 X \equiv \underline{D}. \quad (14)$$

If D satisfies (14), it is always true that $R > \frac{z}{D} + \frac{X}{RD}$ for $R \geq \beta^{-1}$. This means that the promised return on the bank deposit (R) is greater than the expected return on the deposit in the case of the bank run. Thus the derivative of $G(R, D; \overline{D}_{t+1})$ satisfies

$$G_2(R, D; \overline{D}) = -\left\{ R - \left(\frac{z}{D} + \frac{X}{RD}\right) \right\} Rf(RD - \overline{D}) - \left(\frac{z}{D^2} + \frac{X}{RD^2}\right) \int_0^{RD-\overline{D}} f(A)dA.$$

Since $G(\beta^{-1}, \beta\overline{D}; \overline{D}) = \beta^{-1}$, the solution to (13) is no less than $\beta\overline{D}_{t+1}$. Therefore, for $D > \beta\overline{D}_{t+1}$ and $R \geq \beta^{-1}$,

$$G_2(R, D; \overline{D}_{t+1}) < 0. \tag{15}$$

For $\overline{D}$, which is the solution to (12), the following statements hold: $\max_R G(R, \overline{D}; \overline{D}) = \beta^{-1}$; $\max_R G(R, D; D) > \beta^{-1}$ for $D < \overline{D}$; and $\max_R G(R, D; D) < \beta^{-1}$ for $D > \overline{D}$. These conditions and (15) imply that

$$\overline{D}_t(\overline{D}_{t+1}) > \overline{D}_{t+1} \quad \text{for} \quad \overline{D}_{t+1} < \overline{D},$$
$$\overline{D}_t(\overline{D}_{t+1}) < \overline{D}_{t+1} \quad \text{for} \quad \overline{D}_{t+1} > \overline{D}.$$

Therefore, the value $\overline{D}$ given by (12) is the unique threshold that is consistent with the rational expectations of the depositors. (QED)

With the equilibrium condition (4), the above condition (12) uniquely determines $\overline{D}$ and $X = \theta\overline{D}$, given the depositors' bargaining power θ in debt restructuring.

For numerical calculations, we set $\beta = 0.96$, $E[A] = 1$, and $z = 0.1$ throughout this section. Figure 1 shows the relationship between θ and $\overline{D}$ in the case where A_t follows the log-normal distribution with mean 1 and variance 1. $\overline{D}$ is larger as the bargaining power of depositors, θ, is larger. The full value of the land is obviously $\frac{\beta E[A]}{1-\beta}$, which is 24 for the parameter values in Figure 1. $\overline{D}$ is much smaller than $\frac{\beta E[A]}{1-\beta}$ even if $\theta = 1$, because a substantial rent goes to the bank.

Figure 1: Debt capacity, $\overline{D}$, and depositors' bargaining power, θ ($\nu^2 = 1$).

It is shown numerically that the debt capacity is negatively correlated with the variance of A_t. Figure 2 shows that in the case where θ is fixed at 1, the debt capacity $\overline{D}$ decreases as the variance ν^2 increases. The intuition is as follows: Given R and D, the probability of the bank run in (12) increases as ν^2 increases, and the maximand of (12) decreases for all D. Therefore, $\overline{D}$, the solution to (12), decreases as ν^2 increases.

Figure 2: Debt capacity, $\overline{D}$, and variance, ν^2 ($\theta = 1$).

Lemma 1 implies that if D_t is given, R_t is uniquely pinned down by the following equation:

$$R_t \int_{R_t D_t - \overline{D}}^{\infty} f(A_{t+1}) dA_{t+1} + \left(\frac{z}{D_t} + \frac{\theta \overline{D}}{R_t D_t} \right) \int_0^{R_t D_t - \overline{D}} f(A_{t+1}) dA_{t+1} = \beta^{-1}.$$

The probability, as of period t, of occurrence of the bank run in period $t+1$ is

$$Pr(D_t) \equiv \int_0^{R_t D_t - \overline{D}} f(A) dA. \tag{16}$$

We have the following lemma.

Lemma 3 *If $D_t \le \beta \overline{D}$, then $R_t = \beta^{-1}$ and $Pr(D_t) = 0$. If $\beta \overline{D} < D_t \le \overline{D}$, then $R_t > \beta^{-1}$ and $Pr(D_t) > 0$.*

Proof is obvious. Figure 3 illustrates the implication of the above lemma on the evolution of D_t.

Figure 3. The amount of the bank debt and the risk of the bank run.

In Figures 4 and 5, we show R_t and $Pr(D_t)$ as functions of D_t in the case where A_t follows the log-normal distribution with mean 1 and variance 1. We show the cases corresponding to $0 < \theta < \beta$ and $\beta < \theta < 1$, respectively.

Figure 4. R_t and $Pr(D_t)$ in the case where $\theta = 0.95 < \beta$

Figure 5. R_t and $Pr(D_t)$ in the case where $\beta < \theta = 0.99$

This figure shows that there is a trade-off concerning θ. If the depositors' bargaining power is strong, i.e., θ is larger than β, the debt capacity of the bank $\overline{D}$ is large, as Figure 1 shows. Meanwhile, since $X = \theta \overline{D} > \beta \overline{D}$ in this case, Figure 3 shows that after the debt restructuring subsequent to a bank run the bank debt stays in the region of a positive risk of occurrence of another bank run. If θ is small, the debt capacity $\overline{D}$ is also small, while $X = \theta \overline{D} < \beta \overline{D}$ and the bank debt decreases sufficiently after the bank run such that the risk of another bank run becomes zero (Figure 2). The dynamics of D_t is characterized by the FOC:

$$U'(C_t) + \beta R_t V'(R_t D_t) + \lambda(A_t) - \eta(A_t) +$$
$$\mu(A_t)\left[-\left\{R_t - \left(\frac{z}{D_t} + \frac{X}{R_t D_t}\right)\right\} R_t f(R_t D_t - \overline{D}) - \left(\frac{z}{D_t^2} + \frac{X}{R_t D_t^2}\right)\int_0^{R_t D_t - \overline{D}} f(A)dA\right] = 0,$$

where $C_t = A_t - R_{t-1}D_{t-1} + D_t$ and $\lambda(A_t)$ and $\eta(A_t)$ are the Lagrange multipliers for (9) and (10), respectively; and the envelope condition:

$$V'(R_{t-1}D_{t-1}) = -\int_{R_{t-1}D_{t-1}-\overline{D}}^{\infty} U'(C_t)f(A_t)dA_t - \beta V(\overline{RD})f(R_{t-1}D_{t-1} - \overline{D}) - \int_0^x \lambda(A_t)dA_t.$$

The dynamics of D_t depend crucially on the form of $U(\,\cdot\,)$. If $U(C)$ is sufficiently concave, D_t may exhibit a feature of the mean-reversion around a specific value (incomplete). The following example simplifies the dynamics.

2.4 Example

If the utility of the banks is linear, that is, $U(C) = C$, then the dynamics become simple.

Lemma 4 *If $U(C) = C$, then $D_t = \hat{D}_t$, where $\hat{D}_t = \max\{0, R_{t-1}D_{t-1} - A_t\}$, if $\hat{D}_t \leq \overline{D}$; and $D_t = \theta\overline{D}$ if $\hat{D}_t > \overline{D}$.*

See Appendix C for proof. This lemma says that if $U(C) = C$, the bank sets its consumption at zero as long as the remaining debt is positive and reduces the debt as fast as possible; and that only when the remaining debt becomes zero, it consumes the net profits, $A_t - R_{t-1}D_{t-1}$.

