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Choice of market in the monetary economy

Ryoji Hiraguchi^{*†} Keiichiro Kobayashi^{‡§¶}

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Abstract

We investigate a monetary model à la Lagos and Wright (2005), in which there are two kinds of decentralized markets, and each agent stochastically chooses which one to participate in by expending effort. In one market, the pricing mechanism is competitive, whereas in the other market, the terms of trade are determined by Nash bargaining. It is shown that the optimal monetary policy may deviate from the Friedman rule. As the nominal interest rate deviates from zero, buyers expend more effort because a higher interest rate increases the gain for buyers from entering the competitive market, while the marginal increase in social welfare by entering the competitive market is also positive.

Keywords: Friedman rule; effort; search; competitive pricing; bargaining

JEL classification code: E1

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1 Introduction

Analysis of monetary economy and policy should be affected crucially by the market structure posited in the model. Lagos and Wright (2005) propose an extremely tractable market structure, in which the decentralized and centralized transactions occur alternately. For simplicity, they assume that monetary transactions are conducted only in the single decentralized market, whereas analysis of the optimal monetary policy may be altered by changing this assumption. In this study, we analyze a model of monetary economy in which two distinct decentralized markets for monetary transactions exist simultaneously, and agents can (stochastically) choose the market they enter by expending effort. Specifically, we consider that prices are given competitively in one market and by bargaining in the other. We show that the optimal monetary policy can be affected crucially by this “choice-of-market” structure.

Choice of markets by buyers and/or sellers are observed universally in reality. The mode of monetary trade differs among different markets in, for example, big cities and small villages. Retail goods are sold competitively in large stores in big cities, while in rural areas, they are often sold in small family shops, where buyers and the sellers can negotiate prices and conditions. We could say the pricing mechanism is mostly competitive in big cities and tends to be decided by bilateral bargaining in small towns. There exist some evidences for that bargaining is common in the real world in some developing economies (Jaleta and Gardebreek 2007; Keniston 2011) or in some industries (Ayers and Siegelman 1995; Morton, Zettelmeyer, and Silva-Russo 2004).

Our model is related to the literature of the new monetarist models pioneered by Lagos and Wright (2005). Rocheteau and Wright (2005) introduce and analyze three distinct pricing mechanisms in the monetary market in the Lagos–Wright model: bargaining, competitive pricing, and competitive search. Rocheteau and Wright (2005) assume that the price is decided by one of these modes in the unique monetary market, whereas we assume that two monetary markets exist simultaneously, that is, a market with competitive pricing and another with bargaining, and that the agents can choose the market they enter. Choice of market is a novel feature that we add

to the literature. Given this market structure, we show that the Friedman rule is not the optimal monetary policy.

Suboptimality of the Friedman rule is an important topic in the literature. Rocheteau and Wright (2005) show that the Friedman rule is suboptimal in the competitive pricing market, given that a search externality is present.¹ Nosal and Rocheteau (2011) overturn this result by showing that the Friedman rule is optimal in the competitive pricing market when a search externality is nonexistent. Our study further overturns Nosal and Rocheteau's (2011) result by showing that the Friedman rule is suboptimal even when a search externality is nonexistent in the competitive-pricing market, given that there is another monetary market.²

The rest of this paper is organized as follows. In Section 2, we construct a model. In Section 3, we characterize the competitive equilibrium and show the suboptimality of the Friedman rule. An example in this section shows that output is smaller when monetary policy deviates from the Friedman rule. In Section 4, we consider the intuition of the model in a case in which only buyers choose the market. Section 5 concludes. In the appendix, we describe a version of the model in which only sellers choose the market and both welfare and output are larger when monetary policy deviates from the Friedman rule.

2 Model

2.1 Set-up

Time is discrete and changes from $t = 0$ to $+\infty$. There is a continuum of infinitely lived agents with unit measure. Each date is divided into day and night. The day

¹In addition, Hiraguchi and Kobayashi (2014) show the suboptimality of the Friedman rule when a search externality is present in the monetary market with competitive pricing.

²Nosal and Rocheteau (2011, Subsection 6.6) demonstrate that the Friedman rule can be suboptimal in a model in which an agent can choose to become either a buyer or seller in the unique monetary market, whereas in our model, an agent chooses the monetary market in which to participate. The extensive margin is the key factor in both models, which produces suboptimality of the Friedman rule.

market is decentralized and the night market is centralized. In each period, the agent becomes a buyer or seller with probability 0.5.

In the day market, there are two kinds of decentralized markets: the competitive-pricing market (C-market) and the search market (S-market). In the C-market, there is no search friction, and sellers and buyers trade under competitive price p . In the S-market, there are search and matching frictions and the terms of trades are determined by Nash bargaining.

At the beginning of the day market, the buyer chooses effort level e^b . With probability e^b , the buyer enters the C-market, and with probability $1 - e^b$, the buyer enters the S-market. Similarly, the seller can move from the S-market to the C-market with probability e^s if the seller expends effort e^s . The utility cost of the buyer's effort $g(e)$ satisfies $g(0) = g'(0) = 0$, $g'(e) > 0$, and $g''(e) < 0$ if $e > 0$. Similarly, the utility cost of the seller's effort $h(e)$ satisfies $h(0) = h'(0) = 0$, $h'(e) > 0$, and $h''(e) < 0$ if $e > 0$.

