Appendices: Fragility in modeling consumption tax revenue

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Abstract

This appendix contains supplemental results on "Fragility in modeling consumption tax revenue" by Kazuki Hiraga and Kengo Nutahara.

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Contents

A Static economy

A.1 Static model with consumption and labor income taxes

Representative households supply labor n to firms and earn wage rate w. They also receive government transfers s. Let τ^c and τ^n denote consumption and labor income taxes, respectively. The budget constraint of households is

$$(1 + \tau^c)\, c \leq (1 - \tau^n) wn + s,$$

where c denotes consumption.

The firms are perfectly competitive. Their production function is

$$y = n,$$

where y denotes output.

The government budget constraint is

$$s + g = T,$$

where g is government consumption. Total tax revenue T is defined as

$$T = \tau^c c + \tau^n wn.$$

Since there is no investment, the resource constraint of this closed economy is

$$y = c + g.$$

Three types of utility functions are considered.

$$\begin{aligned}
U^{KPR} &= \frac{1}{1-\eta}\left\{c^{1-\eta}\left[1 - \kappa(1-\eta)n^{1+\lambda}\right]^{\eta} - 1\right\}. \\
U^{AS} &= \frac{c^{1-\eta} - 1}{1-\eta} - \kappa n^{1+\lambda}. \\
U^{GG} &= \frac{1}{1-\eta}\left\{\left(c - \kappa n^{1+\lambda}\right)^{1-\eta} - 1\right\}.
\end{aligned}$$

A.2 Total tax revenue curve for consumption tax

In the main text, the consumption tax revenue curve is investigated. Here, we investigate the total tax revenue curve, which also includes labor income tax revenue.

In the case of the total tax revenue curve, fiscal policy schemes are the following.

Definition A. 1. *Scheme (1′): Tax revenue is used as a lump-sum transfer to households.*

$$s = \tau^c c + \tau^n wn, \qquad g = 0$$

Definition A. 2. *Scheme (2′): Tax revenue is used as government consumption.*

$$g = \tau^c c + \tau^n wn, \qquad s = 0$$

A.2.1 Scheme (1′): Tax revenue is used as a lump-sum transfer

Propositions A.1, A.2, and A.3 are the analogues of Propositions 1, 2, and 3 in the main text.

Proposition A. 1. *Suppose that the utility function is KPR; U^{KPR}. The total tax revenue curve for consumption tax under Scheme (1′) is monotonically increasing. The total tax revenue curve is unbounded except for $\lambda = 0$.*

Proof. By the optimization condition for the consumption–labor choice,

$$\eta(1+\lambda)\left(\frac{\kappa c n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}\right) = \frac{1-\tau^n}{1+\tau^c}w,$$

it follows that

$$c = (1-\tau^n)^{1/(1+\lambda)}\left[\tau^c\eta\kappa(1+\lambda) + \kappa(\eta\lambda+1) - \tau^n\kappa(1-\eta)\right]^{-1/(1+\lambda)}.$$

The total tax revenue is

$$\begin{aligned} T &= \tau^c c + \tau^n wn \\ &= (\tau^c + \tau^n)(1-\tau^n)^{1/(1+\lambda)}\left[\tau^c\eta\kappa(1+\lambda) + \kappa(\eta\lambda+1) - \tau^n\kappa(1-\eta)\right]^{-1/(1+\lambda)}. \end{aligned}$$

Then,

$$\frac{dT}{d\tau^c} = (1-\tau^n)^{1/(1+\lambda)}\left[\tau^c\eta\kappa(1+\lambda)+\kappa(\eta\lambda+1)-\tau^n\kappa(1-\eta)\right]^{-1/(1+\lambda)-1}$$
$$\times\left[\tau^c\eta\kappa\lambda+\kappa(\eta\lambda+1-\tau^n)\right] > 0.$$

Since the consumption tax revenue is bounded if and only if $\lambda = 0$, as in Proposition 1 in the main text, the total tax revenue is also bounded if and only if $\lambda = 0$. □

Proposition A. 2. *Suppose that the utility function is additively separable, U^{AS}. The total tax revenue curve for consumption tax under Scheme (1′) is hump shaped if and only if $\tau^n < \eta + \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$. If $\eta + \lambda \leq \tau^n < 1$, the total tax revenue curve for consumption tax is monotonically decreasing. Otherwise, the total tax revenue curve for consumption tax is monotonically increasing. The total tax revenue is bounded if and only if $\eta + \lambda \leq 1$. Otherwise, it is unbounded.*

Proof. The optimization condition for the consumption–labor choice,

$$\kappa(1+\lambda)c^\eta n^\lambda = \frac{1-\tau^n}{1+\tau^c}w,$$

indicates that

$$c = \left[\frac{\kappa(1+\lambda)}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\eta+\lambda}}.$$

The total tax revenue is

$$\begin{aligned} T &= \tau^c c + \tau^n wn \\ &= (\tau^c+\tau^n)\left[\frac{\kappa}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\eta+\lambda}}, \end{aligned}$$

and thus,

$$\frac{dT}{d\tau^c} = \left[\frac{\kappa(1+\lambda)}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\eta+\lambda}-1}\left(\frac{\kappa(1+\lambda)}{1-\tau^n}\right)\left[\tau^c\left(\frac{\eta+\lambda-1}{\eta+\lambda}\right)+\frac{\eta+\lambda-\tau^n}{\eta+\lambda}\right].$$

Suppose that $\eta + \lambda = 1$; then, $\frac{dT}{d\tau^c} > 0$.

Suppose that $\eta + \lambda \neq 1$; then,

$$\frac{dT}{d\tau^c} = \left[\frac{\kappa(1+\lambda)}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\eta+\lambda}-1}\left(\frac{\kappa(1+\lambda)}{1-\tau^n}\right)\left(\frac{\eta+\lambda-1}{\eta+\lambda}\right)\left[\tau^c - \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}\right].$$

Suppose that $\eta + \lambda > 1$; then, $\frac{dT}{d\tau^c} > 0$.

Suppose that $\eta + \lambda < 1$.

$\frac{dT}{d\tau^c} > 0$ for $\tau^c < \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$,
$\frac{dT}{d\tau^c} = 0$ for $\tau^c = \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$, and
$\frac{dT}{d\tau^c} < 0$ for $\tau^c > \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$.
Since the consumption tax revenue is bounded if and only if $\eta + \lambda \leq 1$, as in Proposition 2, the total tax revenue is also bounded if and only if $\eta + \lambda \leq 1$. □

Proposition A. 3. *Suppose that the utility function is additively separable, U^{GHH}. The total tax revenue curve for consumption tax under Scheme (1′) is hump shaped if and only if $\tau^n < \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\lambda-\tau^n}{1-\lambda}$. If $\lambda \leq \tau^n < 1$, the total tax revenue curve for consumption tax is monotonically decreasing. Otherwise, the total tax revenue curve for consumption tax is monotonically increasing. The total tax revenue is bounded if and only if $\lambda \leq 1$. Otherwise, it is unbounded.*

Proof. The optimization condition for the consumption–labor choice,

$$\kappa(1+\lambda)n^\lambda = \frac{1-\tau^n}{1+\tau^c}w,$$

indicates that

$$c = \left[\frac{\kappa(1+\lambda)}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\lambda}}.$$

The total tax revenue is

$$\begin{aligned} T &= \tau^c c + \tau^n wn \\ &= (\tau^c + \tau^n)\left[\frac{\kappa}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\lambda}}, \end{aligned}$$

and thus,

$$\frac{dT}{d\tau^c} = \left[\frac{\kappa(1+\lambda)}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\lambda}-1}\left(\frac{\kappa(1+\lambda)}{1-\tau^n}\right)\left[\tau^c\left(\frac{\lambda-1}{\lambda}\right)+\frac{\lambda-\tau^n}{\lambda}\right].$$

Suppose that $\lambda = 1$; then, $\frac{dT}{d\tau^c} > 0$.

Suppose that $\lambda \neq 1$; then,

$$\frac{dT}{d\tau^c} = \left[\frac{\kappa(1+\lambda)}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\lambda}-1}\left(\frac{\kappa(1+\lambda)}{1-\tau^n}\right)\left(\frac{\lambda-1}{\lambda}\right)\left[\tau^c-\frac{\lambda-\tau^n}{1-\lambda}\right].$$

Suppose that $\lambda > 1$; then, $\frac{dT}{d\tau^c} > 0$.

Suppose that $\lambda < 1$.

$\frac{dT}{d\tau^c} > 0$ for $\tau^c < \frac{\lambda-\tau^n}{1-\lambda}$,
$\frac{dT}{d\tau^c} = 0$ for $\tau^c = \frac{\lambda-\tau^n}{1-\lambda}$, and
$\frac{dT}{d\tau^c} < 0$ for $\tau^c > \frac{\lambda-\tau^n}{1-\lambda}$.
Since the consumption tax revenue is bounded if and only if $\lambda \leq 1$, as in Proposition 3 of the main text, the total tax revenue is also bounded if and only if $\lambda \leq 1$. □

As for the consumption tax revenue curve, the total tax revenue curve for consumption tax is monotonically increasing. in the case of the KPR utility function. In the case of the additively separable utility function U^{AS}, the condition $\eta + \lambda < 1$ is necessary for a hump-shaped total tax revenue curve for consumption tax. Note that the total tax revenue curve might be monotonically decreasing if the labor income tax rate is sufficiently high ($\eta + \lambda \leq \tau^n$). This is interpreted as the case in which there is a negative peak consumption tax rate that maximizes the total tax revenue ($\tau^c = \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$) of the hump-shaped total tax revenue curve. In the case of GHH utility, the condition $\lambda < 1$ is still necessary for a hump-shaped tax revenue curve. The total tax revenue curve also might be monotonically decreasing if the labor income tax rate is sufficiently high ($\lambda \leq \tau^n$), since the peak tax rate is negative.

A.2.2 Scheme (2′): Tax revenue is used as government consumption

Under Scheme (2′), since $g = \tau^c c + \tau^n wn$ and $n = c + g$, the following is obtained:

$$n = \frac{1 + \tau^c}{1 - \tau^n} c.$$

Propositions A.4, A.5, and A.6 are analogues of Propositions 4, 5, and 6 in the main text.

Proposition A. 4. *Suppose that the utility function is KPR, U^{KPR}. The total tax revenue curve for consumption tax under Scheme (2′) is monotonically increasing. The total tax revenue is bounded.*

Proof. By the optimization condition for the consumption–labor choice, the following is obtained:

$$c = \{[\eta(1 + \lambda) + (1 - \eta)]\kappa\}^{-\frac{1}{1+\lambda}} (1 - \tau^n)(1 + \tau^c)^{-1}.$$

Then,

$$\begin{aligned} n &= \frac{1 + \tau^c}{1 - \tau^n} c \\ &= \{[\eta(1 + \lambda) + (1 - \eta)]\kappa\}^{-\frac{1}{1+\lambda}} \end{aligned}$$

This implies that labor supply is independent from both τ^c and τ^n. The total tax revenue is

$$\begin{aligned} T &= \tau^c c + \tau^n wn \\ &= \tau^c c + \tau^n \{[\eta(1 + \lambda) + (1 - \eta)]\kappa\}^{-\frac{1}{1+\lambda}} \end{aligned}$$

Therefore, the shape of the total tax revenue curve is the same as that of the consumption tax revenue curve.