3 Generalization of the Model

We introduce the labor input in this economy and show that the equilibrium with collective bank run can be defined in this generalized model consistently.

3.1 Environment

In addition to the consumers and the banks, we introduce the firms that produce the consumption goods using the labor input (l_t) and the capital input (k_t), where l_t is supplied by the consumers and k_t is the capital service generated from the banks' land. We assume that the firms' technology is Cobb-Douglas and maximizes its profits in each period in a perfectly competitive environment after the realization of productivity, A_t. Therefore the firms' problem is as follows: Given $\tilde{r}_t$ and $\tilde{w}_t$,

$$\max_{l_t,k_t} A_t k_t^{\alpha} l_t^{1-\alpha} - \tilde{r}_t k_t - \tilde{w}_t l_t.$$

Since the total supply of land is unity, $k_t = 1$ in equilibrium. Thus,

$$\tilde{r}_t = \alpha A_t l_t^{1-\alpha},$$
$$\tilde{w}_t = (1-\alpha) A_t l_t^{-\alpha},$$

unless the bank run occurs. If the bank run occurs, the banks are forced to withdraw their land from production by the firms and to give z and the land to their depositors. Therefore, if the bank runs occur for all banks, production by the firms becomes infeasible. (If the bank runs occur only for some fraction of banks, the amount of k in the Cobb-Douglass production becomes the amount of land of the banks that are not run on.) We assume that in each period t, the consumers supply labor l_t *before* A_t is revealed. Supplying l_t incurs utility cost $c(l_t)$ to the consumer. Therefore, the consumer's problem is

$$\max_{c_t,l_t,h_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t \{u(c_t) - c(l_t) - h_t\} \right],$$

subject to

$$c_t + b_t + d_t = (1 + r_{t-1}) b_{t-1} + \tilde{R}_t d_t + \tilde{w}_t l_t + h_t,$$

where l_t is chosen before $\tilde{w}_t$ is realized. The FOC for l_t is

$$c'(l_t) = E_t[\tilde{w}_t]. \tag{17}$$

3.2 Bank's problem and equilibrium

Similar to the basic model, we consider the equilibrium in which the bank runs occur if the banks' debt exceeds a threshold $\overline{D}$. At the beginning of period t, the state variables for a bank is x_{t-1} and l_t, where $x_{t-1} = R_{t-1}D_{t-1}$. Since the revenue of the bank is $r_t = \alpha A_t l_t^{1-\alpha}$, the Bellman equation for a bank entering period t is as follows:

$$V(x_{t-1}, l_t) = \int_{A(x_{t-1},l_t)}^{\infty} \max_{R_t, D_t} \left\{ U(\alpha A_t l_t^{1-\alpha} - x_{t-1} + D_t) + \beta V(R_t D_t, l_{t+1}) \right\} f(A_t) dA_t, \tag{18}$$

where $A(x_{t-1}, l_t) \equiv \frac{x_{t-1} - \overline{D}}{\alpha l_t^{1-\alpha}}$, subject to

$$\max\{0, x_{t-1} - \alpha A_t l_t^{1-\alpha}\} \leq D_t, \tag{19}$$

$$D_t \leq \overline{D}, \tag{20}$$

$$R_t \int_{A(R_t D_t, l_{t+1})}^{\infty} f(A_{t+1}) dA_{t+1} + \left(\frac{z}{D_t} + \frac{X}{R_t D_t} \right) \int_0^{A(R_t D_t, l_{t+1})} f(A_{t+1}) dA_{t+1} \geq \beta^{-1}. \tag{21}$$

Note that l_{t+1} is not a choice variable for the bank and the banks take l_{t+1} as given when they choose R_t and D_t. From (17), the equilibrium condition that determines l_{t+1} is

$$c'(l_{t+1}) = \int_{A(R_t D_t, l_{t+1})}^{\infty} (1-\alpha) A_{t+1} l_{t+1}^{-\alpha} f(A_{t+1}) dA_{t+1}. \tag{22}$$

Note that in (22) the consumers take $R_t D_t$ as given. We assume (4) for X as in the basic model. $\overline{D}$ is determined by

$$\overline{D} = \max \left\{ D \,\middle|\, \max_R R \int_{\frac{(R-1)D}{\alpha l^{1-\alpha}}}^{\infty} f(A) dA + \left(\frac{z}{D} + \frac{\theta}{R} \right) \int_0^{\frac{(R-1)D}{\alpha l^{1-\alpha}}} f(A) dA \geq \frac{1}{\beta} \right\}, \tag{23}$$

where l is determined by

$$c'(l) = \int_{\frac{(\overline{R}-1)\overline{D}}{\alpha l^{1-\alpha}}}^{\infty} (1-\alpha) A l^{-\alpha} f(A) dA. \tag{24}$$

Solving (23) and (24) for $\overline{D}$ and l simultaneously, $\overline{D}$ is determined.

Equilibrium: The equilibrium is the sequence $\{A_t, l_t, D_t, R_t\}$, where D_t and R_t are the solution to (18)–(21), given A_t and l_t; and l_{t+1} satisfies (22), given R_t and D_t.

Bank runs in the generalized model (Incomplete): In the basic model in the previous section, the bank's revenue is A_t, which is exogenously given and does not change depending on the agents' actions, while in the generalized model the bank's revenue is $r_t = \alpha A_t k_t^{\alpha-1} l_t^{1-\alpha}$, which may vary depending on the other agents' actions. In this case, the threshold of the bank run may not be well defined. For example, suppose that the realization of A_t is very small. If all depositors equally run on all banks, all banks goes bankrupt; on the other hand, if only one bank is not run on but the other banks are run on, the rental price of capital r_t for this bank goes up because the other banks withdraw capital services from the firms due to the bank runs; in this case, it is optimal for the depositors of this bank not to run; then this bank survives and the depositors of this bank obtain the full amount of deposits. Therefore, even when the realization of A_t is identical, the bank run may or may not occur at a given bank. It is not difficult, though, to construct a structure of the game among the depositors such that in equilibrium all banks are run on if and only if

$$A_t < \frac{x_{t-1} - \overline{D}}{\alpha l_t^{1-\alpha}}. \tag{25}$$

For example, because the banks are forced to withdraw the capital services from the firms as a result of the depositors' run, if the depositors of all banks must simultaneously choose whether to refinance their banks or to refuse to refinance, the prisoners' dilemma among the depositors (within a bank and between banks) results in the bank runs for all banks when A_t is in the range of (25). Note that the right-hand side of (25) can be positive only if

$$D_{t-1} > \beta \overline{D}.$$

Therefore, Lemma 3 also applies to the generalized model. When $D_t \leq \beta \overline{D}$, the labor supply is determined by $l_{t+1} = l^*$, where

$$c'(l^*) = (1 - \alpha) E[A] (l^*)^{-\alpha}. \tag{26}$$

When $D_t > \beta \overline{D}$, the labor supply l_{t+1}, determined by (22), is less than l^*.

