Liquidity, Assets and Business Cycles

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1. What Do I Try to Do?

- Formulate a hypothesis on the role of asset market liquidity in the business cycle

- Calibrate the model to evaluate the hypothesis

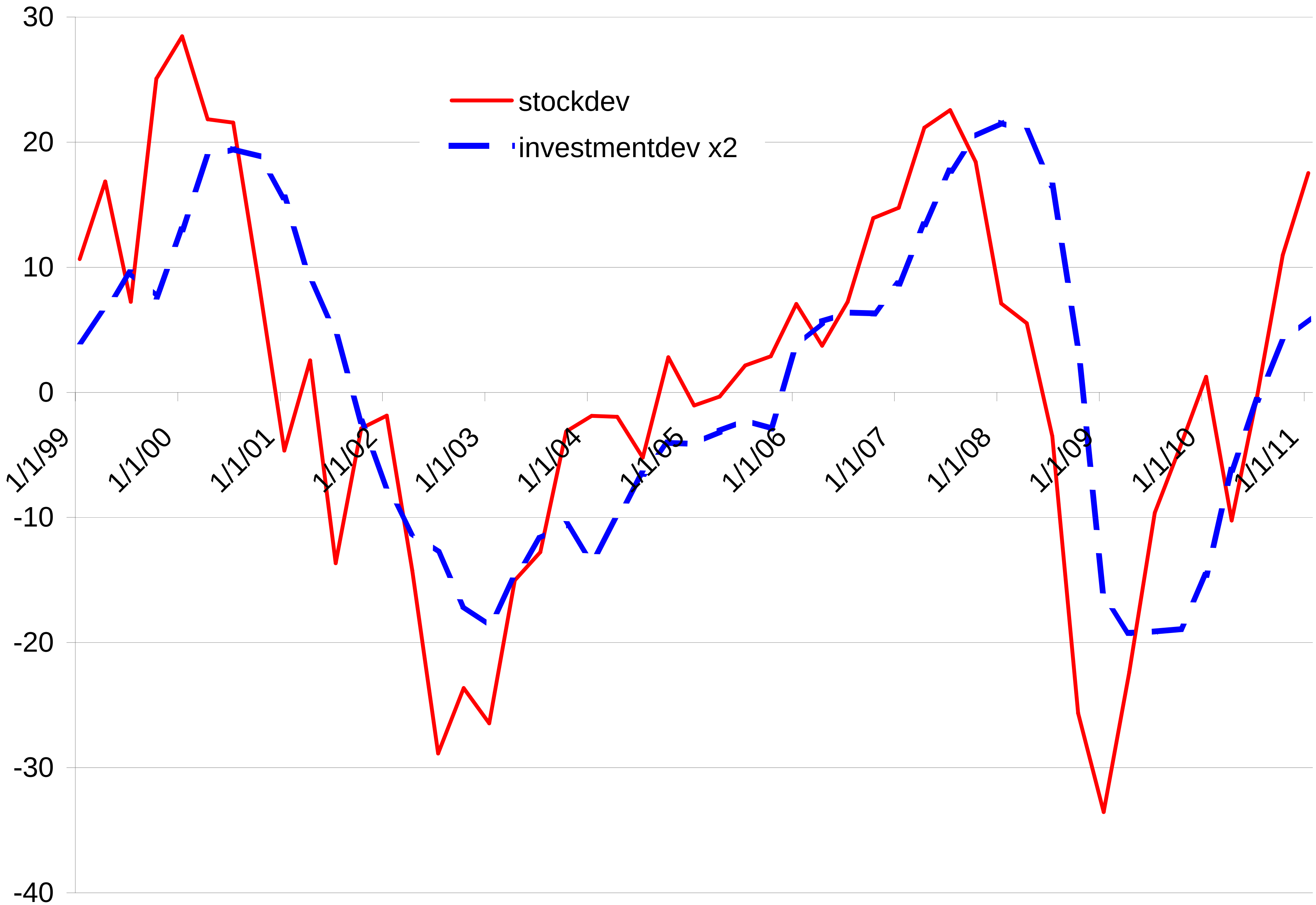

Figure 1.1. Deviation of stock price and investment from trend (%)

