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Abstract

By using a model incorporating a crisis risk triggered by the accumulation of government debt, we provide a new perspective to explain the driving forces behind secular stagnation associated with a persistent decrease in interest rates. According to the model, the fear of the imposition of a large-scale capital levy in the face of a crisis helps explain Japan's decades of persistent stagnation by almost one quarter. As government debt accumulates, not only the level but also the growth rate of output declines persistently, while the government bond yield decreases. We then discuss the possible mechanisms that induce people to share the expectation of a capital levy at the time of a government debt crisis from the historical, theoretical, and political points of view. The model also shows that a permanent increase in consumption tax, which prevents a government debt crisis from occurring, increases social welfare.

JEL Classification Numbers: E32, E62, G18, H12, H63
Keywords: default; government bonds; capital levy; lost decades

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1 Introduction

There is growing concern about a secular stagnation (Hansen, 1939) in the aftermath of the Great Recession in the late 2000s (Gordon, 2012; Krugman, 2013; Summers, 2013). The major concern is that the US and European economies may stagnate persistently in the coming decades. A precedent in this respect is Japan, which has been experiencing its so-called lost decades since the collapse of the asset market bubble around 1990. While there are many possible driving forces behind the secular stagnation,[1] we focus on the fact that this concern arose when the government debt in these countries started to expand substantially, while the interest rate on government bonds did not necessarily increase.

The contribution of our study is to offer a new explanation of the factors behind the secular stagnation as well as the lowered government bond yield. The key factor is *the government debt crisis*, namely the complete loss of market confidence in government debt that forces the government to collect extremely large tax revenues. When a crisis hits, the government is forced to reduce its debt to a certain level by imposing a once-and-for-all increase in various taxes, particularly a heavy tax on capital stock (capital levy) and on government bonds (a partial default of government debt). On the contrary, in normal periods, taxes are low. The government budget is not balanced and thus government debt continues to rise over time. A crisis occurs with an exogenous probability that increases with the outstanding amount of government debt. By using a simple neoclassical closed-economy model, we show that a persistent economic slowdown occurs in normal times because of the fear of a government debt crisis. Furthermore, the expectations of a heavier tax on capital than on government bonds at the crisis explains the puzzling fact in Japan that the yield of government bonds decreased despite the accumulation of government debt.

Our simulation based on a model calibrated to Japan demonstrates that the expectation of the imposition of taxes on capital stock and government bonds outstanding (GBOs) can account for the *ex ante* secular stagnation fairly well. The fear of a tax on capital stock raises the required return on capital and discourages capital investment. This adverse effect is intensified by the accumulation of government debt because the probability of a government debt crisis increases. Moreover, the tax distortion at the crisis also increases because the government imposes higher tax rates to repay its debt. Consequently, the growing risk of a government debt crisis depresses not only the level but also the growth rate of output persistently. Our result that the growing risk of a government debt crisis lowers the economic growth rate is thus a novel contribution to the literature, as previous studies have tended to find that such a disaster risk is constant and lowers only the level of output (e.g., Kozlowski, Veldkamp, and Venkateswaran, 2015). Indeed, our model can account for about one quarter of the output decline in Japan.

The model also shows that the expectation of a capital levy lowers the government bond yield, which is consistent with Japanese data. Because people expect that their own capital

[1]Examples include the slowdown in innovations (Gordon, 2012), a demand shortfall (Summers, 2013; Eggertsson and Mehrotra, 2014), and pessimism (Benigno and Fornaro, 2015). For Japan, Hayashi and Prescott (2002) point out the influence of the decrease in the total factor productivity (TFP) growth rate, while Caballero, Hoshi, and Kashyap (2008) emphasize the malfunctioning financial sector.

will be heavily taxed at a crisis, whereas their bond holdings will not be taxed so heavily, they consider that capital investment is riskier than investment in government bonds and choose to buy more bonds and less capital. This increase in demand for bonds lowers the bond yield despite the accumulation of GBOs.

The spirit of our analytical framework may be considered to be in line with that of business cycle accounting (Chari, Kehoe, and McGrattan, 2007) in the sense that we account for the secular stagnation by exogenous tax wedges. A notable difference is that tax wedges are contemporaneous with economic fluctuations in the literature on business cycle accounting, whereas we consider that tax wedges appear at the time of a crisis in the future and affect the current economy *ex ante* through expectations. Our methodology is as follows. A government debt crisis affects economic activity through various channels, which can be represented by "tax wedges" in a reduced-form neoclassical model. The objective of this study is therefore to identify a suitable combination of tax wedges that best accounts for an *ex ante* slowdown in economic growth and a low government bond yield.

Our model does not formally address the question as to what causes the emergence of a capital levy at the time of a crisis. It seems problematic to posit that the fear of a capital levy is widespread when government debt balloons because the possibility of a capital levy is admittedly arguable even during a government debt crisis. In Section 4, we therefore discuss the capital levy extensively from historical, theoretical, and political perspectives, providing evidence that justifies our assumption of the *ex ante* expectations of a once-and-for-all capital levy. For example, history reveals that there were active debates on the use of a capital levy in Europe during the interwar period and that a capital levy was actually introduced in postwar Japan. Tax theory states that given that the government cannot commit beforehand, the optimal policy is a once-and-for-all capital levy. The fear of a capital levy can also be interpreted as a reduced form of the fear of fire sales of capital by banks at the time of a crisis. Further, as a political charm, the capital levy is effective at reducing wealth inequality.

We also examine the preemptive tax hike scenario in which the government introduces a distortionary consumption tax in normal times to prevent a debt crisis from occurring. We find that this preemptive tax hike increases social welfare.

Background Facts

Economic slowdowns coincide with debt increases in many economies. Figure 1 shows the trend in real GDP per capita and government debt for Japan, the United States, the Euro Area of 15 countries, and Italy. The beginning year is 1975 for Japan and 1992 for the rest, which represents 15 years before the financial crisis in each region (i.e., 1990 and 2007, respectively, shown as the vertical dashed line). The thick solid line represents the logarithm of real GDP per capita shown on the left axis, while the thin solid line represents its linear trend. On the right axis, the line with crosses and the line with circles represent the ratio of gross and net government debt to nominal GDP, respectively. The figure shows that real GDP decreased compared with its trend after these financial crises, while government debt increased in all regions. Notably, Japan's gross government debt now exceeds 200% of nominal GDP.

While this government debt accumulation is commonly cited as the result rather than the cause of the stagnation, Figure 2 shows that increased government debt indeed causes anxiety in Japan. According to the household survey conducted by Japan's Cabinet Office, an increasing number of Japanese are worried in their everyday lives and about their prospects and one-third of them answered that the fiscal balance is one of the reasons for this.[2] There exist surveys about the subjective probability of the government debt crisis in Japan. Morikawa (2016, 2017) finds that Japanese consumers and firm managers consider that the debt crisis will take place by 2030 with the probability around 24% (consumers) or 27% (firm managers) on average, although the survey question allows various interpretations on the side of respondents.

Figure 3 shows the number of occurrences of specific words in the morning and evening editions of the Nihon Keizai Shinbun, Japan's financial newspaper, for the years from 1981 to 2016. The words that suggest a government debt crisis, "fiscal failure (zaisei hatan)" or "fiscal crisis (zaisei kiki)," have been used more and more frequently since 1981. Moreover, the combination of the words "tax increase (zozei)" with either "fiscal failure" or "default" have been used more and more frequently, too, which seems to suggest the mounting fear of a tax increase with a government debt crisis.[3]

A noteworthy puzzle is that the price (yield) of government bonds is high (low) except for Italy around 2012. Figure 4 shows the developments in two types of interest rates for Japan and the United States. The thick and thin lines represent the credit spread and real government bond yield, respectively.[4] The figure hardly suggests the mounting risk of public default, as it shows that bond yields have decreased, while credit spreads have increased to some extent.[5]

Literature Review

Empirical studies report that the economy tends to stagnate when government debt is large.[6] Reinhart, Reinhart, and Rogoff (2012) name this fact the *public debt overhang*. They review

[2] Health and natural disasters are the top two reasons, followed by concerns about public services. This worry is also considered to be related to the accumulation of government debt, which may be preventing the government from providing sufficient public services in the future such as pensions, medical services, and nursing care.

[3] The surge in the number of occurrences around 2010 reflects the Euro crisis. However, we confirm a steady increase even when we exclude the number of occurrences of the words used with "Europe." The word "default" does not necessarily mean that of government bonds; however, when it is used with "tax increase," the word can be considered to be associated with government bond default.

[4] The credit spread is defined as the bank loan rate with one-year maturity or longer minus the government bond yield with five-year maturity for Japan, while it is defined as the corporate bond spread (BAA) with 10-year maturity for the United States. The government bond yield in real terms is defined as that with five-year maturity minus the annual CPI inflation rate in the next year for Japan, while it is defined as that with 10-year maturity minus the annual PCE inflation rate in the next year for the United States.

[5] Against this backdrop, Hoshi and Ito (2014) point out that Japanese government bonds are predominantly held by Japanese institutional investors and argue that they have a strong home bias. Sakuragawa and Sakuragawa (2016) explain the low yield of government bonds by the absence of safe assets. Caballero and Simsek (2017) point out a secular increase in risk intolerance.

[6] See Reinhart and Rogoff (2010), Reinhart, Reinhart, and Rogoff (2012), Checherita-Westphal and Rother (2012), and Baum, Checherita-Westphal, and Rother (2013) for the negative effect of government debt on output. Barro and Sala-i-Martin (1995) show that government consumption has a negative impact on output. Fischer (1991) shows that a fiscal deficit has a negative impact on output.

26 cases of the high accumulation of government debt in advanced countries and report that in 23 of those cases, economic growth remained stagnant for more than a decade. They argue for the existence of causality from the increase in government debt to lower economic growth based on their finding of a nonlinear correlation between larger debt and lower growth.

Our study is also related to the literature in the 1990s on the non-Keynesian effect of fiscal policy developed by Giavazzi and Pagano (1990), Alesina and Perotti (1996), Alesina and Ardagna (1998), and Perotti (1999). Perotti (1999) shows theoretically and empirically that fiscal consolidation has an expansionary effect on consumption when government debt is large. Alesina and Perotti (1996) and Alesina and Ardagna (1998) show that government expenditure cuts have a longer-lasting effect on improving the economy than do tax increases. Our study leads to similar implications in that an increase in government debt has a contractionary effect. However, our focus is more on long-term growth in line with the recent work of Reinhart, Reinhart, and Rogoff (2012) rather than a short-term effect, which is heavily studied in the literature on the non-Keynesian effect of fiscal policy.

Theoretically, the model presented in this paper is a neoclassical model of rare disaster following the work of Rietz (1988), Barro (2006, 2009), Gabaix (2012), and Gourio (2012, 2013). Specifically, our model is a greatly simplified version of Gourio's (2013) model. However, the property of disaster is different in the sense that a debt crisis is an abrupt redistribution of wealth, whereas a disaster destroys resources, so we call it crisis rather than disaster. In our model, crisis matters only for the household through taxes, while disaster in Gourio (2013) influences firms through changes in their productivity and capital values.[7] The literature on sovereign default, such as Arellano (2008) and Arellano, Bai, and Mihalache (2017), is unquestionably related to our study. In particular, Arellano, Bai, and Mihalache (2017) propose a model in which a sovereign default risk causes a persistent recession. Their model is similar to ours in spirit but distinct in details: their model is a small-open economy where the government and household are the same agent and sovereign debt is external, not the domestic debt as in our model.

The organization of the remainder of this paper is as follows. Section 2 introduces the basic model and specifies the equilibrium. Section 3 presents the results of the numerical simulation and Section 4 discusses the plausibility of the expectations of a capital levy. Section 5 presents several extensions of our analyses and Section 6 concludes.

2 Basic Model

The model presented herein is simple and standard except for the occurrence of a crisis. It consists of a representative household, a firm, and a government. For simplicity, we assume that, in normal times, the government collects no tax, while it spends.[8] Thus, government debt

[7]Kozlowski, Veldkamp, and Venkateswaran (2015) use Gourio's model to analyze the exogenous disaster risk of a financial crisis, while Isore and Szczerbowicz (2017) extend it to the New Keynesian model. Kunieda (2015) applies Barro's (2006) model to the risk of a natural disaster.