The matching function at the S-market is given by $\zeta(\mu^b, \mu^s)$ where μ^b (μ^s) is the measure of buyers (sellers). The matching function ζ has constant returns to scale and is increasing and concave. In the following, we suppose that $\zeta(\mu^b, \mu^s) = z(\mu^b)^\alpha (\mu^s)^{1-\alpha}$ with $\alpha \in (0, 1)$. For a buyer, the probability of matching with a seller is $(1 - e^b) \frac{\zeta(\mu^b, \mu^s)}{\mu^b}$. For each seller, the probability of meeting with a buyer is $(1 - e^s) \frac{\zeta(\mu^b, \mu^s)}{\mu^s}$. In competitive equilibrium, $\mu^b = 1 - \bar{e}^b$ and $\mu^s = 1 - \bar{e}^s$, where \bar{e}^b (\bar{e}^s) is the average search intensity of the buyer (seller).

In both the C-market and S-market, the buyer obtains utility $u(q)$ from consuming q units of output. The function u satisfies $u(0) = 0$, $u' > 0$, $u'' < 0$, and $u'(0) = +\infty$. The seller loses utility $c(q)$ by producing q units of output. The function c satisfies $c(0) = 0$, $c' > 0$, and $c'' > 0$. In the night market, each agent obtains utility $U(x)$ from consuming x units of goods and obtains linear disutility z from producing z units of goods. $q^* > 0$ and $x^* > 0$ exist such that $u'(q^*) = c'(q^*)$ and $U'(x^*) = 1$. $s^* = u(q^*) - c(q^*)$ denote the maximized surplus.

Money is divisible and storable. Buyers need money to pay in the day market. A central bank controls the money supply M . Its growth rate is τ .

2.2 Night market

We follow Lagos and Wright (2005) and focus on the degenerate stationary equilibrium in which the level of consumption is the same across all agents and output is constant. We index consecutive period variables by $+1$.

Let $V(m)$ denote the value function of the agent at the beginning of each period. In addition, let $W(m)$ denote the value function of the individual at night, who holds m units of money. At the night market, the agents solve

$$\begin{aligned} W(m) &= \max_{C,h,m_{+1}} \{U(C) - h + \beta V_{+1}(m_{+1})\}, \\ \text{s.t. } C &= h + \phi(m + T - m_{+1}), \end{aligned}$$

where $\beta > 0$ is a discount factor, C is consumption, h is production, ϕ is the value of money in terms of the general good, and T is a transfer from the government. From the quasilinearity of the utility function, we obtain $W(m) = \phi m + W(0)$. The first-order conditions (FOCs) are

$$\phi = \beta \frac{\partial V_{+1}}{\partial m_{+1}}, \quad (1)$$

$$U'(C) = 1. \quad (2)$$

Therefore, trade at the night market is efficient.

2.3 Day markets

There are two decentralized markets in the daytime: the S-market and C-market. The buyer chooses the probability e^b of entering the C-market. Similarly, the seller chooses the probability e^s to enter the C-market. In the C-market, the price of the good p is given and the buyer maximizes her surplus $s^b \equiv u(q^b) - \phi p q^b$ subject to the constraint $p q^b \leq m$. Similarly, the seller maximizes her surplus $s^s \equiv \phi p q^s - c(q^s)$ by choosing q^s . In the C-market, the quantity supplied is equal to $e^s q^s$ and the quantity demanded is $e^b q^b$. In competitive equilibrium, $e^s q^s = e^b q^b$.

The buyer and seller enter the S-market with probability $1 - e^b$ and $1 - e^s$, respectively. In the S-market, the buyer and seller trade bilaterally with Nash bargaining as follows.

$$\max_{d \leq m} [u(\hat{q}) - \phi d]^\theta [\phi d - c(\hat{q})]^{1-\theta}, \quad (3)$$

where $\theta \in (0, 1]$ denotes the bargaining power of the buyer and (\hat{q}, d) represents the terms of trade.

The value function of the buyer who holds m dollars is

$$\begin{aligned} V^b(m) &= \max_{e^b} [-g(e^b) + e^b \max_{pq^b \leq m} \{u(q^b) - \phi p q^b\} \\ &\quad + (1 - e^b) \frac{\zeta(\mu^b, \mu^s)}{\mu^b} \{u(\hat{q}) - \phi d\} + W(m)]. \end{aligned}$$

If we denote the buyer's surplus $u(\hat{q}) - \phi d$ as \hat{s}^b , the FOCs for $V^b(m)$ are

$$\begin{aligned} u'(q^b) &\geq \phi p, \\ g'(e^b) &= s^b - \frac{\zeta(\mu^b, \mu^s)}{\mu^b} \hat{s}^b, \\ \frac{\partial V^b}{\partial m} &= e^b \frac{\partial s^b}{\partial m} + (1 - e^b) \frac{\partial \hat{s}^b}{\partial m} + \phi. \end{aligned}$$

The value function of a seller who holds m dollars is

$$\begin{aligned} V^s(m) &= \max_{e^s} [-h(e^s) + e^s \max_{q^s} [\phi p q^s - c(q^s)] \\ &\quad + (1 - e^s) \frac{\zeta(\mu^b, \mu^s)}{\mu^s} [\phi d - c(\hat{q})] + W(m)] \end{aligned}$$

If we allow \hat{s}^s to denote the seller's surplus $\phi d - c(\hat{q})$, the FOCs for $V^s(m)$ are

$$\begin{aligned}\phi p &= c'(q^s), \\ h'(e^s) &= c'(q^s)q^s - c(q^s) - \frac{\zeta(\mu^b, \mu^s)}{\mu^s} \hat{s}^s \\ \frac{\partial V^s}{\partial m} &= \phi.\end{aligned}$$

The function V satisfies $V(m) = 0.5V^s(m) + 0.5V^b(m)$.