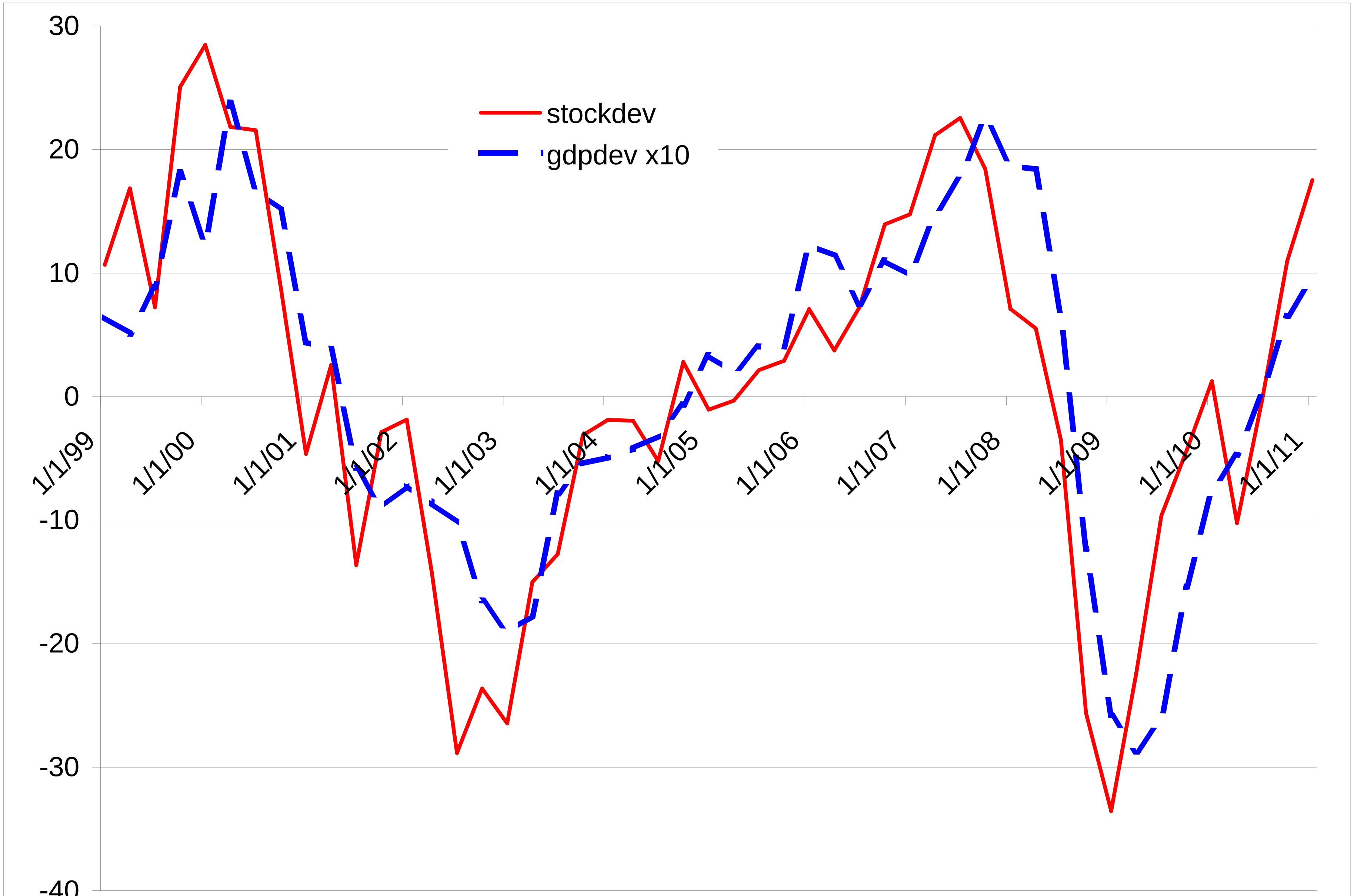

Figure 1.2. Deviation of stock price and GDP from trend (%)

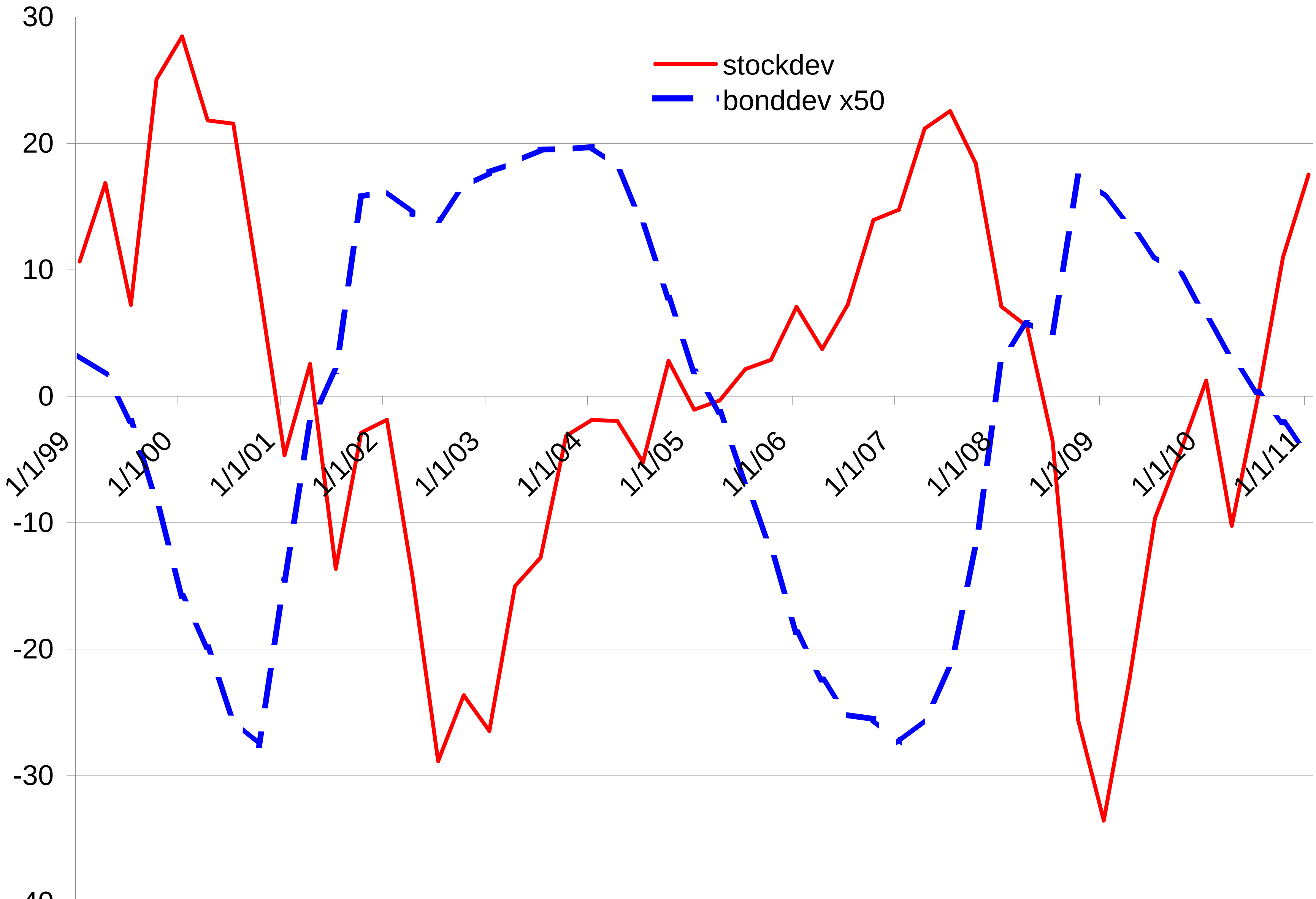

Figure 1.3. Deviation of stock price and bond price from trend (%)

An intuitive explanation/hypothesis:

Liquidity shocks in asset market are an independent cause of the business cycle.