Simulation result: The numerical simulations are shown in Figures 6–9. Parameter values are set that $\beta = 0.96$, $E[A] = 1$, $z = 0.1$, $\alpha = 0.3$, and $c(l) = \frac{l^2}{2}$. The results are qualitatively the same as those for the basic model. The probability of bank runs, which is

$$\int_0^{(R_t D_t - \overline{D})/(\alpha l_{t+1}^{1-\alpha})} f(A_{t+1}) dA_{t+1}, \tag{27}$$

tends to be much higher in the generalized model than in the basic model because $\alpha l_{t+1}^{1-\alpha}$ is considerably smaller than one (Figures 8 and 9). It is also shown that the labor supply decreases as D_t increases when $D_t > \beta\overline{D}$ (Figures 8 and 9).

4 Conclusion

We develop a simple dynamic model of bank runs, in which the banking sector is subject to the refinance risk in each period. We show the existence and uniqueness of the equilibrium in which the banks can accumulate the debt over time and the depositors run on the banks if the amount of the banks' debt exceeds a threshold value, which is determined endogenously. There is a trade-off concerning the debt restructuring after the bank run. If the depositors' bargaining power is strong, the threshold of debt for the bank runs is high, whereas once the bank run occurs it is likely to repeat. If the depositors' bargaining power is weak, the threshold is low, whereas once the bank run occurs, it is not likely to repeat.

The framework of this model may be useful to study the financial crisis further. For example, if the price of land, X, includes the bubble component and the asset-price bubble collapsed suddenly, then the debt capacity $\overline{D}$ may go down suddenly and trigger a bank run. Thus it may be a promising topic for future research to analyze the interaction between the asset-price bubble and the bank runs. The distortion in the real allocations during a financial crisis may be an another promising topic for future research.[2] The

[2] Ohanian (2001) and Hayashi and Prescott (2002) point out the productivity declines during the financial crises. Chari, Kehoe and McGrattan (2007) and Kobayashi and Inaba (2006) show the deteriorations in the labor wedge during the financial crises.

model shows that if D_t is large the probability of occurrence of the bank runs become positive, while it is zero if D_t is smaller than a threshold. In the generalized model, the risk aversion of the consumers leads to a decrease in the labor supply, or a widening of the labor wedge. In addition, if the risk aversion affects the choice of production technology, the aggregate productivity may decline when the banks' debt D_t increases and enters the region of positive risk of the bank runs. The model might be useful to analyze the interaction between the balance-sheet deteriorations of financial institutions and the monetary policy during the financial crises. The model implies that the real interest rate becomes higher when D_t exceeds $\beta\overline{D}$. Under the nominal rigidities the central bank can lower the real interest rate by lowering the nominal interest rate. Thus it may be efficient for the central bank to set a lower rate when $D_t > \beta\overline{D}$ than when $D_t \leq \beta\overline{D}$. The model may imply that the monetary policy should respond to the balance-sheet variables, such as the leverage of the financial institutions.

Appendix A: Proof of Proposition 1

We consider an operator T mapping a space $C[0, \overline{RD}]$ of bounded continuous functions into itself, where $\overline{R}$ and $\overline{D}$ are the solutions to (12). $Tv(x)$ is defined as follows:

$$Tv(x) = \int_{x-\overline{D}}^{\infty} \max_{R,\ D}\{U(A - x + D) + \beta v(RD)\} f(A) dA, \tag{28}$$

subject to

$$\max\{0, x - A\} \leq D, \tag{29}$$

$$D \leq \overline{D}, \tag{30}$$

$$R \int_{RD-\overline{D}}^{\infty} f(A')dA' + \left(\frac{z}{D} + \frac{X}{RD}\right) \int_{0}^{RD-\overline{D}} f(A')dA' \geq \beta^{-1}. \tag{31}$$

The proof of Proposition 1 is to show the existence and uniqueness of the fixed point of the operator T. We define F, a subset of $C[0, \overline{RD}]$, by

$$F = \left\{ v(x) \,\middle|\, v(x) \in C[0, \overline{RD}] \text{ and } 0 \leq v(x) \leq \frac{\beta E[U(A)]}{1-\beta} \text{ for all } x \in [0, \overline{RD}] \right\}.$$

F is closed and bounded. Following Stokey and Lucas (1989) we call T *monotone* if $v, w \in F$ and $v \geq w$ implies $Tv \geq Tw$, where $v \geq w$ means that $v(x) \geq w(x)$, all $x \in [0, \overline{RD}]$. It is also easily shown that T, defined by (28)–(31), is continuous and monotone.

Definition 1 *A subset S of $C[0, \overline{RD}]$ is equicontinuous if for every $\epsilon > 0$ there exists $\delta > 0$ such that $|x - y| < \delta$ implies $|v(x) - v(y)| < \epsilon$, for all $v \in S$.*

Lemma 5 *The family T(F) is equicontinuous.*

(Proof) Since $U(c)$ is a concave function with $U(0) = 0$, for every $\epsilon > 0$ there exists $\delta > 0$ such that $|x - y| < \delta$ implies $|U(x) - U(y)| < \epsilon$. Pick a pair (ϵ, δ) that satisfies this property. For a given x, denote $R(A, x)$ and $D(A, x)$ as the solution to the optimization of (28)–(31).

$$Tv(x + \delta) = \int_{x+\delta-\overline{D}}^{\infty} \max_{R,\ D}\{U(A - x - \delta + D) + \beta v(RD)\} f(A) dA,$$

subject to $\max\{0, x + \delta - A\} \leq D \leq \overline{D}$ and (31). $\{R(A, x), D(A, x)\}$ is feasible for this problem and obviously $Tv(x + \delta) \leq Tv(x)$ and

$$Tv(x + \delta) \geq \int_{x-\overline{D}}^{\infty} \{U(A - x - \delta + D(A, x)) + \beta v(R(A, x) D(A, x))\} f(A) dA.$$

Therefore,

$$\begin{aligned} 0 \leq Tv(x) - Tv(x + \delta) &\leq \int_{x-\overline{D}}^{\infty} \{U(A - x + D(A, x)) - U(A - x - \delta + D(A, x))\} f(A) dA \\ &< \epsilon \int_{0}^{\infty} f(A) dA = \epsilon. \end{aligned}$$

We have shown for every $\epsilon > 0$ there exists $\delta > 0$ such that $|x - y| < \delta$ implies $|Tv(x) - Tv(y)| < \epsilon$, for all $v \in F$. Therefore, $T(F)$ is equicontinuous. (QED)

To show the existence of the fixed point of T, we apply Theorem 17.7 in Stokey and Lucas (1989):

Theorem 1 *(Stokey-Lucas, Theorem 17.7) Let $X \subset R^l$ be a bounded set; let $C(X)$ be the space of bounded continuous functions on X, with the sup norm; and let $F \subset C(X)$ be closed and bounded. Assume that the operator $T: F \to F$ is continuous and monotone and that $T(F)$ is an equicontinuous family. Suppose there exists $f_0 \in F$ such that either $f_0 \leq Tf_0$ or $f_0 \geq Tf_0$. Then the limit $f = \lim T^n f_0$ exists, f is in F, and f is a fixed point of T.*

To apply this theorem to T, we pick $v_0(x) = 0$ and $w_0(x) = \frac{\beta E[U(A)]}{1-\beta}$. Then $v_0 \leq Tv_0$ and $w_0 \geq Tw_0$. Applying the theorem, it is proven that there exist fixed points v and w, where $v = \lim T^n v_0$ and $w = \lim T^n w_0$. As the corollary of Theorem 17.7 in Stokey and Lucas (1989) shows, since v_0 and w_0 are minimal and maximal elements of F, every fixed point h of T satisfies

$$\lim T^n v_0 = v \leq h \leq w = \lim T^n w_0.$$

The uniqueness of the fixed point of T is established by the following lemma:

