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Abstract

We investigate a monetary model with two kinds of decentralized markets and where each agent stochastically chooses the market in which to participate. In one market, the pricing mechanism is competitive, whereas in the other, the terms of trade are determined by Nash bargaining. We show the sub-optimality of the Friedman rule, which is already demonstrated by existing models, where the setting of search externality in the competitive market is not completely satisfactory. We show this result in the more plausible setting when the competitive market does not have a search externality.

Keywords: Friedman rule; effort; search; competitive pricing; bargaining

JEL classification code: E1

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1 Introduction

Analyzing monetary economics and associated policy depends crucially on the market structure posited in the model used for analysis. Lagos and Wright (2005) propose an extremely tractable market structure, in which decentralized and centralized transactions occur alternately. For simplicity, they assume that monetary transactions occur only in the single decentralized market. However, the optimal monetary policy analysis changes by altering this assumption. In this study, we analyze a model of the monetary economy in which two distinct decentralized markets for monetary transactions exist simultaneously, and agents can (stochastically) choose the market they enter by expending effort. Specifically, prices are given competitively in one market and by bargaining in the other. We show that the optimal monetary policy depends crucially on this "choice-of-market" structure.

The buyers' and sellers' choice of markets occurs universally in reality. The mode of monetary trade varies among different markets, for example, in big cities and small villages. Retail goods are sold competitively in large stores in big cities, while in rural areas, they are often sold in small family shops where buyers and sellers can negotiate the prices and conditions. We could say that the pricing mechanism is mostly competitive in big cities and tends to be decided by bilateral bargaining in small towns. Some evidence suggests that bargaining is common in some developing economies (Jaleta and Gardebroek 2007; Keniston 2011) or industries (Ayers and Siegelman 1995; Morton, Zettelmeyer, and Silva-Russo 2004).

Our model relates to the literature of the new monetarist models pioneered by Lagos and Wright (2005). Rocheteau and Wright (2005) introduce and analyze three distinct pricing mechanisms in a monetary market in the Lagos–Wright model: bargaining, competitive pricing, and competitive search. Rocheteau and Wright (2005) assume that one of these three modes determines the price in a unique monetary market. On the other hand, we assume that two monetary markets exist simultaneously; that is, a market with competitive pricing and another with bargaining, and that agents can choose which market to enter. Given this market structure, we show that the Friedman rule is not the optimal monetary policy. The sub-optimality of the Friedman rule is an important topic in the literature. Rocheteau and Wright (2005) show that the Friedman rule is not optimal in the competitive pricing market given the presence of a search externality.[1] Nosal and Rocheteau (2011) show that the Friedman rule is optimal in a competitive pricing market when a search externality does not exist. Our study shows that the Friedman rule is not optimal, even when a search externality does not exist in the competitive pricing market, given that another monetary market exists.[2]

Showing the sub-optimality of the Friedman rule through our model is a theoretical contribution to the literature for the following reasons. We believe that a setting with a search externality in a competitive pricing market is not completely satisfactory in the existing models that show the Friedman rule to be a suboptimal monetary policy. In Section 4 of Rocheteau and Wright (2005), for example, the probability of entering a competitive market for an agent depends on the number of other agents who enter the market, and once

[1]Hiraguchi and Kobayashi (2014) also show that the Friedman rule may not be optimal when a search externality exists in a monetary market with competitive pricing.

[2]Nosal and Rocheteau (2011, Subsection 6.6) demonstrate that the Friedman rule may not be optimal in a model in which the agent can choose to become either a buyer or seller in a unique monetary market. In our model, an agent chooses the monetary market in which to participate. The extensive margin is the key factor in both models, which makes some inflation in excess of the Friedman Rule optimal.

entering the market, the agent can always trade. However, this assumption is not necessarily plausible because there is no sound explanation for why one agent's probability of entering a competitive market depends (positively) on the number of other people who enter the same market. It would be a plausible assumption if Rocheteau and Wright (2005) posited that one's probability is independent of the number of others. In our study, we posit exactly this assumption: an individual agent's search effort determines the probability of his entering the market, but is independent of the effort or number of other agents. Thus, the main theoretical contribution of our paper is that it shows the sub-optimality of the Friedman rule in a more plausible setting than those assumed in the existing literature.[3]

The rest of this paper proceeds as follows. In Section 2, we construct a model, characterize the competitive equilibrium, and show the sub-optimality of the Friedman rule. In Section 3, we consider a case in which sellers choose the market; both welfare and output are higher when monetary policy deviates from the Friedman rule. Section 4 concludes. In the appendix, we describe a version of the model in which buyers and sellers choose the market.