- sudden drop in liquidity depresses equity price
- tightens financing constraints on investment
- investment and output fall
- demand for liquid assets rises; bond price increases

Policy implication of this hypothesis:

Central banks should and can supply liquidity to the asset market to reduce or eliminate recessions.

Examples:

bailouts, QE1, QE2, ...

Hypothesis formulated by N. Kiyotaki and J. Moore (08):

- two frictions in the equity market:
  - difficulty in issuing new equity
  - difficulty in re-selling equity
- liquidity shocks occur in the resale market for equity

Calibrated versions:
Ajello (10): liquidity shocks are important for business cycles
Del Negro et al. (10): Fed policy prevented a greater recession

The tasks:

- simplify the model to capture Kiyotaki-Moore hypothesis:
  - to facilitate aggregation
  - to construct a recursive competitive equilibrium
- calibrate the model to evaluate the hypothesis

What do I find?

- shocks to equity market liquidity can generate large fluctuations in investment, output and employment
- but not all the effects are what one may expect

2. The Model

2.1. The model environment

A large representative household:

- many members share assets at the beginning of a period

- in the period, members are separated from each other, and realize the role as entrepreneurs or workers

- carry out household's instructions that maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \quad \pi \underbrace{u(c_t^e)}_{\text{entrepreneur's}} \quad +(1-\pi) \underbrace{[U(c_t^w) - h(\ell_t)]}_{\text{worker's u}} \}$$

A worker has:

one unit of labor; no investment project

An entrepreneur has:

- no labor endowment

- an investment project:
  one unit of good as input $\Longrightarrow$ one unit of capital

- financing/liquidity constraints (specified later)

Snapshots at different points of time in a period:

- Beginning of the period:
  - aggregate state of the economy is realized
  - a household has:
    physical capital: k_t;   equity claims: s_t;   liquid assets: b_t
  - a household:
    divides assets among the members;   gives instructions
  - then members are separated until beginning of next period

- Investment/production stage:
  - each member realizes whether he is an entrepreneur (prob π) or a worker (prob $1-\pi$)
  - a worker supplies labor ℓ_t to produce goods:

$$y_t = A_t\ F(k_t^d, \ell_t^d)$$

  - an entrepreneur raises funds for investment i_t

- Consumption stage:
  - worker: consumes c_t^w and holds portfolio (s_{t+1}^w, b_{t+1}^w)
  - entrepreneur: consumes c_t^e and holds portfolio (s_{t+1}^e, b_{t+1}^e)

Equity market frictions (Kiyotaki-Moore, 08):

- only $\theta \in (0, 1)$ of investment can be financed by new equity
- only a fraction $\phi_t \in (0, 1)$ of existing equity can be re-sold

Equity liquidity constraint:

$$\underbrace{s^e_{t+1}}_{\substack{\text{equity at}\\ \text{the end}}} \geq \underbrace{(1-\theta)\, i_t}_{\substack{\text{unsold}\\ \text{new equity}}} + \underbrace{(1-\phi_t)\,\sigma\, s_t}_{\substack{\text{unsold}\\ \text{old equity}}}$$

2.2. A household's dynamic programing problem

- **Combined liquidity constraint** (shadow price λ^e):

$$\underbrace{(r + \phi\,\sigma q)\; s}_{\substack{\text{rental and}\\ \text{resale}}} + \underbrace{(b - p_b\; b^e_{+1})}_{\substack{\text{adjust}\\ \text{liquid assets}}} \; - \tau \geq \underbrace{(1 - \theta q)\; i}_{\substack{\text{downpayment}\\ \text{on investment}}} \; + c^e$$

Optimal investment:

$$\underbrace{q - 1}_{\substack{\text{benefit of}\\ \text{new equity}}} \quad = \quad \underbrace{(1 - \theta q)\; \lambda^e}_{\substack{\text{cost of}\\ \text{downpayment}}}$$

2.3. Recursive competitive equilibrium

- components:
  - asset price functions: $(q, p_b)(K, Z)$
  - factor price functions: $(r, w)(K, Z)$
  - policy functions: $x(s, b; K, Z)$, $x \in (i, c^e, s^e_{+1}, b^e_{+1}, \ell, c, s_{+1}, b_{+1})$
  - value function: $v(s, b; K, Z)$
- requirements:
  - optimization by individual households and firms
  - clearing of markets for goods, labor, capital, and assets
  - dynamics of aggregate capital: $K_{+1} = \sigma K + \pi i(K, B; K, Z)$

3. Equilibrium responses to shocks

3.1. Calibration

$$U(c^w) = \frac{(c^w)^{1-\rho} - 1}{1-\rho}, \qquad u(c^e) = u_0 U(c^e)$$

$$h(\ell) = h_0 \ell^\eta, \quad F(K, (1-\pi)\ell) = K^\alpha \left[(1-\pi)\ell\right]^{1-\alpha}$$

$$\log A_{t+1} = (1-\delta_A) \log A^* + \delta_A \log A_t + \varepsilon_{A,t+1}$$

$$\begin{aligned} -\log(\frac{1}{\phi_{t+1}} - 1) = \; & -(1-\delta_\phi) \log\left(\frac{1}{\phi^*} - 1\right) \\ & -\delta_\phi \log\left(\frac{1}{\phi_t} - 1\right) + \varepsilon_{\phi,t+1} \end{aligned}$$

parameter	value	calibration target
π: prob of investment	0.06	annual fraction of investing firms $= 0.24$
B: stock of liquid assets	2.020	fraction of liquid assets in portfolio $= 0.12$
ϕ^*: steady st. resaleability	0.276	annual return to liquid assets $= 0.02$
θ: finance by new equity	0.276	set to equal to ϕ^*
δ_ϕ: ϕ persistence	0.9	exogenously chosen
other		standard targets

