Does the New Keynesian Model Have A Uniqueness Problem?

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Introduction

- NK models have been enormously influential in terms of their policy implications.

- Models' implications for fiscal policy are particularly striking when ZLB is binding.

- Key results:
  - When ZLB binds, output fall is potentially very large.
  - The multiplier is larger when the ZLB binds than when it doesn't.
  - The more binding is the ZLB the larger is the drop in output and the larger is the multiplier.

- These results generated using linearized version of NK model, e.g. EW, CER.

Non-uniqueness and policy

- Non-linear NK models have multiple equilibria.
- Policy prescriptions can vary a lot across equilibria (Mertens and Ravn, Braun et. al. (2012), Cochrane (2015).
  - At some ZLB equilibria, multiplier is small or even negative.
  - At other ZLB equilibria, mutliplier very large.
- So, in principle non-uniqueness of equilibria poses an enormous challenge for policy analysis in NK models.

Is non-uniqueness a substantive problem?

- Yes, if there's no compelling way to select among different equilibria that give different answers to critical policy questions.

- Our argument starts from presumption that the assumption of RE *obviously* wrong.

- But it can be a useful modeling strategy for thinking about a world where RE isn't literally true.

  "... the model described above 'assumes' that agents know a great deal about the structure of the economy and perform some non-routine computations. It is in order to ask, then: will an economy with agents armed with 'sensible' rules-of-thumb, revising these rules from time to time so as to claim observed rents, tend as time passes to behave as described..." Lucas (1978)

Selecting among equilibria

- Suppose agents make a 'small' error in forming expectations about variables relative to their values in a particular REE.
  - Does economy converge to that REE ?
  - If yes, the RE equilibrium is *stable-under-learning*.

- In our view, learnability is a necessary condition for an REE to be empirically interesting.

- Non-learnable REE equilibria are best viewed as mathematical curiosities.

Non-linear Calvo model with binding ZLB

- Apply learnability criterion to standard fully non-linear NK model with Calvo pricing frictions.
- Unlike linearized NK models, ZLB REE can't be characterized by a set of numbers.
  - There's an endogenous state variable (past price dispersion), so ZLB REE is a set of *functions*.
  - Must think about how agents learn about these functions.

Key Results...

- There's multiple REE, including sunspot equilibria (Mertens and Ravn).
  - When we consider fundamental shocks that trigger ZLB episodes, we find two minimum state variable ZLB equilibria.
  - These equilibria converge to different inflation rates if the ZLB episode lasts forever.

- Impact of government consumption can be very different in the different ZLB equilibria.
  - For example, there are ZLB REE in which multiplier is negative.

Key Results...

- There exists a unique interior ZLB REE that's stable-under-learning.
  - That REE that converges to a relatively low ZLB deflation rate.
- Controversial predictions of linearized NK model about fiscal policy in the ZLB, are satisfied at unique learnable ZLB REE.
- We conclude Calvo model doesn't have a substantive uniqueness problem, at least for analysis of fiscal policy.

What about the Rotemberg model?

- Many authors used non-linear versions of the Rotemberg model to proxy for Calvo model.
  - Much easier to work with, no endogenous state variables in ZLB.
  - ZLB REE is a set of numbers (not functions).

- Linearized versions of these models are the same.

- But non-linear versions of the two models are potentially very different.

The Rotemberg model...

- Some properties of non-linear Rotemberg model are very sensitive to how you formulate adjustment costs for prices.
- Number of ZLB REE and their stability properties depend on whether and exactly how you scale adjustment costs for growth.
- But there always exists a unique ZLB REE that's stable-under-learning.
- At that equilibrium, impact of fiscal policy in the ZLB are same as those implied by log-linear NK model.

By-product: Linear Approximations

- Use non-linear Calvo model to assess robustness of log-linear approximations.
- Log–linear approximations work reasonably well for analysis of ZLB and fiscal policy.
- Evidence that quality of linear approximations is poor rests on examples where output deviates by more than 15 percent from its steady state.

The Neo-Fisherian View

- Conclude talk with remarks about neo-Fisherian view of monetary policy.
  - To acheive a high inflation rate, the monetary should target a high nominal interest rate.

- Theoretical foundation for that view collapses in the face of stability-under-learning criterion.

- Non-linear flexible price (BSGU) and NK models (Calvo, Rotemberg) have unique RE equilibrium that's stable-under-learning.
  - First result eliminates BSGU based arguments for neo Ficherian view
  - Second result eliminates Cochrane arguments based on NK model.