Since the consumption tax revenue is bounded if and only if $\lambda = 0$, as in Proposition 4 in the main text, the total tax revenue is also bounded if and only if $\lambda = 0$. □

Proposition A. 5. *Suppose that the utility function is additively separable, U^{AS}. The total tax revenue curve for consumption tax under Scheme (2′) is hump shaped if and only if*

$$(1+\lambda)\tau^n - \lambda < \eta < 1,$$

and the revenue is maximized at $\tau^c = \frac{(1+\lambda)\tau^n-(\eta+\lambda)}{\eta-1}$.
Otherwise, the total tax revenue curve is

- *monotonically increasing if $\eta > 1$ and $(1+\lambda)\tau^n - \lambda \leq \eta$;*
- *U shaped if $\eta > 1$ and $(1+\lambda)\tau^n - \lambda > \eta$;*
- *monotonically decreasing if $\eta = 1$ and $(1+\lambda)\tau^n - \lambda > \eta$;*
- *monotonically increasing if $\eta = 1$ and $(1+\lambda)\tau^n - \lambda < \eta$;*
- *flat if $\eta = 1$ and $(1+\lambda)\tau^n - \lambda = \eta$; and*
- *monotonically decreasing if $\eta < 1$ and $(1+\lambda)\tau^n - \lambda \geq \eta$.*

The total tax revenue is bounded if and only if $\eta \leq 1$. Otherwise, it is unbounded. Otherwise, the total tax revenue curve for consumption tax is monotonically increasing. The total tax revenue is bounded if and only if $\eta + \lambda \leq 1$. Otherwise, it is unbounded.

Proof. The optimization condition for the consumption–labor choice,

$$\kappa(1+\lambda)c^{\eta}n^{\lambda} = \frac{1-\tau^n}{1+\tau^c}w,$$

and $n = (1+\tau^c)/(1-\tau^n)c$ indicate that

$$c = [\kappa(1+\lambda)]^{-\frac{1}{\eta+\lambda}}(1-\tau^n)^{\frac{1+\lambda}{\eta+\lambda}}(1+\tau^c)^{-\frac{1+\lambda}{\eta+\lambda}}.$$

The total tax revenue is

$$\begin{aligned} T &= \tau^c c + \tau^n wn \\ &= [\kappa(1+\lambda)]^{-\frac{1}{\eta+\lambda}}(1-\tau^n)^{\frac{1+\lambda}{\eta+\lambda}}(1+\tau^c)^{-\frac{1+\lambda}{\eta+\lambda}}\left[\frac{\tau^c+\tau^n}{1-\tau^n}\right] \end{aligned}$$

and thus,

$$\frac{dT}{d\tau^c} = [\kappa(1+\lambda)]^{-\frac{1}{\eta+\lambda}} (1-\tau^n)^{\frac{1+\lambda}{\eta+\lambda}} (1+\tau^c)^{-\frac{1+\lambda}{\eta+\lambda}-1} \left[\frac{1}{1-\tau^n}\right] \times$$
$$\left\{\left[\frac{\eta-1}{\eta+\lambda}\right]\tau^c - \frac{1+\lambda}{\eta+\lambda}\tau^n + 1\right\}$$

Suppose that $\eta = 1$.

If $\tau^n < \frac{\eta+\lambda}{1+\lambda}$, then $dT/d\tau^c > 0$.

If $\tau^n > \frac{\eta+\lambda}{1+\lambda}$, then $dT/d\tau^c < 0$.

If $\tau^n = \frac{\eta+\lambda}{1+\lambda}$, then $dT/d\tau^c = 0$.

Suppose that $\eta \neq 1$. The following is obtained:

$$\frac{dT}{d\tau^c} = [\kappa(1+\lambda)]^{-\frac{1}{\eta+\lambda}} (1-\tau^n)^{\frac{1+\lambda}{\eta+\lambda}} (1+\tau^c)^{-\frac{1+\lambda}{\eta+\lambda}-1} \left[\frac{1}{1-\tau^n}\right] \times$$
$$\left[\frac{\eta-1}{\eta+\lambda}\right]\left\{\tau^c - \frac{(1+\lambda)\tau^n - (\eta+\lambda)}{\eta-1}\right\}.$$

Suppose that $\eta > 1$.

If $\tau^n < \frac{\eta+\lambda}{1+\lambda}$, then $\left|\frac{dT}{d\tau^c}\right| > 0$.

If $\tau^n > \frac{\eta+\lambda}{1+\lambda}$, then $\frac{dT}{d\tau^c}|_{\tau^c=0} < 0$ and $\frac{dT}{d\tau^c} > 0$ for $\tau^c > \frac{(1+\lambda)\tau^n-(\eta+\lambda)}{\eta-1}$.

Suppose $\eta < 1$.

If $\tau^n < \frac{\eta+\lambda}{1+\lambda}$, then $\frac{dT}{d\tau^c}|_{\tau^c=0} > 0$ and $\frac{dT}{d\tau^c} < 0$ for $\tau^c > \frac{(1+\lambda)\tau^n-(\eta+\lambda)}{\eta-1}0$.

If $\tau^n \geq \frac{\eta+\lambda}{1+\lambda}$, then $\frac{dT}{d\tau^c}$.

Finally, the condition $\tau^n < \frac{\eta+\lambda}{1+\lambda}$ is rewritten as

$$\tau^n < \frac{\eta+\lambda}{1+\lambda}$$
$$\iff \quad \eta > (1+\lambda)\tau^n - \lambda.$$

Since the consumption tax revenue is bounded if and only if $\eta \leq 1$, as in Proposition 3 in the main text, the total tax revenue is also bounded if and only if $\eta \leq 1$. □

Proposition A. 6. *Suppose that the utility function is additively separable, U^{GHH}. The total tax revenue curve for consumption tax under Scheme (2′) is hump shaped if and only if*

$$\lambda > \frac{\tau^n}{1-\tau^n}$$

and the revenue is maximized at $\tau^c = \lambda - (1+\lambda)\tau^n$. Otherwise, the total tax revenue curve is monotonically decreasing. The total tax revenue is bounded.

Proof. The optimization condition for the consumption–labor choice,

$$\kappa(1+\lambda)n^{\lambda} = \frac{1-\tau^n}{1+\tau^c}w,$$

and $n = (1+\tau^c)/(1-\tau^n)c$ indicate that

$$c = [\kappa(1+\lambda)]^{-\frac{1}{\lambda}}(1-\tau^n)^{\frac{1+\lambda}{\lambda}}(1+\tau^c)^{-\frac{1+\lambda}{\lambda}}.$$

The total tax revenue is

$$\begin{aligned} T &= \tau^c c + \tau^n wn \\ &= [\kappa(1+\lambda)]^{-\frac{1}{\lambda}}(1-\tau^n)^{\frac{1+\lambda}{\lambda}}(1+\tau^c)^{-\frac{1+\lambda}{\lambda}}\left[\frac{\tau^c+\tau^n}{1-\tau^n}\right] \end{aligned}$$

and thus,

$$\begin{aligned} \frac{dT}{d\tau^c} &= -[\kappa(1+\lambda)]^{-\frac{1}{\lambda}}(1-\tau^n)^{\frac{1+\lambda}{\lambda}}(1+\tau^c)^{-\frac{1+\lambda}{\lambda}-1}\left[\frac{1}{1-\tau^n}\right]\times\left\{\left[\frac{1}{\lambda}\right]\tau^c - \left(1-\frac{1+\lambda}{\lambda}\tau^n\right)\right\} \\ &= -[\kappa(1+\lambda)]^{-\frac{1}{\lambda}}(1-\tau^n)^{\frac{1+\lambda}{\lambda}}(1+\tau^c)^{-\frac{1+\lambda}{\lambda}-1}\left[\frac{1}{1-\tau^n}\right]\left(\frac{1}{\lambda}\right)(\tau^c - [\lambda-(1+\lambda)\tau^n]). \end{aligned}$$

If $\lambda - (1+\lambda)\tau^n \leq 0$, then $\frac{dT}{d\tau^c} < 0$.
If $\lambda - (1+\lambda)\tau^n > 0$, then
$\frac{dT}{d\tau^c} > 0$ for $\tau^c < \lambda - (1+\lambda)\tau^n$,
$\frac{dT}{d\tau^c} = 0$ for $\tau^c = \lambda - (1+\lambda)\tau^n$, and
$\frac{dT}{d\tau^c} < 0$ for $\tau^c > \lambda - (1+\lambda)\tau^n$.

Finally, the condition $\lambda - (1+\lambda)\tau^n >$ is rewritten as

$$\lambda > \frac{\tau^n}{1-\tau^n}.$$

The total tax revenue is also bounded. $\square$

A.3 Alternative fiscal policy schemes

Schemes (1) and (2) consider that all tax revenue is used as a lump-sum transfer or government consumption. Here, some relaxed versions are investigated. In this subsection, for the simplicity of analysis, the labor income tax rate is set to zero, and the consumption tax revenue curve is the point of focus.

The following two schemes are one of the analogues of Schemes (1) and (2).

Definition A. 3. *Scheme (1*): The ratio of government consumption to output, g/y, is constant and positive. The rest of tax revenue is used as a lump-sum transfer to households.*

$$s = \tau^c c - g, \qquad g/y = \bar{\phi}_{gy}$$

Definition A. 4. *Scheme (2*): The ratio of lump-sum transfer to output, s/y, is constant and positive. The rest of tax revenue is used as government consumption.*

$$g = \tau^c c - s, \qquad s/y = \bar{\phi}_{sy}$$

The following two schemes are other options, and are based on similar assumptions employed by Trabandt and Uhlig (2011).

Definition A. 5. *Scheme (1**): Government consumption, g, is constant and positive. The rest of tax revenue is used as a lump-sum transfer to households.*

$$s = \tau^c c - g, \qquad g = \bar{g}$$

Definition A. 6. *Scheme (2**): The lump-sum transfer, s, is constant and positive. The rest of tax revenue is used as government consumption.*

$$g = \tau^c c - s, \qquad\qquad s = \bar{s}$$

A.3.1 Scheme (1*): g/y is constant and changes in tax revenue are adjusted by a lump-sum transfer

By the resource constraint,

$$\frac{c}{y} + \frac{g}{y} = 1.$$

Then, c/y is constant (and independent from τ^c) under Scheme (1*). The following holds.

Remark A. 1. *The elasticity of consumption with respect to the consumption tax rate equals that of output:*

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = \left|\frac{dy/y}{d\tau^c/\tau^c}\right|.$$

Proposition A.7 is about the case of KPR utility.

Proposition A. 7. *Suppose that the utility function is KPR, U^{KPR}. The consumption tax revenue curve for consumption tax under Scheme (1*) is monotonically increasing. The consumption tax revenue is unbounded except for $\lambda = 0$.*

Proof. The optimization condition for the consumption–labor choice,

$$\eta(1+\lambda)\left\{\frac{\kappa c n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}\right\} = \frac{1}{1+\tau^c},$$

yields

$$y = (\kappa)^{-1/(1+\lambda)}\left[(1-\eta) + \left(\frac{c}{y}\right)\eta(1+\lambda)(1+\tau^c)\right]^{-1/(1+\lambda)}.$$

Since $\tilde{y} > 0$ for $\tau^c \geq 0$,

$$(1-\eta) + \left(\frac{c}{y}\right)\eta(1+\lambda) > 0.$$

By Remark A.1, it follows that

$$\frac{dc/c}{d\tau^c/\tau^c} = \frac{dy/y}{d\tau^c/\tau^c} = -\frac{\left(\frac{c}{y}\right)\eta\tau^c}{(1-\eta) + \left(\frac{c}{y}\right)\eta\,(1+\tau^c)\,(1+\lambda)}.$$

Letting

$$\Psi = (1-\eta) + \left(\frac{c}{y}\right)\eta\,(1+\tau^c)\,(1+\lambda) > 0,$$

it follows that

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = -\frac{1}{\Psi}\left\{(1-\eta) + \left(\frac{c}{y}\right)\eta(1+\lambda) + \left(\frac{c}{y}\right)\eta\tau^c\lambda\right\} \leq 0.$$

The consumption tax revenue is unbounded if $\lambda > 0$, since $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| > 0$. In the case of $\lambda = 0$, the consumption tax revenue is given by

$$\begin{aligned}\tau^c c &= \phi\tau^c y \\ &= (\kappa)^{-1/(1+\lambda)}\left[(1-\eta) + \left(\frac{c}{y}\right)\eta(1+\lambda)\right]^{-1/(1+\lambda)}\frac{\tau^c}{1+\tau^c},\end{aligned}$$

where $\phi = c/y$. This converges to $(\kappa)^{-1/(1+\lambda)}\left[(1-\eta) + \left(\frac{c}{y}\right)\eta(1+\lambda)\right]^{-1/(1+\lambda)}$. □

Proposition A.8 is about the case of additively separable utility.

