Why is the shape of the Laffer curve for consumption tax different from that for labor income tax?

Kazuki Hiraga
Tokai University
Kengo Nutahara
Senshu University/The Canon Institute for Global Studies

September 13, 2018

Why is the shape of the Laffer curve for consumption tax different from that for labor income tax?

Kazuki Hiraga [*]   Kengo Nutahara [† ‡]

September 13, 2018

---

[*]Tokai University. Address: 4-1-1 Kitakaname, Hiratsuka, Kanagawa 259-1292, Japan.

[†]Senshu University. Address: 2-1-1 Higashimira, Tama-ku, Kawasaki, Kanagawa 214-8580, Japan. E-mail: nutti@isc.senshu-u.ac.jp, Tel.: 044-911-1230, Fax: 044-911-1231.

[‡]The Canon Institute for Global Studies. Address: 5-1-1 Marunouchi, Chiyoda-ku, Tokyo 100-6511, Japan.

Abstract

Recent studies on Laffer curves (tax revenue curves) find that the Laffer curve for consumption tax might not be hump-shaped, but monotonically increasing, whereas the Laffer curve for labor income tax is hump-shaped. This study investigates the cause of this difference in the shapes of two Laffer curves, by decomposing the effects of an increase in a tax rate on the tax base as a product of two parts: (i) the effects on the relative price of leisure (RPL), and (ii) the substitution and income effects. It is shown that the first effect with respect to the consumption tax rate is completely different from that with respect to the labor income tax rate, while the second effect is common among the taxes and depends on the functional form of the utility. The elasticity of the RPL from an increasing consumption tax rate is at most 1, whereas it can be infinity as the tax rate increases in the case of labor income tax.

Keywords: Laffer curve; tax revenue; consumption tax; labor income tax; relative price of leisure

JEL classifications: E62; H20; H30

1 Introduction

In the public finance literature, it is well known that the equivalence between consumption and labor income taxes holds. This implies that any equilibrium allocation (like output, labor supply, and consumption) in an economy with a consumption tax can also be achieved in an economy with a labor income tax. However, the equivalence on the tax revenues between consumption and labor income taxes does not hold. Recent studies on Laffer curves (tax revenue curves) that use dynamic general equilibrium models find that the Laffer curve for consumption tax is not hump-shaped, but monotonically increasing, whereas the Laffer curve for labor income tax is hump-shaped.

The main objective of this study is to investigate the theoretical cause of this difference in the shapes of the Laffer curves for consumption and labor income taxes. For this purpose, this study decomposes the effects of an increase in a tax rate on the tax base as a product of two parts: (i) the effects on the relative price of leisure (RPL), and (ii) the substitution and income effects. It is shown that the first effect with respect to the consumption tax rate is completely different from that with respect to the labor income tax rate; meanwhile, the second effect is common among the taxes and depends on the functional form of the utility. The elasticity of the RPL from an increasing consumption tax rate is at most 1, whereas it can be infinity as the tax rate increases in the case of labor income tax. If a 1% increase in the tax rate reduces the tax base by more than 1%, an increase in the tax rate will reduce the tax revenue. This result then implies that the shape of the Laffer curve for consumption tax depends on the functional form of utility, whereas the Laffer curve for labor income tax is hump-shaped.

The baseline model is a simple static frictionless general equilibrium model. However, the assumption of a static economy is not crucial. The main result is applicable to a dynamic model with investment. The introduction of frictions and distortions might change the shape of Laffer curves. For example, Baydur and Yilmaz (2017) show that the Laffer curve for a value-added tax (VAT) can be hump-shaped once one considers

that financial institutions are exempt from the VAT. However, from the viewpoint of this study, frictions and distortions can be collectively understood as an additional source that changes the RPL.