3.2. Response to a negative liquidity shock

Experiment:

- at $t = 0$: economy is in non-stochastic steady state
- at the beginning of $t = 1$:
  ϕ falls from $\phi^* = \mathbf{0.276}$ to $\phi_1 = \mathbf{0.05}$
- for all $t \geq 2$: ϕ_t follows the process with $\varepsilon_{\phi,t} = 0$
- A is fixed at A^*, and θ is fixed, throughout

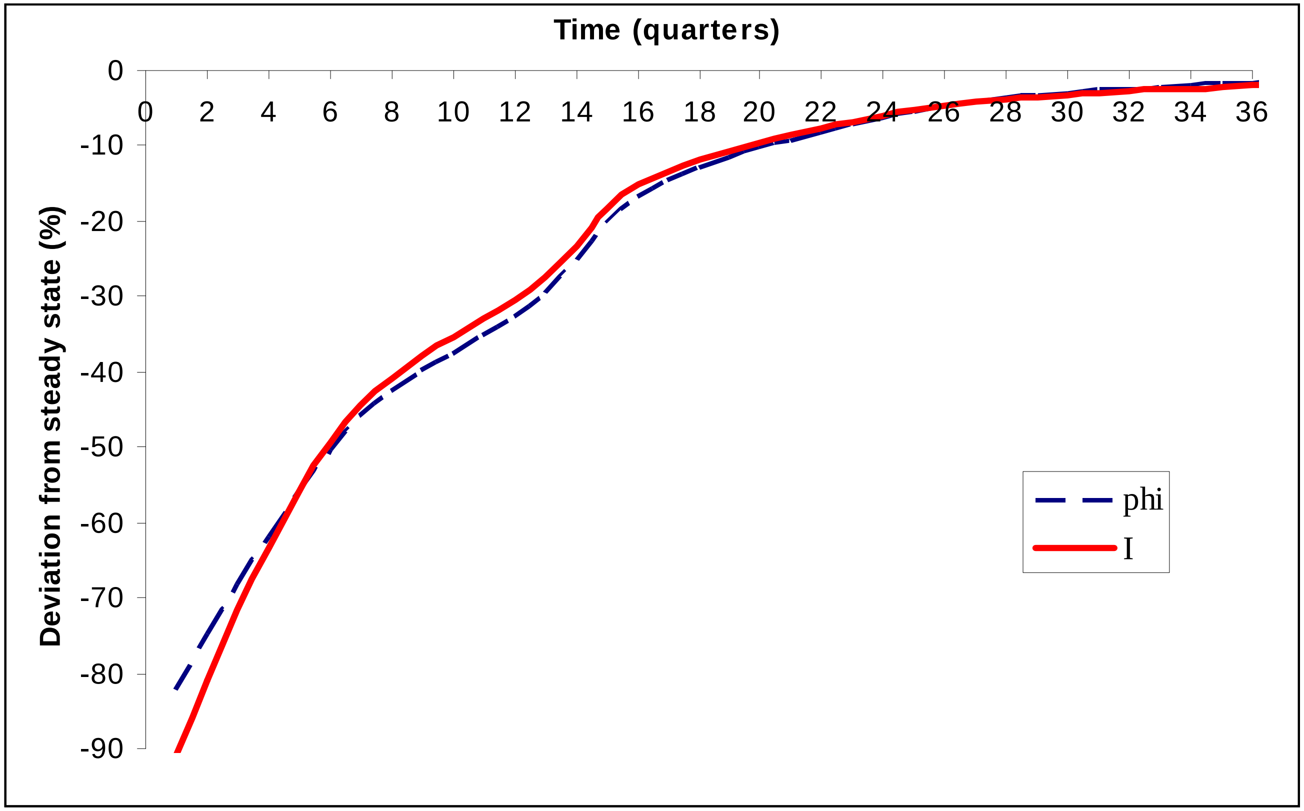

Figure 2.1. Equity resaleability and investment

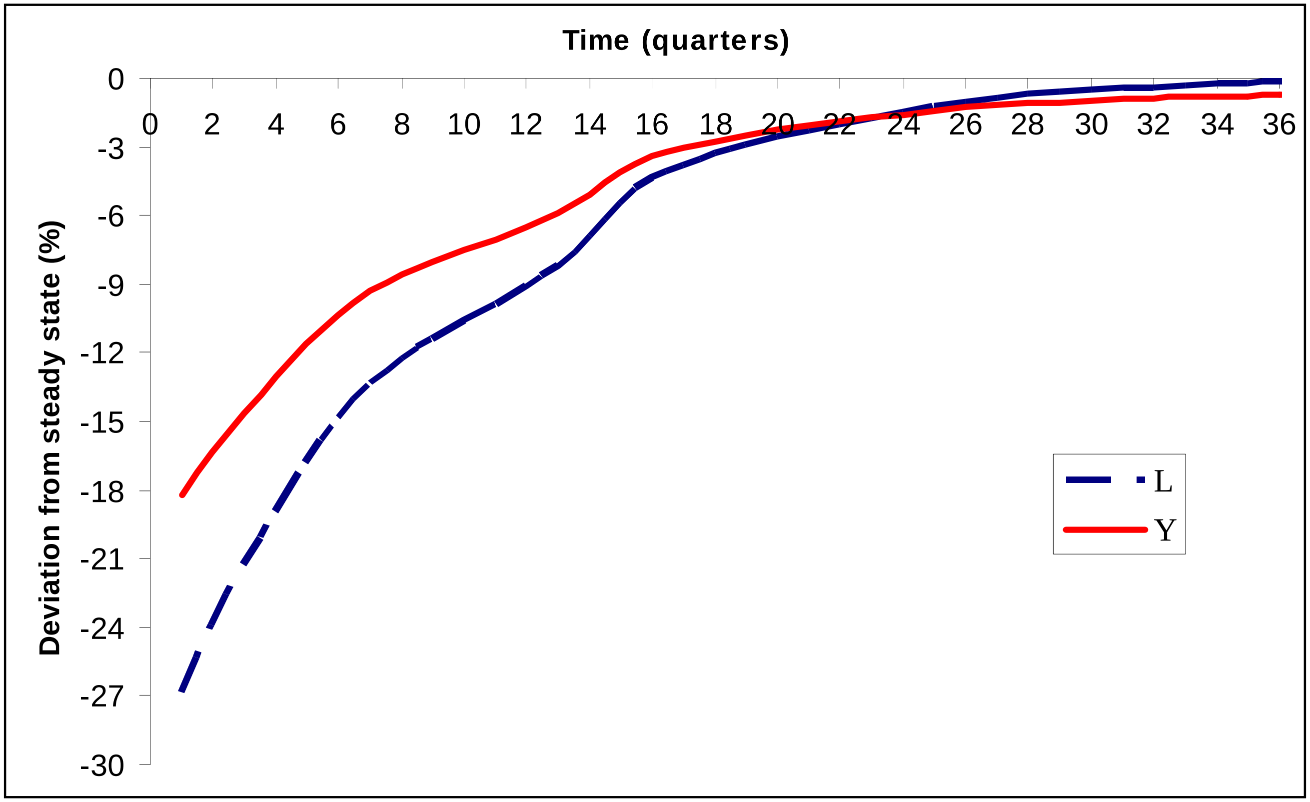

Figure 2.2. Employment and output

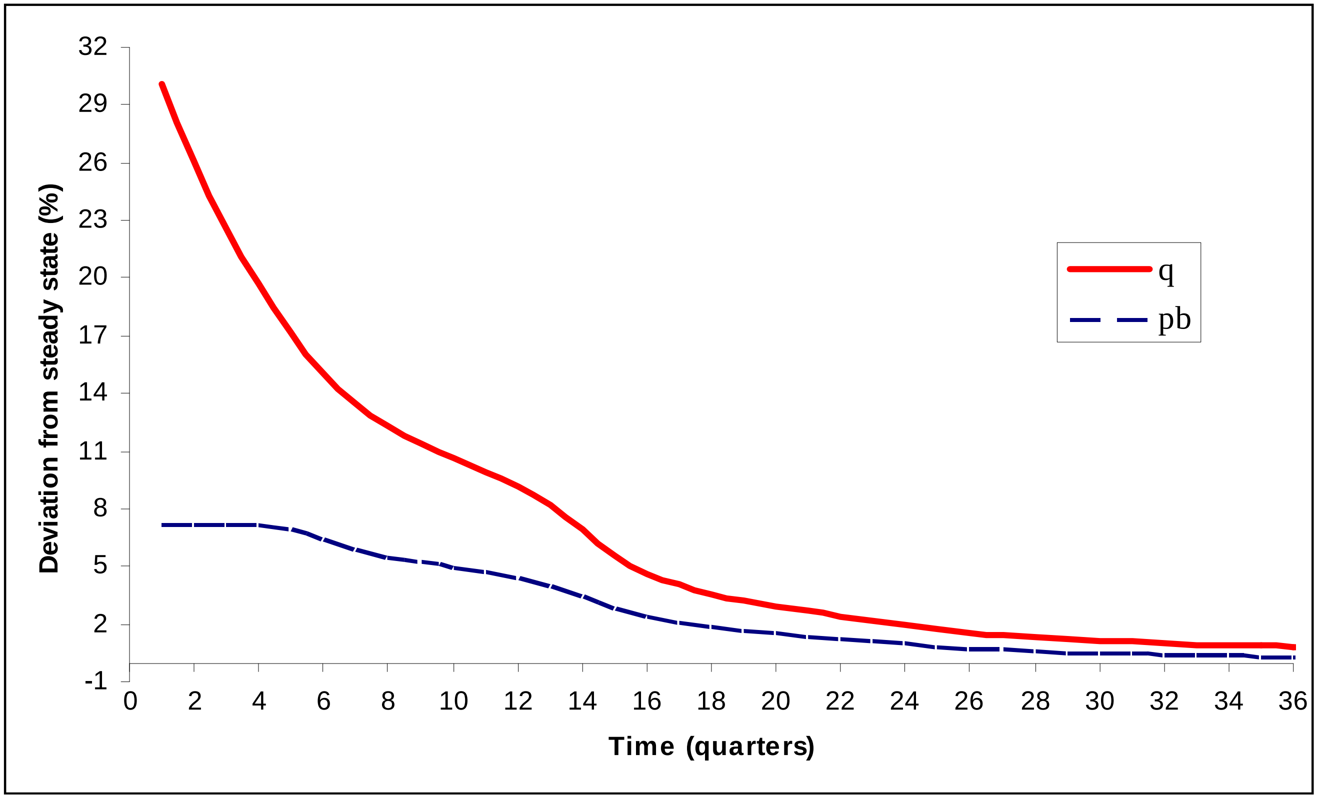

Figure 2.3. Equity price and bond price

A large and persistent negative shock to equity liquidity generates:

