

The Home Market Effect and Patterns of Trade Between Rich and Poor Countries

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Introduction

- Empirically rich (poor) countries tend to export high (low) income elastic products
 - Standard trade models assume *homothetic preferences* to focus on the supply side determinants of the patterns of trade
 - Just adding *nonhomothetic preferences* in the standard models would, *ceteris paribus*, make rich countries *import* high income elastic goods
 - *Virtually* all models of trade with nonhomothetic preferences *assume* that the rich have CA in high income elastic goods.
 - ✓ **Ricardian:** Flam-Helpman (1987), Stokey (1991), Matsuyama (2000), Fielser (2011)
 - ✓ **Factor endowment:** Markusen (1986), Caron-Fally-Markusen (2014)
- These models suggest that the rich export high income elastic goods *despite* they demand relatively more of them.
- Here, we *explain why* the rich have CA in high income elastic goods based on *Home Market Effect*, suggesting that the rich export high income elastic goods *because* they demand relatively more of them.

Home Market Effect (HME): Krugman's (1980) example

- Two Dixit-Stiglitz monopolistic competitive sectors, α & β , with iceberg trade costs
- One factor of production (labor)
- Two countries of equal size, A & B, *mirror-images* of each other
 - A is a nation of α -lovers; with the minority of β -lovers.
 - B is a nation of β -lovers, with the minority of α -lovers.

In equilibrium,

- **Under autarky**, proportionately large share of firms in A operates in sector α .
- **Under trade**, disproportionately large share of firms in A operates in sector α .
- A becomes a net-exporter in α ; B a net exporter in β .

Key Insight: With scale economies and positive but finite trade costs, a relatively larger domestic market is a source of comparative advantage.

Notes: In Krugman (1980),

- Demand composition differs across countries due to *exogenous variations in taste*
- The mirror image setup obscures crucial factors of HME. Also restricts comparative static exercises

This Paper: Krugman-type HME model with demand composition difference due to nonhomothetic preferences. Also dispenses with the mirror-images setup.

- *Continuum* of Dixit-Stiglitz monopolistic competitive sectors with iceberg trade costs
- Two countries; may differ only in *per capita labor endowment* and *population size*.
- Preferences across sectors: ***Implicitly Additively Separable Nonhomothetic CES***
 - Sectors indexed such that their income elasticity is increasing in the index.
 - The Rich has relatively larger domestic market than the Poor in the higher indexed

Under Trade Equilibrium, HME implies

- The Rich's share of firms are disproportionately larger in higher-indexed sectors
- The Rich run trade surpluses (deficits) in higher (lower)-indexed sectors.

Comparative Statics: *Due to endogenous demand compositions*, uniform productivity improvement and a trade cost reduction cause

- *Product cycles:* The Rich switches from a net exporter to a net importer in the middle
- *Welfare gaps to widen (narrow)*, when different sectors produce substitutes (complements)
- When two countries differ in size, a trade cost reduction has additional effects due to the ToT change; *Leapfrogging* and *Reversal of the patterns of trade*

Explicitly vs. Implicitly Additive Separability: Hanoch (1975)

Explicit Additivity: $u = \int_0^1 f_s(c_s) ds;$ CES if $u = \int_0^1 \omega_s(c_s)^{1-1/\eta} ds$

Pigou's Law: $\frac{\text{Income Elasticity of Good } s}{\text{Price Elasticity of Good } s} = \text{constant}$

Two Problems:

- i) Empirically false (Deaton 1974 and others)
- ii) Conceptually impossible to disentangle the effects of income elasticity differences from those of price elasticity differences

Implicit Additivity: $\int_0^1 f_s(u, c_s) ds = 1;$ CES if $\int_0^1 \omega_s(u)(c_s)^{1-1/\eta} ds = 1$

- i) Price elasticities & income elasticities can be separate parameters.
- ii) *Nonhomothetic CES* if $\frac{\partial \log \omega_s(u)}{\partial u}$ varies with s . When we can index s to make it monotone increasing in s , $\frac{\partial^2 \log \omega_s(u)}{\partial s \partial u} > 0$, *log-supermodularity*

Fajgelbaum-Grossman-Helpman (2011); FGH

- A monopolistic competitive sector producing indivisible products with trade costs, with two segments, H&L, across which products are *vertically* differentiated.
- A competitive outside sector producing the divisible numeraire to pin down the ToT
- Each household consumes one unit of a particular product from either H or L.
 - A *discrete choice model* a la McFadden, a *nested-logit demand structure*
 - The rich consumers more likely to choose an H-product if marginal utility of the numeraire is higher when combined with an H-product
- The Rich (Poor) becomes a net-exporter of high-quality H (low-quality L) products.

FGH focuses on specialization along the quality dimension within a single industry. Our model focuses on specialization across a broader range of industries.

Some Advantages of Our Framework

- A minimum departure from the standard HME models
- Parsimonious and yet flexible
 - Comparative statics with any number of sectors and the ToT effect
 - Income elasticities are separate parameters from price elasticities
 - Different sectors may produce substitutes, as in Flam-Helpman (1987), Stokey (1991), and FGH (2011), or complements, as in Matsuyama (2000)

Organization of the Paper

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Home Market Effect with Nonhomothetic Preferences

One Nontradeable Factor (Labor)

Two Countries: (j or $k = 1$ or 2)

N^j identical households with labor endowment h^j , supplied inelastically at w^j .

- $w^j h^j = E^j$: Household Income (and Expenditure)
- $L^j = h^j N^j$; Total Labor Supply in j

N^j and h^j are the only possible sources of heterogeneity across the two countries.

Tradeable Goods:

- A continuum of monopolistically competitive sectors, $s \in [0,1]$,
- Each sector produces a continuum of tradable differentiated goods, $v \in \Omega_s = \Omega_s^1 + \Omega_s^2$,

Ω_s^j : Disjoint sets of differentiated goods in sector s produced in country j in equilibrium

Household Preferences: Two-Tier structure

Lower-level, usual Dixit-Stiglitz aggregator (Homothetic within each sector)

$$\tilde{C}_s^k \equiv \left[\int_{\Omega_s} (c_s^k(v))^{1-\frac{1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}} ; \sigma > 1, s \in [0,1]$$

Upper-level, $\tilde{U}^k = U(\tilde{C}_s^k, s \in [0,1])$, *implicitly* given by

$$\int_0^1 (\beta_s)^{\frac{1}{\eta}} (\tilde{U}^k)^{\frac{\varepsilon(s)-\eta}{\eta}} (\tilde{C}_s^k)^{\frac{\eta-1}{\eta}} ds \equiv 1; \beta_s > 0 \text{ and } \sigma > \eta \neq 1$$