[3] In our setting, the probability of entering the market is similar to that of Lagos and Rocheteau (2005). Our model can be seen as a model that extends the Lagos–Rocheteau model by introducing choice of market. The Friedman rule is optimal in Lagos and Rocheteau, whereas it is suboptimal when we introduce choice of market, as in this paper.

2 A model with buyers' search

2.1 Set-up

Time is discrete and changes from $t = 0$ to $+\infty$. There is a continuum of infinitely lived agents with measure two. We divide each day into day and night. The day market is decentralized and the night market is centralized. In each period, the agent becomes a buyer or seller with probability 0.5. Thus, the measure of buyers and sellers equals one.

During the day time, two kinds of decentralized markets (DM) open: the competitive pricing market (DM-C) and the search market with Nash-bargaining (DM-S). In the DM-C, there is no search friction, and buyers and sellers trade under competitive price p. In the DM-S, there are search and matching frictions and the terms of trade are determined by Nash bargaining. We assume that the buyer's bargaining power is one. In the DMs, individuals are anonymous and must use money to trade. During the night time, the centralized market (CM) with no frictions opens.

At the beginning of the day market, the buyer enters the DM-C or DM-S with probability σ or $1-\sigma$, respectively. The variable σ depends on the buyer's effort level g. When the buyer makes no effort, the buyer enters the DM-C with probability $\underline{\sigma}$, where $\underline{\sigma} \in (0, 1/2)$ is a constant. The utility cost of the buyer's effort $g(\sigma)$ satisfies $g(\underline{\sigma}) = g'(\underline{\sigma}) = 0$, $g'(\sigma) > 0$ if $\sigma > \underline{\sigma}$, $g''(\sigma) > 0$ and $g'(1-\underline{\sigma}) = +\infty$. [4]. Thus, along any feasible allocation, $\underline{\sigma} \leq \sigma \leq 1-\underline{\sigma}$, and regardless of the buyer's effort, the probability of entering the DM-S and DM-C are larger than $\underline{\sigma}$. The sellers make no effort, and are divided among the two markets with equal probability, 0.5.

[4]The example of such a function is $\frac{(\sigma-\underline{\sigma})^2}{1-\underline{\sigma}-\sigma}$.

The matching function in the DM-S is given by $\zeta(\mu^b, \mu^s)$, where μ^b (μ^s) is the measure of buyers (sellers) in the DM-S. The matching function ζ has constant returns to scale and is strictly increasing and concave. We assume that it satisfies $\zeta(0, \mu^s) = \zeta(\mu^b, 0) = 0$, $\zeta_1(0, \mu^s) = \zeta_2(\mu^b, 0) = +\infty$, where $\zeta_1 = \frac{\partial \zeta}{\partial x}$ and $\zeta_2 = \frac{\partial \zeta}{\partial y}$ and $\frac{\zeta(\mu^b, \mu^s)}{\mu^b} = \zeta(1, \mu^s/\mu^b) \in (0, 1)$ for any μ^b and μ^s.

In the DM-S, each buyer matches with a seller with probability $\frac{\zeta(\mu^b, \mu^s)}{\mu^b}$; for each seller, the probability of meeting a buyer is $\frac{\zeta(\mu^b, \mu^s)}{\mu^s}$. In the degenerate competitive equilibrium where all buyers choose the same effort level $g(\sigma)$, $\mu^b = 1 - \sigma$. On the other hand, sellers with measure 0.5 enter the DM-C and the rest enter the DM-S. Thus, $\mu^s = 0.5$.

In the DM-C and DM-S, the buyer obtains utility $u(q)$ from consuming q units of output, and the seller loses utility $c(q)$ by producing q units of output. We assume that the function u satisfies $u' > 0$, $u'' < 0$, $u(0) = 0$, $u'(0) = +\infty$, $\lim_{q \to \infty} u'(q) = 0$ and $\lim_{q \to \infty}\{u(q) - u'(q)q\} = \infty$. The function c satisfies $c(0) = 0$, $c' > 0$, and $c'' > 0$. In the night market, each agent obtains utility $U(x)$ from consuming x units of goods and obtains linear disutility z from producing z units of goods. The function U satisfies $U' > 0$, $U'' < 0$, $U(0) = 0$, $U'(0) = +\infty$ and $\lim_{q \to \infty} U'(q) = 0$. We suppose that $q^* > 0$ and $x^* > 0$ such that $u'(q^*) = c'(q^*)$ and $U'(x^*) = 1$. We let $S^* = u(q^*) - c(q^*)$ denote the maximized surplus.