In the following, we focus on the case in which the S-market is efficient when the nominal interest rate is close to zero. In this case, the constraint $d \leq m$ does not bind in the S-market and the quantity is $\hat{q} = q^*$. The equilibrium transfer is $\phi \hat{d} = \theta c(q^*) + (1 - \theta)u(q^*)$ and then, the S-market is efficient if

$$\phi m > \theta c(q^*) + (1 - \theta)u(q^*). \quad (4)$$

When the S-market is efficient, $\frac{\partial s^b}{\partial m} = 0$. Then, given the nominal interest rate $i = (1 + \tau)/\beta - 1$, Eq. (1) implies

$$i = 0.5e^b \frac{\partial s^b}{\phi \partial m}.$$

If the constraint $d \leq m$ binds in the C-market, $s^b = u(m/p) - \phi m$. Thus,

$$\frac{\partial s^b}{\phi \partial m} = \frac{u'(q^b)}{c'(q^s)} - 1.$$

Therefore, under the Friedman rule where $i = 0$, we obtain $u'(q^b) = c'(q^s)$.

3 Competitive equilibrium

In this section, we characterize competitive equilibrium in the case in which the nominal interest rate is sufficiently low so that the S-market is efficient. Then, the buyer's surplus equals $\hat{s}^b = \theta s^*$ and the seller's surplus equals $\hat{s}^s = (1 - \theta)s^*$. The

stationary equilibrium allocation $\{q^b, q^s, e^b, e^s, \phi\}$ is determined by

$$i = 0.5e^b \left(\frac{u'(q^b)}{c'(q^s)} - 1 \right), \quad (5)$$

$$g'(e^b) = u(q^b) - c'(q^s)q^b - \frac{\zeta\theta s^*}{1 - e^b}, \quad (6)$$

$$h'(e^s) = c'(q^s)q^s - c(q^s) - \frac{\zeta(1 - \theta)s^*}{1 - e^s}, \quad (7)$$

$$\phi M = c'(q^s)q^b > \theta c(q^*) + (1 - \theta)u(q^*), \quad (8)$$

$$e^s q^s = e^b q^b, \quad (9)$$

where $\zeta = \zeta(1 - e^b, 1 - e^s)$. The stationary welfare is

$$W = -g(e^b) - h(e^s) + \zeta(1 - e^b, 1 - e^s)s^* + e^b u(q^b) - e^s c(e^b q^b / e^s).$$

If $\frac{dW}{di} > 0$ at $i = 0$, then the Friedman rule is not optimal. We can show the following proposition.

Proposition 1 *Suppose that $\alpha < \theta$ and that when $i = 0$,*

$$g'' + \frac{\zeta\theta s^*}{(1 - e^b)^2} - \frac{\zeta_1\theta s^*}{1 - e^b} - \frac{e^s \zeta_1(1 - \theta)s^*}{e^b(1 - e^s)} > 0, \quad (10)$$

$$h'' + \frac{\zeta(1 - \theta)s^*}{(1 - e^s)^2} - \frac{\zeta_2(1 - \theta)s^*}{1 - e^s} - \frac{e^b \zeta_2\theta s^*}{e^s(1 - e^b)} > 0, \quad (11)$$

where $\zeta = \zeta(1 - e^b, 1 - e^s)$, $\zeta_1(x, y) = \frac{\partial \zeta}{\partial x}$ and $\zeta_2(x, y) = \frac{\partial \zeta}{\partial y}$. In this case, the Friedman rule is not optimal.

Proof. See the Appendix. ■

In the following, we provide a numerical example in which Eqs. (10) and (11) hold. Suppose that $u(q) = 2\sqrt{q}$, $c(q) = 0.5q^2$, $\zeta(x, y) = 0.5\sqrt{xy}$, $g(e^b) = 0.5A(e^b)^2$, and $h(e^s) = 0.5B(e^s)^2$. Then, $q^* = 1$ and $s^* = 1.5$. First, we find A and B such that the equilibrium levels of effort are $e^b = 0.1$ and $e^s = 0.5$ at the Friedman rule. As $u'(q^b) = c'(e^b q^b / e^s)$ under the Friedman rule, we obtain $q^b = \left(\frac{e^s}{e^b}\right)^{\frac{2}{3}} = 5^{\frac{2}{3}}$ and