Lemma 6 *The minimal and maximal fixed points, v and w, are identical.*

(Proof) Proof is by contradiction. Suppose that $v \neq w$. Then there should exist $x_0 \in [0, \overline{RD}]$ such that $w(x_0) > v(x_0)$. Let $\Delta = w(x_0) - v(x_0)$. We can choose n such that

$$\delta \equiv \beta^n \frac{\beta E[U(A)]}{1-\beta} < \frac{\Delta}{2}.$$

Since w and v are the fixed points of T, applying the operator T for n times yields

$$w(x_0) = \tilde{E}_0 \left[\sum_{i=0}^{n-1} \beta^i U(\tilde{c}_i) \right] + \tilde{E}_0[\beta^n w(\tilde{x}_n)],$$
$$v(x_0) = \hat{E}_0 \left[\sum_{i=0}^{n-1} \beta^i U(\hat{c}_i) \right] + \hat{E}_0[\beta^n v(\hat{x}_n)]$$

where $\tilde{c}_i = A_i - \tilde{x}_{i-1} + \tilde{D}_i$ and $\tilde{x}_i = \tilde{R}_i \tilde{D}_i$ are the variables on the optimal path corresponding to the operation Tw; $\hat{c}_i = A_i - \hat{x}_{i-1} + \hat{D}_i$ and $\hat{x}_i = \hat{R}_i \hat{D}_i$ are those corresponding to Tv; and

$$\tilde{E}_0[U(\tilde{c}_i)] = \int_{R_{-1}D_{-1}-\overline{D}}^{\infty} \left[\cdots \left[\int_{\tilde{R}_{i-2}\tilde{D}_{i-2}-\overline{D}}^{\infty} \left[\int_{\tilde{R}_{i-1}\tilde{D}_{i-1}-\overline{D}}^{\infty} U(\tilde{c}_i) df(A_i) \right] df(A_{i-1}) \right] \cdots \right] df(A_0),$$
$$\hat{E}_0[U(\hat{c}_i)] = \int_{R_{-1}D_{-1}-\overline{D}}^{\infty} \left[\cdots \left[\int_{\hat{R}_{i-2}\hat{D}_{i-2}-\overline{D}}^{\infty} \left[\int_{\hat{R}_{i-1}\hat{D}_{i-1}-\overline{D}}^{\infty} U(\hat{c}_i) df(A_i) \right] df(A_{i-1}) \right] \cdots \right] df(A_0).$$

Since $0 \leq \hat{E}_0[\beta^n v(\hat{x}_n)],\ \tilde{E}_0[\beta^n w(\tilde{x}_n)] \leq \delta$, we have

$$\begin{aligned} \tilde{E}_0 \left[\sum_{i=0}^{n-1} \beta^i U(\tilde{c}_i) \right] - \hat{E}_0 \left[\sum_{i=0}^{n-1} \beta^i U(\hat{c}_i) \right] &= w(x_0) - v(x_0) - \tilde{E}_0[\beta^n w(\tilde{x}_n)] + \hat{E}_0[\beta^n v(\hat{x}_n)] \\ &> \Delta - \delta. \end{aligned} \tag{32}$$

Since $\{\tilde{c}_i, \tilde{x}_i\}$, the optimal path corresponding to Tw, satisfies the constraints (29)–(31) for Tv, we have the following inequality:

$$v(x_0) = \hat{E}_0\left[\sum_{i=0}^{n-1}\beta^i U(\hat{c}_i)\right] + \hat{E}_0[\beta^n v(\hat{x}_n)] \geq \tilde{E}_0\left[\sum_{i=0}^{n-1}\beta^i U(\tilde{c}_i)\right] + \tilde{E}_0[\beta^n v(\tilde{x}_n)]$$
$$\geq \tilde{E}_0\left[\sum_{i=0}^{n-1}\beta^i U(\tilde{c}_i)\right], \tag{33}$$

where the first inequality is because $\{\tilde{c}_i, \tilde{x}_i\}$ is not the optimal path corresponding to Tv. Inequalities (32) and (33) imply that

$$\delta \geq \hat{E}_0[\beta^n v(\hat{x}_n)] \geq \tilde{E}_0\left[\sum_{i=0}^{n-1}\beta^i U(\tilde{c}_i)\right] - \hat{E}_0\left[\sum_{i=0}^{n-1}\beta^i U(\hat{c}_i)\right] \geq \Delta - \delta.$$

This inequality implies that $2\delta \geq \Delta$, which is a contradiction because we set δ such that $2\delta < \Delta$. Therefore, it must be the case that $v = w$. (QED)

From this lemma, the uniqueness of the fixed point of T is established.

Appendix B: Proof of Lemma 1

The FOC for the Bellman equation (7)–(11) with respect to R_t is

$$\beta D_t V'(R_t D_t) + \mu(A_t)\left[\int_{R_t D_t - \overline{D}}^{\infty} f(A)dA - \left\{R_t - \left(\frac{z}{D_t} + \frac{X}{R_t D_t}\right)\right\} D_t f(R_t D_t - \overline{D}) - \frac{X}{R_t^2 D_t}\int_0^{R_t D_t - \overline{D}} f(A)dA\right] = 0, \tag{34}$$

where $\mu(A_t)$ is the Lagrange multiplier for (8). The envelope condition implies

$$V'(x) = -\int_{x-\overline{D}}^{\infty} U'(A_t - x + D_t) f(A_t)dA_t - \beta V(\overline{RD}) f(x - \overline{D}) - \int_0^x \lambda(A_t)dA_t,$$
$$\leq -\int_{x-\overline{D}}^{\infty} U'(A_t - x + D_t) f(A_t)dA_t < 0, \tag{35}$$

where $\lambda(A)$ is the Lagrange multiplier for (9). The inequalities follow from that $V(x)$ and $\lambda(A)$ are nonnegative and $f(A)$ and $U'(A - x + D)$ are strictly positive for some A ($> x - \overline{D}$). The above conditions imply that $\mu(A_t) \neq 0$ because if $\mu(A_t) = 0$ (34) does not hold because (35) implies that $V'(R_t D_t) < 0$. Therefore, since $\mu(A_t)$ is nonnegative,

$\mu(A_t) > 0$ for all A_t, implying that (8) is always binding. Since $V'(R_tD_t) < 0$, for any given D_t the bank chooses the smallest value of R_t from the feasible values. Hence if (8) with equality has two or more solutions, the bank chooses the smallest one.

Appendix C: Proof of Lemma 4 (Outline)

We set $U(C) = C$ in the Bellman equation (7)–(11). The FOC with respect to D_t is

$$\begin{aligned}&1 + \beta R_t V'(R_tD_t) + \lambda(A_t) - \eta(A_t) + \\ &\mu(A_t)\left[-\left\{R_t - \left(\frac{z}{D_t} + \frac{X}{R_tD_t}\right)\right\} R_t f(R_tD_t - \overline{D}) - \left(\frac{z}{D_t^2} + \frac{X}{R_tD_t^2}\right)\int_0^{R_tD_t-\overline{D}} f(A)dA\right] = 0,\end{aligned} \tag{36}$$

where $\eta(A_t)$ is the Lagrange multiplier for (10). Substituting (34) for $\mu(A_t)$ in (36) implies that LHS, the left-hand side of (36), can be written as

$$\begin{aligned}LHS =& 1 + \lambda(A_t) - \eta(A_t) \\ &+ \frac{\beta V'(R_tD_t)\left[R_t \int_{R_tD_t-\overline{D}}^{\infty} f(A)dA + \frac{z}{D_t}\int_0^{R_tD_t-\overline{D}} f(A)dA\right]}{\left[\int_{R_tD_t-\overline{D}}^{\infty} f(A)dA - \left\{R_t - \left(\frac{z}{D_t} + \frac{X}{R_tD_t}\right)\right\} D_t f(R_tD_t - \overline{D}) - \frac{X}{R_t^2D_t}\int_0^{R_tD_t-\overline{D}} f(A)dA\right]}.\end{aligned}$$

We prove that $\lambda(A_t) > 0$ in the case where $D_t > \beta\overline{D}$, and then we prove it in the case where $D_t \le \beta\overline{D}$.