This study closely relates to the literature on the Laffer curves that uses dynamic general equilibrium models. Schmitt-Grohè and Uribe (1997), Trabandt and Uhlig (2011, 2013), Holter, Krueger, and Stepanchuk (2014), and Nutahara (2015) each found that the Laffer curve for labor income tax is hump-shaped, whereas Trabandt and Uhlig (2011, 2013), Holter, Krueger, and Stepanchuk (2014), Kobayashi (2014), Nutahara (2015), and Fève, Matheron, and Sahuc (2018) found that the Laffer curve for consumption tax is not hump-shaped. The main result points to why the two Laffer curves of these studies are different. The current study also relates to that of Hiraga and Nutahara (2018), who found that the shape of the Laffer curve for consumption tax is sensitive to the functional form of the utility. While the concern of Hiraga and Nutahara (2018) differs from that of the current study, their findings are consistent with the results herein.

The current study also relates to the literature on tax structure and economic activity; this body of literature includes the study of Diamond and Mirrlees (1971). More recently, Knellera, Bleaney, and Gemmell (1999), Arnold (2008), and Johansson et al. (2008) empirically found that in terms of economic growth, consumption tax is less harmful than personal income tax. Hansen and Imrohoroglu (2018) found that in Japan, the replacement of income tax with consumption tax improves output and welfare; they determined this through the use of a dynamic general equilibrium model. According to the main finding of the current study, the elasticities of the RPL with respect to consumption is at most 1, whereas that with respect to labor income tax could be infinity. This could serve as a possible theoretical explanation for their results.[1]

The remainder of the current paper is organized as follows. Section 2 introduces the

---

[1] In a distorted economy, a labor income tax might be less harmful than a consumption tax, as Nishiyama and Smetters (2005) show. However, their result is beyond the scope of this paper.

baseline model. Section 3 shows the main results, and Section 4 concludes.

2 Model

For analytical simplicity, a simple static and frictionless general equilibrium model with representative households and competitive firms is considered.

The representative households supply labor n to firms and earn a real wage rate w. They also receive government transfers s. Let τ^c and τ^n denote consumption and labor income taxes, respectively, and suppose that $\tau^c \geq 0$ and $0 \leq \tau^n < 1$. The budget constraint of households is

$$(1 + \tau^c)\, c \leq (1 - \tau^n)wn + s, \tag{1}$$

where c denotes consumption.

The firms are perfectly competitive. Their production function is linear in labor input:

$$y = n, \tag{2}$$

where y denotes output.

The government budget constraint is

$$s = \tau^c c + \tau^n wn. \tag{3}$$

Since there is no investment and there are no government purchases, the resource constraint of this closed economy is

$$y = c. \tag{4}$$

Finally, the utility function of the household is $U(c, n)$, and the standard assumptions are applied: $U_c > 0$, $U_{cc} < 0$, $U_n < 0$, and $U_{nn} \leq 0$.

3 Main Results

It is useful to consider necessary conditions for hump-shaped tax revenue curves, as in Lemma 1.

Lemma 1. *A necessary condition for a hump-shaped consumption tax revenue curve for consumption tax is*

$$\frac{\partial c/c}{\partial \tau^c/\tau^c} < -1.$$

A necessary condition for a hump-shaped labor income tax revenue curve for labor income tax is

$$\frac{\partial n/n}{\partial \tau^n/\tau^n} < -1.$$

Proof. In the case of the consumption tax revenue curve, it is obvious, because the consumption tax revenue is $\tau^c c$. It is also obvious in the case of the labor income tax revenue curve, because the labor income tax revenue is $\tau^n wn$ and the equilibrium wage rate w is independent of τ^c and τ^n. ($w = 1$ in this static economy.) □

To understand the difference in the shape of the tax revenue curves for consumption and labor income taxes, it is useful to focus on the consumption–labor supply choice optimization condition of the household:

$$-\frac{U_n}{U_c} = RPL, \tag{5}$$

where *RPL* is *the relative price of leisure*, which is given by

$$RPL \equiv \frac{1 - \tau^n}{1 + \tau^c} w. \tag{6}$$

An increase in consumption tax, τ^c, or labor income tax, τ^n, reduces the RPL, and it reduces both consumption and labor supply through the substitution effects. Proposition 1 decomposes the changes in consumption and labor supply as a product of these two effects.