$q^s = e^b q^b / e^s = 5^{-\frac{1}{3}}$. Then, $0 < \frac{\zeta(1-e^b, 1-e^s)}{1-e^b} < 1$ and $0 < \frac{\zeta(1-e^b, 1-e^s)}{1-e^s} < 1$. Eqs. (6) and (7) imply that A and B are determined by $A = 10(5^{\frac{1}{3}} - 0.75\theta/\sqrt{1.8})$ and $B = 5^{-\frac{2}{3}} - 1.5\sqrt{1.8}(1-\theta)$. The condition (8) is written as

$$c'(q^s)q^b = q^s q^b = 5^{\frac{1}{3}} > 0.5\theta + 2(1-\theta). \quad (12)$$

Eqs. (10) and (11) are expressed as

$$A + \left(\frac{\theta}{0.9} - \frac{1-\theta}{0.1} \right) 0.25 \sqrt{\frac{0.5}{0.9}} \times 1.5 > 0,$$

$$B + \left[0.5(1-\theta) - \frac{0.05}{0.9}\theta \right] \sqrt{\frac{0.9}{0.5}} \times 1.5 > 0.$$

It is easy to check that if $\theta = 1$, the constants A and B satisfy the abovementioned inequalities. Figure 1 shows stationary welfare as a function of the nominal interest rate when $\theta = 1$.

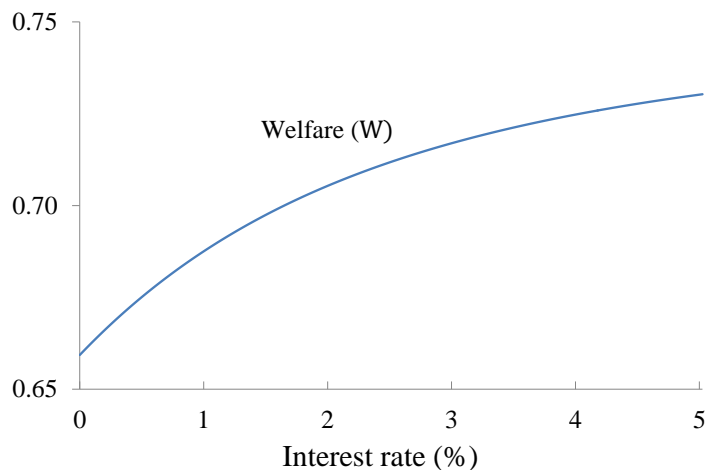


Figure 1: Inflation and welfare

In this model, the output may or may not be larger when monetary policy deviates from the Friedman rule. In the example shown in Figure 1, it is easy to check that output is maximized by the Friedman rule. We can construct a version of the model

in which both welfare and output are larger when monetary policy deviates from the Friedman rule. Such an example is given in Appendix B.

4 Case in which only buyers expend effort

It is difficult to understand clearly how buyers' and sellers' efforts interact with each other in our model to produce the suboptimality of the Friedman rule. If we focus only on one of these factors, we are able to understand the intuition of its workings. In this section, we investigate a case in which only buyers expend effort to choose the market. As before, the buyers choose the probability e^b of entering the C-market. However, here, we suppose that sellers are divided into the two markets with equal probability, 0.5. The value function of a buyer who holds m dollars is

$$\begin{aligned} V^b(m) = & \max_{e^b} [-g(e^b) + e^b \max_{pq^b \leq m} \{u(q^b) - \phi pq^b\} \\ & + (1 - e^b) \frac{\zeta(\mu^b, 1/2)}{\mu^b} \{u(\hat{q}) - \phi d\} + W(m)]. \end{aligned}$$

The value function of a seller who holds m dollars is

$$V^s(m) = 0.5 \max_{q^s} [\phi pq^s - c(q^s)] + \zeta(\mu^b, 1/2) \{\phi \hat{d} - c(\hat{q})\} + W(m).$$

The goods-market equilibrium condition in the C-market is $e^b q^b = 0.5 q^s$. Given a nominal interest rate i , the stationary equilibrium allocation $\{q^b, q^s, e^b\}$ is determined by

$$\frac{2i}{e^b} = \frac{u'(q^b)}{c'(2e^b q^b)} - 1, \quad (13)$$

$$g'(e^b) = u(q^b) - c'(2e^b q^b) q^b - \frac{\zeta(1 - e^b, 1/2)}{1 - e^b} \theta_{s^*}, \quad (14)$$

$$q^s = 2e^b q^b. \quad (15)$$

As before, the condition on the efficiency in the S-market is Eq. (8). The next proposition shows that the effort level is an increasing function of the nominal interest rate as long as this rate is small.

Proposition 2 $de^b/di > 0$ around the Friedman rule.

Proof. See the Appendix. ■

The intuition of this proposition can be explained as follows. A higher nominal interest rate makes q^b smaller because of more severe monetary friction. A smaller q^b makes the buyer's surplus $s^b(q^b) \equiv u(q^b) - pq^b = u(q^b) - c'(q^s)q^b$ larger as long as the nominal interest rate is small. This is because under the Friedman rule, trade is efficient (i.e., $u'(q^b) = c'(q^s)$) and then, if we fix e^b , the surplus satisfies

$$\frac{\partial s^b}{\partial q^b} = \{u(q^b) - c'(2e^b q^b)q^b\}' = -2e^b c''(q^s)q^b < 0.$$

Suppose that monetary policy deviates from the Friedman rule. If the price is fixed, then the effect of deviating from the rule on the buyer's surplus is zero because $\{u(q^b) - pq^b\}' = u'(q^b) - c'(q^s) = 0$. However, this lowers the equilibrium price and then, the deviation increases the surplus. The larger surplus makes entering the C-market more attractive, and buyers expend more effort.