Case 1. $D_t > \beta\overline{D}$**:** Since $f(R_tD_t - \overline{D}) > 0$ in this case, equation (35) directly implies that

$$V'(R_tD_t) < -\int_{R_tD_t-\overline{D}}^{\infty} f(A)dA. \tag{37}$$

Lemmas 1 implies that

$$\int_{R_tD_t-\overline{D}}^{\infty} f(A)dA = \frac{1}{\beta R_t} - \left(\frac{z}{R_tD_t} + \frac{X}{R_t^2D_t}\right)\int_0^{R_tD_t-\overline{D}} f(A)dA.$$

We can show from (37) and the above equation that

$$LHS < 1 + \lambda(A_t) - \eta(A_t) - \Gamma,$$

where

$$\Gamma \equiv \frac{1-\beta\left(\frac{z}{D_t}+\frac{2X}{R_tD_t}\right)\int_0^{R_tD_t-\overline{D}} f(A)dA+\beta^2\left(\frac{z}{D_t}+\frac{X}{R_tD_t}\right)\frac{X}{R_tD_t}\left(\int_0^{R_tD_t-\overline{D}} f(A)dA\right)^2}{1-\beta\left(\frac{z}{D_t}+\frac{2X}{R_tD_t}\right)\int_0^{R_tD_t-\overline{D}} f(A)dA-\beta R_tD_t\left\{R_t-\left(\frac{z}{D_t}+\frac{X}{R_tD_t}\right)\right\}f(R_tD_t-\overline{D})}.$$

Note that $\Gamma \geq 1$, because $\frac{z}{D_t}+\frac{X}{R_tD_t}$ is the expected return for a depositor in the bank run, which is no greater than R_t. Therefore, if $\lambda(A_t)=0$, the above inequality implies that $LHS<0$ for D_t that satisfies $f(R_tD_t-\overline{D})>0$, i.e., $D_t>\beta\overline{D}$. Since (36) implies that $LHS=0$, it is a contradiction. Therefore, $\lambda(A_t)>0$ and $D_t=\hat{D}_t$.

Case 2. $D_t \leq \beta\overline{D}$**:** If $V'(R_tD_t) < -\int_{R_tD_t-\overline{D}}^{\infty} f(A)dA$, then we can prove that $\lambda(A_t)>0$ by the same logic as in Case 1. Therefore, the following claim is true.

Claim 1. If $\lambda(A_t)=0$*, then* $V'(R_tD_t)=-\int_{R_tD_t-\overline{D}}^{\infty} f(A_{t+1})dA_{t+1}$.

We prove $V'(R_tD_t) < -\int_{R_tD_t-\overline{D}}^{\infty} f(A)dA$ by contradiction. Suppose that $V'(R_tD_t) = -\int_{R_tD_t-\overline{D}}^{\infty} f(A_{t+1})dA_{t+1}$. (35) implies that it must hold that $\int_0^x \lambda(A_{t+1})dA_{t+1}=0$, where $x=R_tD_t$. Claim 1 with forwarding one period implies that $V'(R_{t+1}D_{t+1}) = -\int_{R_{t+1}D_{t+1}-\overline{D}}^{\infty} f(A_{t+2})dA_{t+2}$ for all $R_{t+1}D_{t+1}$ that corresponds to each realization of $A_{t+1}\in[0,R_tD_t]$. Then for each $R_{t+1}D_{t+1}$, it must hold that $\int_0^x \lambda(A_{t+2})dA_{t+2}=0$, where $x=R_{t+1}D_{t+1}$. Iterating this process, we have

$$\int_0^{R_{t+j}D_{t+j}} \lambda(A_{t+j+1})dA_{t+j+1}=0 \tag{38}$$

for all $j=0,1,\cdots$ and $A_{t+j+1}\in[0,R_{t+j}D_{t+j}]$, where $R_{t+j}D_{t+j}$ is attainable from $R_{t-1}D_{t-1}$. But (38) cannot hold, because if A_{t+i} $(0\leq i\leq j-1)$ is close to zero for sufficiently many consecutive periods then D_{t+j} must exceed $\beta\overline{D}$. Then from Case 1, $\lambda(A_{t+j+1})>0$. The probability, as of period t, of occurrence of the event that $D_{t+j}>\beta\overline{D}$ is strictly positive. Therefore, $\int_0^x \lambda(A_{t+j+1})dA_{t+j+1}>0$ with a strictly positive probability. This contradicts (38). Therefore, the initial assumption that $V'(R_tD_t) = -\int_{R_tD_t-\overline{D}}^{\infty} f(A_{t+1})dA_{t+1}$ does not hold. Thus, we have shown that $\lambda(A_t)>0$ for $D_t<\beta\overline{D}$. Combining Case 1 and Case 2, we have shown that $\lambda(A_t)>0$ for $D_t\leq\overline{D}$.

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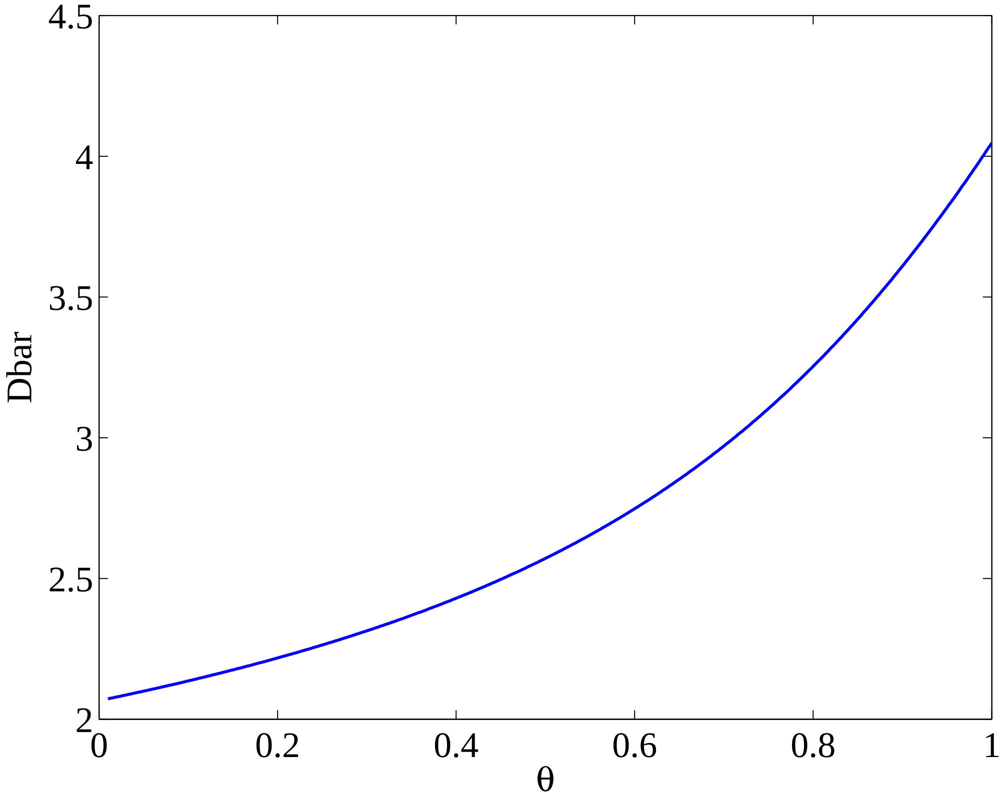

Figure 1: Debt capacity, $\overline{D}$, and depositors' bargaining power, θ ($\nu^2 = 1$).