[8]This assumption is for simplicity. In reality, there are at least four issues regarding government policy in normal times: (i) the government may increase tax to reduce debt. Bohn (1998) and Lo and Rogoff (2015) report

keeps increasing. When a crisis occurs, the government imposes once-and-for-all taxes on the household to repay its debt. The crisis probability is given exogenously and increases as GBOs increase. We assume the model is of a closed economy, as around 90% of government bonds are held by domestic investors in Japan.[9] The asset market is incomplete, meaning that the crisis risk is not insured. See Appendix A for more details on the calculation of the equilibrium.

Firm

A firm faces perfect competition. Its production function is expressed as $Y_t = K_t^{\alpha}(z_t N_t)^{1-\alpha}$, where Y_t, K_t, and N_t represent output, capital stock, and employment, respectively. Productivity (TFP) z_t is given by $\log z_{t+1} = \log z_t + \mu + \sigma_e e_{t+1}$ and $e_{t+1} \sim N(0,1)$. The static profit maximization is

$$\pi(K_t, z_t; W_t) = \max_{K_t, N_t \geq 0} \{K_t^{\alpha}(z_t N_t)^{1-\alpha} + (1-\delta)K_t - R_t^K K_t - W_t N_t\}, \tag{1}$$

which yields the return on capital $R_t^K = 1 - \delta + \alpha Y_t / K_t$ and $N_t = K_t \left(z_t^{1-\alpha}(1-\alpha)/W_t\right)^{\frac{1}{\alpha}}$, where W_t denotes real wages.

Crisis Risk

Define a crisis indicator x_t by $x_t = 0$ in normal times and $x_t = 1$ at the point of the government debt crisis. The variable x_t is an exogenous sunspot shock to the economy. The government debt crisis is an event where holders of government debt lose confidence in government debt and rush to exchange it for real assets and goods, which is exogenous to the government actions. Consequently, the government is forced to raise a substantial amount of tax revenue at the occurrence of the crisis.

The probability that a crisis occurs in period $t+1$ is denoted by $p_t = p(B_t^G/z_t) = \Pr(x_{t+1} = 1 | B_t^G/z_t)$, where B_t^G is the quantity of government bonds. This probability depends positively on GBOs divided by TFP.[10] To be more precise, we assume the form of

$$p_t = p(B_t^G/z_t) = d_0 \exp(d_1 B_t^G/z_t), \tag{2}$$

empirically that governments tend to improve the fiscal balance in response to an increase in their debt; (ii) the government imposes a number of distortionary taxes; (iii) government spending is counter-cyclical and tax revenue is pro-cyclical; and (iv) G_t is not necessarily a transfer, but government spending on goods. However, such tax or spending policy in normal times does not affect our main results qualitatively, because our interest is in how changes in tax policy from normal times to the crisis event influence economic activity. Quantitatively, issue (iii) is considered to strengthen our results. In other words, the risk of a government debt crisis causes a larger decrease in output and investment because GBOs increase more rapidly. Issue (iv) may affect the implications for the optimality of tax smoothing. However, this does not matter for our main results either, because our main analysis is not normative.

[9] Reinhart and Rogoff (2010) emphasize the importance of domestic debt.

[10] A crisis is more likely to occur as GBOs increase (Reinhart and Rogoff, 2010). Arellano (2008) constructs a small open-economy model and shows that default is more likely to occur in recessions, which is consistent with the data. She points out that the incomplete asset market leads to this result, while models based on the complete asset market tend to predict the opposite result. D'Erasmo and Mendoza (2016) show theoretically that default on domestic government debt is more likely when debt is larger and tax revenue is smaller.

where d_0 represents the crisis probability in the next period when there is no government debt today and d_1 represents the elasticity of the crisis probability to the change in government debt.

Household

A representative household has the nonseparable lifetime utility U_t as

$$U_t^{1-\psi} = (1-\beta)(C_t^{\nu}(1-N_t)^{1-\nu})^{1-\psi} + \beta E_t(U_{t+1}^{1-\psi}), \tag{3}$$

where β represents a discount factor, ψ represents the intertemporal elasticity of the substitution of consumption, and ν represents a utility weight on consumption. The utility function takes the standard Cobb–Douglas form, which is consistent with balanced growth. The budget constraint is

$$\begin{aligned} & (1+x_t\tau_t^C)C_t + K_{t+1} + q_t^G B_{t+1}^G + x_t T_t \\ \leq \;& W_t N_t + (1-x_t\tau_t^K)R_t^K K_t + (1-x_t\tau_t^G)B_t^G + G_t, \end{aligned} \tag{4}$$

where q_t^G is the price of government bonds, T_t is the lump-sum tax, and G_t is the lump-sum transfer from the government.

The government imposes a once-and-for-all tax only at the time of the crisis ($x_t = 1$), where τ_t^C is the consumption tax rate, τ_t^K is the tax rate on capital stock, and τ_t^G is the tax rate on GBOs. An important note is that the latter two taxes are wealth taxes, not taxes on net returns from holding these assets. The tax on GBOs is essentially equivalent to a partial default (full default when $\tau^G = 1$).[11] The tax on capital stock, which is also called a capital levy, may need further explanation. In Section 4, we discuss the capital levy when the fiscal crisis strikes from the theoretical, political, and historical points of view.

The stochastic discount factor becomes

$$M_{t+1} = \beta\left(\frac{1+x_t\tau_t^C}{1+x_{t+1}\tau_{t+1}^C}\right)\left(\frac{C_{t+1}}{C_t}\right)^{\nu(1-\psi)-1}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\nu)(1-\psi)}. \tag{5}$$

The Euler equation is written as

$$E_t\left(M_{t+1}(1-x_{t+1}\tau_{t+1}^K)R_{t+1}^K\right) = 1, \tag{6}$$

$$E_t\left(M_{t+1}(1-x_{t+1}\tau_{t+1}^G)\right) = q_t^G. \tag{7}$$

We define $q_t^F \equiv E_t\left[M_{t+1}(1-x_{t+1}\tau_{t+1}^K)\right]$, which represents the price of an imaginary private bond that pays out one unit of capital stock, including tax payment, in period $t+1$ with probability one. We also define $R_t^F \equiv 1/q_t^F$ and $R_t^G \equiv 1/q_t^G$ and call $R_t^F - R_t^G$ and $R_t^G - 1$ the credit spread and government bond yield, respectively. Conceptually, this credit spread should be interpreted as an excess return on an imaginary corporate bond that is free from the default risk, and thus it differs from the credit spread shown in Figure 4. In Section 5.2, we construct a

[11] Although our model is a real model without any explicit role for the nominal variables, a partial default in our model can be interpreted as a debt reduction by inflation tax or seigniorage.

richer model based on Gourio (2013), where firms issue both defaultable corporate bonds and equity and the price of corporate bonds is used to calculate the credit spread.

The price of government bonds, q_t^G, is normally less than one because M_{t+1} is less than one, so the government bond yield is positive and higher as τ^G is higher. However, as we will see below, the government bond yield may be negative when τ^K is very high. In other words, the household is eager to hold the government bond rather than capital, which makes the government bond yield negative owing to the no-arbitrage condition between capital and government bonds.

Government

The government spends by way of a lump-sum transfer $G_t > 0$, whose ratio to TFP z_t is constant for all x_t, while tax is zero if $x_t = 0$ and nonnegative if $x_t = 1$. The government budget constraint is given by

$$q_t^G B_{t+1}^G + x_t \tau_t^C C_t + x_t \tau_t^K R_t^K K_t + x_t \tau_t^G B_t^G + x_t T_t = B_t^G + G_t. \tag{8}$$

To determine the tax rates, we assume tax weight ω^i $(i = C, K, G, T)$, which is exogenous and satisfies

$$\begin{aligned} \tau_t^C C_t &= \omega^C (B_t^G + G_t), & (9)\\ \tau_t^K R_t^K K_t &= \omega^K (B_t^G + G_t), & (10)\\ \tau_t^G B_t^G &= \omega^G (B_t^G + G_t), & (11)\\ T_t &= \omega^T (B_t^G + G_t), & (12)\\ 0 &< \omega^C + \omega^K + \omega^G + \omega^T \leq 1. & (13) \end{aligned}$$

If $\omega^C + \omega^K + \omega^G + \omega^T = 1$, the government issues no new bonds; in other words, it owes nothing after the crisis period. Note that the government in our model implements its tax policy mechanically, taking the parameters ω^i $(i = C, K, G, T)$ and realization of the crisis x_t as given. We adopt this assumption because the government has almost no degree of freedom in optimizing the tax decision, as it is subject to various constraints imposed by stakeholders such as the politicians of ruling and opposition parties, relevant agencies within the government, business communities, lobbyists, and the mass media.

Market Clearing

The goods market is cleared when

$$Y_t = C_t + I_t, \tag{14}$$

where investment I_t equals $K_{t+1} - (1 - \delta) K_t$.

3 Simulation

3.1 Simulation Method

In the model, although the state variables are $\{K_t, B_t^G, x_t, z_t\}$, the equilibrium can be expressed by using $\{k_t = K_t/z_t, b_t^G = B_t^G/z_t, x_t\}$. In what follows, we denote the variables divided by z_t by their lower case letters (e.g., $y_t = Y_t/z_t$) with the exception of u_t, which is defined as $U_t^{1-\psi}/z_t^{\nu(1-\psi)}$. Note that the GBO, b_t^G, does not explode, although there is no deterministic steady state. Because of equation (2), the crisis probability increases exponentially with b_t^G provided $d_1 > 0$, which stabilizes b_t^G.[12]

Following Gourio (2013), we solve the model by using a projection method. The policy functions are approximated by two-dimensional Chebychev polynomials with respect to k_t and b_t^G whose degree is five in each dimension.

3.2 Parameterization

Table 1 shows the benchmark parameter values we use for the simulation. Most of the parameter values are standard and based on Gourio (2013). The trend growth rate of TFP, μ, is 0.0179, which equals the actual annual growth rate of real GDP per capita from 1990 to 2016 for Japan (see Figure 1).[13] Government spending $g = G_t/z_t$ is chosen as 0.02 to be consistent with the speed of Japan's government debt accumulation.

The important parameters that could be controversial are those associated with the crisis. For the crisis probability, we set $d_0 = 0.05$ and $d_1 = 1$, which means that the government imposes taxes to reduce its debt about once every 20 years if government debt is sufficiently low. This probability may be considered to be too low if one refers to past default episodes (e.g., Reinhart and Rogoff, 2010); however, it is not necessarily too low if we focus on developed countries such as Japan and the United States (e.g., Morikawa, 2016, 2017). As we discuss below using Figure 5, these parameter values suggest that the crisis probability is about 0.08 and 0.13 when the ratio of government debt to GDP, b_t^G/y_t, is one and three, respectively.

Even more challenging parameters for the calibration are the tax weights in the crisis period, ω^i $(i = C, K, G, T)$. In the benchmark simulation, we choose $\omega^K = 0.267$, $\omega^G = 0.133$, and $\omega^T = 0.100$ to fit the actual paths of the credit spread and government bond yield for Japan. We set ω^C to zero. In the next section, we provide the rationale behind this choice, particularly for the capital levy, by showing that our benchmark simulation result is altered by changes in the tax weights and discussing which tax scenario when the crisis occurs is most consistent with the data. In other words, we infer from the data what Japanese people anticipate given the framework of our model.

[12] For the numerical calculation, we set that there is an upper bound for the value of b_t^G. This implies that a lump-sum tax is imposed when necessary to keep b_t^G within the upper bound, and hence satisfying the transversality condition.

[13] As Gourio (2012, 2013) and Isore and Szczerbowicz (2017) argue, how we calculate a trend from data is affected by occasional disasters. However, the government debt crisis did not occur from 1975 to 2016 for Japan, so we do not exclude any sample observations to calculate a trend. Changing the definition of the trend does not alter our main results.

3.3 Simulation Results Based on the Benchmark Model

Moments of Variables: Is the Fit of the Model Good?