Standard NK model

- Household maximizes

$$E_0 \sum_{t=0}^{\infty} d_t \left[\log\left(C_t\right) - \frac{\chi}{2} h_t^2 \right]$$

$$d_t = \prod_{j=0}^{t} \left(\frac{1}{1 + r_{j-1}} \right)$$

- Discount factor r_t can take on two values: 'normal value' r and r^ℓ, where $r^\ell < 0$.
  - If $r_t = r^\ell$, it stays at that value with prob p.
  - Once you switch back to r, you stay there forever.

$$P_t C_t + B_t \leq \left(1 + R_{t-1}\right) B_{t-1} + W_t h_t + \Pi_t.$$

Model

- Final homogeneous good, Y_t, produced by competitive and identical firms:

$$Y_t = \left[\int_0^1 \left(Y_{j,t} \right)^{\frac{\varepsilon}{\varepsilon - 1}} dj \right]^{\frac{\varepsilon - 1}{\varepsilon}}, \ \varepsilon > 1.$$

- Input j produced by firm j using technology $Y_{j,t} = h_{j,t}$.
  - competitive in factor markets
  - monopolist in product market.

Model

- Monopolist j choosing $\tilde{P}_t$ to maximize

$$E_t \sum_{k=0}^{\infty} \beta^k \lambda_{t+k} \left((1+v) \tilde{P}_t - P_{t+k} s_{t+k} \right) Y_{j,t+k}$$

  - v is a subsidy that removes steady state distortions owing to monopoly power.

- Monopolist j sets price, $P_{j,t}$, subject to demand curve for its good and Calvo sticky price friction

$$P_{j,t} = \begin{cases} P_{j,t-1} & \text{with probability } \theta \\ \tilde{P}_t & \text{with probability } 1-\theta. \end{cases}$$

Model

- Aggregate output

$$Y_t = p_t^* h_t$$

- p_t^* is a measure of price dispersion

$$p_t^* = \left[(1-\theta) \left[\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta} \right]^{\frac{-\varepsilon}{1-\varepsilon}} + \theta\pi_t^{\varepsilon}(p_{t-1}^*)^{-1} \right]^{-1}.$$

- Aggregate resource constraint

$$C_t + G_t \leq Y_t.$$

- Monetary policy rule

$$R_t = \max\left\{1, 1 + r + \alpha\left(\pi_t - 1\right)\right\}$$

  - Max operator reflects ZLB and $\alpha > 1 + r$.

Solving the model

- Can reduce equilibrium conditions to four non-linear equations.

- There's an endogenous state variable, p_{t-1}^*, and an exogenous state variable, r_t.

- So a solution to the model is a set of *functions* which satisfy these conditions.

- Stage 1: solve for the equilibrium functions that obtain when $r_t = r$, i.e. after the economy has exited the ZLB.

$$Y(p_{t-1}^*), \pi(p_{t-1}^*), F(p_{t-1}^*), p^*(p_{t-1}^*)$$

- Stage 2: solve for equilibrium functions that obtain when r_t is equal to r_ℓ

$$Y_\ell(p_{t-1}^*), \pi_\ell(p_{t-1}^*), F_\ell(p_{t-1}^*) \text{ and } p_\ell^*(p_{t-1}^*)$$

ZLB REE Steady State

- Consider limit as $t \to \infty$ when the economy stays in the ZLB.

- p_t^* converges to a number, $\hat{p}$, for any interior equilibrium.

- System of equations collapses to a system of equations in four unknowns,
$$\pi_\ell(\hat{p}),\ Y_\ell(\hat{p}),\ p_\ell^*(\hat{p}), \text{and } F_\ell(\hat{p}).$$

Solving for steady state ZLB

- Equations defining an interior steady-state ZLB equilibrium collapse into one equation one unknown

$$f(\pi_\ell) = 0.$$

- In a slight abuse of notation we drop explicit dependence of π_ℓ on $\hat{p}$.

- A *necessary* condition for ZLB REE equilibrium to be unique
  - There's a unique solution to this equation.

Parameterizing the model

- Benchmark values:

$$\begin{aligned} \varepsilon &= 7.0,\ \beta = 0.99,\ \alpha = 2.0,\ p = 0.75, \\ r^{\ell} &= -0.02/4,\ \theta = 0.85,\ \eta_g = 0.2. \end{aligned}$$

- Steady state output is normalized to 1 by setting $\chi = 1.25$.

- Sensitivity analysis in appendix.