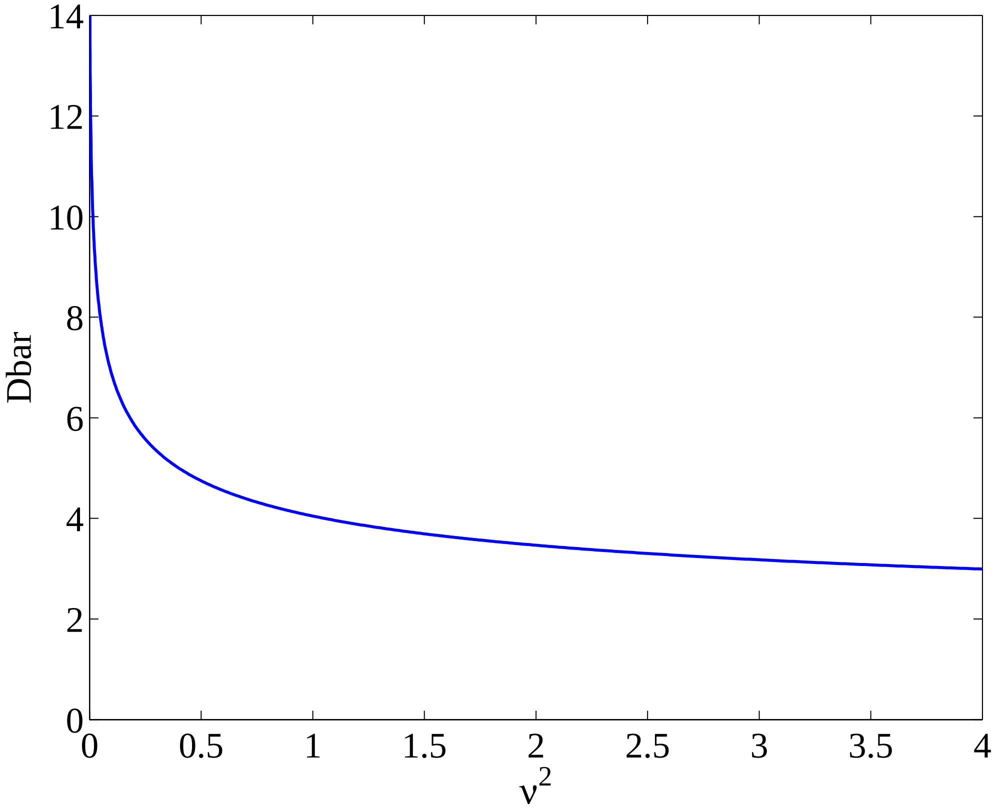

Figure 2: Debt capacity, $\overline{D}$, and variance, ν^2 ($\theta = 1$).

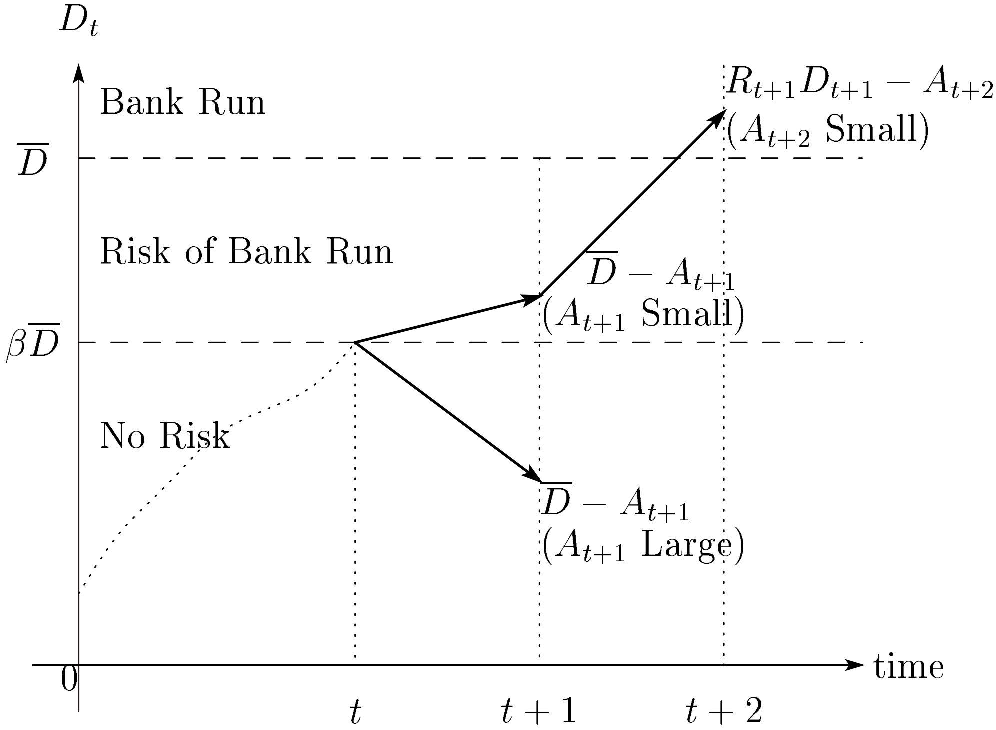

Figure 3: Illustration of Path of Bank Debt.