In what follows, we show two kinds of simulation results. The first simulation aims to check the fit of our model with the data. We conduct a stochastic simulation and tabulate the key first and second moments of the variables such as the mean of the debt to output ratio (b_t^G/y_t) and the correlation coefficient between the change in output ($\Delta \log Y_t$) and the debt to output ratio in the previous period (b_{t-1}^G/y_{t-1}). We calculate these moments by generating the time-series path of TFP, z_t, for $t = 1, 2, \cdots, 2000$ and discarding the first one-third periods. For comparison purposes, we tabulate the actual moment values for Japan from 1975 to 2016, where we use net government debt, not gross, for B_t^G, except for the figures in parentheses. Because a crisis event greatly influences the second moments of the variables and Japan did not experience a crisis from 1975 to 2016, the second moments we report are calculated by excluding the crisis periods.

The second row in Table 2 shows the result of the stochastic simulation based on our benchmark model. The fit of the model is reasonably good. The correlation coefficients are particularly well fitted with respect to both their signs and their sizes. For example, the model correctly yields the negative correlation between output growth and the debt to output ratio. Further, output growth is negatively correlated with the credit spread, while the former is positively correlated with the government bond yield. Although the mean of the simulated credit spread, $R_t^F - R_t^G$, is considerably lower than the actual mean, the former becomes considerably higher in the richer models that we explain in Section 5.2.

Thus, the main caveat in this table is that the simulated government bond yield, $R_t^G - 1$, in both the benchmark and the richer models, is considerably higher than the actual mean. In reality, government bonds act as a medium of exchange like money in that they serve as collateral in short-term transactions in the interbank market. This should add a liquidity premium onto the price of government bonds, decreasing the government bond yield. One of the major causes of the discrepancy between the simulated and actual bond yields may be that our model does not take account of such a liquidity premium.

Time-series Path: Does the Model Explain a Persistent Stagnation?

As the second exercise, we generate the time-series paths of the economic variables by assuming an exogenous path of the crisis indicator x_t and discuss whether our model can explain Japan's persistent stagnation. Specifically, we assume normal times, $x_t = 0$, from $t = 1$ to 39 years, turning to crisis periods, $x_t = 1$, at $t = 40$ and back to $x_t = 0$ thereafter.[14] We set the initial value of k_t as the mean of capital stock in the stochastic simulation, while we set the initial value of b_t^G to zero because the actual value in 1990 was almost zero. We then simulate the time-series paths of the economic variables. For comparison purposes, we also plot the actual

[14] The agents in our model do not know this predetermined event, and instead they form expectations on the crisis probability based on equation (2). Thus, the following simulated paths until the period of $t = 39$ years do not change as long as the crisis occurs at $t = 40$ or later. The longer the delay in the timing of the crisis, the larger the crisis effect because the government introduces larger tax increases.

paths for Japan from 1990 to 2016, where real GDP and investment per capita are shown in deviations from the linear trend of real GDP per capita from 1975 to 2016 (see Figure 1), which is 1.79% annual growth.

Figure 5 shows the simulation result. The simulated path of the government debt to GDP ratio (b_t^G/y_t) depicted in the solid line is similar to the actual path depicted in the solid line with circles. This result is not surprising because we choose government expenditure g to make the two paths match. As government debt accumulates, our model predicts decreases in both output (y_t) and investment (i_t). This is because the crisis probability increases, and the agents in the model are more prepared for a future crisis as well as the resulting high taxes. They are thus discouraged from investing, particularly in capital stock, because of the high capital levy, $\omega^K = 0.267$, which leads to an increase in the credit spread $R_t^F - R_t^G$.[15] There is a discrepancy between the actual and simulated paths of the credit spread in our benchmark case, whereas the path of the simulated credit spread becomes much closer to the actual path in the simulation using richer models, that are explained in Section 5.2.

Two remarks are worth making. First, for the reason we just explained, the decreases in output and investment occur not only in their levels but also in their growth rates as government debt accumulates. Thus, the gap from their trend widens over time, consistent with the actual persistent stagnation in Japan. Quantitatively, our benchmark model seems to explain the actual decrease in output by almost one quarter. Regarding investment, the benchmark model explains the actual decrease almost perfectly.

Second, the simulated bond yield $R_t^G - 1$ decreases rather than increases with government debt. This is because given $\omega^K = 0.267$ and $\omega^G = 0.133$, agents invest more of their savings in government bonds and less in capital. Because a tax on government bonds is virtually a default, this simulation result implies that Japanese do not anticipate a large-scale government bond default, although they do fear the risk of tax increases when a crisis strikes. These patterns are consistent with what we observe from the Japanese data.

The level of the bond yield is higher than the actual one by about 2% points throughout our simulation periods. As we noted in the discussion of Table 2, this may arise from the fact that our model does not take account of a liquidity premium for government bonds.

3.4 What Happens if People Have Different Expectations about Tax Scenarios in the Crisis Period?

In the benchmark model, we assumed tax weights of $\omega^K = 0.267$, $\omega^G = 0.133$, and $\omega^T = 0.100$ as the governmental tax policy following the government debt crisis. In this subsection, we investigate how our simulation results change when Japanese people have alternative expectations about tax scenarios at the point of the crisis.

First, we consider a first-best case in which the government imposes a lump-sum tax (T_t)

[15] The *ex ante* slowdown in output can be thus interpreted as an extreme version of the Ricardian equivalence. In the model, we assume for simplicity that a crisis ends in one period, unless x_t happens to be one in two or more consecutive periods. As our results can be interpreted as those of the Ricardian equivalence, this assumption should not be essential for our results. Our results will hardly change even if we assume that a crisis continues for more than one period as long as the discounted present value of tax revenue during a crisis is similar.

only and the size of the tax collection is the same as that in the benchmark case, namely $\omega^T = 0.6$. Such a large-scale imposition of a lump-sum tax is hypothetical, but a part of this reflects a decrease in government transfers (e.g., a decrease in pension obligations or medical care) during a crisis or an imposition of a poll tax. The results are shown as the "T tax" in Table 3 and Figure 6. For comparison purposes, we show the path of consumption in the bottom-left panel of Figure 6, while we divide output, investment, and consumption by the mean of output in the benchmark model. The rightmost column in Table 3 shows the mean of lifetime utility U_t, which is normalized to zero in the benchmark model and expressed in the unit of permanent consumption change from that based on the benchmark. Because the lump-sum tax is not distortionary, lifetime utility increases, amounting to a 1.2% increase in consumption. The means of output and investment also increase. However, the debt to GDP ratio has no effect on real economic activity, as Figure 6 shows.

Next, as the second and third cases, we consider a tax on either capital ($\omega^K = 0.267$, $\omega^G = 0$, and $\omega^T = 0.233$) or government bonds ($\omega^K = 0$, $\omega^G = 0.133$, and $\omega^T = 0.367$) with positive ω^T to compensate for the lack of tax revenues at the time of the crisis. The results of these two cases are shown as the "K tax" and "G tax," respectively in Table 3 and Figures 6 and 7. Figure 6 shows that the K tax dampens *ex ante* output, just as in the benchmark case, while the G tax has virtually no effect on real economic activity. Interestingly, however, in the G tax case, the government bond yield rises as government debt increases. This lowers the price of government bonds, leading to a faster increase in government debt and a higher crisis probability compared with the benchmark model. However, such a default of government bonds in the case of a positive ω^G has almost no effect on real economic activity *per se* because government bonds play no distortionary role in our model. As a result, lifetime utility is almost as high as that in the T tax case. The credit spread decreases because the government bond yield increases.

Although the G tax is almost neutral in terms of its effect on real economic activity *per se*, it does magnify the effect of the K tax on the real economy. Figure 7 compares the path of the economic variables based on the benchmark model (K+G tax) with that based on the model without a tax on government bonds (K tax). It shows that the presence of a tax on government bonds indeed magnifies the decreases in output and investment. Because the bond price falls, government debt increases more rapidly. Along with the increase in the crisis probability, this contributes to the further decrease in output and investment. For this reason, lifetime utility in the benchmark model (K+G tax) is worse than that in the K tax case.

Finally, we consider the case in which the government imposes a consumption tax when the crisis hits. We assume a lower value of ω^C, as low as 0.2 with $\omega^T = 0.3$, because we are unable to find an equilibrium for higher values. If we assume a high ω^C, when government debt is large, the government must raise the tax rate on consumption to an unlimitedly high level. However, even with such a high tax rate, the government cannot collect the necessary tax revenue to repay its debt as an upper limit of tax revenue exists because households decrease consumption in response to a temporary tax hike.[16] The simulation result for consumption tax ($\omega^C = 0.2$,

[16]See Hiraga and Nutahara (2017) for the study of the Laffer curve regarding consumption tax. They show that consumption tax revenue can be arbitrarily large at the steady state under a certain condition, whereas we

$\omega^K = \omega^G = 0$, $\omega^T = 0.3$) is shown as "C tax" in Table 3 and Figure 6. We find that not only output and investment but also consumption decreases with an increase in government debt. As government debt increases, the expected consumption tax rate in the next period increases. Hence, intertemporal substitution makes the household tend to consume more in the current period than in the next period. However, the negative wealth effect of the expected increase in tax distortion in the next period discourages output, investment, and consumption in the current period. Our simulation shows that the overall effect on current consumption is negative.

There are many alternatives to the tax policies considered above. One of the most realistic options is income tax, where tax is imposed on the return on capital (e.g., $R_t^K - 1$ in our model), the return on government bonds (e.g., $R_t^G - 1$), or labor income (e.g., $W_t N_t$). However, such taxes on incomes (flow variables) are insufficient to repay government debt because their revenues cannot exceed the respective incomes. Thus, expectations of income taxes have small effects on pre-crisis output quantitatively. Qualitatively, the effect of a capital income tax is similar to that of the K tax, while the effect of a tax on income from interests on government bonds is similar to that of the G tax. The effect of a labor income tax is similar to that of the C tax, both qualitatively and quantitatively.

4 Discussion on the Capital Levy

As we saw in the numerical simulation, the fear of a government debt crisis can cause persistent stagnation beforehand only if people share the expectation that a capital levy, a one-time tax on all wealth holders with the goal of retiring government debt, will be imposed at the time of a government debt crisis. Our theoretical exercise similar to business cycle accounting to find the best combination of tax wedges is completed as we found $\omega^K = 0.267$, $\omega^G = 0.133$, and $\omega^T = 0.100$. What we do in this section is to briefly sketch the possible mechanisms that can be ingredients of the models that justify this finding that people should share the expectations of a capital levy at the time of a debt crisis.

4.1 Learning from History

People know that the tax decision at the time of a crisis is not a simple optimization by the government, but rather a result of complex political and economic interactions among policy stakeholders. Given this political and random nature of the tax decision when a government debt crisis occurs, people may posit that ω^i $(i = C, K, G, T)$ are given exogenously, and form the belief on the values of ω^i by learning from the historical episodes. The episodes in interwar Europe and postwar Japan indicate that people may learn that ω^K is positive.

History: Europe during the Interwar Period As Eichengreen (1989) summarizes, prominent British economists and policymakers debated the use of a capital levy in the 1920s when the UK government was suffering from the public debt overhang due to the war debt of World

show numerically that it is bounded from above when focusing only on a temporary tax hike.

War I. This active debate on the capital levy exemplifies the strength of the *ex post* temptation for policymakers to introduce a one-time capital levy when government debt builds up.[17] Similar debates were also active in Italy, Czechoslovakia, Austria, Hungary, Germany, and France. Although Eichengreen concludes that the capital levies in those countries failed,[18] if history repeats itself or people learn from previous events, these episodes suggest that some people anticipate the capital levy at the point a crisis strikes. Moreover, there exists a noticeable example of successful implementation: post-World War II Japan.