Stationary welfare W is

$$W = -g(e^b) + \zeta(1 - e^b, 1/2)s^* + e^b u(q^b) - (1/2)c(2e^b q^b).$$

We obtain the following proposition on the nondesirability of the Friedman rule.

Proposition 3 *The Friedman rule is not optimal if $\alpha < \theta$ and the following inequality holds:*

$$c'(2e^b q^b)q^b > \theta c(q^*) + (1 - \theta)u(q^*). \quad (16)$$

Proof. See the Appendix. ■

The condition $\alpha < \theta$ ensures that the private cost for buyers of exiting the S-market is larger than the social cost, that is,

$$\frac{\zeta(1 - e^b, 1/2)}{1 - e^b} \theta s^* > \zeta_1(1 - e^b, 1/2) s^*.$$

This externality makes the effort level at the Friedman rule strictly lower than the socially optimal level. Thus, an increase in the effort level improves social welfare. Therefore, deviation from the Friedman rule increases the amount of effort and, in turn, increases social welfare. This result implies that the externality associated with the extensive margin of exiting the S-market is crucial for the suboptimality of the Friedman rule in our model.

5 Conclusion

In this study, we investigate a monetary model in which there are two decentralized markets, and each agent chooses which one to participate in by expending effort. In one market, the pricing mechanism is competitive, whereas in the other market, the terms of trade are determined by Nash bargaining. It is shown that the optimal monetary policy may deviate from the Friedman rule, even though the search externality is nonexistent in the competitive-pricing market. The intuition for the suboptimality of the Friedman rule is given as follows. As the nominal interest rate deviates from zero, the buyers expend more effort because a higher interest rate increases the gain for buyers from entering the competitive-pricing market, while the marginal increase in social welfare by entering the competitive-pricing market is also positive.

Appendix

In Appendix A, we provide proofs for propositions. Appendix B describes a version of the model in which output increases when monetary policy deviates from the Friedman rule.

A Proofs

A.1 Proof of Proposition 1

The system is reduced to the following three equations for the three unknowns $\{e^b, e^s, q^b\}$, given i :

$$2i = e^b \left(\frac{u'(q^b)}{c'(e^b q^b / e^s)} - 1 \right), \quad (17)$$

$$g'(e^b) = u(q^b) - c' \left(\frac{e^b q^b}{e^s} \right) q^b - \frac{\zeta(1 - e^b, 1 - e^s)}{1 - e^b} \theta s^*, \quad (18)$$

$$h'(e^s) = c' \left(\frac{e^b q^b}{e^s} \right) \frac{e^b q^b}{e^s} - c \left(\frac{e^b q^b}{e^s} \right) - \frac{\zeta(1 - e^b, 1 - e^s)}{1 - e^s} (1 - \theta) s^*, \quad (19)$$

Stationary welfare depends on three unknowns, e^b, e^s , and q^b :

$$W(e^b, e^s, q^b) = -g(e^b) - h(e^s) + \zeta(1 - e^b, 1 - e^s) s^* + e^b u(q^b) - e^s c(e^b q^b / e^s)$$

Thus, we obtain $\frac{dW}{di} = \frac{\partial W}{\partial e^b} \frac{de^b}{di} + \frac{\partial W}{\partial e^s} \frac{de^s}{di} + \frac{\partial W}{\partial q^b} \frac{dq^b}{di}$. In what follows, we let $\tilde{\theta} = 1 - \theta$, $\mu^b = 1 - e^b$, and $\mu^s = 1 - e^s$. Differentiate Eqs. (17), (18), and (19) by i to obtain

$$\begin{aligned} 2 &= -X_1 \frac{de^b}{di} + X_2 \frac{de^s}{di} - X_3 \frac{dq^b}{di}, \\ X_4 \frac{de^b}{di} &= -X_5 \frac{dq^b}{di} + X_6 \frac{de^s}{di}, \\ X_7 \frac{de^s}{di} &= X_8 \frac{dq^b}{di} + X_9 \frac{de^b}{di}, \end{aligned}$$

where $X_1 = \frac{e^b q^b u' c''}{(c')^2 e^s}$, $X_2 = \frac{(e^b)^2 q^b u' c''}{(e^s)^2 (c')^2}$, $X_3 = e^b \left(\frac{u' c'' e^b}{(c')^2 e^s} - \frac{u''}{c'} \right)$, $X_4 = g'' + \frac{(q^b)^2 c''}{e^s} + \frac{\zeta \theta s^*}{(\mu^b)^2} - \frac{\zeta_1 \theta s^*}{\mu^b}$, $X_5 = \frac{c' e^b q^b}{e^s}$, $X_6 = \frac{(q^b)^2 e^b c''}{(e^s)^2} + \frac{\zeta_2 \theta s^*}{\mu^b}$, $X_7 = h'' + \frac{(q^b)^2 (e^b)^2 c''}{(e^s)^3} + \frac{\zeta \tilde{\theta} s^*}{(\mu^s)^2} - \frac{\zeta_2 \tilde{\theta} s^*}{\mu^s}$, $X_8 = \frac{c'' (e^b)^2 q^b}{(e^s)^2}$ and $X_9 = \frac{(q^b)^2 e^b c''}{(e^s)^2} + \frac{\zeta_1 \tilde{\theta} s^*}{\mu^s}$. We obtain