- $(\varepsilon(s) - \eta)/(1 - \eta) > 0$ for global monotonicity & quasi-concavity
- $\int_0^1 \varepsilon(s) ds = 1$, without loss of generality.
- If $\varepsilon(s) = 1$ for all $s \in [0,1]$, standard homothetic CES
- If $\varepsilon(s) \neq 1$, *nonhomothetic*. Index sectors so that $\varepsilon(s)$ is *increasing* in $s \in [0,1]$. Then,

$$\omega(s, \tilde{U}^k) \equiv (\beta_s)^{\frac{1}{\eta}} (\tilde{U}^k)^{\frac{\varepsilon(s)-\eta}{\eta}} \text{ is } \textit{log-supermodular} \text{ in } s \text{ and } \tilde{U}^k.$$

Lemma 1: For a positive value function, $\hat{g}(\bullet; x): [0,1] \rightarrow \mathbb{R}_+$, with a parameter x , define

$$g(s; x) \equiv \frac{\hat{g}(s; x)}{\int_0^1 \hat{g}(t; x) dt} \text{ (a density function) and } G(s; x) \equiv \int_0^s g(t; x) dt = \frac{\int_0^s \hat{g}(t; x) dt}{\int_0^1 \hat{g}(t; x) dt} \text{ (its}$$

cumulative distribution function).

If $\hat{g}(s; x)$ is *log-supermodular* in s and x , i.e. $\frac{\partial^2 \log \hat{g}(s; x)}{\partial s \partial x} > 0$,

- i) $\frac{g(s; x)}{g(s; x')}$ is decreasing in s for $x < x'$; **Monotone Likelihood Ratio (MLR)**
- ii) $G(s; x) > G(s; x')$ for $x < x'$. **First-Order Stochastic Dominance (FSD)**

The happier households put more weights on the higher-indexed goods.

Household Maximization: Two-Stage Budgeting

1st Stage (Lower-level) Problem: Chooses $c_s^k(v)$ for $v \in \Omega_s$ to:

$$\text{Max } \tilde{C}_s^k \equiv \left[\int_{\Omega_s} (c_s^k(v))^{1-\frac{1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}}, \text{ subject to } \int_{\Omega_s} p_s^k(v) c_s^k(v) dv \leq E_s^k,$$

$p_s^k(v)$ & $c_s^k(v)$: the unit consumer price and consumption of variety $v \in \Omega_s$;

E_s^k : Expenditure allocated to sector-s, taken as given.

Solution:

$$c_s^k(v) = \left(\frac{p_s^k(v)}{P_s^k} \right)^{-\sigma} C_s^k = \frac{(p_s^k(v))^{-\sigma}}{(P_s^k)^{1-\sigma}} E_s^k, \text{ where } P_s^k \equiv \left[\int_{\Omega_s} (p_s^k(v))^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}$$

C_s^k : the maximized value of \tilde{C}_s^k , satisfying $E_s^k = P_s^k C_s^k$.

2nd stage (Upper Level) Problem: Choose $E_s^k = P_s^k C_s^k$ to:

$$\text{Max } \tilde{U}^k, \text{ subject to } \int_0^1 (\beta_s)^\eta (\tilde{U}^k)^{\frac{\varepsilon(s)-\eta}{\eta}} (C_s^k)^{\eta-1} ds \equiv 1 \text{ and } \int_0^1 P_s^k C_s^k ds = \int_0^1 E_s^k ds \leq E^k.$$

Solution: *the share of sector- s in k 's expenditure, m_s^k*

$$m_s^k \equiv \frac{E_s^k}{E^k} \equiv \frac{P_s^k C_s^k}{E^k} = \frac{\beta_s (U^k)^{\varepsilon(s)-\eta} (P_s^k)^{1-\eta}}{\int_0^1 \beta_t (U^k)^{\varepsilon(t)-\eta} (P_t^k)^{1-\eta} dt},$$

where U^k is the maximized value of \tilde{U}^k , given implicitly by:

$$(E^k)^{1-\eta} \equiv \int_0^1 \beta_s (U^k)^{\varepsilon(s)-\eta} (P_s^k)^{1-\eta} ds. \quad (U^k \text{ is strictly increasing in } E^k.)$$

Notes:

- $\partial \log(m_s^k / m_{s'}^k) / \partial \log(U^k) = \varepsilon(s) - \varepsilon(s')$. Higher-indexed more income elastic; Income elasticity differences are constant across different per capita income levels.
- $\beta_s (U^k)^{\varepsilon(s)-\eta} (P_s^k)^{1-\eta}$ is log-supermodular in s and U^k . From **Lemma 1**, for fixed prices, a higher E^k (and U^k) shifts the expenditure share towards higher-indexed.

The Rest of the model: Deliberately kept the same with Krugman (1980).

Iceberg Trade Costs: Only $1/\tau < 1$ fraction of exports survives shipping, reducing the export revenue to its fraction, $\rho \equiv (\tau)^{1-\sigma} < 1$

CES Demand for each good; $D_s(v) = A_s^j (p_s^j(v))^{-\sigma}$, $v \in \Omega_s^j$, where

$A_s^j \equiv b_s^j + \rho b_s^k$ ($k \neq j$): Aggregate demand shifter for the producers in j in s

$b_s^k \equiv \beta_s (E^k)^\eta (U^k)^{\varepsilon(s)-\eta} N^k (P_s^k)^{\sigma-\eta}$; k 's demand shifter for sector s

Standard CES demand curve, but U^k affects b_s^k and hence A_s^j differently across s .

Constant Mark-Up: ψ_s units of labor to produce one unit of each variety in sector- s

$$p_s^j(v) = \frac{w^j \psi_s}{1-1/\sigma} \equiv p_s^j \text{ for } v \in \Omega_s^j$$

Free Entry (Zero-Profit) Condition: ϕ_s units of labor per variety to set up in sector- s .

Labor Market Equilibrium: $\int_0^1 f_s^j ds = 1$, f_s^j : sectoral share in employment (and value-

added) and, if appropriately normalized, in the measure of firms (and varieties).

Autarky Equilibrium ($\rho = 0$):

Standard-of-Living: $U_0^k = u(x_0^k)$ where $x_0^k \equiv (h^k)^\sigma N^k = (h^k)^{\sigma-1} L^k$

where $u(x)$ is defined implicitly by $(x)^{\frac{1-\eta}{\sigma-\eta}} \equiv \int_0^1 \left(\beta_s(u(x))^{\varepsilon(s)-\eta} \right)^{\frac{\sigma-1}{\sigma-\eta}} ds$.

- $U_0^k = u(x_0^k)$ is increasing both in h^k and in N^k . **Aggregate increasing returns**
- Even if $h^1 > h^2$, $U_0^1 < U_0^2$ holds when $L^1 / L^2 < (h^1 / h^2)^{1-\sigma} < 1$.