Proposition A. 8. *Suppose that the utility function is additively separable, U^{AS}. The consumption tax revenue curve for consumption tax under Scheme (1*) is hump shaped if and only if $\eta + \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta-\lambda}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\eta + \lambda \leq 1$. Otherwise, it is unbounded.*

Proof. In the case of additively separable utility, the consumption–labor choice condition is

$$\begin{aligned} & \kappa(1+\lambda)c^{\eta}n^{\lambda} = \frac{1}{1+\tau^c}w \\ \iff \qquad & \kappa(1+\lambda)\left(\frac{c}{y}\right)^{\eta}\left(\frac{n}{y}\right)^{\lambda} y^{\eta+\lambda} = \frac{1}{1+\tau^c}, \end{aligned}$$

then

$$y = (1+\tau^c)^{-1/(\eta+\lambda)}\left[\frac{1}{\kappa(1+\lambda)}\left(\frac{c}{y}\right)^{-\eta}\right]^{1/(\eta+\lambda)}.$$

By Remark A.1, it follows that

$$\frac{dc/c}{d\tau^c/\tau^c} = \frac{dy/y}{d\tau^c/\tau^c} = -\frac{1}{\eta+\lambda}\cdot\frac{\tau^c}{1+\tau^c}.$$

Then,

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \frac{1}{\eta+\lambda}\cdot\frac{1}{1+\tau^c}\Big\{(1-\eta-\lambda)\tau^c - (\eta+\lambda)\Big\}.$$

Suppose $\eta+\lambda = 1$. In this case, $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 < 0$.

Suppose $\eta+\lambda \neq 1$. In this case,

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \frac{1-\eta-\lambda}{\eta+\lambda}\cdot\frac{1}{1+\tau^c}\Big\{\tau^c - \frac{\eta+\lambda}{1-\eta-\lambda}\Big\}.$$

If $\eta+\lambda \geq 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \geq 0$.

If $\eta+\lambda < 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \leq (\eta+\lambda)/(1-\eta-\lambda)$, and $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| > 1$ for $\tau^c > (\eta+\lambda)/(1-\eta-\lambda)$.

By the elasticity of consumption, it is obvious that the consumption tax revenue is bounded if $\eta+\lambda < 1$ and unbounded if $\eta+\lambda > 1$. In the case of $\eta+\lambda = 1$, the consumption tax revenue is

$$\begin{aligned} \tau^c c &= \phi\tau^c y \\ &= \phi\frac{\tau^c}{1+\tau^c}\left[\frac{1}{\kappa(1+\lambda)}\left(\frac{c}{y}\right)^{-\eta}\right]^{1/(\eta+\lambda)}. \end{aligned}$$

where $\phi = c/y$. This converges to $\phi\left[\frac{1}{\kappa(1+\lambda)}\left(\frac{c}{y}\right)^{-\eta}\right]^{1/(\eta+\lambda)}$ as $\tau^c \to \infty$. □

Proposition A.9 is about the case of GHH utility.

Proposition A. 9. *Suppose that the utility function is additively separable, U^{GHH}. The consumption tax revenue curve for consumption tax under Scheme (1*) is hump shaped if and only if $\lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\lambda}{1-\lambda}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\lambda \leq 1$. Otherwise, it is unbounded.*

Proof. In the case of additively separable utility, the consumption–labor choice condition is

$$
\begin{aligned}
& \kappa(1+\lambda)n^{\lambda} = \frac{1}{1+\tau^c}w \\
\Longleftrightarrow \qquad & \kappa(1+\lambda)\left(\frac{n}{y}\right)^{\lambda} y^{\lambda} = \frac{1}{1+\tau^c},
\end{aligned}
$$

then

$$
y = (1+\tau^c)^{-1/\lambda}\left[\frac{1}{\kappa(1+\lambda)}\right]^{1/\lambda}.
$$

By Remark A.1, it follows that

$$
\frac{dc/c}{d\tau^c/\tau^c} = \frac{dy/y}{d\tau^c/\tau^c} = -\frac{1}{\lambda}\cdot\frac{\tau^c}{1+\tau^c}.
$$

Then,

$$
\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \frac{1}{\lambda}\cdot\frac{1}{1+\tau^c}\Big\{(1-\lambda)\tau^c - \lambda\Big\}.
$$

Suppose $\lambda = 1$. In this case, $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 < 0$.

Suppose $\lambda \neq 1$. In this case,

$$
\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \frac{1-\lambda}{\lambda}\cdot\frac{1}{1+\tau^c}\Big\{\tau^c - \frac{\lambda}{1-\lambda}\Big\}.
$$

If $\lambda \geq 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \geq 0$.

If $\lambda < 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \leq \lambda/(1-\lambda)$, and $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| > 1$ for $\tau^c > \lambda/(1-\lambda)$.

By the elasticity of consumption, it is obvious that the consumption tax revenue is bounded if $\lambda < 1$ and unbounded if $\lambda > 1$. In the case of $\lambda = 1$, the consumption tax revenue is

$$\begin{aligned}\tau^c c &= \phi\tau^c y \\ &= \phi\frac{\tau^c}{1+\tau^c}\left[\frac{1}{\kappa(1+\lambda)}\right]^{1/+\lambda}.\end{aligned}$$

where $\phi = c/y$. This converges to $\phi\left[\frac{1}{\kappa(1+\lambda)}\right]^{1/\lambda}$ as $\tau^c \to \infty$. □

Note that Propositions A.7, A.8, and A.9 are the exactly same as Propositions 1, 2, and 3 in the main text. Therefore, Scheme (1*) is a natural extension of Scheme (1).

A.3.2 Scheme (2*): s/y is constant and changes in tax revenue are adjusted by government consumption

By the government budget constraint, it follows that

$$\frac{g}{y} + \frac{s}{y} = \tau^c\frac{c}{y}.$$

Since s/y is constant,

$$\frac{g}{y} = \tau^c\frac{c}{y} - constant.$$

The resource constraint is

$$\begin{aligned}&\frac{c}{y} + \frac{g}{y} = 1 \\ \Longleftrightarrow\quad &\frac{c}{y} + \tau^c\frac{c}{y} - constant = 1,\end{aligned}$$

and then,

$$(1+\tau^c)\frac{c}{y} = constant.$$

Therefore, the following remark holds.

Remark A. 2. *The elasticity of consumption with respect to consumption tax rate equals that of $y/(1+\tau^c)$:*

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = \left|\frac{d(1+\tau^c)^{-1}y/(1+\tau^c)^{-1}y}{d\tau^c/\tau^c}\right|$$

Proposition A. 10. *Suppose that the utility function is KPR, U^{KPR}. The consumption tax revenue curve for consumption tax under Scheme (2*) is monotonically increasing. The consumption tax revenue is bounded.*

Proof. By the consumption–labor choice condition, y is obtained

$$\begin{aligned}
& \eta(1+\lambda)\left\{\frac{\kappa c n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}\right\} = \frac{1}{1+\tau^c} \\
\Longleftrightarrow \quad & \eta(1+\lambda)\left\{\frac{\kappa\left(\frac{c}{y}\right)\left(\frac{n}{\tilde{y}}\right)^{\lambda}}{\tilde{y}^{-1-\lambda}-\kappa(1-\eta)\left(\frac{n}{y}\right)^{1+\lambda}}\right\} = \frac{1}{1+\tau^c} \\
\Longleftrightarrow \quad & y = (\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\left((1+\tau^c)\frac{c}{y}\right)\eta(1+\lambda)\right]^{-1/(1+\lambda)}.
\end{aligned}$$

Then, y is independent from τ^c.

By Remark A.2, the elasticity of $(1+\tau^c)^{-1}y$ is considered. Since

$$(1+\tau^c)^{-1}y = (1+\tau^c)^{-1}(\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\left((1+\tau^c)\frac{c}{y}\right)\eta(1+\lambda)(1+\tau^c)\right]^{-1/(1+\lambda)},$$

it follows that

$$\begin{aligned}
\frac{dc/c}{d\tau^c/\tau^c} &= \frac{d(1+\tau^c)^{-1}y/(1+\tau^c)^{-1}y}{d\tau^c/\tau^c} \\
&= -\frac{\tau^c}{1+\tau^c}.
\end{aligned}$$

$\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is monotonically increasing in τ^c. If $\tau^c = 0$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = 0$. As $\tau^c \to \infty$, $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| \to 1$. Therefore, the consumption tax revenue curve is monotonically increasing.

The boundedness is shown as follows. Letting $(1+\tau^c)c/y = \phi$ yields

$$\begin{aligned}
\tau^c c &= \tau^c \phi y (1+\tau^c)^{-1} \\
&= \phi\frac{\tau^c}{1+\tau^c}(\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\left((1+\tau^c)\frac{c}{y}\right)\eta(1+\lambda)(1+\tau^c)\right]^{-1/(1+\lambda)}.
\end{aligned}$$

As $\tau^c \to \infty$, it converges to $\phi(\kappa)^{-1/(1+\lambda)} \left[(1-\eta) + \left((1+\tau^c)\frac{c}{y}\right)\eta(1+\lambda)(1+\tau^c)\right]^{-1/(1+\lambda)}$.

$\square$

Proposition A. 11. *Suppose that the utility function is additively separable, U^{AS}. The consumption tax revenue curve for consumption tax under Scheme (2*) is hump shaped if and only if $\eta < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is unbounded if and only if $\eta \leq 1$. Otherwise, it is unbounded.*

Proof. By the consumption–labor choice condition,

$$\begin{aligned}
& \kappa(1+\lambda)c^\eta n^\lambda = \frac{1}{1+\tau^c} \\
\Longleftrightarrow \quad & \kappa(1+\lambda)y^{\eta+\lambda}\left(\frac{c}{y}\right)^\eta \left(\frac{n}{y}\right)^\lambda = \frac{1}{1+\tau^c} \\
\Longleftrightarrow \quad & y = (1+\tau^c)^{-(1-\eta)/(\eta+\lambda)} \left[\frac{1}{\kappa(1+\lambda)}\left((1+\tau^c)\frac{c}{y}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}.
\end{aligned}$$

By Remark A.2, the elasticity of $(1+\tau^c)y$ is considered. Since

$$(1+\tau^c)^{-1}y = (1+\tau^c)^{-(1+\lambda)/(\eta+\lambda)}\left[\frac{1}{\kappa(1+\lambda)}\left((1+\tau^c)\frac{c}{y}\right)^{-\eta}\right]^{1/(\eta+\lambda)},$$

it follows that

$$\begin{aligned}
\frac{dc/c}{d\tau^c/\tau^c} &= \frac{d(1+\tau^c)^{-1}y/(1+\tau^c)^{-1}y}{d\tau^c/\tau^c} \\
&= -\frac{1+\lambda}{\eta+\lambda}\cdot\frac{\tau^c}{1+\tau^c}.
\end{aligned}$$

$\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is monotonically increasing in τ^c. If $\tau^c = 0$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = 0$. If $\tau^c \to \infty$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| \to (1+\lambda)/(\eta+\lambda)$. Therefore, a necessary and sufficient condition for a hump-shaped consumption tax revenue curve is $\eta < 1$. The peak tax rate is

$$\tau^c_{max} = \frac{\eta+\lambda}{1-\eta}.$$

By the elasticity of consumption, it is obvious that the tax revenue is bounded if $\eta < 1$ and unbounded if $\eta > 1$. Suppose $\eta = 1$. Letting $(1+\tau^c)c/y = \phi$ yields

$$\begin{aligned}\tau^c c &= \tau^c \phi y (1+\tau^c)^{-1}\\ &= \phi \frac{\tau^c}{1+\tau^c}\left[\frac{1}{\kappa(1+\lambda)}\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-1}\right]^{1/(1+\lambda)}.\end{aligned}$$

As $\tau^c \to \infty$, it converges to $\phi\left[\frac{1}{\kappa(1+\lambda)}\left((1+\tau^c)\frac{c}{y}\right)^{-1}\right]^{1/(1+\lambda)}$.