$$\begin{aligned} \frac{de^s}{di} &= - \frac{X_4 X_8 - X_5 X_9}{X_5 X_7 - X_6 X_8} \frac{de^b}{di}, \\ \frac{de^b}{di} &= \frac{2X_5 (X_5 X_7 - X_6 X_8)}{\Delta}. \end{aligned}$$

where $\Delta = (X_2 X_9 - X_1 X_7) X_5^2 + (X_1 X_6 - X_2 X_4) X_5 X_8 + (X_4 X_7 - X_6 X_9) X_3 X_5$. It is shown that

$$\begin{aligned} X_4 X_8 - X_5 X_9 &= \frac{(e^b)^2 c'' q^b}{(e^s)^2} \left[X_{10} - \frac{e^s \zeta_1}{e^b \mu^s} \tilde{\theta} s^* \right], \\ X_5 X_7 - X_6 X_8 &= \frac{c'' e^b q^b}{e^s} \left[X_{11} - \frac{e^b \zeta_2}{e^s \mu^b} \theta s^* \right], \end{aligned}$$

where $X_{10} = g'' + \frac{\zeta}{(\mu^b)^2} \theta s^* - \frac{\zeta_1}{\mu^b} \theta s^*$ and $X_{11} = h'' + \frac{\zeta}{(\mu^s)^2} \tilde{\theta} s^* - \frac{\zeta_2}{\mu^s} \tilde{\theta} s^*$. In addition, it is shown that

$$\begin{aligned} (X_2 X_9 - X_1 X_7) X_5^2 &= \frac{(q^b)^3 (c'')^3 (e^b)^3 u'}{(e^s)^3 (c')^2} \left[-X_{11} + \frac{e^b \zeta_1}{e^s \mu^s} \tilde{\theta} s^* \right], \\ (X_1 X_6 - X_2 X_4) X_5 X_8 &= \frac{(q^b)^3 (c'')^3 (e^b)^5 u'}{(e^s)^5 (c')^2} \left[-X_{10} + \frac{e^s \zeta_2}{e^b \mu^b} \theta s^* \right], \\ (X_4 X_7 - X_6 X_9) X_3 X_5 &= X_3 X_5 \left(X_{10} X_{11} - \frac{\zeta_1 \zeta_2 \theta \tilde{\theta} (s^*)^2}{\mu^b \mu^s} \right) \\ &\quad + X_{12} \frac{(q^b)^2 c''}{e^s} \left(X_{11} - \frac{e^b \zeta_2 \theta s^*}{e^s \mu^b} \right) \\ &\quad + X_{12} \frac{(q^b)^2 (e^b)^2 c''}{(e^s)^3} \left(X_{10} - \frac{e^s \zeta_1 \tilde{\theta} s^*}{e^b \mu^s} \right) \\ &\quad - (X_2 X_9 - X_1 X_7) X_5^2 - (X_1 X_6 - X_2 X_4) X_5 X_8. \end{aligned}$$

where $X_{12} = (-\frac{u''}{c'}) (\frac{c''(e^b)^2 q^b}{e^s})$. Therefore, the denominator of de^b/di , Δ , satisfies

$$\begin{aligned} \Delta &= X_3 X_5 \left(X_{10} X_{11} - \frac{\zeta_1 \zeta_2 \theta \tilde{\theta} (s^*)^2}{\mu^b \mu^s} \right) + X_{12} \frac{(q^b)^2 c''}{e^s} \left(X_{11} - \frac{e^b \zeta_2 \theta s^*}{e^s \mu^b} \right) \\ &\quad + X_{12} \frac{(q^b)^2 (e^b)^2 c''}{(e^s)^3} \left(X_{10} - \frac{e^s \zeta_1 \tilde{\theta} s^*}{e^b \mu^s} \right) \\ &> X_3 X_5 \left(X_{10} - \frac{e^s \zeta_1 \tilde{\theta} s^*}{e^b \mu^s} \right) \left(X_{11} - \frac{e^b \zeta_2 \theta s^*}{e^s \mu^b} \right) + X_{12} \frac{(q^b)^2 c''}{e^s} \left(X_{11} - \frac{e^b \zeta_2 \theta s^*}{e^s \mu^b} \right) \\ &\quad + X_{12} \frac{(q^b)^2 (e^b)^2 c''}{(e^s)^3} \left(X_{10} - \frac{e^s \zeta_1 \tilde{\theta} s^*}{e^b \mu^s} \right), \end{aligned}$$

where the inequality follows from $XY - xy > (X - x)(Y - y)$ for $X > x > 0$ and $Y > y > 0$.

Since $\theta > \alpha$, we obtain

$$\begin{aligned} \frac{\partial W}{\partial q^b} &= e^b [u'(q^b) - c'(q^s)] = 0, \\ \frac{\partial W}{\partial e^b} &= -g' - \zeta_1 s^* + u(q^b) - q^b c'(q^s) = \frac{\zeta}{\mu^b} \theta s^* - \zeta_1 s^* > 0, \\ \frac{\partial W}{\partial e^s} &= -h' - \zeta_2 s^* - c(q^s) + c'(q^s) q^s = \frac{\zeta}{\mu^s} \tilde{\theta} s^* - \zeta_2 s^* < 0. \end{aligned}$$