- large and persistent reductions in investment

- large and persistent reductions in output and employment

- problem: large and persistent **equity price BOOM**

3.3. What is the source of this problem?

Some suspects:

- glitch in Matlab programs

- shock is too large: non-linearity messed up things

- θ (friction in new equity) is fixed: θ should fall

- model is unrealistic because it omits:
  wage/price rigidity; adjustment costs; habit persistence

The simple reason:

- Optimal investment requires:

$$\underbrace{q-1}_{\text{benefit of equity}} \quad = \quad \underbrace{(1-\theta q)\ \lambda^e}_{\text{cost of downpayment}}$$

- negative liquidity shock tightens the liquidity constraint, and increases the shadow price of the constraint, λ^e

- equity price q must rise to restore the balance

$$q - 1 = (1 - \theta q)\ \lambda^e$$

The equity price boom is even **LARGER** if

- θ falls: difficulty in issuing new equity increases

- wages are sticky:
  rental income falls, tightening liquidity constraint further

- consumption has habit persistence:
  an entrepreneur also needs to maintain high consumption

Adjustment cost in investment won't help much either:

- adjustment in investing i: $i^*\Psi(i/i^*)$
- optimal investment:

$$q - (1 + \Psi') = (1 + \Psi' - \theta q)\lambda^e$$

- Ψ' needs to be large to make a difference, but then
  - investment does not fall by much
  - liquidity constraint is tighter,
  - λ^e increases by a lot, and so q increases

Assumptions that reduced the equity price boom:

- structure of large households:
  - pooling assets at the beginning of a period eliminates persistence in heterogeneity in asset holdings
  - this should reduce tightness of liquidity constraint

- rental income is immediately available to entrepreneurs:
  - this relaxed the liquidity constraint

4. Some Solutions to the Problem

For equity price to fall after a negative liquidity shock,
the equity liquidity constraint must become **LESS** tight.

- Need other shocks to sufficiently reduce the need for investment

- Some candidates:
  - negative shock to productivity A
  - negative shock to quality of capital
  - negative shock to investment opportunities: a fall in π