Market Size (and Firm) Distributions: $f_s^k = m_s^k = \frac{\left(\beta_s(u(x_0^k))^{\varepsilon(s)-\eta} \right)^{\frac{\sigma-1}{\sigma-\eta}}}{\int_0^1 \left(\beta_t(u(x_0^k))^{\varepsilon(t)-\eta} \right)^{\frac{\sigma-1}{\sigma-\eta}} dt}$

Notes:

- In autarky, firms (and labor) are distributed proportionately with market sizes.
- $\left(\beta_s(u(x_0^k))^{\varepsilon(s)-\eta} \right)^{\frac{\sigma-1}{\sigma-\eta}}$ is *log-supermodular* in s and x_0^k . From **Lemma 1**,

With a higher $x_0^k \equiv (h^k)^\sigma N^k$, the household becomes happier and spends relatively more on higher-indexed goods *in equilibrium*.

• Compare $m_s^k = \frac{\left(\beta_s(u(x_0^k))\right)^{(\varepsilon(s)-\eta)} \frac{\sigma-1}{\sigma-\eta}}{\int_0^1 \left(\beta_t(u(x_0^k))\right)^{(\varepsilon(t)-\eta)} \frac{\sigma-1}{\sigma-\eta} dt}$ & $m_s^k = \frac{\beta_s(U^k)^{\varepsilon(s)-\eta} (P_s^k)^{1-\eta}}{\int_0^1 \beta_t(U^k)^{\varepsilon(t)-\eta} (P_t^k)^{1-\eta} dt}$ and

notice $\frac{\sigma-1}{\sigma-\eta} > 1$ iff $\eta > 1$.

Given price indices, $U \uparrow$ shifts the expenditure toward the higher-indexed.

In equilibrium, this causes entries (exits) and hence more (less) varieties in the higher (lower)-indexed sectors, reducing the effective relative prices of higher-indexed goods, which amplifies (moderates) the shift if $\eta > (<) 1$.

• $\frac{d \log u(\lambda x)}{d \log \lambda} = \frac{\lambda x u'(\lambda x)}{u(\lambda x)} = \zeta(\lambda x)$ is increasing (decreasing) in x , if $\eta > (<) 1$. Hence,

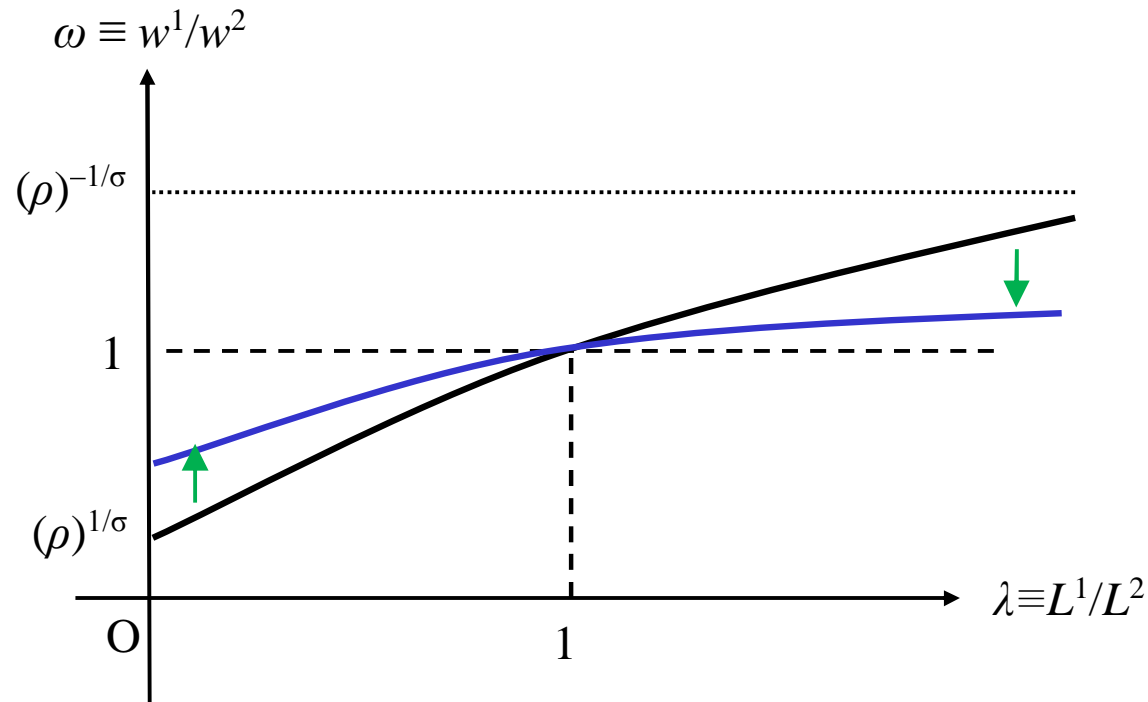
i) If $\eta < 1$, gains from a percentage increase in x is lower at a higher x .

ii) If $\eta > 1$, gains from a percentage increase in x is higher at a higher x .

Trade Equilibrium and Patterns of Trade

Figure 1: (Factor) Terms of Trade Determination

$$\frac{L^1}{L^2} = \Lambda(\omega; \rho) \equiv (\omega)^{2\sigma-1} \frac{1 - \rho(\omega)^{-\sigma}}{1 - \rho(\omega)^\sigma}, \text{ where } \omega \equiv \frac{w^1}{w^2}.$$



- The factor price lower in the smaller economy (Aggregate increasing returns)
- Globalization ($\tau \downarrow$ or $\rho \uparrow$) reduces the smaller country's disadvantage and hence the factor price differences.

Standard-of-Living: summarized by a single index, x_ρ^k

$$U_\rho^1 = u(x_\rho^1), \text{ where } x_\rho^1 \equiv \frac{(1-\rho^2)x_0^1}{1-\rho(\omega)^{-\sigma}} > x_0^1 ; U_\rho^2 = u(x_\rho^2), \text{ where } x_\rho^2 \equiv \frac{(1-\rho^2)x_0^2}{1-\rho(\omega)^\sigma} > x_0^2$$

$u(x)$, defined as before. **Gains from trade**

$$\text{Market Size Distributions: } m_s^k = \frac{\left(\beta_s(u(x_\rho^k))\right)^{(\varepsilon(s)-\eta)} \frac{\sigma-1}{\sigma-\eta}}{\left(x_\rho^k\right)^{\frac{1-\eta}{\sigma-\eta}}} = \frac{\left(\beta_s(u(x_\rho^k))\right)^{(\varepsilon(s)-\eta)} \frac{\sigma-1}{\sigma-\eta}}{\int_0^1 \left(\beta_t(u(x_\rho^k))\right)^{(\varepsilon(t)-\eta)} \frac{\sigma-1}{\sigma-\eta} dt}$$

$\left(\beta_s(u(x_\rho^k))\right)^{(\varepsilon(s)-\eta)} \frac{\sigma-1}{\sigma-\eta}$ is *log-supermodular* in s & x_ρ^k . From **Lemma 1**, if $u(x_\rho^1) < u(x_\rho^2)$

i) **MLR:** $\frac{m_s^1}{m_s^2} = \left(\frac{x_\rho^1}{x_\rho^2}\right)^{\frac{\eta-1}{\sigma-\eta}} \left(\frac{u(x_\rho^1)}{u(x_\rho^2)}\right)^{(\varepsilon(s)-\eta)\frac{\sigma-1}{\sigma-\eta}}$ is strictly decreasing in s :

ii) **FSD:** $\int_0^1 m_t^1 dt > \int_0^1 m_t^2 dt$

The Rich (Poor) has relatively larger domestic markets in higher(lower)-indexed sectors.