□

Proposition A. 12. *Suppose that the utility function is GHH, U^{GHH}. The consumption tax revenue curve for consumption tax under Scheme (2*) is hump shaped if and only if $\eta < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is unbounded if and only if $\eta \leq 1$. Otherwise, it is unbounded.*

Proof. By the consumption–labor choice condition,

$$\begin{aligned}&& \kappa(1+\lambda)n^\lambda &= \frac{1}{1+\tau^c}\\ \iff && \kappa(1+\lambda)y^\lambda\left(\frac{n}{y}\right)^\lambda &= \frac{1}{1+\tau^c}\\ \iff && y &= (1+\tau^c)^{-1/\lambda}\left[\frac{1}{\kappa(1+\lambda)}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}.\end{aligned}$$

By Remark A.2, the elasticity of $(1+\tau^c)y$ is considered. Since

$$(1+\tau^c)^{-1}y = (1+\tau^c)^{-(1+\lambda)/\lambda}\left[\frac{1}{\kappa(1+\lambda)}\right]^{1/\lambda},$$

it follows that

$$\begin{aligned}\frac{dc/c}{d\tau^c/\tau^c} &= \frac{d(1+\tau^c)^{-1}y/(1+\tau^c)^{-1}y}{d\tau^c/\tau^c}\\ &= -\frac{1+\lambda}{\lambda}\cdot\frac{\tau^c}{1+\tau^c}.\end{aligned}$$

$\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is monotonically increasing in τ^c. If $\tau^c = 0$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = 0$. If $\tau^c \to \infty$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| \to (1+\lambda)/\lambda$. Therefore, the tax revenue curve is hump shaped. The peak tax rate is

$$\tau^c_{max} = \lambda.$$

The tax revenue is bounded.

□

A.3.3 Scheme (1**): government consumption g is constant and changes in tax revenue are adjusted by a lump-sum transfer

In the case of KPR utility, the consumption tax revenue curve is monotonically increasing, but tax revenue is bounded under Scheme (1**), as in the following proposition.

Proposition A. 13. *Suppose that the utility function is KPR, U^{KPR}. The consumption tax revenue curve for consumption tax under Scheme (1**) is monotonically increasing. The consumption tax revenue is bounded.*

Proof. By the production function and the resource constraint, it follows that

$$y = n = c + g.$$

Then, the consumption–labor choice condition is

$$\begin{aligned}
& \eta(1+\lambda)\frac{\kappa c n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}} = \frac{1}{1+\tau^c} \\
\iff \quad & \eta(1+\lambda)\frac{\kappa c (c+g)^{\lambda}}{1-\kappa(1-\eta)(c+g)^{1+\lambda}} = \frac{1}{1+\tau^c} \\
\iff \quad & \eta(1+\lambda)\kappa c (c+g)^{\lambda} = \frac{1-\kappa(1-\eta)(c+g)^{1+\lambda}}{1+\tau^c}
\end{aligned}$$

Taking the total derivatives yields

$$
\begin{aligned}
& \eta(1+\lambda)\kappa\left[(c+g)^{\lambda}\frac{dc}{d\tau^c}+\lambda c(c+g)^{\lambda-1}\frac{dc}{d\tau^c}\right] \\
& \quad = -\frac{(1+\tau^c)(1+\lambda)\kappa(1-\eta)(c+g)^{\lambda}\frac{dc}{d\tau^c}+[1-\kappa(1-\eta)(c+g)^{1+\lambda}]}{(1+\tau^c)^2}. \\
\iff \quad & (1+\tau^c)^2\eta(1+\lambda)\kappa(c+g)^{\lambda-1}\left[(c+g)+\lambda c\right]\frac{dc}{d\tau^c} \\
& \quad = -(1+\tau^c)(1+\lambda)\kappa(1-\eta)(c+g)^{\lambda}\frac{dc}{d\tau^c}-[1-\kappa(1-\eta)(c+g)^{1+\lambda}].
\end{aligned}
$$

By the consumption–labor choice condition, it follows that

$$
1-\kappa(1-\eta)(c+g)^{1+\lambda}=(1+\tau^c)\eta(1+\lambda)\kappa c(c+g)^{\lambda}.
$$

Then, the following holds

$$
\begin{aligned}
& (1+\tau^c)^2\eta(1+\lambda)\kappa(c+g)^{\lambda-1}\left[(c+g)+\lambda c\right]\frac{dc}{d\tau^c} \\
& \quad = -(1+\tau^c)(1+\lambda)\kappa(1-\eta)(c+g)^{\lambda}\frac{dc}{d\tau^c}-(1+\tau^c)\eta(1+\lambda)\kappa c(c+g)^{\lambda} \\
\iff \quad & (1+\tau^c)\eta\left[(c+g)+\lambda c\right]\frac{dc}{d\tau^c}=-(1-\eta)(c+g)\frac{dc}{d\tau^c}-\eta c(c+g) \\
\iff \quad & \left\{(1+\tau^c)\eta\left[(c+g)+\lambda c\right]+(1-\eta)(c+g)\right\}\frac{dc}{d\tau^c}=-\eta c(c+g) \\
\iff \quad & \frac{dc}{d\tau^c}=-\frac{\eta c(c+g)}{(1+\tau^c)\eta\left[(c+g)+\lambda c\right]+(1-\eta)(c+g)}.
\end{aligned}
$$

The elasticity of consumption is given by

$$
\begin{aligned}
\left|\frac{dc/c}{d\tau^c/\tau^c}\right| &= \frac{\eta\tau^c(c+g)}{(1+\tau^c)\eta\left[(c+g)+\lambda c\right]+(1-\eta)(c+g)} \\
&= \frac{\eta\tau^c(c+g)}{\eta(c+g)+\tau^c\eta(c+g)+(1+\tau^c)\eta\lambda c+(c+g)-\eta(c+g)} \\
&= \frac{\eta\tau^c(c+g)}{\tau^c\eta(c+g)+(1+\tau^c)\eta\lambda c+(c+g)} \\
&= \left[1+\frac{1}{\eta\tau^c}+\frac{1+\tau^c}{\tau^c}\cdot\frac{\lambda c}{c+g}\right]^{-1} \\
&= \left[1+\frac{1}{\eta\tau^c}+\frac{1+\tau^c}{\tau^c}\cdot\frac{\lambda}{1+g/c}\right]^{-1}.
\end{aligned}
$$

$\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is increasing in τ^c. If $\tau^c = 0$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = 0$. If $\tau^c \to \infty$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| \to 1$, since $c \to 0$.

Therefore, the consumption tax revenue curve is monotonically increasing.

$c \to 0$ as $\tau^c \to \infty$, which is proved as follows. Since the elasticity of consumption is negative, as $\tau^c \to \infty$, c converges to a non-negative value $a \geq 0$. By the consumption–labor choice condition, it follows that

$$\begin{aligned} & \eta(1+\lambda)\kappa c(c+g)^{\lambda} = \frac{1-\kappa(1-\eta)(c+g)^{1+\lambda}}{1+\tau^c} \\ \iff \quad & c = \frac{1-\kappa(1-\eta)(c+g)^{1+\lambda}}{(1+\tau^c)\eta(1+\lambda)\kappa(c+g)^{\lambda}}. \end{aligned}$$

The left-hand side of this equation converges to a as $\tau^c \to \infty$, and the right-hand side converges to zero. Therefore, $c \to 0$.

The consumption tax revenue is given by

$$\tau^c c = \frac{1-\kappa(1-\eta)(c+g)^{1+\lambda}}{\eta(1+\lambda)\kappa(c+g)^{\lambda}} \frac{\tau^c}{1+\tau^c}.$$

As $\tau^c \to \infty$, consumption tax revenue converges to $\frac{1-\kappa(1-\eta)g^{1+\lambda}}{\eta(1+\lambda)\kappa g^{\lambda}}$.

□

The result of Proposition A.13 is consistent with the result of Trabandt and Uhlig (2011). They report that the slope of the tax revenue curve for consumption tax converges to zero as $\tau^c \to \infty$ by numerical simulation under a similar fiscal policy scheme (although their model is dynamic).

Proposition A. 14. *Suppose that the utility function is additively separable, U^{AS}. The consumption tax revenue curve for consumption tax under Scheme (1**) is hump shaped if and only if $\eta < 1$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\eta \leq 1$. Otherwise, it is unbounded.*

Proof. By the resource constraint and the production function, it follows that

$$y = n = c + g.$$

In the case of additively separable utility, the consumption–labor choice condition is

$$\begin{aligned} & \kappa(1+\lambda)c^{\eta}n^{\lambda} = \frac{1}{1+\tau^c}w \\ \Longleftrightarrow \qquad & \kappa(1+\lambda)c^{\eta}(c+g)^{\lambda} = \frac{1}{1+\tau^c} \end{aligned}$$

Taking the total derivatives yields

$$\frac{dc}{d\tau^c} = -\kappa^{-1}(1+\lambda)^{-1}c^{1-\eta}(c+g)^{1-\lambda}\left[\eta(c+g)+\lambda c\right]^{-1}\frac{1}{(1+\tau^c)^2}.$$

By the consumption–labor choice condition, it follows that

$$c^{-\eta} = \kappa(1+\lambda)(c+g)^{\lambda}(1+\tau^c),$$

and then,

$$\begin{aligned} \frac{dc}{d\tau^c} &= -c(c+g)(1+\tau^c)\left[\eta(c+g)+\lambda c\right]^{-1}\cdot\frac{1}{(1+\tau^c)^2} \\ &= -c\frac{c+g}{\eta(c+g)+\lambda c}\cdot\frac{1}{1+\tau^c}. \end{aligned}$$

The elasticity of consumption is

$$\begin{aligned} \left|\frac{dc/c}{d\tau^c/\tau^c}\right| &= \frac{c+g}{\eta(c+g)+\lambda c}\times\frac{\tau^c}{1+\tau^c} \\ &= \frac{1}{\eta+\lambda\frac{c}{c+g}}\times\frac{\tau^c}{1+\tau^c} \\ &= \frac{1}{\eta+\lambda\frac{1}{1+g/c}}\times\frac{\tau^c}{1+\tau^c}. \end{aligned}$$

$|\frac{dc/c}{d\tau^c/\tau^c}|$ is increasing in τ^c. If $\tau^c = 0$, then $|\frac{dc/c}{d\tau^c/\tau^c}| = 0$. If $\tau^c \to \infty$, then $|\frac{dc/c}{d\tau^c/\tau^c}| = 1/\eta$, since $c \to 0$. Therefore, $\eta < 1$ is a necessary and sufficient condition for a hump-shaped consumption tax revenue curve.

$c \to 0$ as $\tau^c \to \infty$, which is proved as follows. Since $\frac{dc}{d\tau^c} < 0$, c converges to a non-negative value $a \geq 0$. By the consumption–labor choice condition, it follows that

$$c = [\kappa(1+\lambda)]^{-1/\eta}(c+g)^{-\lambda/\eta}(1+\tau^c)^{-1/\eta}.$$

The left-hand side converges to a, and the right-hand side converges to zero. Therefore, $c \to 0$.