Proposition 1. *The equilibrium elasticity of consumption with respect to τ^c is given by*

$$\left| \frac{\partial c/c}{\partial \tau^c/\tau^c} \right| = \left| \frac{\partial RPL/RPL}{\partial \tau^c/\tau^c} \right| \times \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1}.$$

The analog of labor supply with respect to τ^n is given by

$$\left| \frac{\partial n/n}{\partial \tau^n/\tau^n} \right| = \left| \frac{\partial RPL/RPL}{\partial \tau^n/\tau^n} \right| \times \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1}.$$

Proof. See Appendix A. □

The first parts ($\left| \frac{\partial RPL/RPL}{\partial \tau^c/\tau^c} \right|$ and $\left| \frac{\partial RPL/RPL}{\partial \tau^n/\tau^n} \right|$) are the negative effects on the RPL of increasing tax rates; meanwhile, the second parts ($\left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1}$) are the sum of substitution and income effects. The substitution and income effects depend on the functional form of utility, and it is common among the cases of τ^c and τ^c.[2] The key difference comes from the elasticities of the RPL with respect to τ^c and τ^n, as in Proposition 2.

Proposition 2. *The elasticities of the RPL with respect to τ^c and τ^n are given by*

$$\left| \frac{\partial RPL/RPL}{\partial \tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c},$$

$$\left| \frac{\partial RPL/RPL}{\partial \tau^n/\tau^n} \right| = \frac{\tau^n}{1 - \tau^n}.$$

These are increasing in τ^c and τ^n, respectively. Their limits are given by

$$\lim_{\tau^c \to \infty} \left| \frac{\partial RPL/RPL}{\partial \tau^c/\tau^c} \right| = 1,$$

$$\lim_{\tau^n \to 1} \left| \frac{\partial RPL/RPL}{\partial \tau^n/\tau^n} \right| = \infty.$$

Proof. These are obvious by $RPL = \frac{1-\tau^n}{1+\tau^c} w$. □

---

[2]It is assumed that an increase in τ^c (τ^n) reduces consumption (labor supply). That is,

$$-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} > 0.$$

Proposition 2 shows that the limits of the elasticity of the RPL to τ^c is completely different from that to τ^n. The elasticity of the RPL by increasing τ^c is at most 1, whereas it becomes infinity in the case of τ^n.

By Propositions 1 and 2, the following proposition is obtained.

Proposition 3. *The limits of elasticity of equilibrium consumption with respect to τ^c and those of equilibrium labor supply with respect to τ^n are given, respectively, by*

$$\lim_{\tau^c \to \infty} \left| \frac{\partial c/c}{\partial \tau^c/\tau^c} \right| = \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1},$$

$$\lim_{\tau^n \to 1} \left| \frac{\partial n/n}{\partial \tau^n/\tau^n} \right| = \infty,$$

as long as $\left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]$ is finite.

Proof. These are obvious by Propositions 1 and 2. □

Proposition 3 shows that the condition for a hump-shaped Laffer curve in Lemma 1 is always satisfied in the case of τ^n, whereas in the case of τ^c it depends on the functional form of utility. This finding closely relates to that of Hiraga and Nutahara (2018), who found that the shape of the tax revenue curve for consumption tax is very sensitive to the functional form of the utility. The following examples are in line with their finding.

Example 1: Additively separable utility   The necessary condition for a hump-shaped Laffer curve for consumption in Lemma 1 can be satisfied if the utility function is additively separable in consumption and labor supply with constant relative risk aversion and constant labor supply elasticity given by

$$U = \frac{c^{1-\eta} - 1}{1 - \eta} - \kappa \frac{n^{1+\lambda}}{1 + \lambda}.$$

The relative risk aversion is η, and the inverse of labor supply elasticity is λ. This implies that the elasticities of both consumption and leisure are increasing in τ^c and τ^n. In this

8

case,

$$-\frac{cU_{cc}}{U_c} = \eta, \qquad \frac{nU_{nn}}{U_n} = \lambda, \qquad \frac{cU_{cn}}{U_n} = 0, \quad -\frac{nU_{cn}}{U_c} = 0,$$

and then,

$$\left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1} = \frac{1}{\eta + \lambda}.$$

Therefore, if $\eta + \lambda < 1$, the tax revenue curve for consumption tax can be hump-shaped.