The assumptions directly lead to $\frac{de^b}{di} > 0$ and $\frac{de^s}{di} < 0$. Therefore, $\frac{dW}{di} = \frac{\partial W}{\partial e^b} \frac{de^b}{di} + \frac{\partial W}{\partial e^s} \frac{de^s}{di} > 0$. This inequality implies that welfare can be improved by raising the nominal interest rate from $i = 0$. ■

A.2 Proof of Proposition 2

Differentiating Eqs. (13) and (14) with respect to i at the Friedman rule, we obtain

$$\begin{aligned} X_{13} \frac{de^b}{di} &= -X_{14} \frac{dq^b}{di}, \\ 2 &= -X_{15} \frac{de^b}{di} - X_{16} \frac{dq^b}{di}, \end{aligned}$$

where

$$\begin{aligned} X_{13} &= g''(e^b) + \frac{\zeta(1 - e^b, 0.5)}{(1 - e^b)^2} \theta_{s^*} - \frac{\zeta'(1 - e^b, 0.5)}{(1 - e^b)} \theta_{s^*} + 2(q^b)^2 c''(2e^b q^b), \\ X_{14} &= 2e^b q^b c''(2e^b q^b) > 0, \\ X_{15} &= \frac{2e^b q^b u'(q^b) c''(2e^b q^b)}{(c'(2e^b q^b))^2}, \\ X_{16} &= \frac{2(e^b)^2 u'(q^b) c''(2e^b q^b)}{(c'(2e^b q^b))^2} - \frac{e^b u''(q^b)}{c'(2e^b q^b)}. \end{aligned}$$

In this case, we obtain $\frac{de^b}{di} = \frac{2X_{14}}{X_{13}X_{16} - X_{14}X_{15}}$, and the denominator is expressed as

$$\begin{aligned} X_{13}X_{16} - X_{14}X_{15} &= \left[g'' + \frac{\zeta \theta_{s^*}}{(1 - e^b)^2} - \frac{\zeta' \theta_{s^*}}{1 - e^b} \right] \frac{2(e^b)^2 u' c''}{(c')^2} \\ &\quad + \left[g'' + \frac{\zeta \theta_{s^*}}{(1 - e^b)^2} - \frac{\zeta' \theta_{s^*}}{1 - e^b} + 2(q^b)^2 c'' \right] \frac{-e^b u''}{c'}, \end{aligned}$$

which is positive. Therefore, we obtain $\frac{de^b}{di} > 0$ at the Friedman rule. ■

A.3 Proof of Proposition 3

At the Friedman rule, the response of social welfare to the interest rate is

$$\frac{dW}{di} = \frac{\partial W}{\partial e^b} \frac{de^b}{di} + \frac{\partial W}{\partial q^b} \frac{dq^b}{di} = \frac{\partial W}{\partial e^b} \frac{de^b}{di}.$$

As $\alpha < \theta$,

$$\begin{aligned}\frac{\partial W}{\partial e^b} &= -g'(e^b) - \zeta'(1 - e^b, 1/2)s^* + u(q^b) - q^b c'(2e^b q^b) \\ &= \frac{\zeta(1 - e^b, 1/2)}{1 - e^b} \theta s^* - \zeta_1(1 - e^b, 1/2)s^* > 0.\end{aligned}$$

It follows that $\frac{dW}{di} > 0$ at the Friedman rule. ■

B Case in which sellers choose the market

To demonstrate that both welfare and output can increase if monetary policy deviates from the Friedman rule, we describe a case in which only sellers choose the market. Sellers can move only from the S-market to the C-market with probability e^s if they expend effort e^s , which causes the utility cost $h(e^s)$. Buyers are divided into the two markets with equal probability, 0.5. Thus, $\mu^b = 0.5$. Here, we assume that the buyer's bargaining power in the S-market is equal to one. In what follows, we let $e^s = e$.

The problem in the night market is the same as before. In the S-market, buyers who have m units of money maximize their surplus $\hat{s}^b = u(\hat{q}) - \phi \hat{d}$:

$$\max_{\hat{q}, \hat{d}} [u(\hat{q}) - \phi \hat{d}] \text{ s.t. } \phi \hat{d} = c(\hat{q}) \text{ and } \hat{d} \leq m,$$

where the first condition describes the participation constraints of the seller. If $\phi m \geq c(q^*)$, then $\phi \hat{d} = c(q^*)$, $\hat{q} = q^*$ and $\hat{s}^b = s^*$. On the other hand, if $\phi m < c(q^*)$, then $\phi \hat{d} = \phi m = c(\hat{q})$. The buyer's value function is

$$V^b(m) = 0.5 \frac{\zeta(\mu^b, \mu^s)}{\mu^b} [u(\hat{q}) - \phi \hat{d}] + 0.5 \max_{pq^b \leq m} [u(q^b) - \phi pq^b] + W(m).$$

As $\mu^b = 0.5$, FOCs for $V^b(m)$ are

$$\begin{aligned} u'(q^b) &\geq \phi p, \\ \frac{\partial V^b}{\partial m} &= \zeta(0.5, \mu^s) \frac{\partial \hat{s}^b}{\partial m} + 0.5 \frac{\partial s^b}{\partial m} + \phi. \end{aligned}$$

The seller solves

$$V^s(m) = \max_e [-h(e) + e \max_{q^s} [\phi p q^s - c(q^s)] + W(m)]$$

FOCs for $V^s(m)$ are

$$\begin{aligned} h'(e) &= c'(q^s)q^s - c(q^s), \\ \phi p &= c'(q^s), \\ \frac{\partial V^s}{\partial m} &= \phi. \end{aligned} \tag{20}$$

Eq. (20) determines e as a function of q^s . Let $e(q)$ denote the function that satisfies $h'(e(q)) = c'(q)q - c(q)$. Obviously, $e'(q) > 0$.