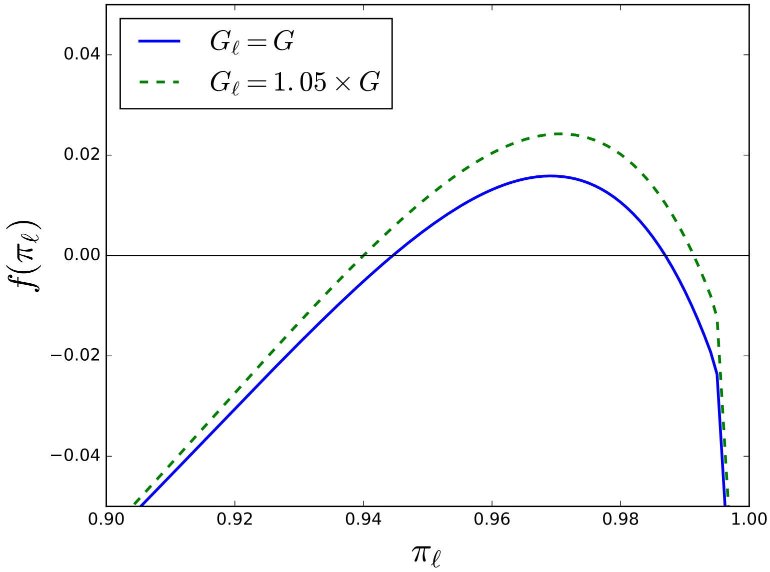
ZLB Steady States

- The function $f(\pi_\ell)$ has inverted U shape so there's either two interior steady-state ZLB equilibria or none.

- Benchmark case: two steady-state ZLB REE equilibria
  - 'high' and 'low' inflation.

- Number of minimum state variable ZLB REE equilibria coincides with number of steady state ZLB REE equilibria.
  - A numerical result, not a theorem.

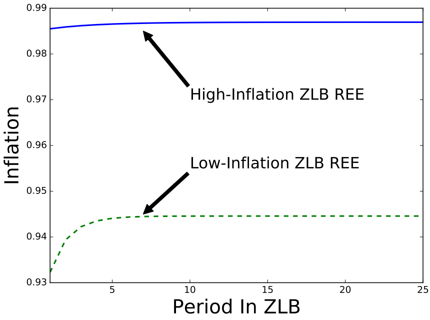
ZLB REEE Steady States

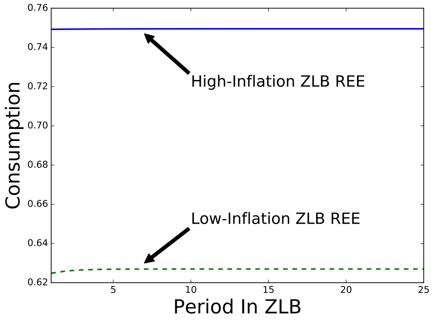


Dynamic Response Functions

- Unlike Rotemberg, ZLB REE isn't a number because of endogenous state variable, p_{t-1}^*.
- Consider dynamic response of π_t and C_t to r_t shock when economy converges to high and low π steady-state ZLB REE.
- Refer to these paths as: high and low inflation ZLB REE.

Dynamic Response Functions





Dynamic Response Functions

- Along high π ZLB REE path,
  - Quarterly π and C initially drop by 1.5 and -6.35 percentage points, respectively.
  - After about 5 quarters π and C declines stabilize at -1.3 and -6.3 percentage points.

- Along low π ZLB REE path, quarterly π and C initially drop by -7.25 and -23.5 percentage points.
  - After about 5 quarters π and C declines stabilize at -6.0 and -23.3 percentage points.

The Multiplier

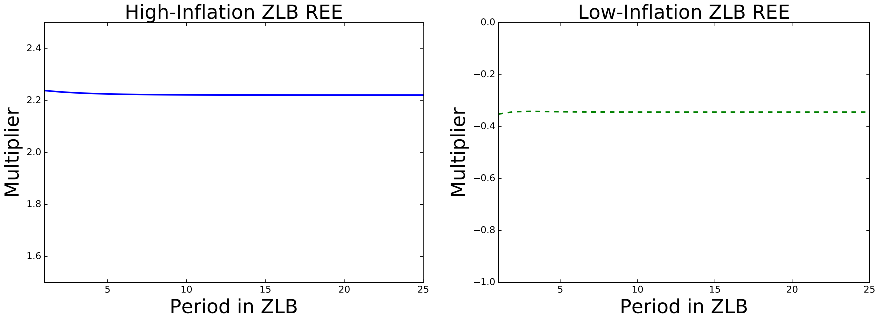
- $G^\ell = 1.05 \times G^h$, i.e. when economy is in ZLB, G rises by 1 percentage of steady state output.