Example 2: King–Plossor–Rebelo utility with constant labor supply elasticity The necessary condition in Lemma 1 is never satisfied if the utility function is of a King–Plossor–Rebelo type with constant labor supply elasticity given by

$$U = \frac{1}{1-\eta}\left\{c^{1-\eta}\left[1 - \kappa(1-\eta)n^{1+\lambda}\right]^\eta - 1\right\},$$

as employed by Shimer (2009) and Trabandt and Uhlig (2011). In this case,

$$-\frac{cU_{cc}}{U_c} = \eta, \qquad \frac{nU_{nn}}{U_n} = \lambda + \frac{(1-\eta)^2\kappa(1+\lambda)n}{1 - \kappa(1-\eta)n^{1+\lambda}},$$

$$\frac{cU_{cn}}{U_n} = 1 - \eta, \qquad -\frac{nU_{cn}}{U_c} = \frac{\eta\kappa(1-\eta)(1+\lambda)n^{1+\lambda}}{1 - \kappa(1-\eta)n^{1+\lambda}}.$$

Hiraga and Nutahara (2018) show that n is decreasing in τ^c and $n \to 0$ as $\tau^c \to \infty$. Then, $\left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1}$ is increasing in τ^c and

$$\lim_{\tau^c \to \infty}\left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1} = \frac{1}{1 + \lambda}.$$

This implies that $\left|\frac{\partial c/c}{\partial \tau^c/\tau^c}\right|$ cannot be greater than 1.

The main results are obtained in a simple static general equilibrium model. However, they are applicable to a dynamic version of the model with investment. Appendix B shows that Propositions 1, 2, and 3 hold even in a dynamic general equilibrium model at a steady state.

9

4 Concluding Remarks

Recent studies on Laffer curves (tax revenue curves) using dynamic general equilibrium models find that the Laffer curve for consumption tax might not be hump-shaped, but monotonically increasing, whereas the Laffer curve for labor income tax is hump-shaped. The current study investigated the cause of this difference in shape between the two Laffer curves by decomposing the effects of an increase in tax rate on the tax base as a product of two parts: (i) the effects on the relative price of leisure (RPL), and (ii) the substitution and income effects. It has been shown that the first effect with respect to the consumption tax rate is completely different from that with respect to the labor income tax rate, while the second effect is common among the taxes and depends on the functional form of the utility. The elasticity of the RPL by increasing the consumption tax rate is at most 1, whereas in the case of labor income tax it can be infinity as the tax rate increases.

Tax structure has received significant attention in recent years, as both political and academic issues. These issues include the effects of tax structure on tax revenue, economic efficiency, and economic performance. The main finding of this paper contributes to our knowledge in these areas.

References

[1] Arnold, J., 2008. Do tax structures affect aggregate economic growth? Empirical evidence from a panel of OECD countries. OECD Economics Department Working Paper No. 643.

[2] Baydur, I., and Yilmaz, F. 2017. VAT treatment of financial institutions: Implications for the real economy. Paper presented at Singapore Economic Review Conference 2017.

[3] Diamond, P. A., and Mirrlees, J. A., 1971. Optimal taxation and public production I: Production efficiency. American Economic Review, 61(1), 8–27.

[4] Fève, P., Matheron, J., and Sahuc, J. 2018. The horizontally S-shaped Laffer curve. Journal of European Economic Association, 16(3), 857–893.

[5] Hansen, G., and Imrohoroglu, S., 2018. Replacing income taxation with consumption taxation in Japan. Journal of the Japanese and International Economies, 48, 15–28.

[6] Hiraga, K., and Nutahara, K., 2018. Fragility in modeling consumption tax revenue. Mimeo.

[7] Holter, Hans A., Krueger, D., and Stepanchuk, S., 2014. How does tax progressivity and household heterogeneity affect Laffer curves? NBER Working Paper No. 20688.

[8] Johansson, A., Heady, C., Arnold, J., Brys, B., and Vartia, L., 2008. Tax and economic growth. OECD Economics Department Working Paper No. 620.

[9] Knellera, R., Bleaney, M. F., and Gemmell, N., 1999. Fiscal policy and growth: Evidence from OECD countries. Journal of Public Economics, 74(2), 171–190.

[10] Kobayashi, K., 2014. There is no natural debt limit with consumption tax. RIETI Discussion Paper 14–E–043.