By the elasticity of consumption, it is obvious that the consumption tax revenue is bounded if $\eta < 1$, and unbounded if $\eta > 1$. Suppose $\eta = 1$. The consumption tax revenue is given by

$$\tau^c c = [\kappa(1+\lambda)]^{-1}(c+g)^{-\lambda}\frac{\tau^c}{1+\tau^c}.$$

This converges to $[\kappa(1+\lambda)]^{-1}g^{-\lambda}$ as $\tau^c \to \infty$.

□

Note that the condition for a hump-shaped consumption tax revenue curve is $\eta < 1$, not $\eta + \lambda < 1$, under Scheme (1^{**}). This is because g/y increases as τ^c increases, and represents downward pressure on consumption (negative income effect).

Proposition A. 15. *Suppose that the utility function is additively separable, U^{GHH}. The consumption tax revenue curve for consumption tax under Scheme (1^{**}) is hump shaped. The consumption tax revenue is bounded.*

Proof. By the resource constraint and the production function, it follows that

$$y = n = c + g.$$

In the case of additively separable utility, the consumption–labor choice condition is

$$\begin{aligned} & \kappa(1+\lambda)n^\lambda = \frac{1}{1+\tau^c}w \\ \iff \quad & \kappa(1+\lambda)(c+g)^\lambda = \frac{1}{1+\tau^c} \end{aligned}$$

Taking the total derivatives yields

$$\frac{dc}{d\tau^c} = -\kappa^{-1}(1+\lambda)^{-1}\lambda(c+g)^{1-\lambda}\frac{1}{(1+\tau^c)^2} < 0.$$

By the consumption–labor choice condition, it follows that

$$\kappa(1+\lambda)(c+g)^{\lambda}(1+\tau^c) = 1,$$

and then,

$$\frac{dc}{d\tau^c} = -\frac{\lambda(c+g)}{1+\tau^c}.$$

The elasticity of consumption is

$$\begin{aligned}\left|\frac{dc/c}{d\tau^c/\tau^c}\right| &= \frac{\lambda(c+g)}{c} \times \frac{\tau^c}{1+\tau^c} \\ &= \lambda(1+g/c) \times \frac{\tau^c}{1+\tau^c}.\end{aligned}$$

$|\frac{dc/c}{d\tau^c/\tau^c}|$ is increasing in τ^c, since $\frac{dc}{d\tau^c} < 0$. If $\tau^c = 0$, then $|\frac{dc/c}{d\tau^c/\tau^c}| = 0$. Because $g/c > 0$, it is obvious that $|\frac{dc/c}{d\tau^c/\tau^c}| > 1$ for sufficiently high τ^c. Therefore, the consumption tax revenue curve is hump shaped.

□

A.3.4 Scheme (2**): Transfer s is constant and changes in tax revenue are adjusted by government consumption

Proposition A. 16. *Suppose that the utility function is KPR, U^{KPR}. The consumption tax revenue curve for consumption tax under Scheme (2**) is monotonically increasing. The consumption tax revenue is bounded.*

Proof. By the government budget constraint, it follows that

$$y = n = c + g = (1+\tau^c)c - s.$$

By the consumption–labor choice condition, the following is obtained:

$$
\begin{aligned}
& \eta(1+\lambda)\frac{\kappa c n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}} = \frac{1}{1+\tau^c} \\
\iff \quad & \eta(1+\lambda)\frac{\kappa c[(1+\tau^c)c-s]^{\lambda}}{1-\kappa(1-\eta)[(1+\tau^c)c-s]^{1+\lambda}} = \frac{1}{1+\tau^c} \\
\iff \quad & \eta(1+\lambda)\kappa c[(1+\tau^c)c-s]^{\lambda} = \frac{1-\kappa(1-\eta)[(1+\tau^c)c-s]^{1+\lambda}}{1+\tau^c}.
\end{aligned}
$$

Taking the total derivatives yields

$$
\begin{aligned}
& \eta(1+\lambda)\kappa\left[\frac{dc}{d\tau^c}[(1+\tau^c)c-s]^{\lambda} + c\lambda[(1+\tau^c)c-s]^{\lambda-1}[c+(1+\tau^c)\frac{dc}{d\tau^c}]\right] \\
& = \frac{-(1+\tau^c)\kappa(1-\eta)(1+\lambda)[(1+\tau^c)c-s]^{\lambda}[c+(1+\tau^c)\frac{dc}{d\tau^c}] - \left\{1-\kappa(1-\eta)[1+\tau^c)c-s)]^{1+\lambda}\right\}}{(1+\tau^c)^2} \\
\iff \quad & (1+\tau^c)^2\eta(1+\lambda)\kappa\left[\frac{dc}{d\tau^c}[(1+\tau^c)c-s]^{\lambda} + c\lambda[(1+\tau^c)c-s]^{\lambda-1}[c+(1+\tau^c)\frac{dc}{d\tau^c}]\right] \\
& = -(1+\tau^c)\kappa(1-\eta)(1+\lambda)[(1+\tau^c)c-s]^{\lambda}[c+(1+\tau^c)\frac{dc}{d\tau^c}] \\
& - \left\{1-\kappa(1-\eta)[(1+\tau^c)c-s)]^{1+\lambda}\right\} \\
\iff \quad & (1+\tau^c)^2\eta(1+\lambda)\kappa[(1+\tau^c)c-s]^{\lambda-1}\left[\frac{dc}{d\tau^c}[(1+\tau^c)c-s] + c\lambda[c+(1+\tau^c)\frac{dc}{d\tau^c}]\right] \\
& = -(1+\tau^c)\kappa(1-\eta)(1+\lambda)[(1+\tau^c)c-s]^{\lambda}[c+(1+\tau^c)\frac{dc}{d\tau^c}] \\
& - \left\{1-\kappa(1-\eta)[(1+\tau^c)c-s)]^{1+\lambda}\right\}.
\end{aligned}
$$

By the consumption–labor choice condition, it follows that

$$
1-\kappa(1-\eta)[(1+\tau^c)c-s)]^{1+\lambda} = \eta(1+\lambda)\kappa c[(1+\tau^c)c-s]^{\lambda}(1+\tau^c),
$$

and then,

$$
\begin{aligned}
\iff \quad & (1+\tau^c)^2\eta(1+\lambda)\kappa[(1+\tau^c)c-s]^{\lambda-1}\left[\frac{dc}{d\tau^c}[(1+\tau^c)c-s]+c\lambda[c+(1+\tau^c)\frac{dc}{d\tau^c}]\right] \\
& = -(1+\tau^c)\kappa(1-\eta)(1+\lambda)[(1+\tau^c)c-s]^{\lambda}[c+(1+\tau^c)\frac{dc}{d\tau^c}] \\
& -\eta(1+\lambda)\kappa c[(1+\tau^c)c-s]^{\lambda}(1+\tau^c) \\
\iff \quad & (1+\tau^c)\eta[(1+\tau^c)c-s]^{-1}\left[\frac{dc}{d\tau^c}[(1+\tau^c)c-s]+c\lambda[c+(1+\tau^c)\frac{dc}{d\tau^c}]\right] \\
& = -(1-\eta)[c+(1+\tau^c)\frac{dc}{d\tau^c}]-\eta c \\
\iff \quad & \eta\frac{dc}{d\tau^c}+\eta\lambda[(1+\tau^c)c-s]^{-1}c^2+\eta\lambda c(1+\tau^c)[(1+\tau^c)c-s]^{-1}\frac{dc}{d\tau^c} \\
& = -\frac{1}{1+\tau^c}\left[(1-\eta)[c+(1+\tau^c)\frac{dc}{d\tau^c}]+\eta c\right] \\
\iff \quad & \eta\frac{dc}{d\tau^c}+\eta\lambda[(1+\tau^c)c-s]^{-1}c^2+\eta\lambda c(1+\tau^c)[(1+\tau^c)c-s]^{-1}\frac{dc}{d\tau^c} \\
& = -\frac{1}{1+\tau^c}\left[(1-\eta)(1+\tau^c)\frac{dc}{d\tau^c}+c\right]. \\
\iff \quad & \frac{dc}{d\tau^c}\left\{\eta+\eta\lambda c(1+\tau^c)[(1+\tau^c)c-s]^{-1}+(1-\eta)\right\} = -\eta\lambda[(1+\tau^c)c-s]^{-1}c^2-\frac{c}{1+\tau^c} \\
\iff \quad & \frac{dc}{d\tau^c}\left\{1+\eta\lambda c(1+\tau^c)[(1+\tau^c)c-s]^{-1}\right\} = -\eta\lambda[(1+\tau^c)c-s]^{-1}c^2-\frac{c}{1+\tau^c} \\
\iff \quad & \frac{dc}{d\tau^c} = -\frac{\eta\lambda[(1+\tau^c)c-s]^{-1}c^2+\frac{c}{1+\tau^c}}{1+\eta\lambda c(1+\tau^c)[(1+\tau^c)c-s]^{-1}}.
\end{aligned}
$$

The elasticity of consumption is given by

$$
\begin{aligned}
\left|\frac{dc/c}{d\tau^c/\tau^c}\right| &= \frac{\eta\lambda c[(1+\tau^c)c-s]^{-1}+\frac{1}{1+\tau^c}}{1+\eta\lambda c(1+\tau^c)[(1+\tau^c)c-s]^{-1}}\times\tau^c \\
&= \frac{\tau^c}{1+\tau^c}\times\frac{\eta\lambda c(1+\tau^c)[(1+\tau^c)c-s]^{-1}+1}{1+\eta\lambda c(1+\tau^c)[(1+\tau^c)c-s]^{-1}} \\
&= \frac{\tau^c}{1+\tau^c}.
\end{aligned}
$$

Therefore, the consumption tax revenue curve is monotonically increasing.

The following results are obtained for boundedness. First, $c \to 0$ as $\tau^c \to \infty$. This is proved as follows. By the consumption–labor choice condition, it follows that

$$
\begin{aligned}
& 1-\kappa(1-\eta)[(1+\tau^c)c-s)]^{1+\lambda} = \eta(1+\lambda)\kappa c[(1+\tau^c)c-s]^{\lambda}(1+\tau^c) \\
\iff \quad & c = \frac{1-\kappa(1-\eta)[(1+\tau^c)c-s)]^{1+\lambda}}{\eta(1+\lambda)\kappa[(1+\tau^c)c-s]^{\lambda}} \times \frac{1}{1+\tau^c}.
\end{aligned}
$$

Suppose that $c \to a$ as $\tau^c \to \infty$. The left-hand side converges to a, and the right-hand side converges to zero. Therefore, $c \to 0$.