- Multiplier:

$$\frac{G^\ell}{\left(C^\ell(p^*_{t-1}) + G^\ell\right)} \frac{\Delta\left(C^\ell(p^*_{t-1}) + G^\ell\right)}{\Delta G^\ell}.$$

- If economy is in high π (low π) ZLB REE for low value of G, it's in high π (low π) ZLB REE for low value of G.
  - A non-trivial assumption.

The Multiplier



Comparisons

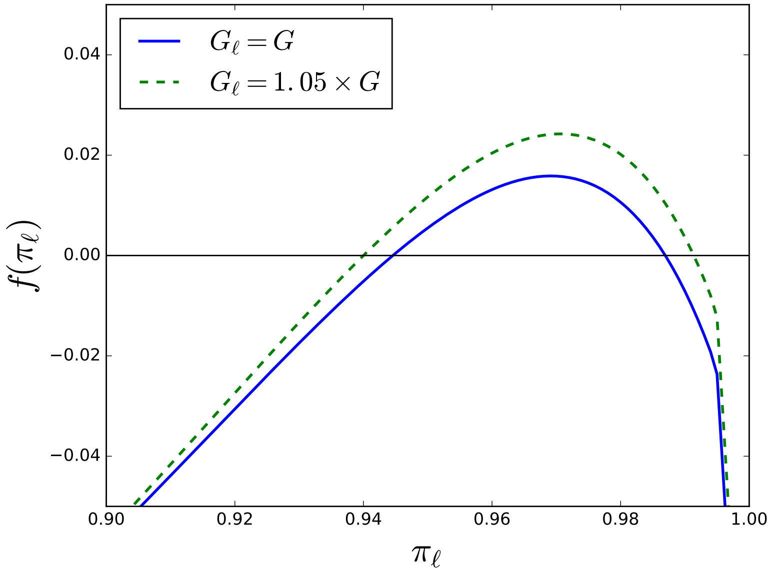
- Multiplier in high-π ZLB REE is large, exceeding two over the time period displayed.

- Multiplier is *negative* in low-π ZLB REE.

- To understand this result, note that an increase in G^{ℓ} shifts $f(\pi_{\ell})$ upwards.
  - So effect of increase in G depends on which equilibrium we focus on.

- Dramatic illustration of basic result in Mertens and Raven (2011) where multiplier in one REE is a lot smaller than in the other REE.

ZLB REEE Steady States



Comparison to linearized model

Impact period of shock

- Responses of linearized model similar tothose in high-π ZLB REE.
- Properties of low-π ZLB REE are very different.

Model	Output	Inflation	Multiplier
Linear	-2.18	-0.0066	1.63
Nonlinear, high-inflation	-2.84	-0.0093	2.24
Nonlinear, low-inflation	-17.87	-0.0734	-0.35

Comparison to linearized model

- Basic qualitative results reported in CER using log-linear approximation hold up when we focus on high-π ZLB REE.
- Multiplier can be much bigger than 1 when ZLB binds.
- When duration of ZLB increases or degree of flexibility of prices increases,
  - Severity of output collapse and multiplier are larger.
- One interesting difference:
  - For parameter values that imply linear multipliers explode, REE ceases to exist in non-linear model.

Stability-under-learning

- In Calvo model, firms choose $P_{j,t}$, based in part on value of P_t.
- But, P_t is a function of firms' collective price decisions.
- Firms can't 'know' P_t when they choose their own price, in sense of actually observing it.
- Standard assumption: firms form a 'belief' about P_t when they make their decision.
- In REE that belief is correct.

Stability-under-learning

- If firms don't have rational expectations, it's not natural to assume they see P_t when they choose their prices.
- But if they don't see P_t, they also don't see C_t.
- Must attribute to firms views about equilibrium functions for current and future aggregate π and C.

Stability-under-learning

- $x_\ell^{e,f}(p_{t-1}^*, t-1)$: firm's belief, formed using information up to time $t-1$, about equilibrium *function* for x_ℓ.

- To make time t decision, firms must forecast values of future variables as p_t^* evolves.
  - FONC's involve objects like $x_\ell^{e,f}(p_{t-1}^*, t-1)$ and $x_\ell^{e,f}(p_{t+j}^*, t-1)$ for $j \geq 0$.
  - So firms must have views about the entire function.