Firm Distributions: $f_s^1 = \frac{m_s^1 - \rho(\omega)^{-\sigma} m_s^2}{1 - \rho(\omega)^{-\sigma}};$ $f_s^2 = \frac{m_s^2 - \rho(\omega)^\sigma m_s^1}{1 - \rho(\omega)^\sigma}$

HME; $\frac{f_s^1}{f_s^2} > \frac{m_s^1}{m_s^2} > 1;$ $\frac{f_s^1}{f_s^2} = \frac{m_s^1}{m_s^2} = 1;$ or $\frac{f_s^1}{f_s^2} < \frac{m_s^1}{m_s^2} < 1.$

Sectoral Trade Balances: From $NX_s^1 = -NX_s^2 \equiv V_s^1 \rho b_s^2 (w^1)^{1-\sigma} - V_s^2 \rho b_s^1 (w^2)^{1-\sigma},$

$$NX_s^1 = -NX_s^2 = \frac{\rho w^2 L^2}{(\omega)^{-\sigma} - \rho} (m_s^1 - m_s^2) = \frac{\rho w^1 L^1}{(\omega)^\sigma - \rho} (m_s^1 - m_s^2) \propto (m_s^1 - m_s^2).$$

Determined by the difference in *the Demand Composition, not in the Market Size.*

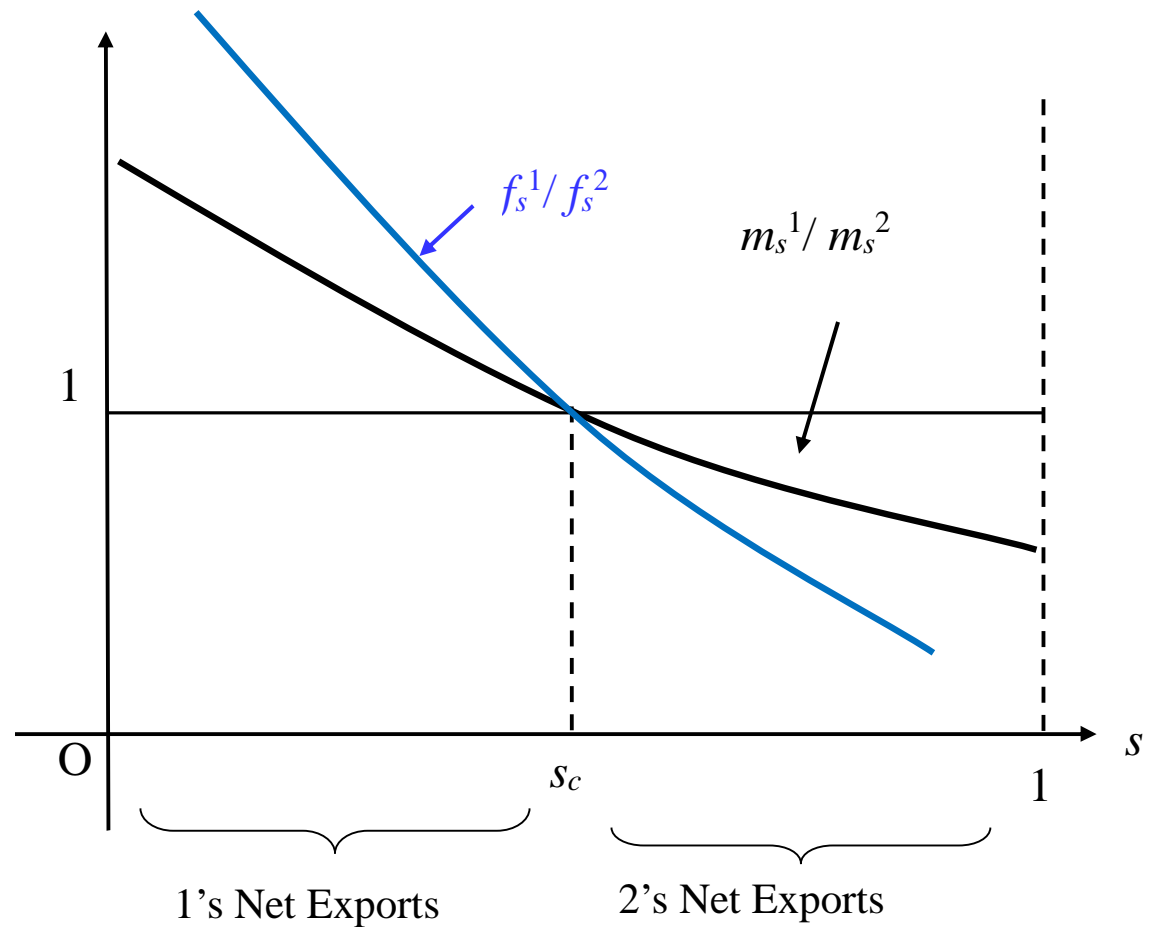
$U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2) \rightarrow m_s^1 / m_s^2$ is strictly decreasing in $s \rightarrow$

a **unique cutoff sector**, $s_c \in (0,1)$, such that

$$NX_s^1 = -NX_s^2 > 0 \text{ for } s < s_c; \quad NX_s^1 = -NX_s^2 < 0 \text{ for } s > s_c.$$