Money is divisible and storable. Buyers need money to pay in the DM-C and DM-S. A central bank controls the money supply M at a growth rate of τ.

2.2 Night market

We follow Lagos and Wright (2005) and focus on the degenerate stationary equilibrium in which the level of consumption is the same across all agents and output is constant. We index consecutive period variables by $+1$. We solve the model backward, and first investigate the centralized night market.

Let $V(m)$ denote the agent's value function at the beginning of each period. In addition, let $W(m)$ denote the individual's value function, who holds m units of money, at night. In the night market, the agents solve

$$W(m) = \max_{C,h,m_{+1}} \{U(C) - h + \beta V_{+1}(m_{+1})\},$$

$$\text{s.t. } C = h + \phi(m + T - m_{+1}),$$

where $\beta > 0$ is a discount factor, C is consumption, h is production, ϕ is the value of money in terms of the general good, and T is a transfer from the government. The first-order conditions (FOCs) are

$$\phi = \beta \frac{\partial V_{+1}}{\partial m_{+1}}, \tag{1}$$

$$U'(C) = 1. \tag{2}$$

Trade in the night market is efficient. From the quasi-linearity of the utility function, we obtain $W(m) = \phi m + W(0)$.

2.3 Competitive-pricing market (DM-C)

There are two kinds of decentralized markets during the day sub-period: the DM-C and the DM-S. We first investigate the DM-C. The value function of the buyer holding m dollars and entering the DM-C is

$$v^b(m) = \max_{q\geq 0}[u(q) + W(m - pq)] \text{ s.t. } pq \leq m, \tag{3}$$

where p is the price of the good. The optimal quantity q depends on m and is given by

$$\begin{aligned} q &= m/p \text{ if } u'(m/p) \geq p\phi, \\ u'(q) &= p\phi \text{ otherwise.} \end{aligned}$$

We let $S(m) = u(q) - \phi pq$ denote the buyer's surplus in the DM-C.

The value function of the seller holding m dollars and entering the DM-C is

$$v^s(m) = \max_{q^s\geq 0}[-c(q^s) + W(m + pq^s)]. \tag{4}$$

The value function W is linear, and thus the seller maximizes the surplus $\phi pq^s - c(q^s)$ by choosing q^s. The first order condition is

$$\phi p = c'(q^s).$$

Thus we have $\frac{1}{\phi}\frac{\partial S}{\partial m} = \frac{u'(q)}{c'(q^s)} - 1$, which is always nonnegative.

2.4 Search market (DM-S)

We next investigate the DM-S in which the buyer and seller trade bilaterally with Nash bargaining. The value function of the buyer who holds m dollars and enters the DM-S is

$$\hat{v}^b(m) = \max_{\hat{q}\geq 0}[u(\hat{q}) + W(m - \hat{d})] \text{ s.t. } \hat{d} \leq m \text{ and } -c(\hat{q}) + W(m^s + \hat{d}) \geq W(m^s), \tag{5}$$

where $(\hat{q}, \hat{d})$ is the terms of trade and the second inequality shows the seller's participation constraint. Since the function W is linear, the buyer simply maximizes its surplus subject to the seller's participation constraint:

$$\max_{\hat{q},\hat{d}}[u(\hat{q}) - \phi\hat{d}] \text{ s.t. } \hat{d} \le m \text{ and } \phi\hat{d} - c(\hat{q}) \ge 0. \tag{6}$$

The participation constraint always binds. Thus, the quantity $\hat{q}$ satisfies

$$\begin{aligned} c(\hat{q}) &= \phi m \text{ if } \phi m \le c(q^*), \\ \hat{q} &= q^* \text{ otherwise.} \end{aligned}$$

The buyer's surplus in the DM-S is $\hat{S}(m) = u(\hat{q}) - c(\hat{q})$. We have $\frac{1}{\phi}\frac{\partial \hat{S}}{\partial m} = \frac{u'(\hat{q})}{c'(\hat{q})} - 1$, which is always nonnegative. The seller's surplus is zero and their value function is $\hat{v}^s(m) = W(m)$.