The consumption tax revenue is given by

$$
\tau^c c = \frac{1-\kappa(1-\eta)[(1+\tau^c)c-s)]^{1+\lambda}}{\eta(1+\lambda)\kappa[(1+\tau^c)c-s]^{\lambda}} \times \frac{\tau^c}{1+\tau^c}.
$$

Suppose that $\tau^c c \to z$ as $\tau^c \to \infty$. This implies

$$
z = \frac{1-\kappa(1-\eta)[z-s)]^{1+\lambda}}{\eta(1+\lambda)\kappa[z-s]^{\lambda}}.
$$

The limit z must satisfy this equation. It is obvious that z is finite (otherwise, the above equation does not hold). □

Proposition A. 17. *Suppose that the utility function is additively separable, U^{AS}. The consumption tax revenue curve for consumption tax under Scheme (2^{**}) is hump shaped if and only if $\eta < 1$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\eta \leq 1$. Otherwise, it is unbounded.*

Proof. By the government budget constraint, it follows that

$$
g = \tau^c c - s.
$$

By the production function and the resource constraint, it follows that

$$
y = n = c + g = (1+\tau^c)c - s.
$$

In the case of additively separable utility U^{AS}, the consumption–labor choice condition is

$$\kappa(1+\lambda)c^{\eta}n^{\lambda} = \frac{1}{1+\tau^c}w$$

$$\iff \qquad \kappa(1+\lambda)c^{\eta}[(1+\tau^c)c-s]^{\lambda} = \frac{1}{1+\tau^c}.$$

Taking the total derivatives yields

$$\kappa(1+\lambda)\left[\eta c^{\eta-1}[(1+\tau^c)c-s]^{\lambda}\frac{dc}{d\tau^c} + \lambda c^{\eta}[(1+\tau^c)c-s]^{\lambda-1}[c+(1+\tau^c)\frac{dc}{d\tau^c}]\right] = -\frac{1}{(1+\tau^c)^2}$$

$$\iff \qquad \kappa(1+\lambda)c^{\eta-1}[(1+\tau^c)c-s]^{\lambda-1}\left[\eta[(1+\tau^c)c-s]\frac{dc}{d\tau^c} + \lambda c[c+(1+\tau^c)\frac{dc}{d\tau^c}]\right] = -\frac{1}{(1+\tau^c)^2}.$$

By the consumption–labor choice condition, it follows that

$$c^{\eta} = (1+\tau^c)^{-1}\kappa^{-1}(1+\lambda)^{-1}[(1+\tau^c)c-s]^{-\lambda},$$

and then,

$$\iff \qquad c^{-1}[(1+\tau^c)c-s]^{-1}\left[\eta[(1+\tau^c)c-s]\frac{dc}{d\tau^c} + \lambda c[c+(1+\tau^c)\frac{dc}{d\tau^c}]\right] = -\frac{1}{1+\tau^c}$$

$$\iff \qquad \left[\eta[(1+\tau^c)c-s]\frac{dc}{d\tau^c} + \lambda c[c+(1+\tau^c)\frac{dc}{d\tau^c}]\right] = -\frac{c[(1+\tau^c)c-s]}{1+\tau^c}$$

$$\iff \qquad \left[\eta[(1+\tau^c)c-s]\frac{dc}{d\tau^c} + \lambda c(1+\tau^c)\frac{dc}{d\tau^c} + \lambda c^2\right] = -\frac{c[(1+\tau^c)c-s]}{1+\tau^c}$$

$$\iff \qquad [\eta[(1+\tau^c)c-s] + \lambda c(1+\tau^c)]\,\frac{dc}{d\tau^c} = -\frac{c[(1+\tau^c)c-s]}{1+\tau^c} - \lambda c^2$$

$$\iff \qquad \frac{dc}{d\tau^c} = -\left[\eta[(1+\tau^c)c-s] + \lambda c(1+\tau^c)\right]^{-1}\left[\frac{c[(1+\tau^c)c-s]}{1+\tau^c} + \lambda c^2\right].$$

The elasticity of consumption with respect to consumption tax rate is given by

$$\begin{aligned}
\left|\frac{dc/c}{d\tau^c/\tau^c}\right| &= \frac{\tau^c}{c}\left[\eta[(1+\tau^c)c-s] + \lambda c(1+\tau^c)\right]^{-1}\left[\frac{c[(1+\tau^c)c-s]}{1+\tau^c} + \lambda c^2\right] \\
&= \frac{\tau^c}{c}\left[\eta[(1+\tau^c)c-s] + \lambda c(1+\tau^c)\right]^{-1}\left[\frac{c[(1+\tau^c)c-s] + \lambda c^2(1+\tau^c)}{1+\tau^c}\right] \\
&= \frac{\tau^c}{1+\tau^c} \times \frac{[(1+\tau^c)c-s] + \lambda c(1+\tau^c)}{\eta[(1+\tau^c)c-s] + \lambda c(1+\tau^c)} \\
&= \frac{\tau^c}{1+\tau^c} \times \frac{[(1+\tau^c) - \frac{s}{c}] + \lambda(1+\tau^c)}{\eta[(1+\tau^c) - \frac{s}{c}] + \lambda(1+\tau^c)}
\end{aligned}$$

If $\eta > 1$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| < 1$.

If $\eta < 1$, then the limit of $\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is greater than $1/\eta$, since $c \to 0$. Therefore, the condition $\eta < 1$ is a necessary and sufficient condition for a hump-shaped consumption tax revenue curve.

$c \to 0$ as $\tau^c \to \infty$, which is proved as follows. By the consumption–labor choice condition, it follows that

$$c = (1+\tau^c)^{-1/\eta}\kappa^{-1/\eta}(1+\lambda)^{-1/\eta}[(1+\tau^c)c - s]^{-\lambda/\eta}.$$

Suppose that $c \to a$ as $\tau^c \to \infty$. The left-hand side of this equation converges to a, while the right-hand side converges to zero. Therefore, $c \to 0$.

By the elasticity of consumption, it is obvious that tax revenue is unbounded if $\eta > 1$, and it is bounded if $\eta < 1$. Suppose $\eta = 1$. The consumption tax revenue is given by

$$\tau^c c = \frac{\tau^c}{1+\tau^c}\kappa^{-1}(1+\lambda)^{-1}[(1+\tau^c)c - s]^{-\lambda}$$

Let $\tau^c c \to z$. The left-hand side converges to z, while the right-hand side converges to $\kappa^{-1}(1+\lambda)^{-1}[z-s]^{-\lambda}$. Therefore, z must satisfies the following:

$$z = \kappa^{-1}(1+\lambda)^{-1}[z-s]^{-\lambda}.$$

It is obvious that z is finite. (Otherwise, this equality does not hold.)

□

Proposition A. 18. *Suppose that the utility function is additively separable, U^{GHH}. The consumption tax revenue curve for consumption tax under Scheme (2^{**}) is hump shaped The consumption tax revenue is bounded.*

Proof. By the government budget constraint, it follows that

$$g = \tau^c c - s.$$

By the production function and the resource constraint, it follows that

$$y = n = c + g = (1 + \tau^c)c - s.$$

In the case of additively separable utility U^{AS}, the consumption–labor choice condition is

$$\begin{aligned} & \kappa(1+\lambda)n^\lambda = \frac{1}{1+\tau^c} w \\ \iff \qquad & \kappa(1+\lambda)[(1+\tau^c)c - s]^\lambda = \frac{1}{1+\tau^c}. \end{aligned}$$

Taking the total derivatives yields

$$\kappa(1+\lambda)\lambda[(1+\tau^c)c - s]^{\lambda-1}\left(c + (1+\tau^c)\frac{dc}{d\tau^c}\right) = -\frac{1}{(1+\tau^c)^2}.$$

By the consumption–labor choice condition, it follows that

$$\kappa(1+\lambda)[(1+\tau^c)c - s]^\lambda(1+\tau^c) = 1,$$

and then,

$$\begin{aligned} & \lambda[(1+\tau^c)c - s]^{-1}\left(c + (1+\tau^c)\frac{dc}{d\tau^c}\right) = -\frac{1}{1+\tau^c} \\ \iff \qquad & \frac{dc}{d\tau^c} = -\frac{1}{(1+\tau^c)^2}\lambda^{-1}[(1+\tau^c)c - s] - \frac{c}{1+\tau^c} < 0. \end{aligned}$$

The elasticity of consumption is given by

$$\begin{aligned} \left|\frac{dc/c}{d\tau^c/\tau^c}\right| &= \frac{\tau^c}{1+\tau^c} \times \frac{1}{\lambda} \times \frac{(1+\tau^c)c - s}{(1+\tau^c)c} + \frac{\tau^c}{1+\tau^c} \\ &= \frac{\tau^c}{1+\tau^c}\left[\frac{1}{\lambda}\left(1 - \frac{s}{(1+\tau^c)c}\right) + 1\right]. \end{aligned}$$

$\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ if $\tau^c = 0$. $\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is greater than one for sufficiently high τ^c because $(1+\tau^c)c - s = n > 0$. $\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is increasing in τ^c because

$$\frac{d\left|\frac{dc/c}{d\tau^c/\tau^c}\right|}{d\tau^c} = \frac{1}{(1+\tau^c)^2}\left[\frac{1}{\lambda}\left(1 - \frac{s}{(1+\tau^c)c}\right) + 1\right] + \frac{\tau^c}{1+\tau^c}\left(\frac{1}{\lambda}\right)\frac{s(c + 1 + \tau^c)}{(1+\tau^c)^2c^2} > 0.$$

Therefore, the consumption tax revenue curve is hump shaped.

□

B Dynamic economy à la Trabandt and Uhlig (2011)

B.1 Model

Representative households hold capital stock k_{t-1} and debt b_{t-1} as assets at the beginning of the period. They supply labor n_t and capital stock k_{t-1} to firms, and earn wage rate w_t, rental rate of capital d_t, and interest rate on debt R_t^b. They also receive government transfers s_t and transfers from abroad m_t. Let τ_t^c, τ_t^n, and τ_t^k denote the consumption tax, labor tax, and capital tax rates, respectively. The budget constraint of households is

$$(1+\tau_t^c)c_t + x_t + b_t \leq (1-\tau_t^n)w_t n_t + (1-\tau_t^k)(d_t-\delta)k_{t-1} + \delta k_{t-1} + R_t^b b_t + s_t + m_t,$$

where c_t denotes consumption, δ the depreciation rate of capital, and x_t investment. The capital stock evolves according to the following equation.

$$k_t = (1-\delta)k_{t-1} + x_t.$$

The firms are perfectly competitive. Their production function is

$$y_t = \xi^t k_{t-1}^{\theta} n_t^{1-\theta},$$

where ξ denotes the technology growth rate and θ the capital share of production. The profit maximization problem implies

$$w_t = (1-\theta)\frac{y_t}{n_t} \qquad \text{and}$$
$$d_t = \theta\frac{y_t}{k_{t-1}}.$$

The government budget constraint is

$$g_t + s_t + R_t^b b_{t-1} \leq b_t + T_t,$$

where g_t denotes government consumption. The total tax revenue T_t is defined as

$$T_t = \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k (d_t-\delta)k_{t-1}.$$

The resource constraint of this economy is

$$y_t = c_t + x_t + g_t - m_t.$$

The KPR utility function for this dynamic economy is

$$\mathbb{U}^{KPR} = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\eta} \left\{ c_t^{1-\eta} \left[1 - \kappa(1-\eta) n_t^{1+\lambda} \right]^{\eta} - 1 \right\} + v(g_t) \right],$$

where $v(\cdot)$ is an increasing function. The additively separable utility function is

$$\mathbb{U}^{AS} = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\eta} - 1}{1-\eta} - \kappa \psi^{t(1-\eta)} n_t^{1+\lambda} + v(g_t) \right].$$

The preference over labor supply shifts with the level of technology, $\psi^{t(1-\eta)}$, to guarantee the existence of a balanced growth path, as utilized by Erceg, Guerrieri, and Gust (2006). The GHH utility function is

$$\mathbb{U}^{GHH} = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\bar{\eta}} \left\{ \left(c_t - \kappa \psi^t n_t^{1+\lambda} \right)^{1-\bar{\eta}} - 1 \right\} + v(g_t) \right].$$

The preference shift parameter for the balanced growth path is $\psi^{t(1-\eta)}$ in this case.

The tax revenue curve for consumption tax is given by the relationship between the tax revenue and The tax rate on the balanced growth path. With regard to the use of tax revenue, the following two schemes are considered.