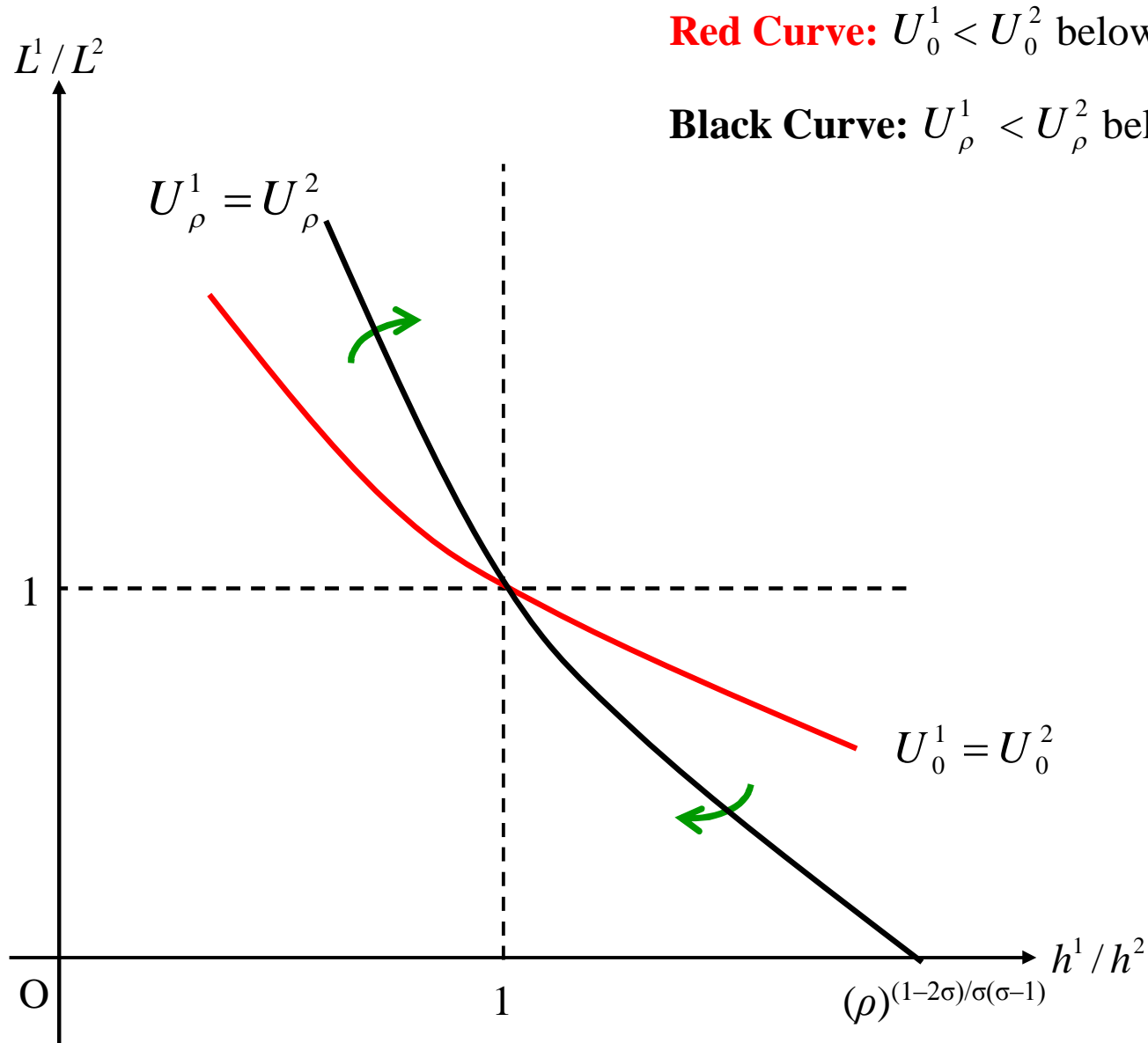
Figure 2: Home Market Effect and Patterns of Sectoral Trade Balances:

For $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$



The Rich (Poor) runs surpluses in the higher-(lower-) indexed sectors, which produce with higher (lower) income elastic goods.

Figure 3: Ranking the Countries



Red Curve: $U_0^1 < U_0^2$ below, $U_0^1 > U_0^2$ above

Black Curve: $U_\rho^1 < U_\rho^2$ below, $U_\rho^1 > U_\rho^2$ above

Comparative Statics

Uniform Productivity Improvement: ($\partial \log(h^1) = \partial \log(h^2) \equiv \partial \log(h) > 0$)

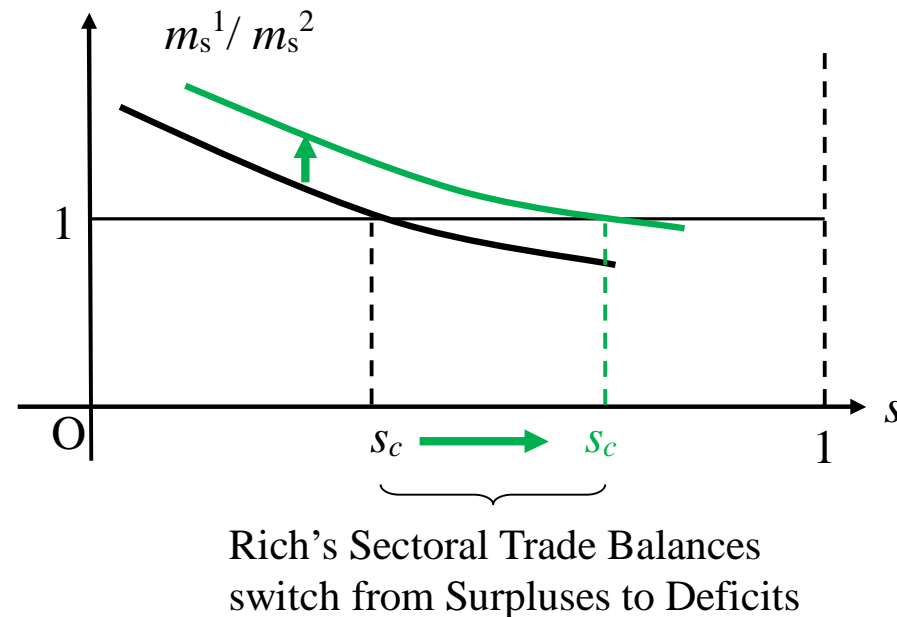
h^1 / h^2 , L^1 / L^2 , $\omega = w^1 / w^2$, x_0^1 / x_0^2 , x_ρ^1 / x_ρ^2 all unchanged, with $\partial \log(x_\rho^1) = \partial \log(x_\rho^2) = \sigma \partial \log(h) > 0$.

- Both $U_\rho^1 = u(x_\rho^1)$ and $U_\rho^2 = u(x_\rho^2)$ go up. Since $\left(\beta_s(u(x_\rho^k))^{(\varepsilon(s)-\eta)}\right)^{\frac{\sigma-1}{\sigma-\eta}}$ is *log-supermodular* in s and x_ρ^k , from **Lemma 1**, the market size distributions shift toward higher-indexed sectors in both countries, in the sense of MLR and FSD.

- $\text{sgn} \frac{\partial \log(U_\rho^1 / U_\rho^2)}{\partial \log(h)} = \text{sgn}(\eta - 1) \text{sgn}(x_\rho^1 - x_\rho^2)$, from **Lemma 2**.

Welfare gaps widen (narrow) if sectors produce substitutes (complements).

- $\text{sgn} \frac{\partial \log(m_s^1 / m_s^2)}{\partial \log(h)} = \text{sgn}(x_\rho^2 - x_\rho^1) \rightarrow s_c$ goes up.