History: Postwar Japan In Japan, the government debt inherited from wartime amounted to 267% of national income in 1944, more than 99% of which was internal debt (Ministry of Finance (MOF), 1976 and Kawamura, 2013). Although Eichengreen (1989) emphasizes that the absolute power of the Supreme Commander for the Allied Powers that occupied Japan was crucial in the successful implementation of the capital levy of 1946–47, MOF (1976) and Kawamura (2013) show memorandums that prove that it was the MOF that decided to impose a capital levy to avoid the outright default of government debt. The Supreme Commander actually recommended to declare a partial default. The capital levy, or wealth tax, in Japan was a tax on all the real and financial assets of Japanese residents, such as land, houses, government bonds, bank deposits, and machinery. Tax rates varied progressively from 25% to 90% depending on the income class of taxpayers. The capital levy worked effectively to reduce wealth inequality among Japanese.[19] "With important elements of democracy in suspension, the levy could be quickly and effectively implemented" (Eichengreen, 1989), with the deposit blockade and withdrawal of the legal tender status of old yen. The package of these policies, which could not have been implemented in normal times, helped the Japanese government to seize domestic wealth efficiently.[20]

[17] Eichengreen (1989) points out that "[i]n modern times, capital levies have come under consideration following every period of major military expenditure and rapidly rising debt/income ratios. (...) None of these proposals was adopted. For examples where capital levies were actually implemented, we must turn to the 20th century."

[18] Eichengreen (1989) finds that Italy and Czechoslovakia were the closest to success. One factor that caused the failure of the capital levies in these countries was the democratic decision-making processes, as the political resistance of property owners led to extreme delays and opportunities of capital flight.

[19] According to MOF (1976), the MOF's main aim of the capital levy was to avoid the default, while the Supreme Commander's main aim was to reduce inequality. Kawamura (2013) points out that although the super rich class was taxed most heavily, the middle class paid the largest proportion of the total revenue of the capital levy overall.

[20] Saito (2017) points out that the exchange of old yen for new yen was effective for the government to seize the private assets concealed on the black market. However, it is also important to note that the tax revenue from the capital levy was below the necessary amount to restore the sustainability of government debt. According to Kawamura (2013), the estimated revenue was 43.5 billion yen, while the amount of GBOs was 140.8 billion yen in 1945. By contrast, Hattori and Oguro (2016) estimate that the seigniorage revenue for the Japanese government resulting from hyperinflation immediately after World War II was as high as almost 29% of GDP. However, this large seigniorage revenue was not an intended one by the government or MOF officials. Rather, the MOF was concerned about the risk of hyperinflation like Germany, which prompted the MOF to introduce the capital levy (MOF, 1976).

4.2 Optimal Taxation under the Lack of Commitment

Another possible mechanism for the expectations of a capital levy to prevail in the rational expectations framework is optimal taxation by the government subject to the lack of commitment.

The government does not have the full ability to commit *ex ante* to or not to impose a certain type of tax when a debt crisis occurs.[21] From the menu of various taxes such as labor income tax, capital income tax, consumption tax, and capital levy on capital stock, Eichengreen (1989) argues that the once-and-for-all capital levy has no distortionary effect on economic activity *ex post facto* in theory, whereas other taxes have more or less distortionary consequences on the economy. Thus, given that the government cannot commit beforehand, the optimal policy for the government would be to impose a capital levy during a crisis.[22] In general, optimal taxation theory (Chamley, 1986; Chari, Christiano, and Kehoe, 1994) shows that the optimal tax rate on capital stock or capital income can be positive only in the first period when the government renews the tax schedule. This is because it does not distort the accumulation of capital stock, which is already predetermined when the new tax is introduced. Given this knowledge of optimal tax theory and lack of the government's commitment to tax policy at a crisis, it can be rational for people to anticipate the imposition of a capital levy following a government debt crisis.

4.3 Reduced Form of a Financial Crisis

The imposition of a once-and-for-all capital levy upon a government debt crisis may be interpreted as a reduced form of a financial crisis associated with an abrupt decline in the real value of government debt. A debt crisis, or an abrupt fall in the value of government debt, may take various forms that include outright default. In a debt crisis, expectations of a default typically prevail in the market, leading to fire sales of government bonds by bond-holders and a fall in bond value. As banks and other financial institutions tend to hold government debt as a large part of their assets, it is straightforward that the fall in government debt will make them insolvent and lead to a financial crisis, causing fire sales of productive capital. Then, the fire sale misallocates capital stock in the economy and makes them used inefficiently, leading

[21] Why does the government lack the ability to commit to the tax policy at a crisis beforehand? At the onset of a government debt crisis, the government should face uncontrollable economic turmoil because investors lose confidence in government debt. They rush to exchange government debt for real assets and goods, leading to a sharp and volatile economic downturn. The government is then forced to use any means to raise a large amount of tax revenue to restore confidence in its ability to repay debt and stabilize the economy. A sufficient amount can be raised only by imposing a capital levy and a tax on GBOs, as our numerical experiment shows. The revenues from other orthodox tax instruments such as income taxes are negligibly small, making them ineffective at restoring market confidence immediately.

[22] Another optimal policy in our model is a tax on government bonds (i.e., defaults). Actually, this is not distortionary either *ex ante* or *ex post*, because government bonds alone do not play a role in real economic activity in our model. However, in reality, government bond defaults might have large economic consequences by causing a financial crisis. There may be also a political reason for the government to choose imposition of a capital levy, and not default of the government bonds: the reputation cost for the government officials. It is said that the MOF officials in the postwar Japan were inclined to avoid the default because it was regarded as a shame for them who were responsible for maintaining the credibility of government debt.

to a reduction in the aggregate value of capital stock, at least temporarily. This reduction in capital value works as if it were a capital levy from the perspective of households. Thus, the fear of a capital levy at the time of a government debt crisis can be seen as a reduced form of the fear of a financial crisis.

4.4 Political Considerations

In a model of political economy, we may be able to show that a capital levy is supported by the majority of people upon a government debt crisis. Then, it may be rational for people to hold the expectations of a capital levy beforehand.

A political charm of a capital levy is that it is effective at reducing wealth inequality. This effect was demonstrated in postwar Japan (Eichengreen, 1989). A debt crisis may worsen the inequality and strengthen people's desire to correct it. Although history tells us that capital levies tended not to be introduced successfully in the early 20th century, their effect on reducing inequality may now convince people to believe that a capital levy will win strong support from the public at the onset of a government debt crisis. Moreover, voting rights have been extended to the poor since the early 20th century, which has increased their political power in the democratic system. These lines of thought may lead people to anticipate that a capital levy will be imposed when a crisis occurs.

4.5 Note on Natural Disasters

Japan frequently experiences natural disasters such as earthquakes and tsunamis. Indeed, Figure 2 shows that the second greatest anxiety for Japanese people is natural disasters. Theoretically, a natural disaster is considered to work as a capital levy in our model because it demolishes the capital stock that private agents hold. Thus, fear of a natural disaster raises the required return on capital and may cause a stagnation.

Although natural disasters have a similar effect to capital levies, there are three important differences. First, natural disasters are by nature local events, whereas a capital levy upon a government debt crisis is a nationwide event. Thus, the risk of a natural disaster should not have a large effect on economic dynamics compared with a government debt crisis. Second, a natural disaster demolishes capital, while a government debt crisis does not. A debt crisis just changes the allocation of capital. The third difference is that the risk of a natural disaster is irrelevant to the size of government debt, making it hard to explain the decline in the growth rate of output as debt accumulated from the early 1990s.[23]

[23] The Great East Japan Earthquake in 2011 and global climate change may cause Japanese to be increasingly concerned about natural disasters. Meanwhile, the accumulation of government debt may prevent the government from spending in natural disaster mitigation. If this is the case, government debt serves to amplify the natural disaster risk.

5 Further Analyses

5.1 A Smaller Probability of Crisis

We examine the robustness of our results to changes in the parameters associated with the crisis probability, d_0 and d_1. Table 3 shows the two cases in which the sensitivity of the crisis probability to government debt, d_1, decreases from 1 to 0.1 and the crisis probability at the time of no government debt, d_0, decreases from 0.05 to 0.01. In both cases, the mean government debt to GDP ratio increases, because the crisis probability decreases. The mean credit spread also decreases, which leads to an increase in the mean output level and welfare. It is important to note that the negative correlation between output growth and the debt to output ratio disappears. In this regard, our benchmark parameters seem better in the model fit.

5.2 Simulation Results Based on the Full Model

In this section, we conduct simulations by using richer models to obtain more reliable quantitative results. Since the increase in the credit spread, $R_t^F - R_t^G$, plays a key role in generating the stagnation, the benchmark model is extended in two ways to improve the fit of the credit spread, using Gourio's (2013) model. First, an extended model incorporates the corporate finance (CF) structure, where firms issue both corporate bonds and equity. The price of corporate bonds is then used to calculate the credit spread. We call this model the "+CF model." Second, the setup is changed so that the household has Epstein-Zin preferences. In the benchmark model, the intertemporal elasticity of the substitution of consumption, ψ, has another meaning: risk aversion. By using Epstein-Zin preferences, we treat them apart, which makes the credit spread in our model more reliable. We call the model with a CF structure and Epstein-Zin preferences the "Full model." See Appendix B for a detailed explanation of its setup. We borrow most of the new parameter values from Gourio (2013): the recovery rate of firm value by corporate bond-holders at firm default, θ, is 0.7; the debt advantage over equity, χ, is 1.042; risk aversion, γ, is 10; and the intertemporal elasticity of the substitution of consumption, ψ, is 0.5. We set a higher value for the standard deviation of idiosyncratic shocks, σ_ε, 0.35, while Gourio (2013) sets it at 0.01925. This modification increases the mean of the firm default probability to 0.02 per year, which is equal to the actual average default probability from 1984 to 2016 in Japan. All the other model setup details such as $\omega^i (i = C, K, G, T)$ are the same as before.

Table 2 and Figure 8 confirm that our previous results are qualitatively robust to changes to the "+CF model" and "Full model." Quantitatively, the fit of the model is improved. The simulated path for the credit spread becomes much closer to the actual path for Japan. Meanwhile, decreases in output, investment, and bond yields are slightly attenuated.

In Appendix C, we also calibrate the full model for the US economy and examine whether the same mechanism works in the United States. The model can explain the country's stagnation in terms of output and investment fairly well.

5.3 Permanent Distortionary Tax to Prevent a Crisis from Occurring

Should the government raise tax rates now to avoid a crisis? Because a crisis event causes not only a depression at the time but also stagnation before it, it may be better to introduce higher tax rates preemptively. To answer this question, we consider a model in which the government always aims to maintain a bounded range of government debt. More specifically, we assume the following tax policy. The government imposes only a consumption tax, where the maximum tax rate for τ_t^C is 50%. It sets the target for $b_t^G = B_t^G/z_t$ at 0.5 and chooses the consumption tax rate endogenously to maintain this.[24] Such a tax policy means that the government sets the 50% tax rate when b_t^G is well over its target, which is above $\bar{b} \sim 0.8$. Then, once b_t^G becomes close to its target of 0.5, i.e., b_t^G is considerably lower than $\bar{b}$, the government reduces the tax rate to around 6% and maintains b_t^G around its target.

Table 4 shows the simulation result. The bottom row shows that although the debt to GDP ratio, b_t^G/y_t, stabilizes at a high level around one (89%), output and investment increase compared with the benchmark. As a result, lifetime utility increases, amounting to a 0.3% increase in consumption.[25] Therefore, preemptive tax increases improve social welfare.

6 Concluding Remarks

We analyzed an economy at risk of a government debt crisis occurring and provided a new perspective to explain a secular stagnation and low interest rates. We demonstrated that the major features of the persistent slowdown in economic growth can be accounted for by the increasing fear of the imposition of a capital levy associated with increasing government debt.

As our framework is simple, many possibilities to extend and enrich our model exist. One way would be to introduce nominal variables. The nominal version of our model would be useful to analyze price dynamics such as hyperinflation and the implications for monetary policy in a secular stagnation. In this context, it is important to note that inflation is essentially equivalent to a tax on GBOs, that is, a default, because inflation decreases the real value of nominal bonds directly, whereas it does not affect the real value of capital very much. Thus, we think that our analysis in this paper concerning the tax on GBOs approximates the effects of high inflation in a nominal model. The second possibility would be to make it an open economy in which policy implications may be altered by the interaction between domestic and external debt as well as an incentive for capital flight. In fact, during Japan's lost decades, Japanese firms have increased foreign direct investment, which may be analyzed properly in an open-economy model. Third, our model could be extended to allow uncertainty about what happens during a crisis. A crisis usually entails a large degree of uncertainty in the market and government responses, which

[24] Because the TFP shock z_t is stochastic, the government cannot perfectly stabilize b_t^G or b_t^G/y_t at its target.