[11] Nishiyama, S., and Smetters, K., 2005. Consumption taxes and economic efficiency with idiosyncratic wage shocks. Journal of Political Economy, 113(5), 1088–1115.

[12] Nutahara, K., 2015. Laffer curves in Japan. Journal of the Japanese and International Economies, 36, 56–72.

11

[13] Shimer, R., 2009. Convergence in macroeconomics: The labor wedge. American Economic Journal: Macroeconomics, 1(1), 280–297.

[14] Schmitt-Grohè, S., and Uribe, M., 1997. Balanced-budget rules, distortionary taxes, and aggregate instability. Journal of Political Economy, 105(5), 976–1000.

[15] Trabandt, M, and Uhlig, H., 2011. The Laffer curve revisited. Journal of Monetary Economics, 58, 305–327.

[16] Trabandt, M, and Uhlig, H., 2013. How do Laffer curves differ across countries? NBER Chapters: Fiscal Policy After the Financial Crisis. Editors A. Alesina and F. Giavazzi, University of Chicago Press.

Appendix

A. Proof of Proposition 1

Since $y = c = n$ at equilibrium in this static economy, then

$$\frac{\partial y/y}{\partial \tau^j/\tau^j} = \frac{\partial c/c}{\partial \tau^j/\tau^j} = \frac{\partial n/n}{\partial \tau^j/\tau^j},$$

for $j = c$ and n.

The consumption–leisure choice optimization condition is

$$-\frac{U_n}{U_c} = RPL.$$

Taking the first-order derivative by τ^j, one can obtain

$$-U_{cn}\frac{\partial c}{\partial \tau^j} - U_{nn}\frac{\partial n}{\partial \tau^j} = \left[U_{cc}\frac{\partial c}{\partial \tau^c} + U_{cn}\frac{\partial n}{\partial \tau^j} \right] RPL + U_c\frac{\partial RPL}{\partial \tau^j}$$

since c and n depend on τ^j. This condition is rewritten as

$$- cU_{cn}\frac{\partial c/c}{\partial\tau^j/\tau^j} - nU_{nn}\frac{\partial n/n}{\partial\tau^j/\tau^j} = \left[cU_{cc}\frac{\partial c/c}{\partial\tau^j/\tau^j} + nU_{cn}\frac{\partial n/n}{\partial\tau^j/\tau^j}\right]RPL + U_cRPL\frac{\partial RPL/RPL}{\partial\tau^j/\tau^j}$$

$$\Longleftarrow \quad - cU_{cn}\frac{\partial y/y}{\partial\tau^j/\tau^j} - nU_{nn}\frac{\partial y/y}{\partial\tau^j/\tau^j} = \left[cU_{cc}\frac{\partial y/y}{\partial\tau^j/\tau^j} + nU_{cn}\frac{\partial y/y}{\partial\tau^j/\tau^j}\right]RPL + U_cRPL\frac{\partial RPL/RPL}{\partial\tau^j/\tau^j}$$

$$\Longleftarrow \quad \frac{\partial y/y}{\partial\tau^j/\tau^j}\left[-cU_{cn} - nU_{nn} - cU_{cc}RPL - nU_{cn}RPL\right] = U_cRPL\frac{\partial RPL/RPL}{\partial\tau^j/\tau^j}$$

$$\Longleftarrow \quad \frac{\partial y/y}{\partial\tau^j/\tau^j}\left[-\frac{cU_{cn}}{U_c} - \frac{nU_{nn}}{U_c} - \frac{cU_{cc}}{U_c}RPL - \frac{nU_{cn}}{U_c}RPL\right] = RPL\frac{\partial RPL/RPL}{\partial\tau^j/\tau^j}.$$