- All these functions have time $t-1$ as argument
  - Reflects our assumption that firms think they're in stationary environment.

Stability-under-learning

- Firms' beliefs evolve according to

$$x_\ell^{e,f}(p_t^*,t) = \omega x_\ell(p_{t-1}^*,t-1) + (1-\omega)x_\ell^{e,f}(p_{t-1}^*,t-1).$$

- For $\omega > 0$, this formulation embodies the heroic assumption that agents know time $t-1$ equilibrium function for x_ℓ.
  - Paper reports sensitivity analysis to simpler rules.

- In Rotemberg model, there's no state variables in ZLB.
  - Replace above rule with assumption that agents' expectations evolve according to simple constant gain algorithm about the values of variables.

Stability-under-learning in the NK model

- When households make their time t consumption decisions, firms' actions have already determined π_t.

- So households can compute the time t equilibrium function for π (again heroic!).

- $\pi_\ell^{e,h}(p^*_{t-1}, t)$: households' belief, at time t, about equilibrium function for π_ℓ.

- Given new information, households beliefs evolve according to

$$\pi_\ell^{e,h}(p^*_t, t+1) = \omega \pi_\ell(p^*_{t-1}, t) + (1-\omega)\pi_\ell^{e,h}(p^*_{t-1}, t).$$

A learning ZLB equilibrium

- Assume agents know REE functions when economy isn't in ZLB.

- A sequence of functions for all of endogenous variables that satisfy
  - Resource constraint,
  - Monetary policy rule,
  - Household and firm optimality conditions for all t,
  - given initial set of beliefs $\pi_{\ell}^{e,f}(\cdot, 0)$, $C_{\ell}^{e,f}(\cdot, 0)$, and $\pi_{\ell}^{e,h}(\cdot, 0)$ that evolve according to above rules.

Stability-under-learning

- A ZLB REE is *stable-under learning* if a learning equilibrium with initial beliefs close to, but not equal to, the REE functions, converges back to the ZLB REE equilibrium.

- If an economy stays in ZLB forever, it will converge to a steady state ZLB REE.

- Learning equilibrium must also approach steady state ZLB REE if initial ZLB REE is stable-under-learning.

- Allows us to eliminate *all* of ZLB REE that lead to low-π steady state ZLB REE.
  - They're not stable-under-learning.

Stability-under-learning

- Consider a firm that believes that
  - Steady state inflation rate is $\pi_\ell^{e,f}$.
  - Economy is in steady state corresponding to that rate of inflation.

- The belief $\pi_\ell^{e,f}$ isn't an REE belief so the steady state associated with it (including p_{t-1}^*) isn't a steady state ZLB REE.

- So $f(\pi_\ell^{e,f})$ isn't equal to zero.

Stability-under-learning

- There's an equivalence between a belief $\pi_\ell^{e,f}$ and a value of $\tilde{p}_\ell^{e,f}$ $= \frac{\tilde{P}_t^{e,f}}{P_{t-1}}$ that will be chosen by firms who can update their price.

- So we use function $f(\pi_\ell^{e,f})$ to define a new function

$$\tilde{f}\left(\tilde{p}_\ell^{e}\right)$$

that must be equal to zero at a steady state ZLB REE.

- Write firms' FONC for $\tilde{p}_t = \tilde{P}_t / P_{t-1}$ can be written, after imposing all of equilibrium conditions, as