Figure 4: Product Cycles Due to Uniform Productivity Improvement

- As everyone becomes more productive, they shift their spending towards the higher-indexed.
- The relative weights of the sectors in which the Rich runs surpluses go up.
- To keep the overall trade account between the two countries in balance, the Rich's trade account in each sector must deteriorate.
- The Rich switches from being the net-exporter to the net-importer in middle sectors.

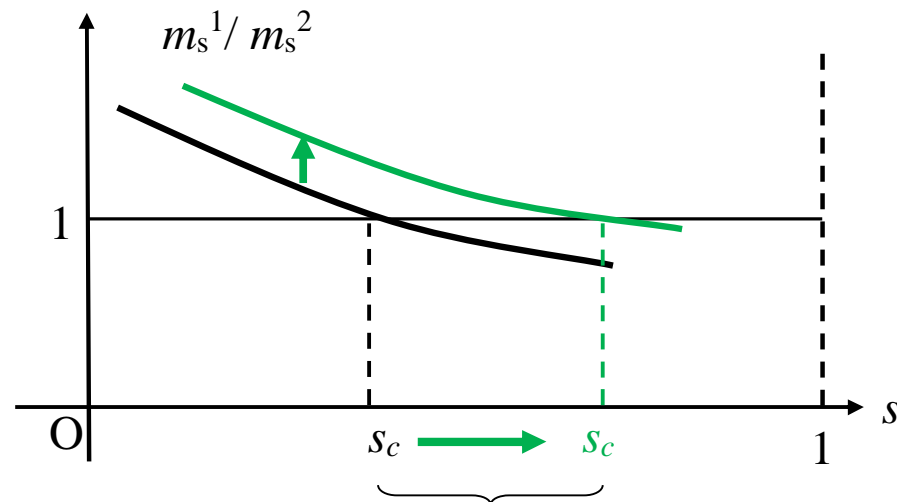
Globalization, a higher $\rho = (\tau)^{1-\sigma}$, when two countries are equal in size: $L^1 = L^2 = L$

$$\omega = 1 \rightarrow x_{\rho}^k = (1 + \rho)x_0^k = (1 + \rho)(h^k)^{\sigma} N^k = (1 + \rho)(h^k)^{\sigma-1} L$$

The relative factor price fixed at $\omega = 1$ and independent of ρ . No ToT change

- The country with higher per capita labor endowment is richer.
- a higher $1 + \rho$ is isomorphic to a uniform increase in h^k .

Figure 4: Product Cycles Due to Globalization



Rich's Sectoral Trade Balances
switch from Surpluses to Deficits

Globalization, a higher $\rho = (\tau)^{1-\sigma}$, when two countries are unequal in size:

Leapfrogging and Reversal of the Patterns of Trade

For $h^1 / h^2 > 1$ and below the Red curve,

$U_\rho^1 < U_\rho^2$ at a low ρ ,

Closer to autarky, Country 1 is poorer due to its disadvantage of being smaller, running surpluses in lower-indexed.

$U_\rho^1 > U_\rho^2$ at a high ρ ,

Globalization leads to a factor price convergence, which makes the smaller but smarter 1 richer, running surpluses in higher-indexed.

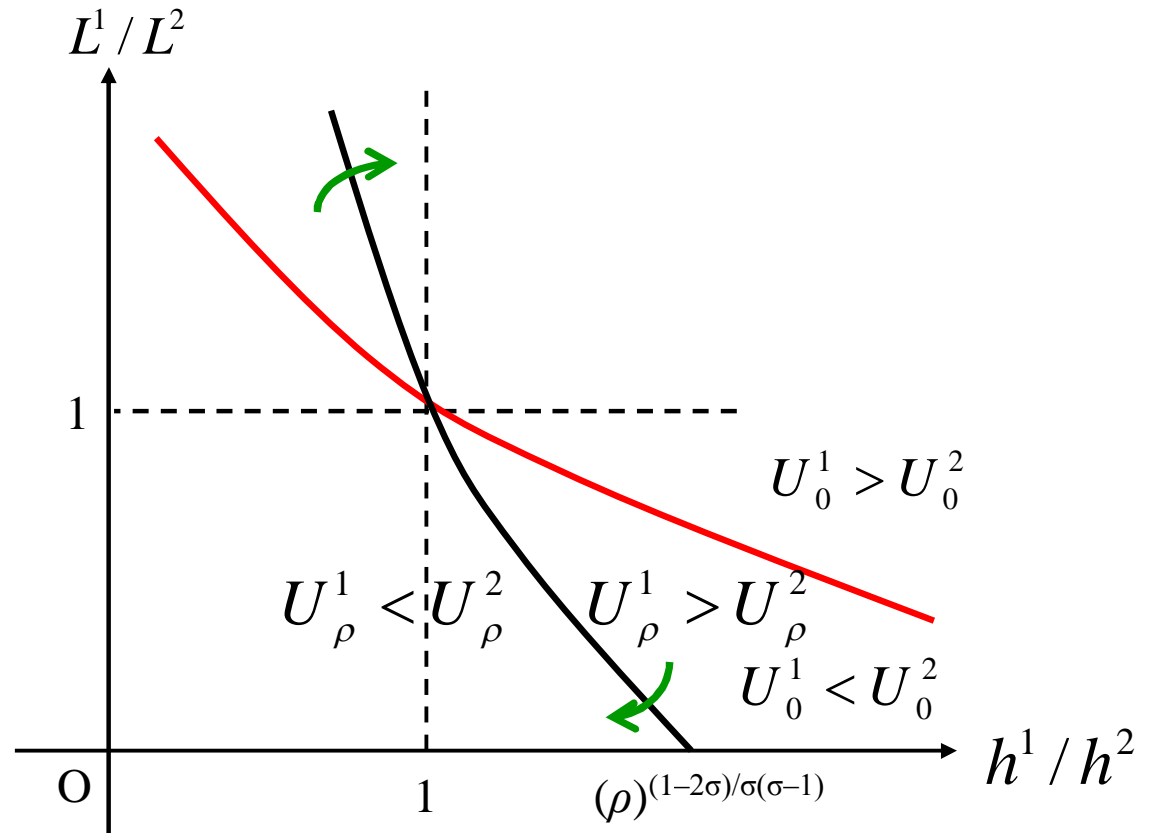


Figure 5

HME with Exogenous Taste Variations: A Comparison

An Extension of Krugman (1980):

Keep the same structure, except the upper-level preferences are homothetic CES,

$$\tilde{U}^k \equiv \left[\int_0^1 (\beta_s^k)^{\frac{1}{\eta}} (\tilde{C}_s^k)^{1-\frac{1}{\eta}} ds \right]^{\frac{\eta}{\eta-1}}, \quad \text{normalized to } \int_0^1 (\beta_s^k)^{\frac{\sigma-1}{\sigma-\eta}} ds = 1$$

with different weights β_s^k , and β_s^1 / β_s^2 strictly decreasing in s .