2.5 Nominal interest rate

We denote the value function of the buyer who holds m dollars and chooses the effort level at the beginning of the day markets with

$$V^b(m) = \max_{\sigma \ge \underline{\sigma}}[-g(\sigma) + \sigma S(m) + (1-\sigma)\bar{\zeta}\hat{S}(m) + W(m)], \tag{7}$$

where $\bar{\zeta} = \frac{\zeta(\mu^b,\mu^s)}{\mu^b}$. The FOCs on σ are

$$g'(\sigma) = S - \bar{\zeta}\hat{S}, \tag{8}$$

if $S \ge \bar{\zeta}\hat{S}$, and $\sigma = \underline{\sigma}$ if $S < \bar{\zeta}\hat{S}$. In the following, we focus on the case where $S \ge \bar{\zeta}\hat{S}$ and the buyer makes a positive effort.

Similarly, the value function of a seller holding m dollars at the beginning of the day market is

$$V^s(m) = 0.5 \max_{q^s}[\phi p q^s - c(q^s)] + W(m).$$

Differentiating V^b with respect to m, yields

$$\frac{\partial V^b}{\partial m} = \sigma \frac{\partial S}{\partial m} + (1-\sigma)\bar{\zeta}\frac{\partial \hat{S}}{\partial m} + \phi. \tag{9}$$

Differentiating V^s with respect to m, yields

$$\frac{\partial V^s}{\partial m} = \phi. \tag{10}$$

Since each individual becomes a buyer or seller with probability 0.5, $V(m) = 0.5\{V^b(m) + V^s(m)\}$ and (1) implies that $\phi = 0.5\beta\{\frac{\partial V^b_{+1}}{\partial m_{+1}} + \frac{\partial V^s_{+1}}{\partial m_{+1}}\}$. Thus, from (9), and (10), the nominal interest rate $i = \frac{\phi}{\beta\phi_{+1}} - 1 = \frac{1}{2\phi}\{\sigma\frac{\partial S}{\partial m} + (1-\sigma)\bar{\zeta}\frac{\partial \hat{S}}{\partial m}\}$ is determined by

$$i = \frac{1}{2\phi}\left[\sigma\left(\frac{u'(q)}{c'(q^s)} - 1\right) + (1-\sigma)\bar{\zeta}\left(\frac{u'(\hat{q})}{c'(\hat{q})} - 1\right)\right]. \tag{11}$$

Since $\frac{\partial S}{\partial m}$ and $\frac{\partial \hat{S}}{\partial m}$ are nonnegative, i is also non-negative.

2.6 Competitive equilibrium

In this section, we characterize the competitive equilibrium. In the DM-C, the buyer's and seller's measures are σ and 0.5, respectively, and the quantity supplied is $0.5q^s$ and the the quantity demanded is σq. Thus, $q^s = 2\sigma q$ in the competitive equilibrium. Moreover, the probability $\bar{\zeta}$ equals $\frac{\zeta(1-\sigma, 0.5)}{1-\sigma}$.

$$i = 0.5\left\{\sigma\left(\frac{u'(q)}{c'(2\sigma q)} - 1\right) + \zeta(1-\sigma, 0.5)\left(\frac{u'(\hat{q})}{c'(\hat{q})} - 1\right)\right\}, \tag{12}$$

The equilibrium price in the DM-C equals $\phi p = c'(2\sigma q)$ and the surplus is written as $S(m) = u(q) - c'(2\sigma q)q$. Thus (8) implies

$$g'(\sigma) = u(q) - c'(2\sigma q)q - \frac{\zeta(1-\sigma, 0.5)}{1-\sigma}\{u(\hat{q}) - c(\hat{q})\}. \tag{13}$$

The stationary equilibrium allocation $\{q, \hat{q}, \sigma, \phi\}$, if it exists, satisfies Eqs. (12) and (13). In the following, we provide two more conditions to characterize the allocation $\{q, \hat{q}, \sigma, \phi\}$. These conditions depend on whether the buyer's feasibility constraints on money in the two DMs bind or not. There are three cases.

Case 1: buyers' constraints bind in the DM-C but not in the DM-S. In this case, the DM-S is efficient, and the allocation $\{q, \hat{q}, \sigma, \phi\}$ is determined by Eqs. (12), (13), and the following equations:

$$\phi M = c'(2\sigma q)q, \tag{14}$$

$$\hat{q} = q^*. \tag{15}$$

Since the DM-S is efficient, $\phi M = c'(2\sigma q)q \geq c(q^*)$.