[25] We confirm that this result holds for a short-run transition. What we compared above is the mean lifetime utility values at the stochastic steady state for the two tax policies. One may wonder whether we should rather compare lifetime utility at t conditional on the state such that the debt to GDP ratio in the previous period, b_{t-1}^G/y_{t-1}, is high. We confirm that this hardly changes our result. Since the discount factor β in our model is close to one, the household is concerned about the long-run state of the economy rather than its short-run transition.

may alter our result quantitatively. Lastly, it would be worth extending our model to the heterogeneous agent model, where taxes on capital stock and government bonds when a crisis hits influence the holding of assets (i.e., both capital stock and government bonds), which plays an important role in the self-insurance of heterogeneous households.

These extensions may help us explore whether the increasing risk of a crisis causes other economic difficulties in addition to the persistent stagnation demonstrated in this study.

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Table 1: Parameterization in the Benchmark Model

Parameters	Values	Parameters	Values
IES utility ψ	1.5	Crisis prob d_0	0.05
Capital share α	0.3	Crisis prob d_1	1
Discount factor β	0.995	Gov spending g	0.02
Utility weight on C ν	0.3	Tax weight on K ω^K	0.267
Trend growth of TFP μ	0.0179	Tax weight on C ω^C	0
SD of TFP shock σ_e	0.015	Tax weight on B^G ω^G	0.133
Depreciation δ	0.08	Tax weight on lump-sum ω^T	0.100

Table 2: Data and Simulation Results

	b/y	y	$R^F - R^G$	$R^G - 1$
Data	0.4913 (1.1032)	–	0.0079	0.0213
Model				
Benchmark	1.251	1.000	0.0012	0.0389
+ CF	1.067	1.146	0.0146	0.0391
Full model	0.961	1.218	0.0130	0.0327

Excluding disaster

	s(dY)	s(dI)	s(dC)	cor(dY,B/Y)	cor(dI,B/Y)	cor(dY,$R^F - R^G$)	cor(dY,$R^G - 1$)
Data	0.022	0.065	0.017	-0.349	-0.105	-0.171	0.285
Model							
Benchmark	0.013	0.029	0.010	-0.211	-0.125	-0.160	0.421
+ CF	0.013	0.023	0.010	-0.181	-0.116	-0.128	0.425
Full model	0.014	0.030	0.009	-0.167	-0.097	-0.105	0.438

Notes: The data are the means from 1975 to 2016 for Japan, except for the credit spread that is the mean from 1993 to 2016. The figure in the parenthesis indicates gross, not net, government bonds relative to nominal GDP. CF represents the model with the structure of corporate finance. The full model combines the CF model with the Epstein-Zin preference.

Table 3: Simulation Results for Various Model Specifications

	b/y	y	$R^F - R^G$	$R^G - 1$	U
Data	0.4913 (1.1032)	–	0.0079	0.0213	–
Model					
Benchmark	1.251	1.000	0.0012	0.0389	0
T tax	0.974	1.091	-0.0001	0.0259	0.0123
K tax	1.069	1.020	0.0104	0.0261	0.0054
G tax	1.123	1.091	-0.0127	0.0385	0.0123
C tax	1.003	1.071	-0.0001	0.0256	0.0068
low d_1	1.605	1.030	0.0001	0.0344	0.0079
low d_0	2.019	1.057	0.0014	0.029	0.0112
+CF	1.067	1.146	0.0146	0.0391	0.0109
Full model	0.961	1.218	0.0130	0.0327	–

Excluding disaster

	s(dY)	s(dI)	s(dC)	cor(dY,B/Y)	cor(dI,B/Y)	cor(dY,$R^F - R^G$)	cor(dY,$R^G - 1$)
Data	0.022	0.065	0.017	-0.349	-0.105	-0.171	0.285
Model							
Benchmark	0.013	0.029	0.010	-0.211	-0.125	-0.160	0.421
T tax	0.012	0.021	0.009	-0.065	-0.038	-0.558	0.562
K tax	0.013	0.026	0.010	-0.172	-0.109	-0.141	0.420
G tax	0.012	0.021	0.009	-0.095	-0.065	0.092	0.235
C tax	0.026	0.120	0.109	-0.157	0.175	-0.041	0.086
low d_1	0.012	0.024	0.009	0.008	0.227	0.007	0.459
low d_0	0.012	0.023	0.009	0.047	0.185	0.077	0.520
+CF	0.013	0.023	0.010	-0.181	-0.116	-0.128	0.425
Full model	0.014	0.030	0.009	-0.167	-0.097	-0.105	0.438

Table 4: Comparison with a Permanent Consumption Tax Policy

	b/y	y	$R^F - R^G$	$R^G - 1$	U
Data	0.4913 (1.1032)	–	0.0079	0.0213	–
Model					
Benchmark	1.251	1.000	0.0012	0.0389	0
T tax	0.974	1.091	-0.0001	0.0259	0.0123
Always C tax	0.885	1.266	-0.0001	0.0037	0.003

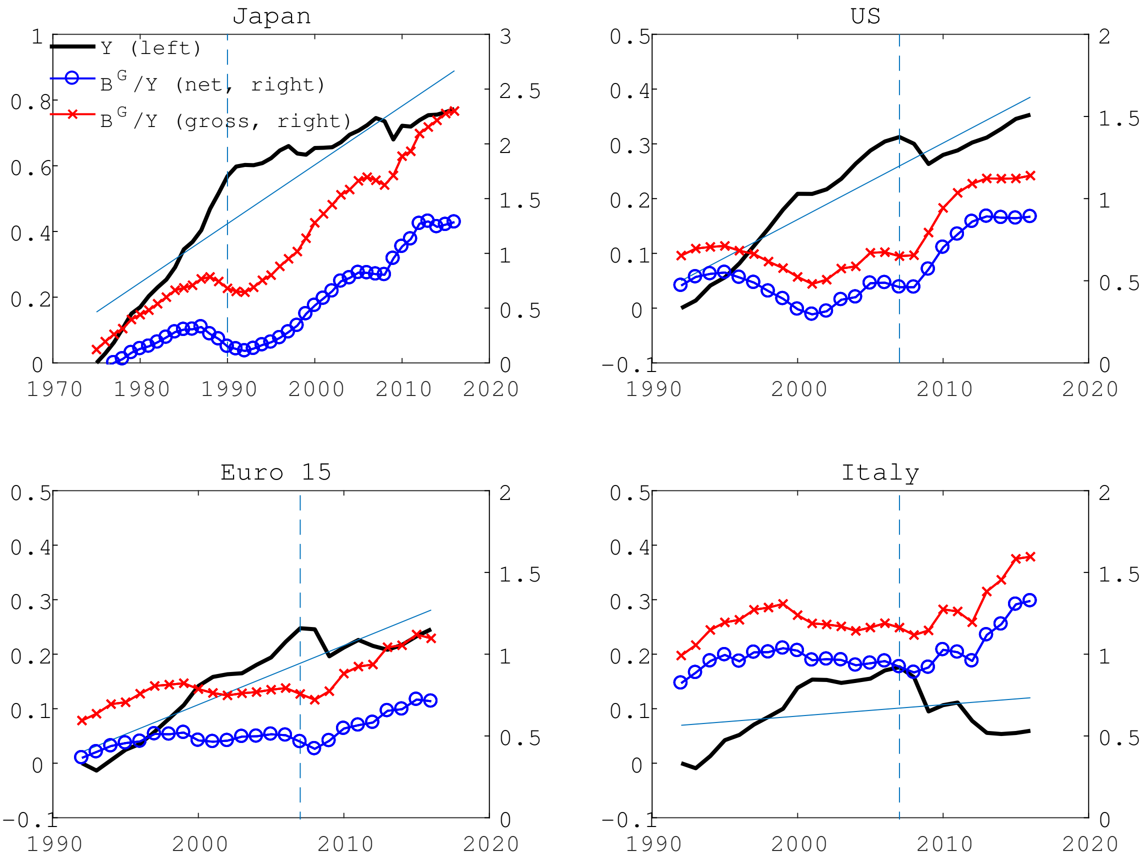

Figure 1: Government Debt and GDP

Note: The beginning year is 1975 for Japan and 1992 for the rest, which represents 15 years before the financial crisis in each region (i.e., 1990 and 2007, respectively, shown as the vertical dashed line). The thick solid line represents the logarithm of real GDP per capita (set to zero in the beginning year) shown on the left axis, while the thin solid line represents its linear trend. On the right axis, the line with crosses and the line with circles represent the ratio of gross and net government debt to nominal GDP, respectively.

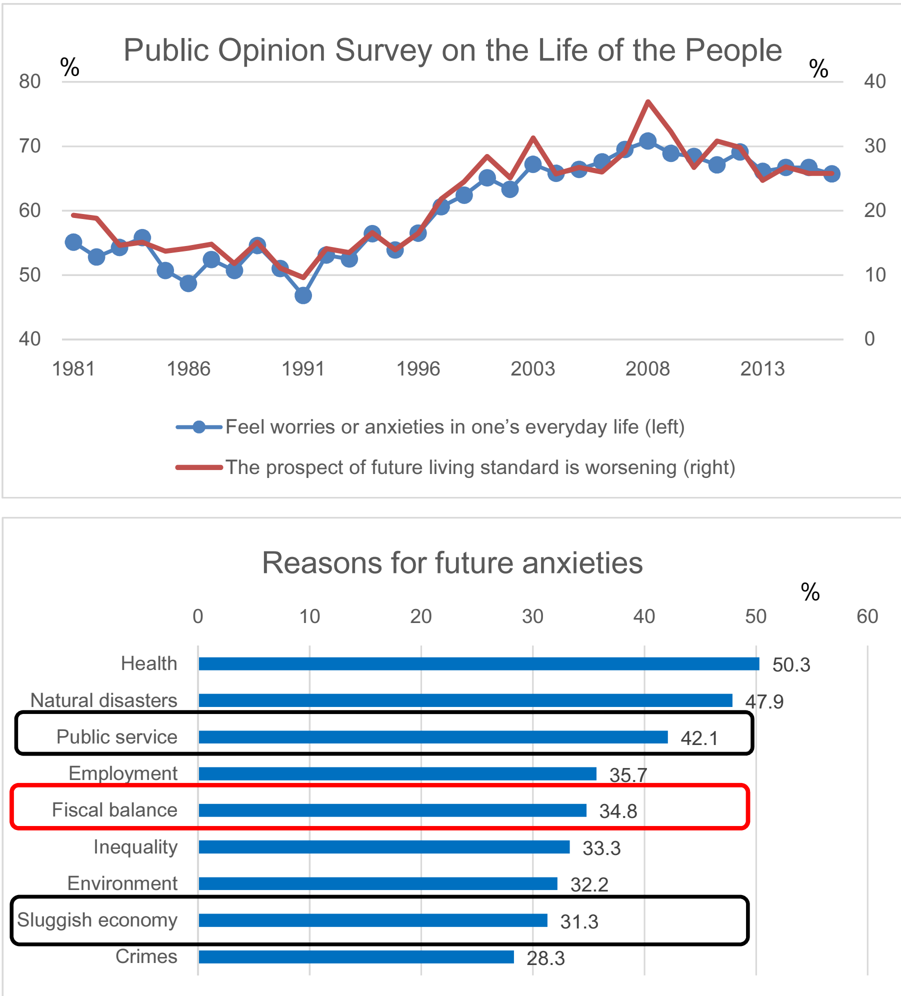

Figure 2: Concerns about Government Debt

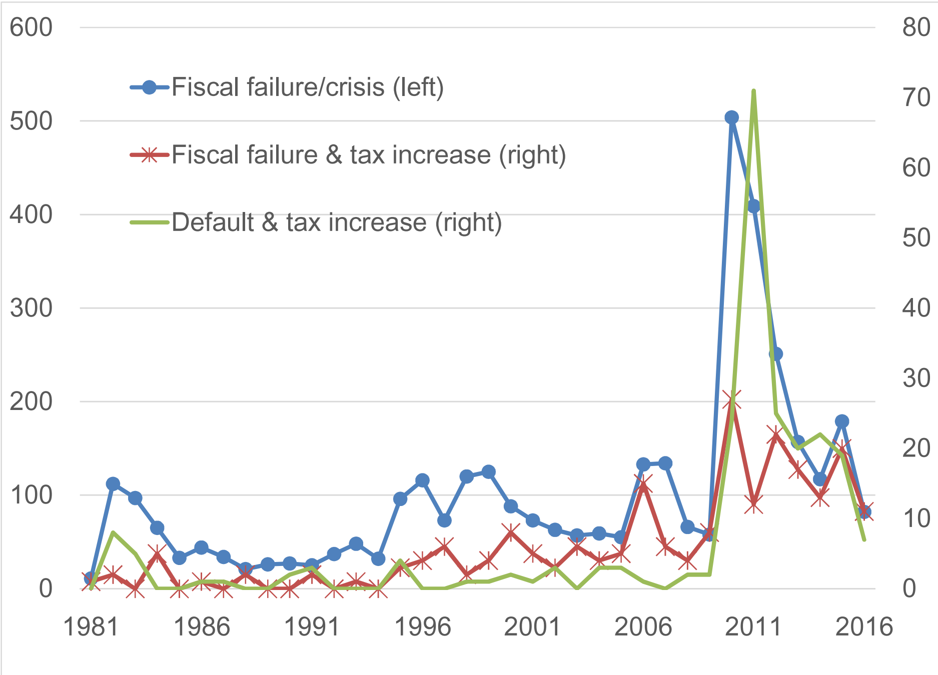

Figure 3: The Number of Occurrences of Specific Words in Newspapers

Note: The figure shows the number of occurrences of specific words in the morning and evening editions of the Nihon Keizai Shinbun, Japan's financial newspaper, for each year.