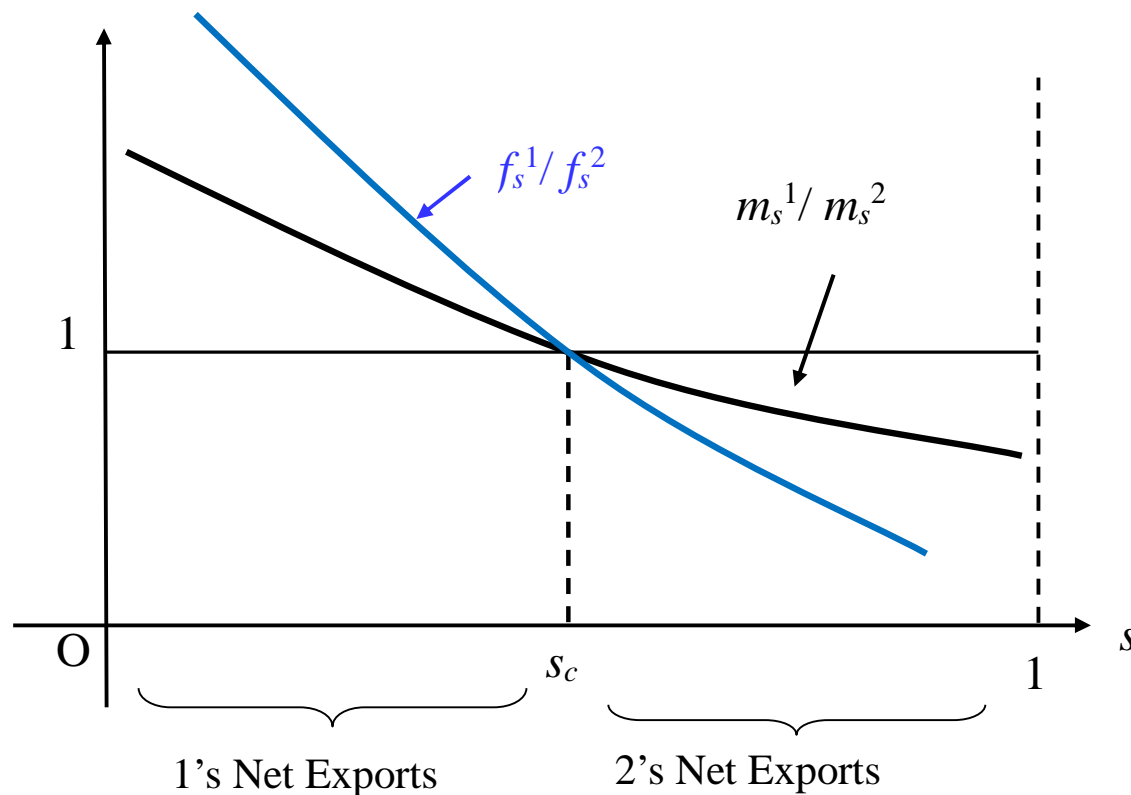
Then,

Standard-of-living: $U_\rho^k = (x_\rho^k)^{\frac{1}{\sigma-1}}$

Market Size Distribution: $m_s^k = (\beta_s^k)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)} \rightarrow m_s^1 / m_s^2 = (\beta_s^1 / \beta_s^2)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}$
strictly decreasing in s .

Otherwise, the same

Figure 2

**Notes:**

- m_s^1 / m_s^2 depends solely on the exogenous preferences parameters. Independent of ρ and h^k . Effects on s_c in the previous model are entirely due to nonhomotheticity.
- Uniform productivity growth cannot change the welfare gap.
- Leapfrogging can occur; Reversal of Patterns of Trade cannot.
- Krugman (1980), a special case with $\eta = 1$, $L^1 = L^2$, and $\beta_s^1 / \beta_s^2 = \gamma > 1$ for $0 \leq s < 1/2$; $\beta_s^1 / \beta_s^2 = 1/\gamma < 1$ for $1/2 < s \leq 1$.

Adding An Outside Goods Sector

An Extension of the Helpman and Krugman (1985) Home Market Effect Model

The same structure as before, except

Homogeneous Good (Numeraire): competitive, CRS (1-to-1), zero trade cost

Household Preferences: Three-Tier structure

$$\text{Lower-level, } \tilde{C}_s^k \equiv \left[\int_{\Omega_s} (c_s^k(v))^{1-\frac{1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}}; \quad \sigma > 1, \quad s \in [0,1]$$

$$\text{Middle-level, } \int_0^1 (\beta_s)^\eta (\tilde{U}^k)^{\frac{\varepsilon(s)-\eta}{\eta}} (\tilde{C}_s^k)^{\frac{\eta-1}{\eta}} ds \equiv 1; \quad \beta_s > 0 \text{ and } \sigma > \eta \neq 1,$$

$$\text{Upper-level, } \tilde{W}^k = (1-\alpha) \log \tilde{C}_0^k + \alpha \log(\tilde{U}^k)$$

\tilde{C}_0^k : Household consumption of the numeraire

α : (Fixed) expenditure share of differentiated goods

With a sufficiently small α , both countries produce the numeraire.

- $L^j - \int_0^1 V_s^j ds > 0$; a positive employment in the numeraire sector.
- $w^j = 1$; (Factor) Terms of Trade uniquely pinned down and independent of ρ .
- Each household earns h^k and spends $E^k = \alpha h^k$ on differentiated goods.

The Equilibrium Conditions would be the same otherwise.

Autarky Equilibrium

Standard-of-Living: $W_0^k = (1 - \alpha) \log((1 - \alpha)h^k) + \alpha \log(u(x_0^k))$,

with $x_0^k \equiv (\alpha h^k)^\sigma N^k = \alpha (\alpha h^k)^{\sigma-1} L^k$

Market Size Distributions: $m_s^k = \frac{\left(\beta_s(u(x_0^k))\right)^{(\varepsilon(s)-\eta)} \frac{\sigma-1}{\sigma-\eta}}{\int_0^1 \left(\beta_t(u(x_0^k))\right)^{(\varepsilon(t)-\eta)} \frac{\sigma-1}{\sigma-\eta} dt}$

Trade Equilibrium:

Standard-of-Living: $W_\rho^k = (1 - \alpha) \log((1 - \alpha)h^k) + \alpha \log(u(x_\rho^k))$,

where $x_\rho^k \equiv (1 + \rho)(\alpha h^k)^\sigma N^k = (1 + \rho)x_0^k$

Market Size Distributions: $m_s^k = \frac{\left(\beta_s(u(x_\rho^k))\right)^{(\varepsilon(s)-\eta)} \left(\frac{\sigma-1}{\sigma-\eta}\right)}{\int_0^1 \left(\beta_t(u(x_\rho^k))\right)^{(\varepsilon(t)-\eta)} \left(\frac{\sigma-1}{\sigma-\eta}\right) dt}$