Because the consumption–leisure choice optimization condition is rewritten as

$$-\frac{1}{U_c} = \frac{RPL}{U_n},$$

one obtains

$$\frac{\partial y/y}{\partial\tau^j/\tau^j}\left[\frac{cU_{cn}}{U_n}RPL + \frac{nU_{nn}}{U_n}RPL - \frac{cU_{cc}}{U_c}RPL - \frac{nU_{cn}}{U_c}RPL\right] = RPL\frac{\partial RPL/RPL}{\partial\tau^j/\tau^j}$$

$$\Longleftarrow \quad \frac{\partial y/y}{\partial\tau^j/\tau^j}\left[\frac{cU_{cn}}{U_n} + \frac{nU_{nn}}{U_n} - \frac{cU_{cc}}{U_c} - \frac{nU_{cn}}{U_c}\right] = \frac{\partial RPL/RPL}{\partial\tau^j/\tau^j}.$$

Finally, the elasticities are given by

$$\frac{\partial y/y}{\partial\tau^j/\tau^j} = \frac{\partial c/c}{\partial\tau^j/\tau^j} = \frac{\partial n/n}{\partial\tau^j/\tau^j} = \frac{\partial RPL/RPL}{\partial\tau^j/\tau^j} \times \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1}$$

B. Robustness in a dynamic model

Consider a dynamic version of the model in Section 2. In this case, the capital stock k_t and investment i_t are introduced to the households' problem.

The representative households' problem is

$$\max_{\{c_t,k_{t+1},i_t,n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\beta^t U(c_t, n_t)$$

$$\text{s.t. } (1 + \tau_t^c)c_t + i_t \le (1 - \tau_t^n)w_t n_t + r_t k_t + s_t,$$

$$k_{t+1} = (1 - \delta)k_t + i_t,$$

where r_t denotes the rental rate of capital.

The firms' problem is

$$\max_{k_t, n_t} \left[y_t - r_t k_t - w_t n_t \right],$$

$$\text{s.t. } y_t = k_t^{\alpha} n_t^{1-\alpha}.$$

The government transfers its tax revenue to the households:

$$s_t = \tau^c c_t + \tau^n w_t n_t.$$

Finally, the resource constraint of this closed economy is

$$c_t + i_t = y_t.$$

Even in this dynamic model, the key equation is still the consumption–labor supply choice condition:

$$-\frac{U_n(t)}{U_c(t)} = RPL_t,$$

where $RPL_t = (1 - \tau_t^n)/(1 + \tau_t^c)w_t$.

Propositions 1, 2, and 3 in Section 3 do hold in this dynamic model at a steady state, given the following Lemmas 2 and 3.

Lemma 2. *The steady-state real wage rate, w, is independent of the consumption tax rate, τ^c, and the labor income tax rates, τ^n.*

Proof. The Euler equation at a steady state is given by

$$1 = \beta \left[1 - \delta + \alpha \frac{y}{k} \right].$$

Then, it is obvious that the capital–output ratio k/y is independent of τ^c and τ^n. It implies the labor–output ratio n/y is also independent of τ^c and τ^n at a steady state, because the production function is rewritten as

$$1 = \left(\frac{k}{y}\right)^{\alpha} \left(\frac{n}{y}\right)^{1-\alpha}.$$

14

Therefore, the steady-state real wage rate w is also independent of τ^c and τ^n, since it is given by

$$w = (1 - \alpha)\frac{y}{n}.$$

$\square$

Lemma 3. *At a steady state, the elasticities of output, consumption, and labor supply with respect to consumption and labor income taxes have the same value:*

$$\frac{\partial y/y}{\partial \tau^j/\tau^j} = \frac{\partial c/c}{\partial \tau^j/\tau^j} = \frac{\partial n/n}{\partial \tau^j/\tau^j}$$

for $j = c$ and n.

Proof. As in the proof for Lemma 2, the labor–output ratio n/y is independent of τ^c and τ^n. Then, it is obvious that $\frac{\partial y/y}{\partial \tau^j/\tau^j} = \frac{\partial n/n}{\partial \tau^j/\tau^j}$ for $j = c$ and n.

Because the capital–output ratio k/y is independent of τ^c and τ^n, then the investment–output ratio i/y is also independent of τ^c and τ^n. Because of the resource constraint of this economy $c + i = y$, the consumption–output ratio c/y is also independent of τ^c and τ^n. Therefore, $\frac{\partial y/y}{\partial \tau^j/\tau^j} = \frac{\partial c/c}{\partial \tau^j/\tau^j}$ for $j = c$ and n $\square$