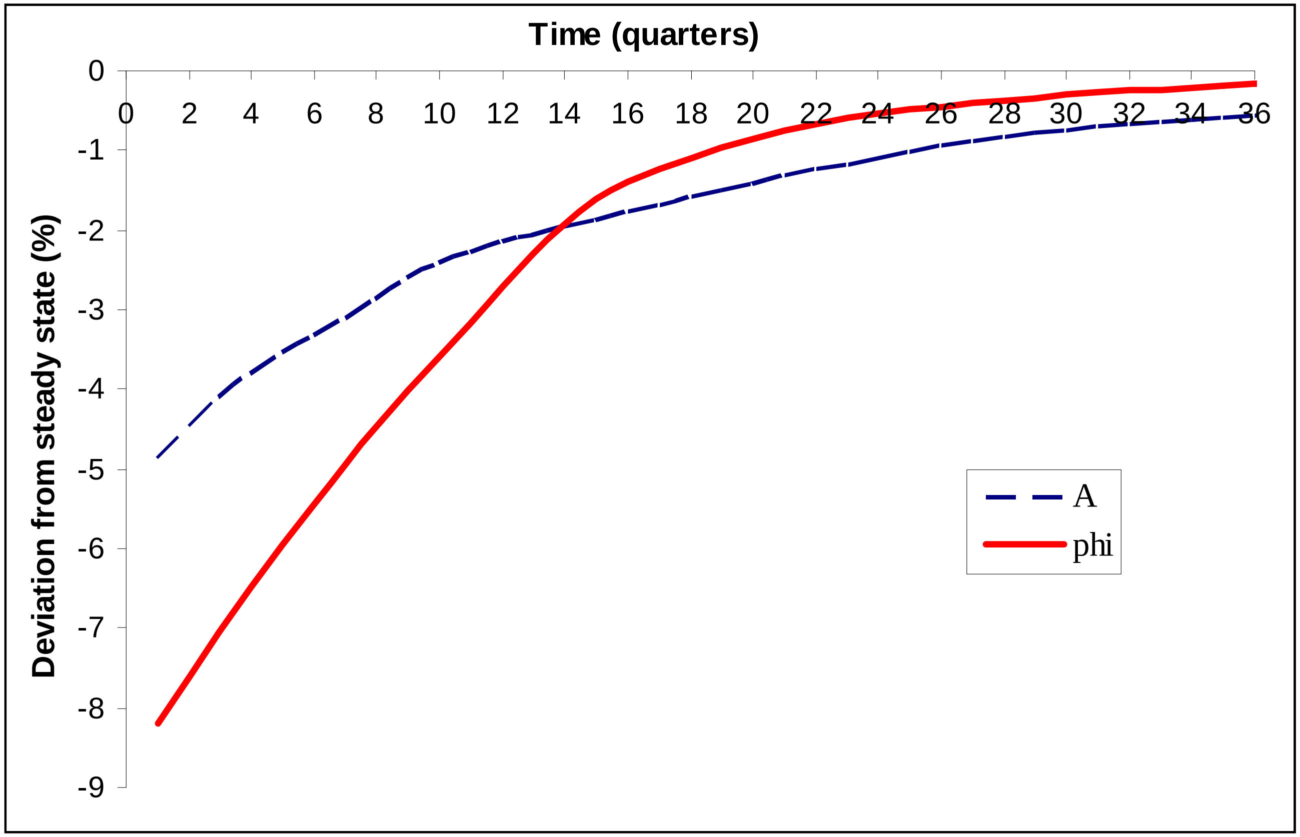

Figure 3.1. Negative shocks to ϕ and A

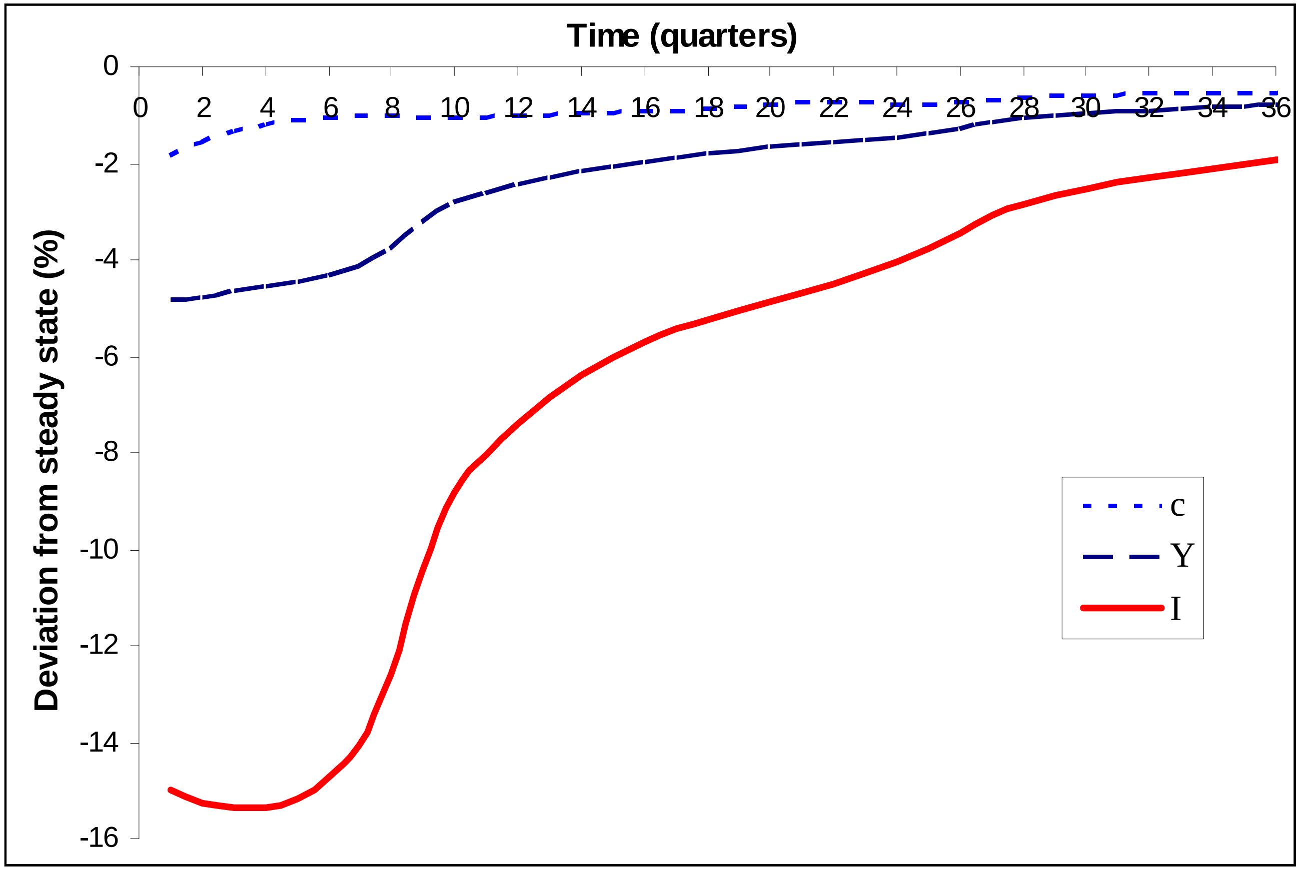

Figure 3.2. Investment, output and consumption

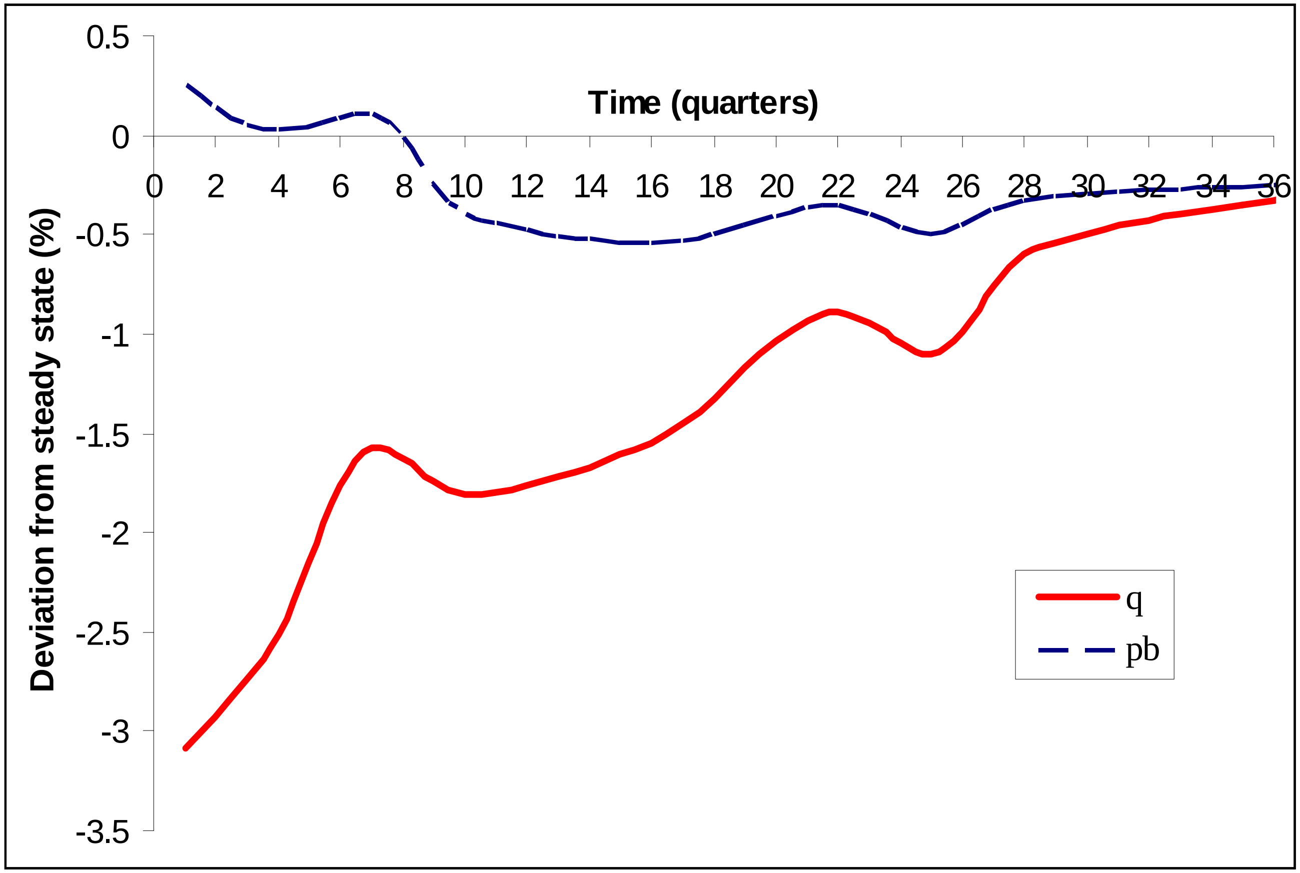

Figure 3.3. Equity price and bond price

5. Conclusion

- Liquidity shocks to the asset market
  - can amplify and propagate business cycles:
    they generate large and persistent changes in macro variables
  - cannot be the primary driving force of business cycles:
    negative liquidity shocks generate equity price boom!
- Other shocks are needed to reduce equity price in recessions
- Problem exists in ALL models where equity financing is important

- Did the Fed policy help?
  - It might have;
  - but it may not be the cure
- Important to model why asset market liquidity fluctuates

2.2. A household's maximization problem

- aggregate state (K, Z), $\quad Z = (A, \phi)$
  A: total factor productivity; $\quad \phi$: equity resaleability

- household's value function: $v(s, b; K, Z)$

- household's choices of:
  - an entrepreneur's
    investment i, consumption c^e, portfolio: (s^e_{+1}, b^e_{+1})
  - quantities per member: c, s_{+1}, b_{+1}
  - a worker's labor supply: ℓ

A household's maximization problem (cont'd):

$$v(s,b;K,Z) = \max \left\{ \begin{array}{l} \pi\ u(c^e) + (1-\pi)\left[U(c^w) - h(\ell)\right] \\ \\ +\beta\ \mathbb{E}v(s_{+1}, b_{+1}; K_{+1}, Z_{+1}) \end{array} \right\}$$

(i) household's resource constraint:

$$\left[\begin{array}{l} (q-1)\pi i + rs + (1-\pi)w\ell \\ \\ +q(\sigma s - s_{+1}) + (b - p_b b_{+1}) - \tau \end{array} \right] \geq c$$

(ii) equity liquidity constraint: $s^e_{+1} \geq (1-\theta)i + (1-\phi)\sigma s$

(iii) an entrepreneur's resource constraint:

$$rs + q(i + \sigma s - s^e_{+1}) + (b - p_b\, b^e_{+1}) - \tau \geq i + c^e$$

New liquidity constraint (eliminate s^e_{+1} from above):

$$\underbrace{(r + \phi\, \sigma q)\, s}_{\substack{\text{rental and} \\ \text{resale}}} + \underbrace{(b - p_b\, b^e_{+1})}_{\substack{\text{adjust} \\ \text{liquid assets}}} - \tau \geq \underbrace{(1 - \theta q)\, i}_{\substack{\text{downpayment} \\ \text{on investment}}} + c^e$$

Price of liquid assets:

$$p_b = \beta \, \mathbb{E}\left[\frac{U'(c^w_{+1})}{U'(c^w)}\left(1+\pi\lambda^e_{+1}\right)\right]$$

Equity price:

$$q = \beta \, \mathbb{E}\left\{\frac{U'(c^w_{+1})}{U'(c^w)}\left[\begin{array}{l} r_{+1}+\sigma q_{+1} \\ +\pi\lambda^e_{+1}\left(r_{+1}+\phi_{+1}\sigma q_{+1}\right) \end{array}\right]\right\}$$

Equity premium: $\frac{r_{+1}+\sigma q_{+1}}{q} - \frac{1}{p_b}$

Compute a recursive equilibrium:

- Step 1: given asset price functions $(q, p_b)(K, Z)$,
  firm's optimal conditions $\Longrightarrow$ factor prices;

  household's optimization $\Longrightarrow$ policy functions

- Step 2:
  asset pricing equations $\Longrightarrow$ new functions $T(q, p_b)(K, Z)$

- Iterate to find a fixed point of mapping T

parameter	value	calibration target
β: discount factor	0.992	exogenously chosen
ρ: risk aversion	2	exogenously chosen
u_0: utility parameter	44.801	capital stock/annual output $= 3.32$
h_0: scale in disutility	17.005	hours of work $= 0.25$
η: curvature of disutility	1.5	labor supply elasticity $1/(\eta - 1) = 2$

parameter	value	calibration target
α: capital share	0.36	labor income share $(1-\alpha) = 0.64$
σ: capital survival	0.981	annual investment/capital $= 0.076$
A^*: steady state TFP	1	normalization
δ_A: TFP persistence	0.95	persistence in TFP $= 0.95$
g: gov't spending	0.193	government spending/GDP $= 0.18$