$$\tilde{F}\left(\tilde{p}_t, \tilde{p}_\ell^{e,f}\right) = 0.$$

Stability-under-learning

- Define the best-response function

$$\tilde{p}_t = g(\tilde{p}_\ell^{e,f}).$$

- This function has the property that,

$$\tilde{F}\left(g(\tilde{p}_\ell^{e,f}), \tilde{p}_\ell^{e,f}\right) = 0.$$

- In a steady state RE ZLB equilibrium

$$\tilde{p}_t = \tilde{p}_\ell^{e,f}$$

Stability-under-learning

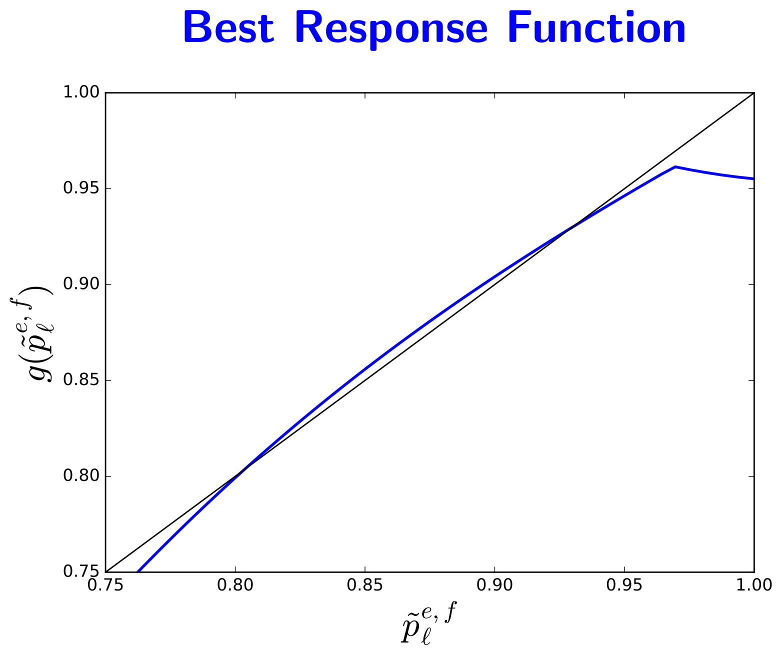
- Following figure plots typical firm's best response function, i.e. $\tilde{p}_t$ as a function of $\tilde{p}_\ell^{e,f}$.

- Steady state ZLB REE equilibria correspond to the two points where best response function intersects 45 degree line.

- Given any belief, $\tilde{p}_\ell^{e,f}$, between RE steady state beliefs, best response $g(\tilde{p}_\ell^{e,f})$ is *greater* than $\tilde{p}_\ell^{e,f}$.

- Follows that realized π will exceed beliefs about π.

- So learning equilibrium will move towards high-π ZLB REE steady state.


Best Response Function
1.00
0.95
0.90
0.85
0.80
0.75
0.75
0.80
0.85
0.90
0.95
1.00
$g(\tilde{p}_\ell^{e,f})$
$\tilde{p}_\ell^{e,f}$

Best Response Function

- Now consider any belief, $\tilde{p}_{\ell}^{e,f}$, that exceeds high-π REE ZLB steady state.
  - Best response function $g(\tilde{p}_{\ell}^{e,f})$ is *less* than $\tilde{p}_{\ell}^{e,f}$.
  - So realized π will be lower than beliefs about π.
  - Learning equilibrium will move towards high-π ZLB REE steady state.

- Finally, consider any belief, $\tilde{p}_{\ell}^{e,f}$, that's less than low-π ZLB REE ZLB steady state.
  - Here best response function $g(\tilde{p}_{\ell}^{e,f})$ is *less* than $\tilde{p}_{\ell}^{e,f}$.
  - So realized π will be lower than beliefs about inflation.
  - Learning equilibrium will move away from the low-π ZLB REE steady state.

Stability-under-learning

- Previous discussion focused on limiting point of ZLB REE.
- To be stable-under-learning, functions defining a learning equilibrium must converge point-wise to functions defining a ZLB REE for every possible for p_t^*, including steady state value of p_ℓ^*.
- Just showed that any ZLB REE that converges to low-π steady state ZLB REE doesn't satisfy this condition.
- So those equilibria aren't stable-under-learning.

Stability-under-learning

- Previous discussion doesn't establish that a ZLB REE that converges to high-π steady state REE ZLB is stable-under-learning.

- Our solution algorithm parameterizes ZLB REE functions with a finite number of parameters, z_t.

- Learning algorithm defines a mapping from current values of those parameters to next period's values:

$$z_{t+1} = s\left(z_t\right).$$

- Define

$$S(\tilde{z}) = \left[\frac{ds_i\left(z\right)}{dz_j}\right]|_{\tilde{z}},$$

for all $i, j < N$ where N is number of parameters.

Stability-under-learning

- Evaluate S for the parameters of the high-π REE.
- Max eigenvalue is less than one in absolute value.
  - So, locally, functions in neighborhood of high-π ZLB REE will converge to those REE functions in a learning equilibrium.
- Repeat analysis for parameters of the low-π ZLB REE.
  - Maximum eigenvalue is greater than one in absolute value.
  - So, locally, functions in neighborhood of low-π ZLB REE, equilibrium will diverge from those REE functions in a learning equilibrium.