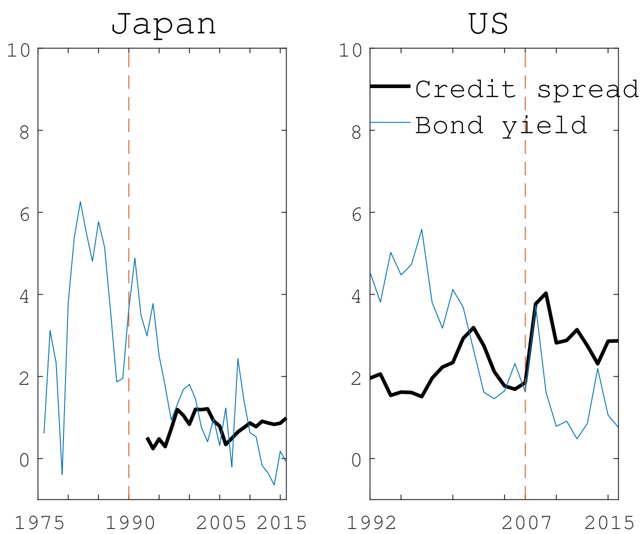

Figure 4: Spreads in Japan and the United States

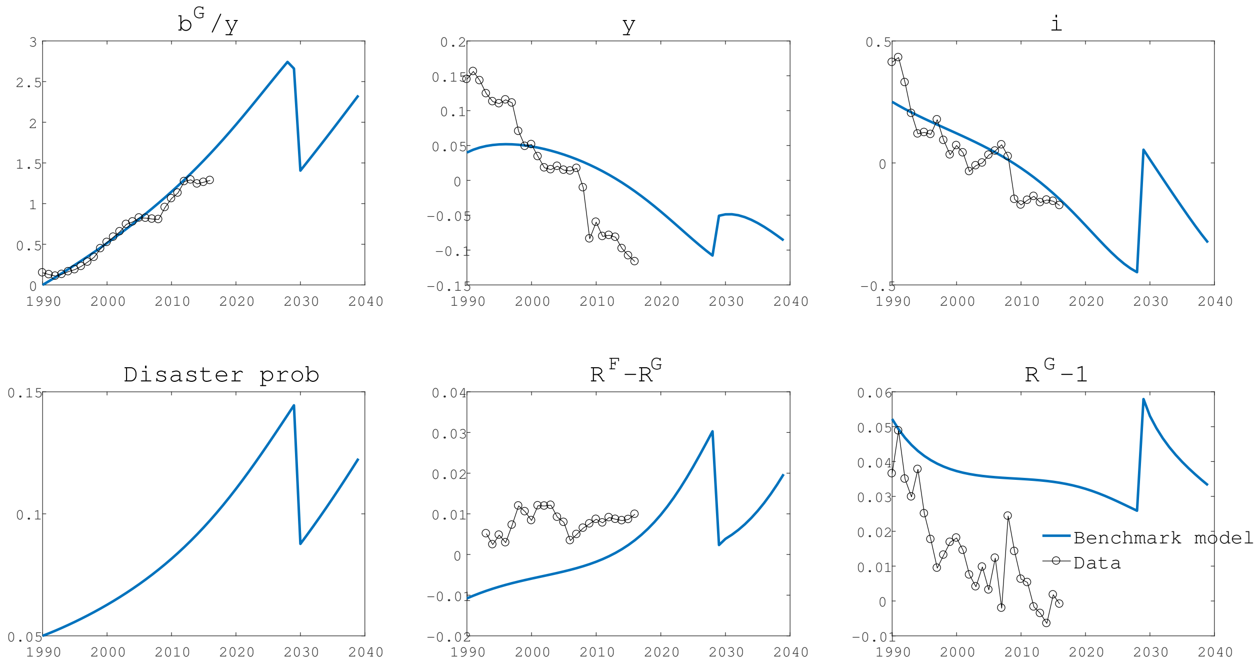

Figure 5: Simulation Results of the Benchmark Model

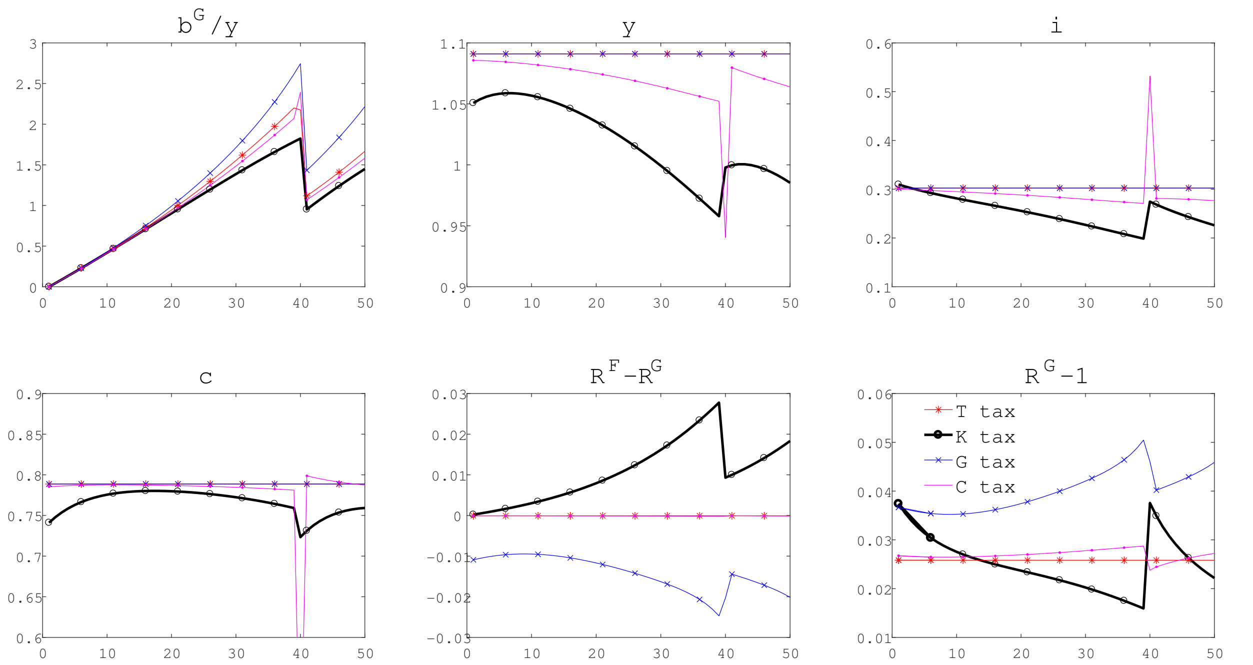

Figure 6: Simulation Results in Different Crisis Scenarios

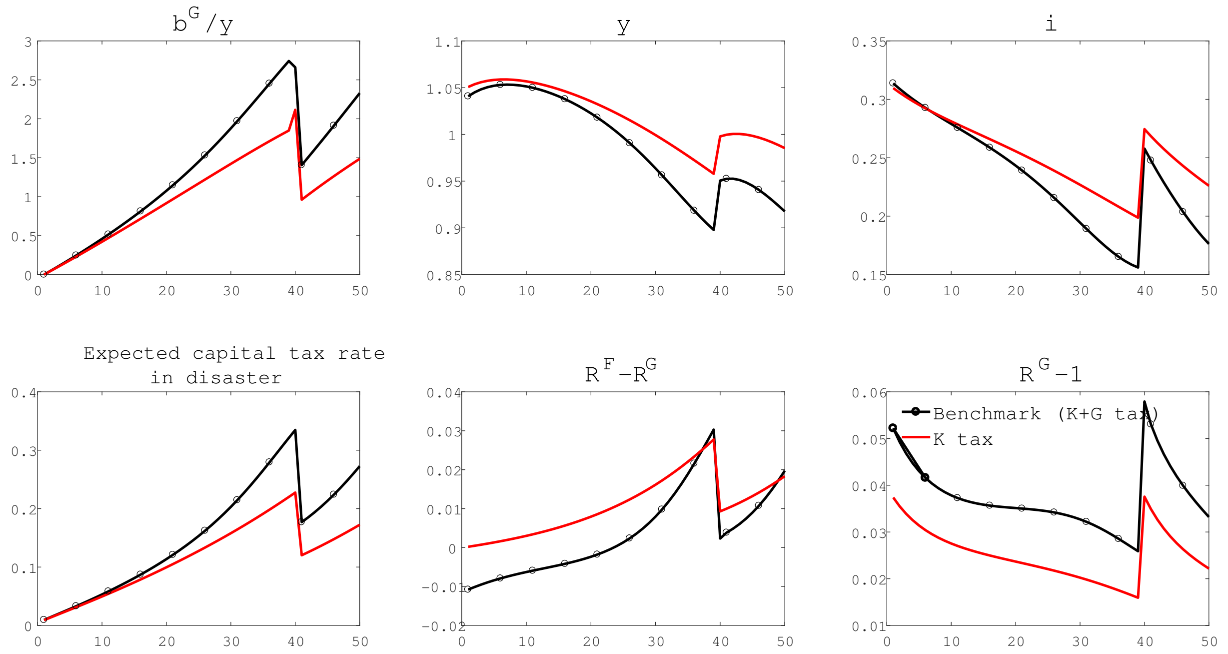

Figure 7: Simulation Results in Different Crisis Scenarios 2

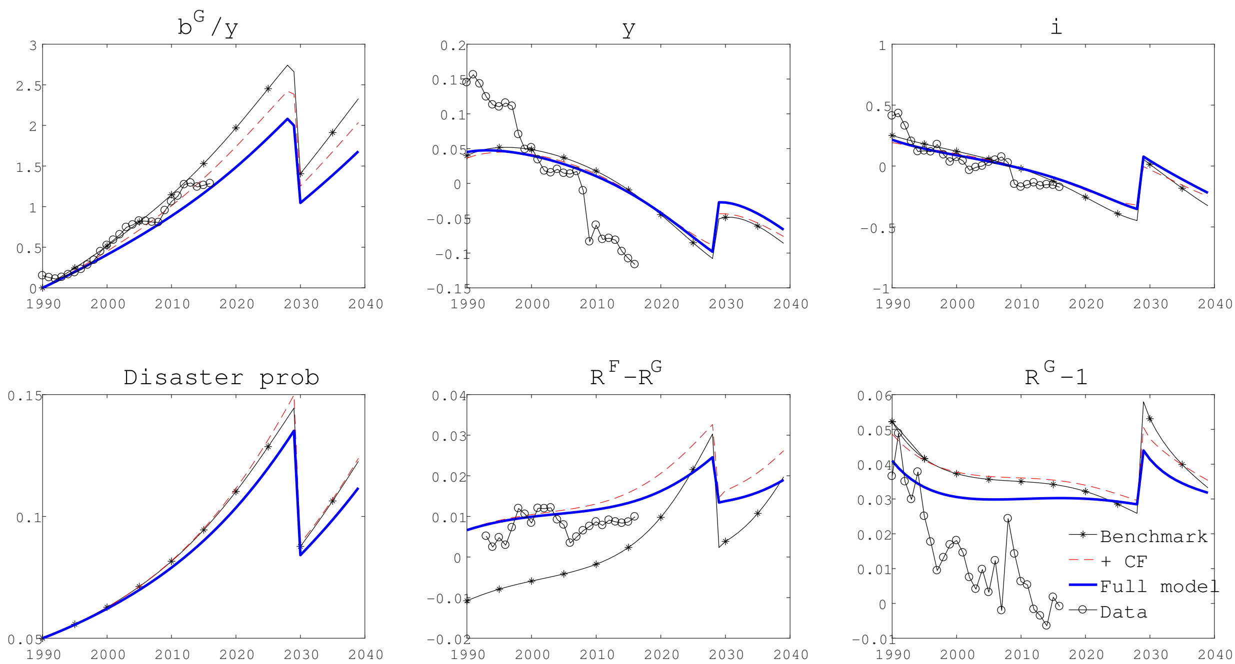

Figure 8: Simulation Results Based on Richer Models

Appendix of "Secular Stagnation under the Fear of a Government Debt Crisis"