Definition A. 7. *Scheme (3): Government bond, b, grows at the balanced growth rate and g/y is constant. The other changes in tax revenue are adjusted by a lump-sum transfer to households.*

$$s = T - (R^b - 1)b - g, \qquad g/y = \phi_{gy}$$

Definition A. 8. *Scheme (4): b/y and s/y are constant. The other changes in tax revenue are adjusted by government consumption.*

$$g = T - (R^b - 1)b - s, \qquad s/y = \phi_{sy}, \qquad b/y = \phi_{by}$$

Assumption A. 1. *The ratio of net imports to GDP, m/y, is constant.*

B.2 Equilibrium system of the dynamic model

The equilibrium system of the dynamic model is

$$
\begin{aligned}
&(1+\tau_t^c)\lambda_t = u_1(c_t, n_t),\\
&\lambda_t(1-\tau_t^n)w_t = -u_2(c_t, n_t),\\
&\lambda_t = \beta E_t\left\{\lambda_{t+1}\left[(1-\delta)+(1-\tau_{t+1}^k)(d_{t+1}-\delta)+\delta\right]\right\},\\
&\lambda_t = \beta E_t\left[\lambda_{t+1}R_{t+1}^b\right],\\
&k_t = (1-\delta)k_{t-1}+x_t,\\
&y_t = \xi^t\left[k_{t-1}\right]^\theta n_t^{1-\theta},\\
&w_t = (1-\theta)\frac{y_t}{n_t},\\
&d_t = \theta\frac{y_t}{k_{t-1}},\\
&y_t = c_t + x_t + g_t - m_t,\\
&T_t = \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k(d_t-\delta)k_{t-1},
\end{aligned}
$$

where, if the utility function is KPR $\mathbb{U}^{KPR}$, marginal utility is defined as

$$
\begin{aligned}
U_c(c_t, n_t) &\equiv (c_t)^{-\eta}\left[1-\kappa(1-\eta)n_t^{1+\lambda}\right]^\eta,\\
U_n(c_t, n_t) &\equiv -\eta\,(1+\lambda)\left\{(c_t)^{1-\eta}\left[1-\kappa(1-\eta)n_t^{1+\lambda}\right]^{\eta-1}\kappa n_t^\lambda\right\}
\end{aligned}
$$

if the utility function is additively separable $\mathbb{U}^{AS}$, by

$$
\begin{aligned}
U_c(c_t, n_t) &\equiv (c_t)^{-\eta},\\
U_n(c_t, n_t) &\equiv -\kappa\psi^{t(1-\eta)}(1+\lambda)n_t^\lambda
\end{aligned}
$$

and if the utility function is GHH $\mathbb{U}^{GHH}$, by

$$
\begin{aligned}
U_c(c_t, n_t) &\equiv (c_t-\kappa\psi^t n_t^{1+\lambda})^{-\bar{\eta}},\\
U_n(c_t, n_t) &\equiv -\kappa\psi^t(1+\lambda)n_t^\lambda(c_t-\psi^t\kappa n_t^{1+\lambda})^{-\bar{\eta}}.
\end{aligned}
$$

The detrended equilibrium system is

$$
\begin{aligned}
&(1+\tau_t^c)\tilde{\lambda}_t = u_1(\tilde{c}_t, n_t),\\
&\tilde{\lambda}_t(1-\tau_t^n)\tilde{w}_t = -u_2(\tilde{c}_t, n_t),\\
&\tilde{\lambda}_t = \beta\psi^{-\eta}E_t\left\{\tilde{\lambda}_{t+1}\left[(1-\delta)+(1-\tau_{t+1}^k)(d_{t+1}-\delta)+\delta\right]\right\},\\
&\tilde{\lambda}_t = \beta\psi^{-\eta}E_t\left[\tilde{\lambda}_{t+1}R_{t+1}^b\right],\\
&\psi\tilde{k}_t = (1-\delta)\tilde{k}_{t-1}+\tilde{x}_t,\\
&\tilde{y}_t = \left[\tilde{k}_{t-1}\right]^\theta n_t^{1-\theta},\\
&\tilde{w}_t = (1-\theta)\frac{\tilde{y}_t}{n_t},\\
&d_t = \theta\frac{\tilde{y}_t}{\tilde{k}_{t-1}}.\\
&\tilde{y}_t = \tilde{c}_t+\tilde{x}_t+\tilde{g}_t-\tilde{m}_t,\\
&\tilde{T}_t = \tau_t^c\tilde{c}_t+\tau_t^n\tilde{w}_t n_t+\tau_t^k(d_t-\delta)\tilde{k}_{t-1}.
\end{aligned}
$$

On the balanced growth path, the system becomes

$$
\begin{aligned}
&(1+\tau^c)\tilde{\lambda} = u_1(\tilde{c}, n),\\
&\tilde{\lambda}(1-\tau^n)\tilde{w} = -u_2(\tilde{c}, n),\\
&1 = \beta\psi^{-\eta}\left[(1-\delta)+(1-\tau^k)(d-\delta)+\delta\right],\\
&1 = \beta\psi^{-\eta}R^b,\\
&\psi\tilde{k} = (1-\delta)\tilde{k}+\tilde{x},\\
&\tilde{y} = \left[\tilde{k}\right]^\theta n^{1-\theta},\\
&\tilde{w} = (1-\theta)\frac{\tilde{y}}{n},\\
&d = \theta\frac{\tilde{y}}{\tilde{k}}.\\
&\tilde{y} = \tilde{c}+\tilde{x}+\tilde{g}-\tilde{m}_t,\\
&\tilde{T} = \tau_t^c\tilde{c}+\tau^n\tilde{w}n+\tau^k(d-\delta)\tilde{k}.
\end{aligned}
$$

Scheme (3): Changes in tax revenue are adjusted by a lump-sum transfer: Under Scheme (3), $\tilde{g}/\tilde{y} = \phi_g$ and $\tilde{m}/\tilde{y} = \phi_m$ are constant. Then, the balanced growth path values are obtained by

$$
\begin{aligned}
R^b &= \frac{\psi^\eta}{\beta}, \\
d &= \frac{1}{1-\tau^k}\left[R^b - 1 + \delta\right], \\
\frac{\tilde{k}}{\tilde{y}} &= \frac{\theta}{d}, \\
\frac{\tilde{x}}{\tilde{y}} &= [\psi - (1-\delta)]\,\frac{\tilde{k}}{\tilde{y}}, \\
\frac{\tilde{c}}{\tilde{y}} &= 1 - \frac{\tilde{x}}{\tilde{y}} - \frac{\tilde{g}}{\tilde{y}} + \frac{\tilde{m}}{\tilde{y}}, \\
\frac{n}{\tilde{y}} &= \left[\frac{\tilde{y}}{\tilde{k}}\right]^{\theta/(1-\theta)}, \\
\tilde{w} &= (1-\theta)\frac{\tilde{y}}{\tilde{n}},
\end{aligned}
$$

From this system, the following lemma and corollary are obtained from the balanced growth path equilibrium system.

Lemma A. 1. *On the balanced growth path, the dividend (d), capital–output ratio ($k/y = \tilde{k}/\tilde{y}$), investment–output ratio ($x/y = \tilde{x}/\tilde{y}$), consumption–output ratio ($c/y = \tilde{c}/\tilde{y}$), and labor–output ratio ($n/\tilde{y}$) are independent from the consumption tax rate (τ^c).*

Remark A. 3. *The elasticity of consumption with respect to the consumption tax rate equals that of output:*

$$
\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| = \left|\frac{d\tilde{y}/\tilde{y}}{d\tau^c/\tau^c}\right|.
$$

Scheme (4): Changes in tax revenue are adjusted by government consumption: By the budget constraint, the following is obtained:

$$
\tilde{T} = \tau^c\tilde{c} + \tau^n\tilde{w}n + \tau^k(d-\delta)\tilde{k}.
$$

Dividing by $\tilde{y}$ yields

$$\frac{\tilde{g}}{\tilde{y}} + \frac{\tilde{s}}{\tilde{y}} + (R^b - 1)\frac{\tilde{b}}{\tilde{y}} = \tau^c \frac{\tilde{c}}{\tilde{y}} + \tau^n \tilde{w}\frac{\tilde{n}}{\tilde{y}} + \tau^k (d - \delta)\frac{\tilde{k}}{\tilde{y}}$$

Since $n/\tilde{y}$ and $\tilde{k}/\tilde{y}$ are independent from τ^c,

$$\frac{\tilde{g}}{\tilde{y}} = \tau^c \frac{\tilde{c}}{\tilde{y}} + constant.$$

The resource constraint can be rewritten as

$$\begin{aligned} & c + i + g - m = y \\ \iff \qquad & \frac{\tilde{c}}{\tilde{y}} + \frac{\tilde{i}}{\tilde{y}} + \tau^c \frac{\tilde{c}}{\tilde{y}} + const - \frac{\tilde{m}}{\tilde{y}} = 1. \end{aligned}$$

Since $\tilde{i}/\tilde{y}$ and $\tilde{m}/\tilde{y}$ is independent from τ^c, it follows that

$$(1 + \tau^c)\frac{\tilde{c}}{\tilde{y}} = constant.$$

Therefore, the following lemma holds.

Lemma A. 2. $(1 + \tau^c)\tilde{c}/\tilde{y}$ *is independent from* τ^c

By Lemma A.2, the following is obtained.

Remark A. 4. *The elasticity of consumption with respect to the consumption tax rate equals that of* $y/(1 + \tau^c)$*:*

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| = \left|\frac{d(1 + \tau^c)^{-1}\tilde{y}/(1 + \tau^c)^{-1}\tilde{y}}{d\tau^c/\tau^c}\right|$$

B.3 Consumption tax revenue curve in the dynamic model

In the main text, the total tax revenue curve is investigated as the tax revenue curve. Here, the results for the consumption tax revenue curve are shown.

B.3.1 Scheme (3): Changes in tax revenue are adjusted by a lump-sum transfer

Propositions A.19, A.20, and A.21 refer to the consumption tax revenue curve under Scheme (3) in the dynamic economy.

Proposition A. 19. *Suppose that the utility function is KPR, $\mathbb{U}^{KPR}$. The consumption tax revenue curve for consumption tax under Scheme (3) is monotonically increasing. The consumption tax revenue is unbounded except for $\lambda = 0$.*

Proof. The optimization condition for the consumption–labor choice,

$$\eta\,(1+\lambda)\left\{\frac{\kappa\tilde{c}n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}\right\} = \frac{1-\tau^{n}}{1+\tau^{c}}(1-\theta)\frac{\tilde{y}}{h},$$

yields

$$\tilde{y} = \left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1+\tau^{c}}{1-\tau^{n}}\right]^{-1/(1+\lambda)}.$$

Since $\tilde{y} > 0$ for $\tau^{c} \geq 0$,

$$(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^{n}} > 0.$$

By Remark A.3, it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^{c}/\tau^{c}} = \frac{d\tilde{y}/\tilde{y}}{d\tau^{c}/\tau^{c}} = -\frac{\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\frac{\tau^{c}}{1-\tau^{n}}}{(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\left(\frac{1+\tau^{c}}{1-\tau^{n}}\right)(1+\lambda)}.$$

Letting

$$\Psi = (1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\left(\frac{1+\tau^{c}}{1-\tau^{n}}\right)(1+\lambda) > 0,$$

it follows that

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^{c}/\tau^{c}}\right| - 1 = -\frac{1}{\Psi}\left\{(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^{n}}+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\frac{\tau^{c}}{1-\tau^{n}}\lambda\right\} \leq 0.$$

The consumption tax revenue is unbounded if $\lambda > 0$, since $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| > 0$. In the case of $\lambda = 0$, the consumption tax revenue is given by

$$\begin{aligned}
\tau^c \tilde{c} &= \phi \tau^c \tilde{y} \\
&= \left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1}\left[(1-\eta) + \frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\frac{1+\tau^c}{1-\tau^n}\right]^{-1}\tau^c \\
&= \left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1}\left[\frac{1-\eta}{\tau^c} + \frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\frac{1+\tau^c}{\tau^c}\frac{1}{1-\tau^n}\right]^{-1},
\end{aligned}$$

where $\phi = c/y$. This converges to $\left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1}\left[\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\frac{1}{1-\tau^n}\right]^{-1}$. □

Proposition A. 20. *Suppose that the utility function is additively separable;* $\mathbb{U}^{AS}$. *The consumption tax revenue curve for consumption tax under Scheme (3) is hump shaped if and only if* $\eta + \lambda < 1$, *and the revenue is maximized at* $\tau^c = \frac{\eta+\lambda}{1-\eta-\lambda}$. *Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if* $\eta + \lambda \leq 1$. *Otherwise, it is unbounded.*

Proof. By the optimization condition for the consumption–labor choice,

$$\kappa(1+\lambda)\tilde{c}^{\eta} n^{\lambda} = \frac{1-\tau^n}{1+\tau^c}\tilde{w},$$

it follows that

$$\tilde{y} = (1+\tau^c)^{-1/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}.$$

By Remark A.3, it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} = \frac{d\tilde{y}/\tilde{y}}{d\tau^c/\tau^c} = -\frac{1}{\eta+\lambda}\cdot\frac{\tau^c}{1+\tau^c}.$$

Then,

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 = \frac{1}{\eta+\lambda}\cdot\frac{1}{1+\tau^c}\Big\{(1-\eta-\lambda)\tau^c - (\eta+\lambda)\Big\}.$$

Suppose $\eta + \lambda = 1$. In this case, $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 < 0$.