Learning equilibria

Dynamic Paths

- At $t = -1$ economy is in high- π ZLB REE steady state where $p^*_{-1} = p_\ell$.

- At time 0,

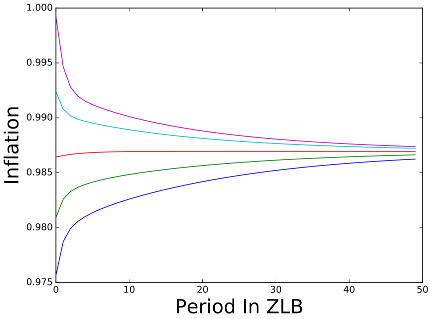
$$x_\ell^{e,f}(p^*_{-1}, -1) = x_\ell(p^*_{-1}) + \bar{x}_\ell,\ \bar{x}_\ell > 0.$$

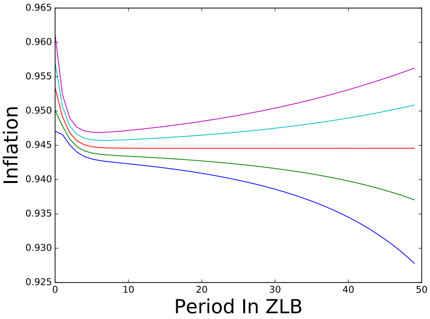
- Agents think that if ZLB ends, economy will be in REE that converges to high-π steady state.
  - Also assume parameter $\omega = 1$ (tomorrow will be like today).

- $\bar{x}_\ell =$ (-.02,-.01, 0, 0.01, 0.02).

Learning equilibria

Red line: REE value in high inflation ZLB REE





Learning equilibria

- Regardless of value of $\bar{x}_\ell$, π converges to high$-\pi$ ZLB REE.
- Establishes that learning equilibrium converges to equilibrium function defining an REE when evaluated at the p_ℓ^*.
- Second panel: begin from the low-π ZLB REE.
- Inflation diverges from that equilibrium in the learning equilibrium.

Learning equilibria

- Till now we've assumed that agents belief that once ZLB is over, economy will go to REE that converges to the high-π steady state.

- Redo analysis assuming agents think that post-ZLB, economy will go to REE that converges to low-π steady state.

- All of qualitative results hold, e.g.REE that converge to low π steady state are not stable-under-learning.

Fiscal policy in learning equilibria

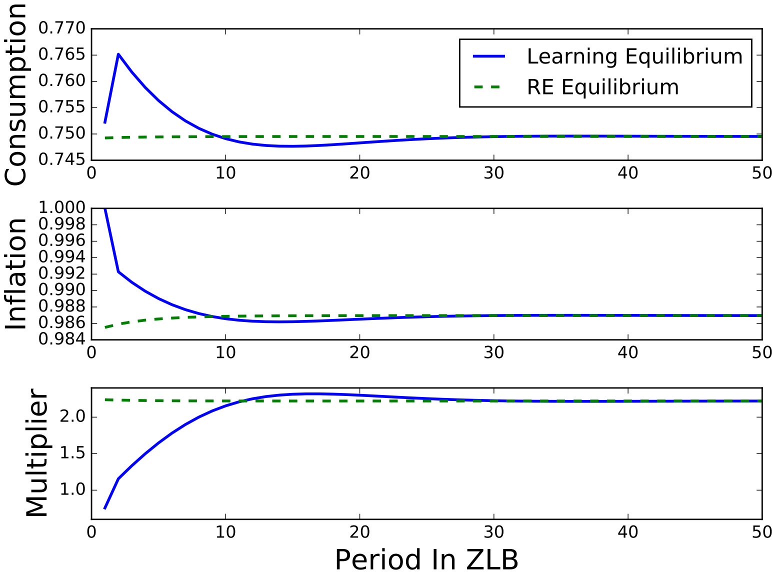
- Initially assume that agents think that when ZLB episode is over, economy goes to REE that converges to high-π steady state.

- Economy begins in high-π steady state.

- At time 0, $p^*_{-1} = 1$, r falls to r_ℓ.

- Firms obey learning laws discussed above.

Fiscal policy in learning equilibria



Fiscal policy in learning equilibria

- C and and π converge to high π ZLB REE from above.
- Reason that they initially take on higher values is that initial expectations about higher future π and C spur demand now.
- As expectations adjust downward with realized π and C, they push C and π down further.
- Multiplier starts out low because ZLB isn't binding in first few periods.
- Once ZLB starts to bind, the multiplier quickly rises above 1.