Keiichiro Kobayashi* Kozo Ueda†

A Equilibrium in the Basic Model

The labor market is cleared when

$$(1-\alpha)\frac{Y_t}{N_t} = W_t = \frac{1-\nu}{\nu}\frac{(1+\tau_t^C)C_t}{1-N_t}. \quad (1)$$

The goods market is cleared when

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t. \quad (2)$$

In the economy, the state variables are $\{K_t, B_t^G, x_t, z_t\}$; however, as in Gourio (2013), the equilibrium can be expressed by using $\{k_t = K_t/z_t, b_t^G = B_t^G/z_t, x_t\}$. Similarly, we denote the variables divided by z_t by their lower case letters (e.g., $y_t = Y_t/z_t$) with the exception of $u_t = U_t^{1-\psi}/z_t^{\nu(1-\psi)}$.

In summary, we have 15 equations for 15 unknown endogenous variables $\{c_t,\ k_{t+1},\ R_{t+1}^K,\ N_t,\ y_t,\ M_{t+1},\ u_t,\ x_{t+1},\ \tau_t^K,\ q_t^G,\ q_t^F,\ b_{t+1}^G,\ t_t,\ \tau_t^C,\ \tau_t^G\}$:

$$(1-\alpha)\frac{y_t}{N_t} = \frac{1-\nu}{\nu}\frac{(1+x_t\tau_t^C)c_t}{1-N_t}, \quad (3)$$

$$1 = E_t\left[M_{t+1}(1-x_{t+1}\tau_{t+1}^K)R_{t+1}^K\right], \quad (4)$$

$$q_t^G = E_t\left[M_{t+1}(1-x_{t+1}\tau_{t+1}^G)\right], \quad (5)$$

$$q_t^F = E_t\left[M_{t+1}(1-x_{t+1}\tau_{t+1}^K)\right], \quad (6)$$

$$R_{t+1}^K = 1-\delta+\alpha\frac{y_{t+1}}{k_{t+1}}, \quad (7)$$

$$y_t = k_t^\alpha N_t^{1-\alpha}, \quad (8)$$

$$\begin{aligned} M_{t+1} &= \beta\left(\frac{1+x_t\tau_t^C}{1+x_{t+1}\tau_{t+1}^C}\right)\left(\frac{C_{t+1}}{C_t}\right)^{\nu(1-\psi)-1}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\nu)(1-\psi)} \\ &= \beta\left(\frac{1+x_t\tau_t^C}{1+x_{t+1}\tau_{t+1}^C}\right)e^{(\nu(1-\psi)-1)(\mu+\sigma_e e_{t+1})}\left(\frac{c_{t+1}}{c_t}\right)^{\nu(1-\psi)-1}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\nu)(1-\psi)}, \end{aligned} \quad (9)$$

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$$u_t = (1-\beta)c_t^{\nu(1-\psi)}(1-N_t)^{(1-\nu)(1-\psi)} + \beta E_t\left(e^{\nu(1-\psi)(\mu+\sigma_e e_{t+1})}u_{t+1}\right), \tag{10}$$

$$y_t = c_t + k_{t+1}e^{\mu+\sigma_e e_{t+1}} - (1-\delta)k_t, \tag{11}$$

$$p_t = p(b_t^G) = \Pr(x_{t+1} = 1|b_t^G) = d_0 \exp(d_1 b_t^G), \tag{12}$$

$$\begin{aligned} & q_t^G b_{t+1}^G e^{\mu+\sigma_e e_{t+1}} + x_t\tau_t^C c_t + x_t\tau_t^K k_t \\ & + x_t\tau_t^G b_t^G + x_t t_t = b_t^G + g, \end{aligned} \tag{13}$$

$$\tau_t^K R_t^K K_t = \omega^K(b_t^G + g), \tag{14}$$

$$\tau_t^C c_t = \omega^C(b_t^G + g), \tag{15}$$

$$\tau_t^G b_t^G = \omega^G(b_t^G + g), \tag{16}$$

$$t_t = \omega^T(b_t^G + g), \tag{17}$$

where $e_{t+1} \sim N(0,1)$.

B Full Model

Following Gourio (2013), the model is extended to allow for a richer CF structure and Epstein-Zin preferences.

Firms – Two-period lived

Firms are heterogeneous in their capital values. The production function is given by

$$Y_{it} = K_{it}^{\alpha}(z_t N_{it})^{1-\alpha}, \tag{18}$$

where productivity is

$$\log z_{t+1} = \log z_t + \mu + \sigma_e e_{t+1}, \qquad \text{where} \quad e_{t+1} \sim N(0,1). \tag{19}$$

There is an idiosyncratic shock ε_{it} on capital depreciation. Thus, capital equals

$$K_{it} = K_{it}^{w}\varepsilon_{it}, \tag{20}$$

where K_{it}^w is the amount of capital purchased in period t.

The static profit maximization is

$$\pi(K_{it}, z_t; W_t) = \max_{N_{it}\geq 0}\{K_{it}^{\alpha}(z_t N_{it})^{1-\alpha} - W_t N_{it}\}, \tag{21}$$

which yields

$$N_{it} = K_{it}\left(\frac{z_t^{1-\alpha}(1-\alpha)}{W_t}\right)^{\frac{1}{\alpha}}. \tag{22}$$

The value of the firm is

$$\begin{aligned} V_{it+1} &= \pi_{it+1} + (1-\delta)K_{it+1} \\ &= K_{it+1}\left(1-\delta+\alpha\frac{Y_{t+1}}{K_{t+1}}\right) \\ &\equiv \varepsilon_{it+1}R^K_{t+1}K^w_{t+1}. \end{aligned} \tag{23}$$

The corporate bond price is given by

$$q^F_t = E_t\left[M_{t+1}(1-x_{t+1}\tau^K_{t+1})\left(\int_{\varepsilon^*_{t+1}}^{\infty} dH(\varepsilon) + \frac{\theta}{B^F_{t+1}}\int_0^{\varepsilon^*_{t+1}} \varepsilon R^K_{t+1}K^w_{t+1}dH(\varepsilon)\right)\right], \tag{24}$$

where τ^K_t is the capital tax rate imposed only during a crisis, the threshold value for a default ε^*_{t+1} is given by

$$\varepsilon^*_{t+1} \equiv \frac{B^F_{t+1}}{R^K_{t+1}K^w_{t+1}}, \tag{25}$$

and M_{t+1} and θ represent the stochastic discount factor and recovery parameter when bankruptcy occurs ($\theta < 1$).

The firm decision problem for investment and financing is

$$\max_{B^F_{t+1},K^w_{t+1},S^F_t} \left\{E_t\left[M_{t+1}(1-x_{t+1}\tau^K_{t+1})\max(V_{it+1}-B^F_{t+1},\ 0)\right] - S^F_t\right\}, \tag{26}$$

subject to

$$\chi q^F_t B^F_{t+1} + S^F_t = K^w_{t+1},$$

where $\chi \geq 1$ captures the benefit from issuing corporate debts. The above problem is rewritten as

$$\begin{aligned} &E_t\left[M_{t+1}(1-x_{t+1}\tau^K_{t+1})R^K_{t+1}K^w_{t+1}\right] + (\chi-1)E_t\left[M_{t+1}(1-x_{t+1}\tau^K_{t+1})B^F_{t+1}(1-H(\varepsilon^*_{t+1}))\right] \\ &-(1-\theta\chi)E_t\left[M_{t+1}(1-x_{t+1}\tau^K_{t+1})R^K_{t+1}K^w_{t+1}\Omega(\varepsilon^*_{t+1})\right] - K^w_{t+1}, \end{aligned}$$

where $\Omega(\varepsilon^*_{t+1}) = \int_0^{\varepsilon^*_{t+1}} x dH(x)$. At the equilibrium, the equity price is given by S^F_t, which makes equation (26) zero. Thus,

$$S^F_t = E_t\left[M_{t+1}(1-x_{t+1}\tau^K_{t+1})\max(V_{it+1}-B^F_{t+1},\ 0)\right].$$

With equation (25), this leads to

$$E_t\left[M_{t+1}(1-x_{t+1}\tau^K_{t+1})R^K_{t+1}\lambda_{t+1}\right] = 1, \tag{27}$$

where

$$\lambda_{t+1} = 1 + (\chi-1)\varepsilon^*_{t+1}\left(1-H(\varepsilon^*_{t+1})\right) - (1-\theta\chi)\Omega(\varepsilon^*_{t+1}). \tag{28}$$

The first-order condition with respect to B^F_{t+1} is given by

$$(1-\theta)E_t\left[M_{t+1}(1-x_{t+1}\tau^K_{t+1})\varepsilon^*_{t+1}h(\varepsilon^*_{t+1})\right] = \frac{\chi-1}{\chi}E_t\left[M_{t+1}(1-x_{t+1}\tau^K_{t+1})(1-H(\varepsilon^*_{t+1}))\right]. \tag{29}$$

Household

The representative household has Epstein-Zin preferences

$$U_t = \left((1-\beta)(C_t^{\nu}(1-N_t)^{1-\nu})^{1-\psi} + \beta E_t(U_{t+1}^{1-\gamma})^{\frac{1-\psi}{1-\gamma}}\right)^{\frac{1}{1-\psi}}. \tag{30}$$

The budget constraint is

$$(1+x_t\tau_t^C)C_t+S_t^F+q_t^F B_{t+1}^F+q_t^G B_{t+1}^G+x_tT_t \leq W_tN_t+(1-x_t\tau_t^K)(\varrho_t^F B_t^F+D_t^F)+(1-x_t\tau_t^G)B_t^G+\Gamma_t+G, \tag{31}$$

where τ_t^C is the consumption tax rate, q_t^G is the price of government bonds, B_t^G is the quantity of government bonds, ϱ_t^F is the redemption rate of the corporate debt, D_t^F is the payoff to equity-holders, τ_t^G is the tax rate on GBOs, T_t is the lump-sum tax, Γ_t is the lump-sum transfer that comes from the default costs, and G is the lump-sum transfer from the government.

The stochastic discount factor becomes

$$M_{t+1} = \beta\left(\frac{1+x_t\tau_t^C}{1+x_{t+1}\tau_{t+1}^C}\right)\left(\frac{C_{t+1}}{C_t}\right)^{\nu(1-\psi)-1}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\nu)(1-\psi)}\frac{U_{t+1}^{\psi-\gamma}}{E_t\left(U_{t+1}^{1-\gamma}\right)^{\frac{\psi-\gamma}{1-\gamma}}}. \tag{32}$$

The Euler equation is

$$E_t\left(M_{t+1}\frac{(1-x_{t+1}\tau_{t+1}^K)\varrho_{t+1}^F}{q_t^F}\right) = 1, \tag{33}$$

$$E_t\left(M_{t+1}\frac{1-x_{t+1}\tau_{t+1}^G}{q_t^G}\right) = 1, \tag{34}$$

$$E_t\left(M_{t+1}\frac{(1-x_{t+1}\tau_{t+1}^K)D_{t+1}^F}{S_t^F}\right) = 1, \tag{35}$$

where the first and third equations are equivalent to equation (24) and equation (26), which are equal to zero, respectively.