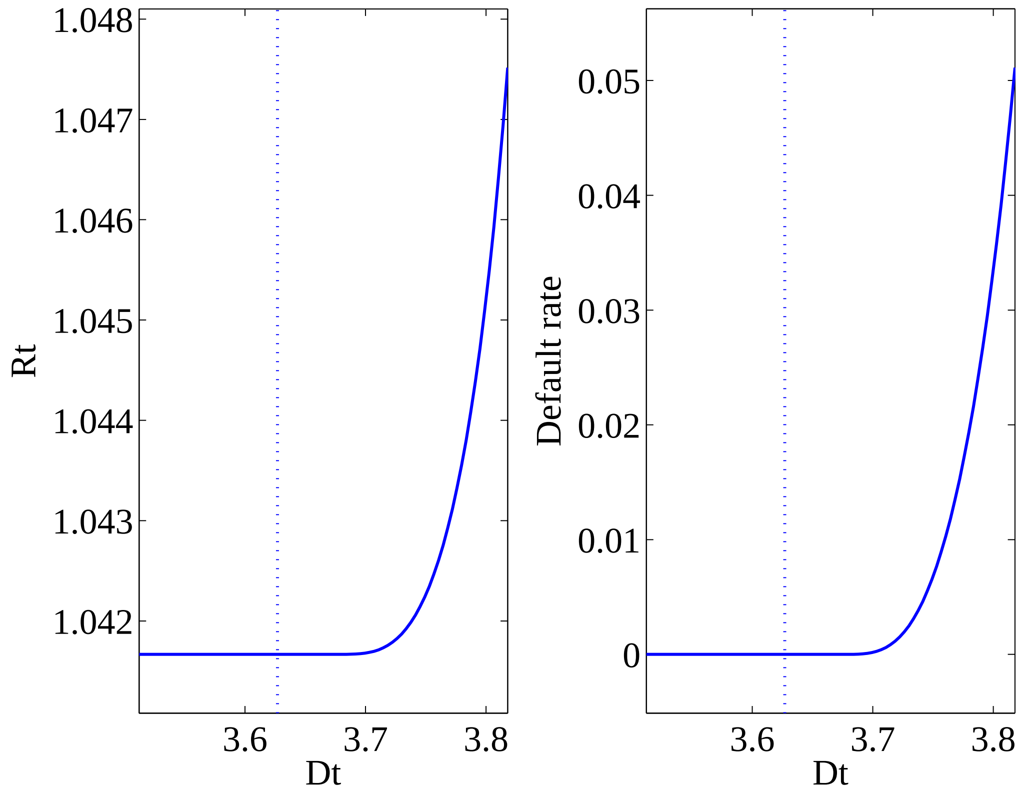

Figure 4: R_t and $Pr(D_t)$ in the case where $\theta = 0.95 < \beta$

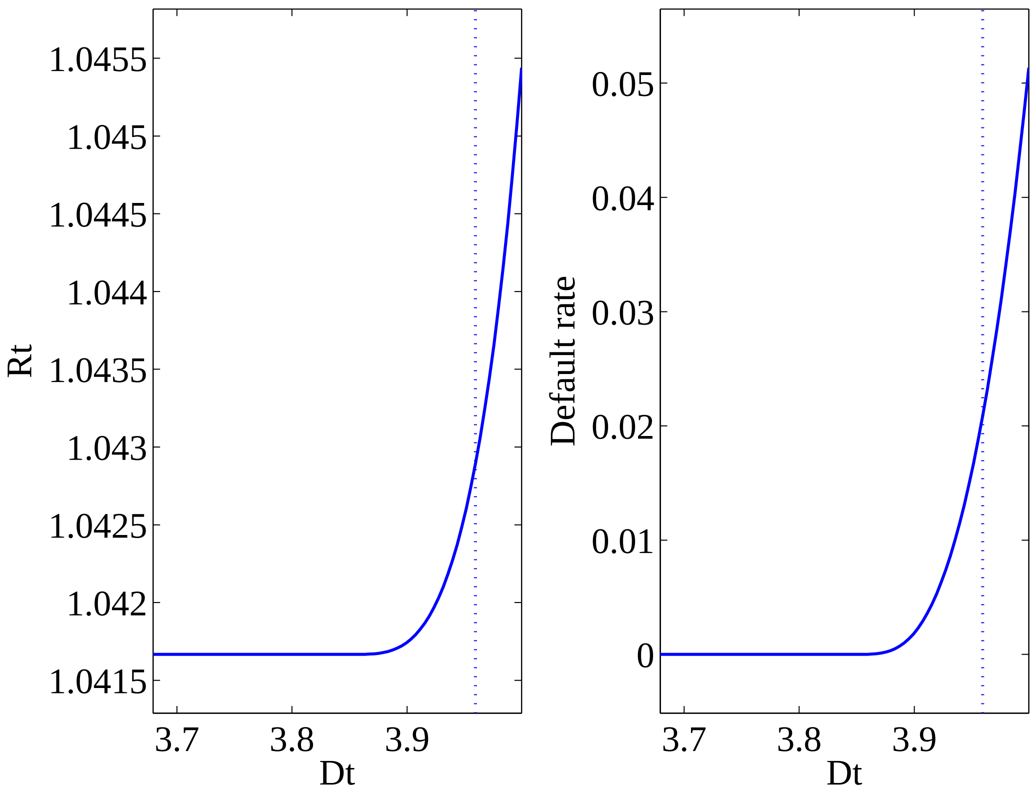

Figure 5: R_t and $Pr(D_t)$ in the case where $\beta < \theta = 0.99$

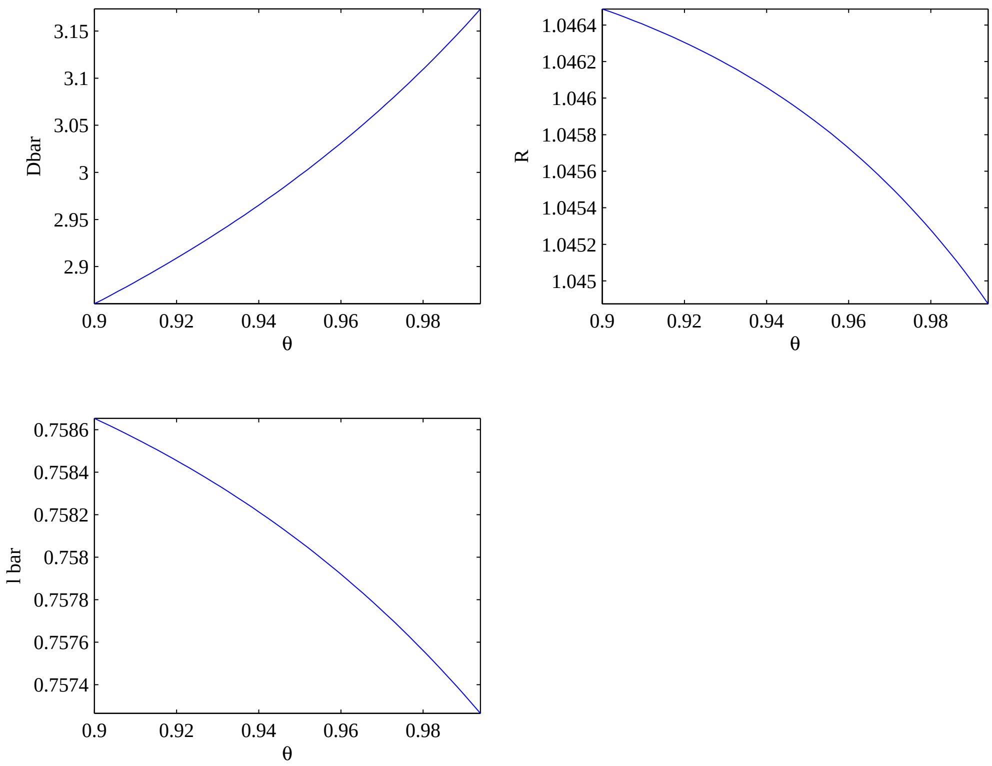

Figure 6: Debt capacity, $\overline{D}$, and bargaining power, θ ($\nu^2 = 0.01$).