Suppose $\eta + \lambda \neq 1$. In this case,

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 = \frac{1-\eta-\lambda}{\eta+\lambda} \cdot \frac{1}{1+\tau^c}\left\{\tau^c - \frac{\eta+\lambda}{1-\eta-\lambda}\right\}.$$

If $\eta + \lambda \geq 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \geq 0$.

If $\eta + \lambda < 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \leq (\eta+\lambda)/(1-\eta-\lambda)$, and $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| > 1$ for $\tau^c > (\eta+\lambda)/(1-\eta-\lambda)$.

By the elasticity of consumption, it is obvious that the consumption tax revenue is bounded if $\eta + \lambda < 1$ and unbounded if $\eta + \lambda > 1$. In the case of $\eta + \lambda = 1$, the consumption tax revenue is

$$\begin{aligned}\tau^c\tilde{c} &= \phi\tau^c\tilde{y} \\ &= \phi\frac{\tau^c}{1+\tau^c}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right].\end{aligned}$$

where $\phi = c/y$. This converges to $\phi\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]$ as $\tau^c \to \infty$. □

Proposition A. 21. *Suppose that the utility function is additively separable, $\mathbb{U}^{GHH}$. The consumption tax revenue curve for consumption tax under Scheme (3) is hump shaped if and only if $\lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\lambda}{1-\lambda}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\lambda \leq 1$. Otherwise, it is unbounded.*

Proof. By the optimization condition for the consumption–labor choice,

$$\kappa(1+\lambda)n^\lambda = \frac{1-\tau^n}{1+\tau^c}\tilde{w},$$

it follows that

$$\tilde{y} = (1+\tau^c)^{-1/\lambda}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}.$$

By Remark A.3, it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} = \frac{d\tilde{y}/\tilde{y}}{d\tau^c/\tau^c} = -\frac{1}{\lambda} \cdot \frac{\tau^c}{1+\tau^c}.$$

Then,

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 = \frac{1}{\lambda} \cdot \frac{1}{1+\tau^c}\Big\{(1-\lambda)\tau^c - \lambda\Big\}.$$

Suppose $\lambda = 1$. In this case, $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 < 0$.
Suppose $\lambda \neq 1$. In this case,

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 = \frac{1-\lambda}{\lambda} \cdot \frac{1}{1+\tau^c}\left\{\tau^c - \frac{\lambda}{1-\lambda}\right\}.$$

If $\lambda \geq 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \geq 0$.
If $\lambda < 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| < 1$ for $\tau^c \leq \lambda/(1-\lambda)$,
$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| = 1$ for $\tau^c = \lambda/(1-\lambda)$, and
$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| > 1$ for $\tau^c > \lambda/(1-\lambda)$.

By the elasticity of consumption, it is obvious that the consumption tax revenue is bounded if $\eta + \lambda < 1$ and unbounded if $\lambda > 1$. In the case of
$lambda = 1$, the consumption tax revenue is

$$\begin{aligned}\tau^c\tilde{c} &= \phi\tau^c\tilde{y} \\ &= \phi\frac{\tau^c}{1+\tau^c}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}.\end{aligned}$$

where $\phi = c/y$. This converges to $\phi\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}$ as $\tau^c \to \infty$. □

Note that these propositions are the same as Propositions 1, 2, and 3 in the main text, while the dynamic economy has a far richer structure (capital, investment, debt evolution, etc.) than the static economy does.

B.3.2 Scheme (4): Changes in tax revenue are adjusted by government consumption

Proposition A. 22. *Suppose that the utility function is KPR, $\mathbb{U}^{KPR}$. The consumption tax revenue curve for consumption tax under Scheme (4) is monotonically increasing. The consumption tax revenue is bounded.*

Proof. By the consumption–labor choice condition, $\tilde{y}$ is obtained

$$
\begin{aligned}
& \eta(1+\lambda)\left\{\frac{\kappa\tilde{c}n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}\right\} = \frac{1-\tau^n}{1+\tau^c}(1-\theta)\frac{\tilde{y}}{h} \\
\Longleftrightarrow \quad & \eta(1+\lambda)\left\{\frac{\kappa\left(\frac{\tilde{c}}{\tilde{y}}\right)\left(\frac{n}{\tilde{y}}\right)^{\lambda}}{\tilde{y}^{-1-\lambda}-\kappa(1-\eta)\left(\frac{n}{\tilde{y}}\right)^{1+\lambda}}\right\} = \frac{1-\tau^n}{1+\tau^c}(1-\theta)\frac{\tilde{y}}{n} \\
\Longleftrightarrow \quad & \tilde{y} = \left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\frac{1}{1-\theta}\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^n}\right]^{-1/(1+\lambda)}.
\end{aligned}
$$

Then, $\tilde{y}$ is independent from τ^c.

By Remark A.4, the elasticity of $(1+\tau^c)^{-1}\tilde{y}$ is considered. Since

$$
(1+\tau^c)^{-1}\tilde{y} = (1+\tau^c)^{-1}\left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\frac{1}{1-\theta}\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^n}\right]^{-1/(1+\lambda)},
$$

it follows that

$$
\begin{aligned}
\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} &= \frac{d(1+\tau^c)^{-1}\tilde{y}/(1+\tau^c)^{-1}\tilde{y}}{d\tau^c/\tau^c} \\
&= -\frac{\tau^c}{1+\tau^c}.
\end{aligned}
$$

$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right|$ is monotonically increasing in τ^c. If $\tau^c = 0$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| = 0$. As $\tau^c \to \infty$, $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \to 1$. Therefore, the consumption tax revenue curve is monotonically increasing.

The boundedness is shown as follows. Let $(1+\tau^c)\tilde{c}/\tilde{y} = \phi$; then,

$$
\begin{aligned}
\tau^c\tilde{c} &= \tau^c\phi\tilde{y}(1+\tau^c)^{-1} \\
&= \phi\frac{\tau^c}{1+\tau^c}\left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\frac{1}{1-\theta}\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^n}\right]^{-1/(1+\lambda)}.
\end{aligned}
$$

As $\tau^c \to \infty$, it converges to $\phi\left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\frac{1}{1-\theta}\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^n}\right]^{-1/(1+\lambda)}$.

□

Proposition A. 23. *Suppose that the utility function is additively separable, $\mathbb{U}^{AS}$. The consumption tax revenue curve for consumption tax under Scheme (4) is hump shaped if and only if $\eta < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\eta \leq 1$. Otherwise, it is unbounded.*

Proof. By the consumption–labor choice condition,

$$\begin{aligned}
& \kappa(1+\lambda)\tilde{c}^{\eta} n^{\lambda} = \frac{1-\tau^n}{1+\tau^c}\tilde{w} \\
\iff \quad & \kappa(1+\lambda)\tilde{y}^{\eta+\lambda}\left(\frac{\tilde{c}}{\tilde{y}}\right)^{\eta}\left(\frac{n}{\tilde{y}}\right)^{\lambda} = \frac{1-\tau^n}{1+\tau^c}(1-\theta)\frac{\tilde{y}}{n} \\
\iff \quad & \tilde{y} = (1+\tau^c)^{-(1-\eta)/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}.
\end{aligned}$$

By Remark A.4, the elasticity of $(1+\tau^c)\tilde{y}$ is considered. Since

$$(1+\tau^c)^{-1}\tilde{y} = (1+\tau^c)^{-(1+\lambda)/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)},$$

it follows that

$$\begin{aligned}
\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} &= \frac{d(1+\tau^c)^{-1}\tilde{y}/(1+\tau^c)^{-1}\tilde{y}}{d\tau^c/\tau^c} \\
&= -\frac{1+\lambda}{\eta+\lambda}\cdot\frac{\tau^c}{1+\tau^c}.
\end{aligned}$$

$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right|$ is monotonically increasing in τ^c. If $\tau^c = 0$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| = 0$. If $\tau^c \to \infty$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \to (1+\lambda)/(\eta+\lambda)$. Therefore, a necessary and sufficient condition for a hump-shaped consumption tax revenue curve is $\eta < 1$. The peak tax rate is

$$\tau^c_{max} = \frac{\eta+\lambda}{1-\eta}.$$

By the elasticity of consumption, it is obvious that the tax revenue is bounded if $\eta < 1$ and unbounded if $\eta > 1$. Suppose $\eta = 1$. Letting $(1+\tau^c)\tilde{c}/\tilde{y} = \phi$, the following is obtained:

$$\begin{aligned}
\tau^c \tilde{c} &= \tau^c \phi \tilde{y}(1+\tau^c)^{-1} \\
&= \phi \frac{\tau^c}{1+\tau^c} \left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-1} \left(\frac{n}{\tilde{y}}\right)^{-1-\lambda} \right]^{1/(1+\lambda)}.
\end{aligned}$$

As $\tau^c \to \infty$, it converges to $\phi \left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-1} \left(\frac{n}{\tilde{y}}\right)^{-1-\lambda} \right]^{1/(1+\lambda)}$.

□

Proposition A. 24. *Suppose that the utility function is additively separable, $\mathbb{U}^{GHH}$. The consumption tax revenue curve for consumption tax under Scheme (4) is hump shaped, and the revenue is maximized at $\tau^c = \lambda$. The consumption tax revenue is bounded.*

Proof. By the consumption–labor choice condition,

$$\begin{aligned}
& \kappa(1+\lambda)n^\lambda = \frac{1-\tau^n}{1+\tau^c}\tilde{w} \\
\iff \quad & \kappa(1+\lambda)\tilde{y}^\lambda \left(\frac{n}{\tilde{y}}\right)^\lambda = \frac{1-\tau^n}{1+\tau^c}(1-\theta)\frac{\tilde{y}}{n} \\
\iff \quad & \tilde{y} = (1+\tau^c)^{-1/\lambda} \left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda} \right]^{1/\lambda}.
\end{aligned}$$

By Remark A.4, the elasticity of $(1+\tau^c)\tilde{y}$ is considered. Since

$$(1+\tau^c)^{-1}\tilde{y} = (1+\tau^c)^{-(1+\lambda)/\lambda} \left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda} \right]^{1/\lambda},$$

it follows that

$$\begin{aligned}
\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} &= \frac{d(1+\tau^c)^{-1}\tilde{y}/(1+\tau^c)^{-1}\tilde{y}}{d\tau^c/\tau^c} \\
&= -\frac{1+\lambda}{\lambda} \cdot \frac{\tau^c}{1+\tau^c}.
\end{aligned}$$

$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right|$ is monotonically increasing in τ^c. If $\tau^c = 0$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| = 0$. If $\tau^c \to \infty$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \to (1+\lambda)/\lambda > 1$. Therefore, the consumption tax revenue curve is hump shaped. The peak tax rate is

$$\tau^c_{max} = \lambda.$$

It is obvious that the tax revenue is bounded.

□

These propositions are the same as Propositions 4, 5, and 6 in the main text, which concern the consumption tax revenue curve in the static economy.