Government

We assume the following governmental policy. The lump-sum transfer $G > 0$ is constant, while taxes are zero in normal times ($x_t = 0$). That is, in normal times, the government cannot cover its expenses by taxes. When $x_t = 1$ (crisis), the government imposes distortionary taxes.

The government budget constraint is given by

$$q_t^G B_{t+1}^G + x_t\tau_t^C C_t + x_t\tau_t^K(\varrho_t^F B_t^F + D_t^F) + x_t\tau_t^G B_t^G + x_tT_t = B_t^G + G. \tag{36}$$

We assume a tax weight ω^i ($i = C, K, G, T$), which satisfies

$$\tau_t^C C_t = \omega^C(B_t^G + G), \tag{37}$$

$$\tau_t^K(\varrho_t^F B_t^F + D_t^F) = \omega^K(B_t^G + G), \tag{38}$$

$$\tau_t^G B_t^G = \omega^G(B_t^G + G), \tag{39}$$

$$T_t = \omega^T(B_t^G + G), \tag{40}$$

$$0 < \omega^C + \omega^K + \omega^G + \omega^T \leq 1. \tag{41}$$

Crisis Risk

The probability that a crisis occurs in period $t+1$ is denoted by $p_t = p(B_t^G/z_t) = \Pr(x_{t+1} = 1|B_t^G/z_t)$. This probability depends on the ratio of GBOs to productivity only.

Equilibrium

The labor market is cleared when

$$(1-\alpha)\frac{Y_t}{N_t} = W_t = \frac{1-\nu}{\nu}\frac{(1+x_t\tau_t^C)C_t}{1-N_t}. \tag{42}$$

The goods market is cleared when

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t. \tag{43}$$

In the economy, the state variables are $\{K_t, B_t^F, B_t^G, x_t, z_t\}$; however, as in Gourio (2013), the equilibrium can be expressed by using $\{k_t = K_t/z_t, b_t^G = B_t^G/z_t, x_t\}$. Similarly, we denote the variables divided by z_t by their lower case letters (e.g., $y_t = Y_t/z_t$) with the exception of $u_t = U_t^{1-\psi}/z_t^{\nu(1-\psi)}$.

In summary, we have 20 equations for 20 unknown endogenous variables $\{c_t$, b_{t+1}^F, k_{t+1}, R_{t+1}^K, N_t, y_t, M_{t+1}, u_t, x_{t+1}, ε_{t+1}^*, τ_t^K, L_t, v_{t+1}, q_t^F, s_t^F, q_t^G, b_{t+1}^G, t_t, τ_t^C, $\tau_t^G\}$:

$$(1-\alpha)\frac{y_t}{N_t} = \frac{1-\nu}{\nu}\frac{(1+x_t\tau_t^C)c_t}{1-N_t}, \tag{44}$$

$$E_t\left[M_{t+1}(1-x_{t+1}\tau_{t+1}^K)R_{t+1}^K\left\{1+(\chi-1)\varepsilon_{t+1}^*\left(1-H(\varepsilon_{t+1}^*)\right)-(1-\theta\chi)\Omega(\varepsilon_{t+1}^*)\right\}\right] = 1, \tag{45}$$

$$(1-\theta)E_t\left[M_{t+1}(1-x_{t+1}\tau_{t+1}^K)\varepsilon_{t+1}^*h(\varepsilon_{t+1}^*)\right] - \frac{\chi-1}{\chi}E_t\left[M_{t+1}(1-x_{t+1}\tau_{t+1}^K)(1-H(\varepsilon_{t+1}^*))\right] = 0, \tag{46}$$

$$q_t^G = E_t\left(M_{t+1}(1-x_{t+1}\tau_{t+1}^G)\right), \tag{47}$$

$$\varepsilon_{t+1}^* = \frac{b_{t+1}^F}{R_{t+1}^K k_{t+1}}\left(=\frac{L_t}{R_{t+1}^K}\right), \tag{48}$$

$$R_{t+1}^K = 1-\delta+\alpha\frac{y_{t+1}}{k_{t+1}}, \tag{49}$$

$$y_t = k_t^\alpha N_t^{1-\alpha}, \tag{50}$$

$$\begin{aligned} M_{t+1} &= \beta\left(\frac{1+x_t\tau_t^C}{1+x_{t+1}\tau_{t+1}^C}\right)\left(\frac{C_{t+1}}{C_t}\right)^{\nu(1-\psi)-1}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\nu)(1-\psi)}\frac{U_{t+1}^{\psi-\gamma}}{E_t\left(U_{t+1}^{1-\gamma}\right)^{\frac{\psi-\gamma}{1-\gamma}}} \\ &= \beta\left(\frac{1+x_t\tau_t^C}{1+x_{t+1}\tau_{t+1}^C}\right)e^{(\nu(1-\gamma)-1)(\mu+\sigma_e e_{t+1})}\left(\frac{c_{t+1}}{c_t}\right)^{\nu(1-\psi)-1}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\nu)(1-\psi)} \\ &\quad\cdot\frac{u_{t+1}^{\frac{\psi-\gamma}{1-\psi}}}{E_t\left(e^{\nu(1-\gamma)(\mu+\sigma_e e_{t+1})}u_{t+1}^{\frac{1-\gamma}{1-\psi}}\right)^{\frac{\psi-\gamma}{1-\gamma}}}, \end{aligned} \tag{51}$$

$$u_t = (1-\beta)c_t^{\nu(1-\psi)}(1-N_t)^{(1-\nu)(1-\psi)} + \beta E_t \left(e^{\nu(1-\gamma)(\mu+\sigma_e e_{t+1})} u_{t+1}^{\frac{1-\gamma}{1-\psi}} \right)^{\frac{1-\psi}{1-\gamma}}, \quad (52)$$

$$y_t = c_t + k_{t+1}e^{\mu+\sigma_e e_{t+1}} - (1-\delta)k_t, \quad (53)$$

$$p_t = p(b_t^G) = \Pr(x_{t+1} = 1|b_t^G) = d_0 \exp(d_1 b_t^G), \quad (54)$$

$$L_t = \frac{b_{t+1}^F}{k_{t+1}}, \quad (55)$$

$$v_{t+1} = R_{t+1}^K k_{t+1}, \quad (56)$$

$$q_t^F = E_t \left[M_{t+1}(1 - x_{t+1}\tau_{t+1}^K) \left(1 - H(\varepsilon_{t+1}^*) + \frac{\theta R_{t+1}^K k_{t+1}}{b_{t+1}^F} \Omega(\varepsilon_{t+1}^*) \right) \right], \quad (57)$$

$$s_t^F \equiv E_t \left[M_{t+1}(1 - x_{t+1}\tau_{t+1}^K) \left((1 - \Omega(\varepsilon_{t+1}^*)) R_{t+1}^K k_{t+1} - (1 - H(\varepsilon_{t+1}^*)) b_{t+1}^F \right) \right], \quad (58)$$

$$q_t^G b_{t+1}^G e^{\mu+\sigma_e e_{t+1}} + x_t \tau_t^C c_t + x_t \tau_t^K \left\{ \theta\Omega(\varepsilon_t^*) + 1 - \Omega(\varepsilon_t^*) \right\} R_t^K k_t + x_t \tau_t^G b_t^G + x_t t_t = b_t^G + g, \quad (59)$$

which is implied by

$$\begin{aligned} & q_t^G B_{t+1}^G + x_t \tau_t^C C_t \\ & + x_t \tau_t^K \left\{ \left(1 - H(\varepsilon_t^*) + \frac{\theta R_t^K K_t}{B_t^F} \Omega(\varepsilon_t^*) \right) B_t^F + (1 - \Omega(\varepsilon_t^*)) R_t^K K_t - (1 - H(\varepsilon_t^*)) B_t^F \right\} \\ & + x_t \tau_t^G B_t^G + x_t T_t \\ = \; & B_t^G + G, \end{aligned}$$

$$\tau_t^K \left\{ \theta\Omega(\varepsilon_t^*) + 1 - \Omega(\varepsilon_t^*) \right\} R_t^K k_t = \omega^K (b_t^G + g), \quad (60)$$

$$\tau_t^C c_t = \omega^C (b_t^G + g), \quad (61)$$

$$\tau_t^G b_t^G = \omega^G (b_t^G + g), \quad (62)$$

$$t_t = \omega^T (b_t^G + g), \quad (63)$$

where $e_{t+1} \sim N(0,1)$.

C Simulation Results for the United States

We investigate whether the same mechanism can explain the stagnation in the US economy. We use the full model and change some of the parameter values to those in Gourio (2013): the discount factor, β, is 0.987; the trend growth rate of TFP, μ, is 0.01; and the standard deviation of idiosyncratic shocks, σ_ε, is 0.01925. Moreover, government spending g is chosen to be slightly lower, 0.015, to make the simulated path of government debt consistent with the actual path for the United States. Finally, we choose $\omega^K = 0.4$, $\omega^T = 0.1$, and $\omega^C = \omega^G = 0$ to fit the actual path of the bond yield. The other parameter values are unchanged.

Figure C.1 shows that our model calibrated to the the United States can explain the country's stagnation in terms of output and investment fairly well, although the simulated credit spread is lower than the actual one.

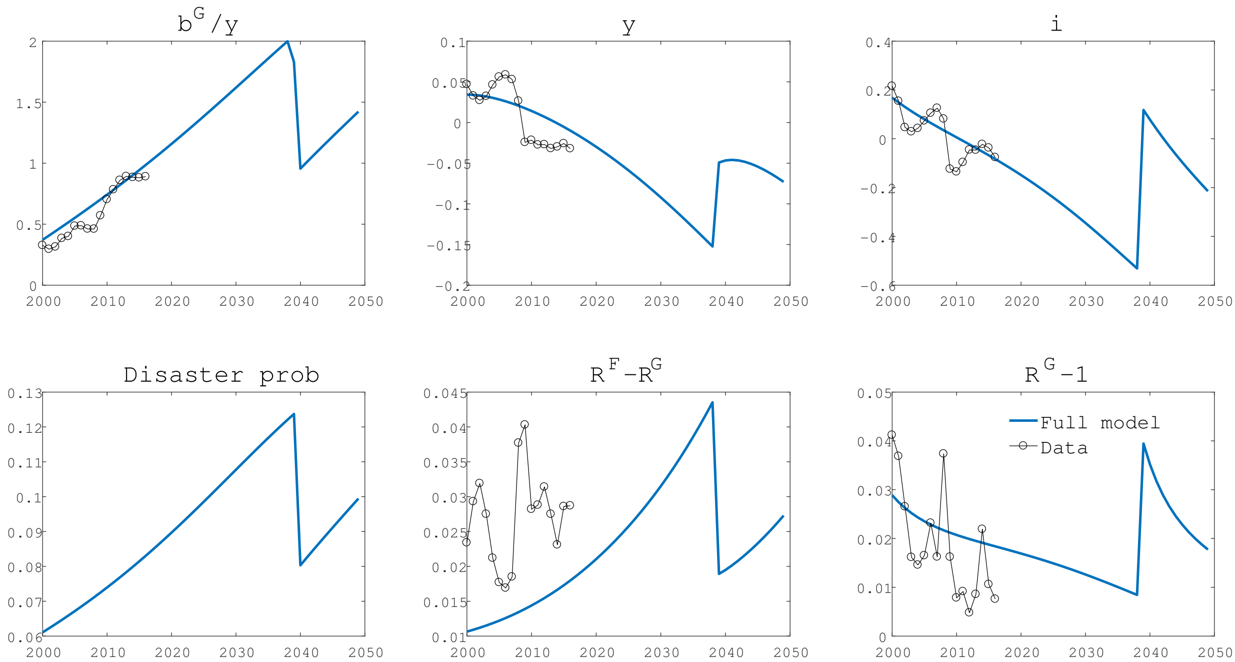

Figure C.1: Simulation Results for the United States