In competitive equilibrium, $\mu^s = 1 - e$ and $0.5q^b = eq^s$. If

$$\phi M = \phi p q^b = c'(q^s)q^b > c(q^*), \tag{21}$$

then the S-market is efficient and the C-market is inefficient. In this case, $\hat{s}^b = s^*$ and the FOCs imply

$$4i = \frac{\partial s^b}{\phi \partial m} = \frac{u'(q^b)}{c'(q^s)} - 1 \geq 0.$$

Therefore, at the Friedman rule where $i = 0$, we obtain $u'(q^b) = c'(q^s)$. Given the nominal interest rate, the quantity at the stationary equilibrium is determined by

$$4ic'(q^s) = u'(2e(q^s)q^s) - c'(q^s). \tag{22}$$

Differentiating both sides of Eq. (22) by q^s , we get

$$4 \frac{di}{dq^s} c'(q^s) + 4i c''(q^s) = 2\{q^s e(q^s)\}' u''(2e(q^s)q^s) - c''(q^s). \quad (23)$$

The right hand side of Eq. (23) is negative. Thus $\frac{dq^s}{di}\big|_{i=0} < 0$ and

$$\frac{de}{di}\bigg|_{i=0} = e'(q^s) \frac{dq^s}{di}\bigg|_{i=0} < 0. \quad (24)$$

Welfare is denoted as

$$W = -h(e(q^s)) + \zeta(0.5, 1 - e(q^s))s^* + 0.5u(2e(q^s)q^s) - e(q^s)c(q^s).$$

Using Eq. (20), we obtain

$$\frac{dW}{dq^s} = -e'(q^s)\zeta_2 s^* + e\{u'(2eq^s) - c'(q^s)\}.$$

Under the Friedman rule, $u'(2eq^s) = c'(q^s)$. This implies that $\frac{dW}{dq^s}\big|_{i=0} = -\zeta_2 s^* e'(q^s) < 0$. Therefore, $\frac{dW}{di}\big|_{i=0} = \frac{dq^s}{di}\big|_{i=0} \frac{dW}{dq^s}\big|_{i=0} > 0$ and a deviation from the Friedman rule improves welfare.

Next, we show that total output may increase when monetary policy deviates from the Friedman rule. To simplify the analysis, we suppose further that $h(e) = A \frac{e^{\rho+1}}{1+\rho}$, $u(q) = B \frac{q^{1-\sigma}}{1-\sigma}$, and $c(q) = \frac{q^{1+\phi}}{1+\phi}$ with $\phi \geq 1$. The efficient quantity is $q^* = B^{\frac{1}{\phi+\sigma}}$. The output is

$$Y = eq^s + \zeta(1/2, 1 - e)q^*. \quad (25)$$

The FOCs are reexpressed as

$$4i(q^s)^\phi = B(2eq^s)^{-\sigma} - (q^s)^\phi, \quad (26)$$

$$Ae^\rho = \frac{\phi}{1+\phi}(q^s)^{1+\phi}. \quad (27)$$

Eq. (27) implies $q^s = \eta e^{\frac{\rho}{1+\phi}}$ with $\eta = (\frac{A(1+\phi)}{\phi})^{\frac{1}{1+\phi}}$. Thus, we obtain

$$\frac{dY}{di} = \left[\frac{\rho + 1 + \phi}{1 + \phi} \eta e^{\frac{\rho}{1+\phi}} (q^*)^{-1} - \zeta_2(1/2, 1 - e) \right] q^* \frac{de}{di}.$$

Eq. (26) implies that

$$B = (2eq^s)^\sigma (q^s)^\phi = 2^\sigma \eta^{\phi+\sigma} e^{\frac{\rho(\phi+\sigma)+\sigma(1+\phi)}{1+\phi}}$$

under the Friedman rule with $i = 0$. Since $q^* = B^{\frac{1}{\phi+\sigma}}$, we obtain $\eta e^{\frac{\rho}{1+\phi}} (q^*)^{-1} = 2^{-\frac{\sigma}{\phi}} \eta^{-\frac{\sigma}{\phi}} e^{-\frac{\rho\sigma+\sigma(1+\phi)}{(1+\phi)\phi}}$.

We now choose the value of A so that $\eta = \epsilon^{-\frac{\rho+1+\phi}{1+\phi} - \frac{\phi}{\sigma}}$ where ϵ is a small constant. Next, we set $B = 2^\sigma \epsilon^{-\phi(2+\phi/\sigma)}$. Then, it is easy to check that $e = \epsilon$ and $\eta e^{\frac{\rho}{1+\phi}} (q^*)^{-1} = 2^{-\frac{\sigma}{\phi}} \epsilon$. In this case, if ϵ is sufficiently small,

$$\frac{\rho + 1 + \phi}{1 + \phi} \eta e^{\frac{\rho}{1+\phi}} (q^*)^{-1} < \zeta_2(1/2, 1 - \epsilon).$$

Using Eq. (24), we obtain $\frac{dY}{di} > 0$ at $i = 0$.

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