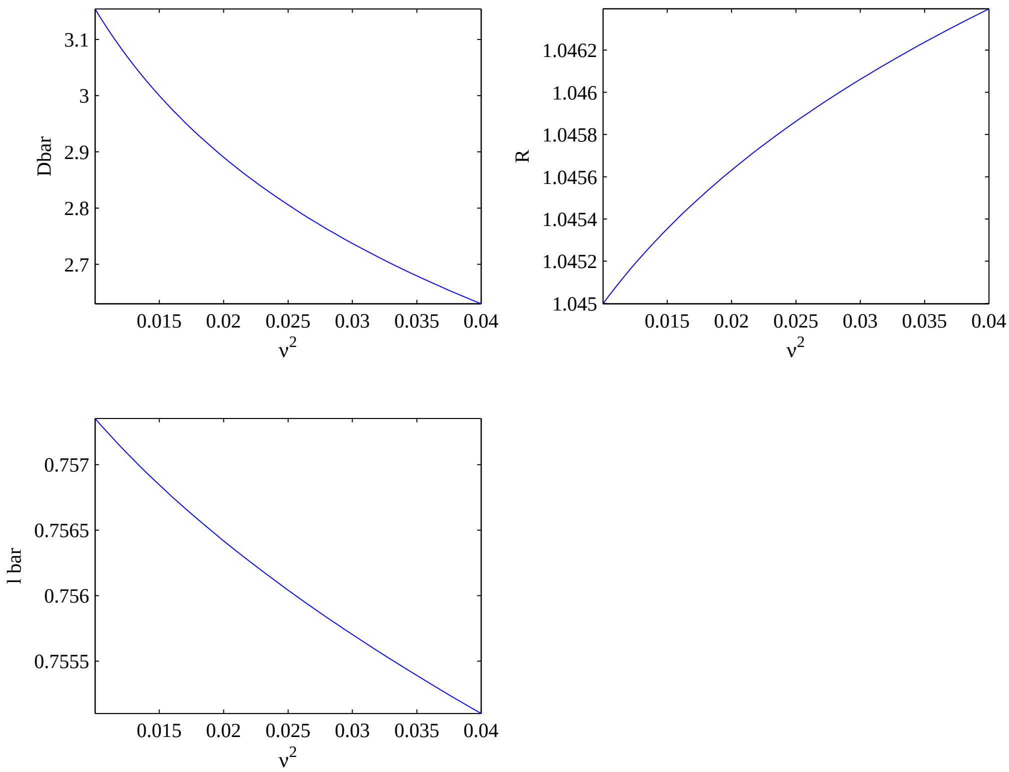

Figure 7: Debt capacity, $\overline{D}$, and variance of productivity, ν^2 ($\theta = 0.99$).

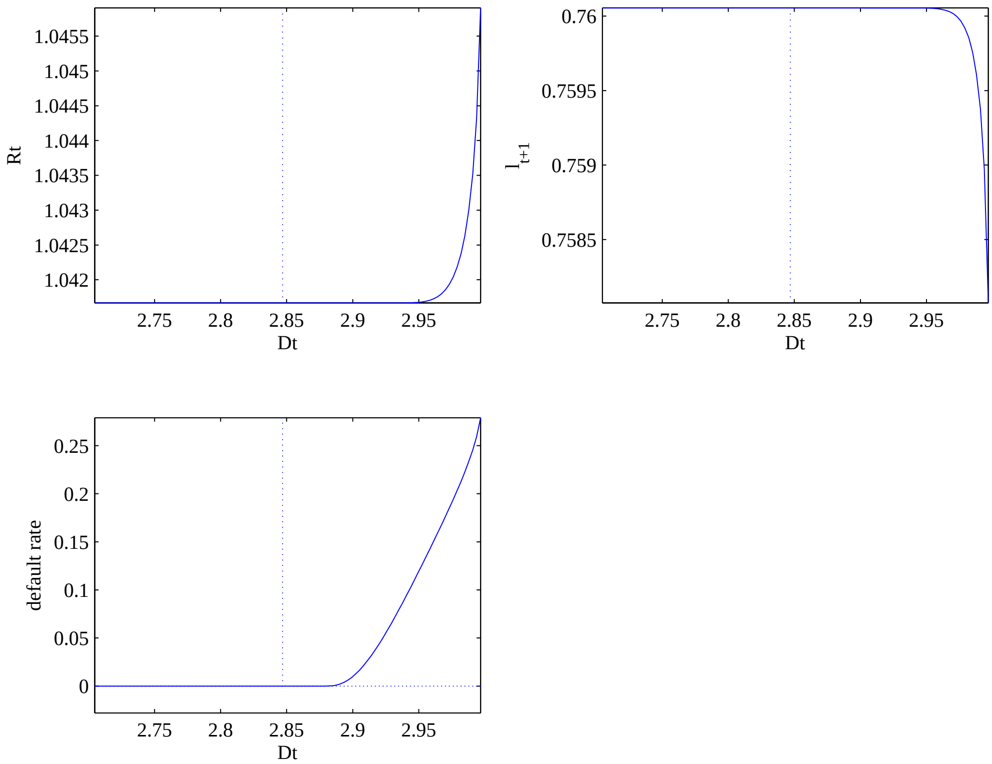

Figure 8: R_t, $Pr(D_t)$, l_{t+1} in the case where $\theta = 0.95 < \beta$.

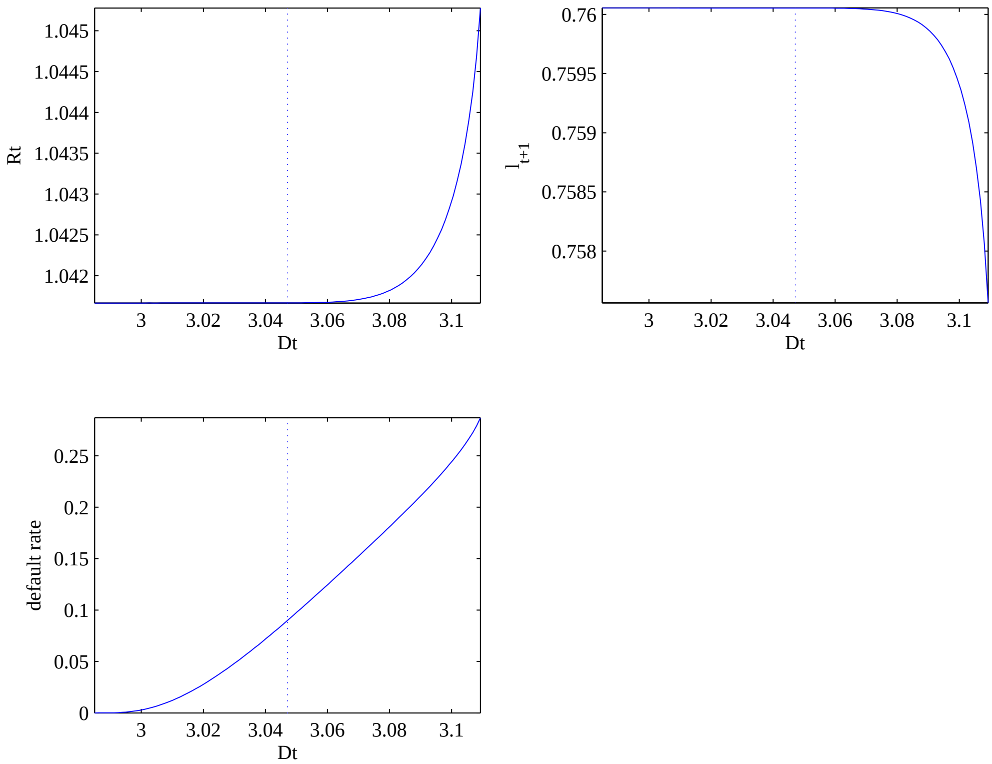

Figure 9: R_t, $Pr(D_t)$, l_{t+1} in the case where $\beta < \theta = 0.99$.