Alternative experiment

- After r_t shock, firms and household have beliefs near the low-π ZLB REE.

- C and π converge to high π ZLB REE from below.

- Reason that they initially take on lower values is that expectations about low future inflation and consumption depress demand in the present.

Alternative experiment

- Multiplier starts out around 1 and then rises after that.
- Multiplier rises because fiscal expansion helps quickly move expectations toward those associated with the high-π ZLB REE.
  - Without change in G , expectations remain close to low-inflation ZLB REE for some time.
- After many periods, the multiplier eventually approaches the high-inflation steady state ZLB REEvalue.

Mertens and Ravn (2015)

- Report multiplier is *small* when they analyze a learning equilibrium near the low inflation steady state RE ZLB equilibrium.

- This result is very different than ours - we just argued that the relevant multiplier is *large*.

- Why?

Mertens and Ravn (2015)

- When we calculate the multiplier we initially consider an economy in which agents initial expectations about inflation differ by ϵ_π from the low π steady state ZLB REE.

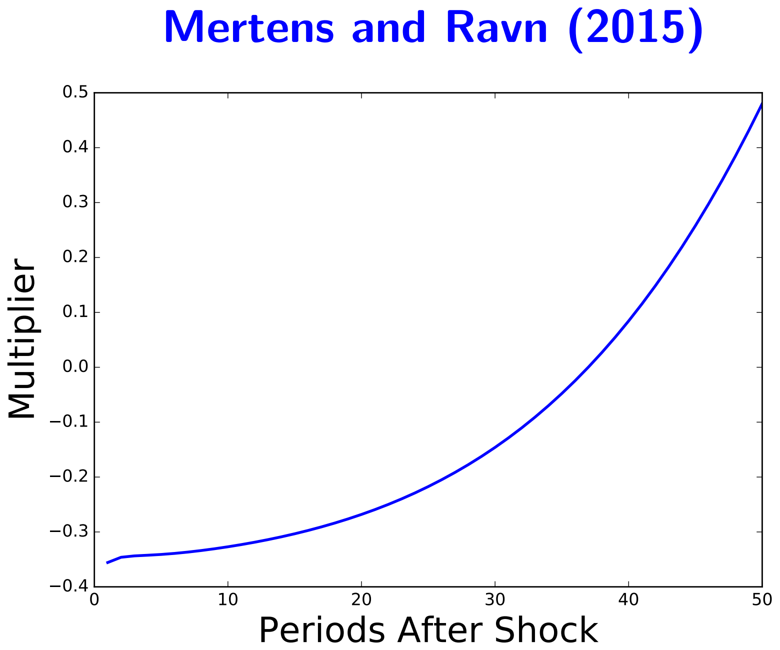
- We then consider a separate economy with shocks that set r to r_ℓ and a shock to G.

- Expectations start in the same place for the two economies.

- We then use difference in output between the two economies to calculate the multiplier.

Mertens and Ravn (2015)

- In their experiment when G increases, the rate of inflation in the steady state of the ZLB REE falls by ϵ'_{π}.
- When they raise G in the learning equilibrium, they also decrease agents' expectations about inflation by ϵ'_{π}.
  - In and of itself this fall in inflation reduces output in the ZLB.
- Next figure displays multiplier if we adopt their assumption.
- We obtain a *negative* multiplier that persists for roughly 10 years.
- Change in expectations is quantitatively much more important than the increase in G.


Mertens and Ravn (2015)
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
Multiplier
0
10
20
30
40
50
Periods After Shock

Rotemberg model

- Scaling term of price-adjustment costs can have large effect on properties of the equilibria that we find that aren't stable-under-learning.

$$\Phi_{t+k}\left(\frac{P_{j,t+k}}{P_{j,t+k-1}}-1\right)^2$$

- But there's always a unique stable under learning equilibrium in our examples.

Adj. Cost	Stable Equilibrium	Unstable Equilibrium
$\Phi_t = \frac{\phi}{2}$	1.56	0.98
$\Phi_t = \frac{\phi}{2}(C_t + G_t)$	1.70	0.36
$\Phi_t = \frac{\phi}{2} Y_t$	1.65	1.07

Concluding Remarks

- Non-uniqueness of equilibria in NK models does not pose a substantive challenge to key conclusions about the efficacy of fiscal policy in ZLB episodes.

- A close derivative of our analysis is that the 'neo-Fisherian' views of monetary policy based on NK models of flexible price model like BSGU are not empirically relevant.