Firms Distributions:

$$\text{From } V_s^1 = \frac{m_s^1(\alpha L^1) - \rho m_s^2(\alpha L^2)}{1 - \rho} > 0; \quad V_s^2 = \frac{m_s^2(\alpha L^2) - \rho m_s^1(\alpha L^1)}{1 - \rho} > 0,$$

$$f_s^1 = \frac{m_s^1 L^1 - \rho m_s^2 L^2}{L^1 - \rho L^2} > 0; \quad f_s^2 = \frac{m_s^2 L^2 - \rho m_s^1 L^1}{L^2 - \rho L^1} > 0 \quad \text{for } \rho < \frac{m_s^1 L^1}{m_s^2 L^2} < \frac{1}{\rho}.$$

Sectoral Trade Balances:

$$NX_s^1 = -NX_s^2 \equiv V_s^1 \rho b_s^2 - V_s^2 \rho b_s^1 = \frac{\rho}{1+\rho} (V_s^1 - V_s^2) = \frac{\alpha\rho}{1-\rho} (m_s^1 L^1 - m_s^2 L^2) \propto (m_s^1 L^1 - m_s^2 L^2)$$

What matters is the cross-country difference in the market size in each sector itself.

$$\text{Trade Balances in Differ. Goods Sectors: } \int_0^1 NX_s^1 ds = -\int_0^1 NX_s^2 ds = \frac{\alpha\rho}{1-\rho} (L^1 - L^2)$$

Instead of having a higher factor price, the larger country runs an overall surplus in the differentiated goods sectors, with a deficit in the outside good sector.

$$\text{Factor Price Equalization Condition; } \alpha < \text{Min} \left\langle \frac{(1-\rho)L^1}{L^1 - \rho L^2}, \frac{(1-\rho)L^2}{L^2 - \rho L^1} \right\rangle$$

Patterns of Trade: Home Market Effect

- m_s^1 / m_s^2 is strictly decreasing in s , for $x_0^1 < x_0^2 \Leftrightarrow L^1 / L^2 < (h^1 / h^2)^{1-\sigma}$
- When L^1 and L^2 are not too different, a **unique cutoff sector**, $s_c \in (0,1)$ such that

$$NX_s^1 = -NX_s^2 = \frac{\alpha\rho L}{1-\rho} (m_s^1 L^1 - m_s^2 L^2) > 0 \text{ for } s < s_c; < 0 \text{ for } s > s_c.$$

Comparative Statics: With a uniform productivity improvement and globalization,

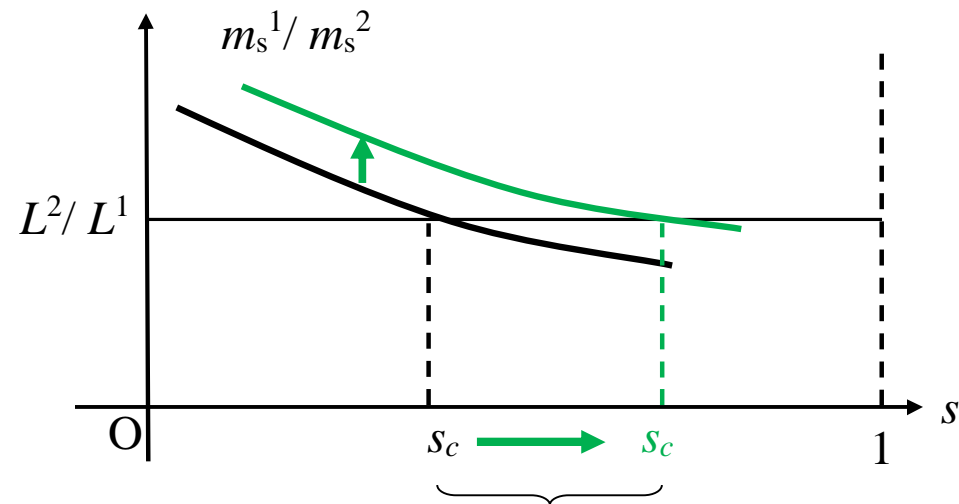
- m_s^k shifts towards the higher-indexed in the sense of MLR and FSD.

$$\bullet \operatorname{sgn} \frac{\partial \log(U_\rho^1 / U_\rho^2)}{\partial \log(h)} = \operatorname{sgn} \frac{\partial \log(U_\rho^1 / U_\rho^2)}{\partial \log(1+\rho)}$$

$$= \operatorname{sgn}(\eta - 1) \operatorname{sgn}(x_\rho^1 - x_\rho^2).$$

$$\bullet \operatorname{sgn} \frac{\partial \log(m_s^1 / m_s^2)}{\partial \log(h)} = \operatorname{sgn} \frac{\partial \log(m_s^1 / m_s^2)}{\partial \log(1+\rho)}$$

$$= \operatorname{sgn}(x_\rho^2 - x_\rho^1) \rightarrow s_c \in (0,1) \text{ moves up.}$$



Rich's Sectoral Trade Balances
switch from Surpluses to Deficits

In Summary:

- With the ToT pinned down by the numeraire good, a higher ρ does not change ToT change, even when the country sizes are different.
- With no ToT change, the effect of a higher ρ is isomorphic to the effects of uniform productivity improvement (an equi-proportional increase in h^k), as in the $L^1 = L^2$ case of the previous model.
- With no ToT change, Leapfrogging and A Reversal of Patterns of Trade cannot occur.

Two Caveats: Unlike in the $L^1 = L^2$ case of the previous model, $L^1 \neq L^2$ generates the possibility:

- $U_\rho^1 < U_\rho^2 \Leftrightarrow L^1 / L^2 < (h^1 / h^2)^{1-\sigma}$ may occur, even if $h^1 > h^2$.
- If L^1 and L^2 are too different, the larger country may run a surplus in all s .

Concluding Remarks

- Empirically, goods differ widely in their income elasticities; rich (poor) countries tend to export goods with high (low) income elasticities.
- We aim to explain *why* the rich (poor) have CA in high (low) income elastic goods with two ingredients, *Nonhomothetic Preferences & Home Market Effect*
- Simple intuition
 - ✓ Demand composition of the Rich (Poor) more skewed towards high (low) income elastic goods
 - ✓ With scale economies and positive but finite trade costs, such cross-country differences in the demand composition become a source of comparative advantage.
- No previous studies capture this intuition in a setup flexible and yet tractable enough to allow for a variety of comparative static exercises, because GE models with *imperfect competition, scale economies, positive but finite trade costs, and nonhomotheticity* would be intractable
 - ✓ **Explicitly additively separable nonhomothetic preferences**, such as Stone-Geary or CRIE, are too restrictive and too intractable
- **Implicitly additively separable nonhomothetic preferences** enables us to overcome this difficulty