Case 2: the buyers' constraints bind in the DM-S but not in the DM-C. In this case, the DM-C is efficient, and the allocation $\{q, \hat{q}, \sigma, \phi\}$ is determined by Eqs. (12), (13), and the following equations:

$$u'(q) = c'(2\sigma q), \tag{16}$$

$$\phi M = c(\hat{q}). \tag{17}$$

Since the DM-C is efficient, $\phi M = c(\hat{q}) \geq u'(q)q$.

Case 3: buyers' constraints bind in the DM-C and DM-S. The allocation $\{q, \hat{q}, \sigma, \phi\}$ is determined by Eqs. (12), (13), and the following equations:

$$\phi M = c'(2\sigma q)q, \tag{18}$$

$$\phi M = c(\hat{q}). \tag{19}$$

2.7 Non-optimality of the Friedman rule

We can easily check that as i goes to zero, $\frac{u'(q)}{c'(2\sigma q)}$ and $\frac{u'(\hat{q})}{c'(\hat{q})}$ go to 1. [5]If we denote the allocation under the Friedman rule of setting i to zero as $\{q^F, \hat{q}^F, \sigma^F, \phi^F\}$, we obtain $\hat{q}^F = q^*$. Eqs. (12), (13) imply that the variables q^F and σ^F are determined by

$$u'(q^F) = c'(2\sigma^F q^F), \tag{20}$$

$$u(q^F) - u'(q^F)q^F = g'(\sigma^F) + \frac{\zeta(1-\sigma^F, 0.5)}{1-\sigma^F} S^*. \tag{21}$$

The price of money under the Friedman rule must satisfy $\phi^F M \geq \max\{c(q^*), c'(2\sigma^F q^F)q^F\}$. The following lemma shows that the equilibrium under the Friedman rule is uniquely determined under several parametric restrictions.

Lemma 1 *Let $\underline{q}$ be a constant satisfying the equation*

$$u(\underline{q}) - u'(\underline{q})\underline{q} = \frac{\zeta(1-\underline{\sigma}, 0.5)}{1-\underline{\sigma}} S^*.$$

If $c'(2\underline{\sigma}\underline{q}) < u'(\underline{q})$, then $\{q^F, \sigma^F\}$ is uniquely determined.

Proof. See the appendix. ■

The following lemma shows that the buyers' constraints continue to bind in both the DM-C and DM-S as i converges to zero only under a special parametric restriction.

Lemma 2 *Case 1 holds in the neighborhood of the Friedman rule when*

$$u'(q^F)q^F > c(q^*). \tag{22}$$

[5]In Eq. (12), $\frac{u'(q)}{c'(2\sigma q)} \geq 1$ and $\frac{u'(\hat{q})}{c'(\hat{q})} \geq 1$ for any i. Moreover, since $\sigma \geq \underline{\sigma}$, and $\zeta(1-\sigma, 0.5) \geq \zeta(\underline{\sigma}, 0.5) > 0$, both σ and $\zeta(1-\sigma, 0.5)$ are away from zero.

Case 2 holds in the neighborhood of the Friedman rule when $u'(q^F)q^F < c(q^*)$. Case 3 holds at the Friedman rule only when $u'(q^F)q^F = c(q^*)$.

Proof. See the Appendix. ■

Generally, $c(q^*)$ differs from $u'(q^F)q^F$, in which case, Case 3 does not hold. In the following, we assume Eq. (22) and we focus on Case 1. We then simplify Eqs. (12) and (13) as

$$i = 0.5\sigma\left(\frac{u'(q)}{c'(2\sigma q)} - 1\right). \tag{23}$$

$$g'(\sigma) = u(q) - c'(2\sigma q)q - \frac{\zeta(1-\sigma, 0.5)}{1-\sigma}S^* \tag{24}$$

The equations characterizing the equilibrium allocation are nonlinear and thus the existence of the equilibrium is uncertain. The next proposition shows that under some parametric restrictions, the competitive equilibrium allocation exists and the inequality above always holds.

Proposition 1 *Suppose $u(q) = \frac{q^{1-\alpha}}{1-\alpha}$ and $c(q) = \frac{q^{\eta+1}}{1+\eta}$ where $0 < \alpha < 1$ and $\eta > \frac{1}{\zeta(1,0.5)}$. Also suppose that parameter $\underline{\sigma}$ satisfies $\underline{\sigma} \in (0, (\alpha/4)^{2/(1-\alpha)})$. If the nominal interest rate i is less than $\underline{\sigma}/2$, then the stationary equilibrium allocation uniquely exists and always corresponds to Case 1.*

Proof. See the Appendix. ■

The next proposition shows that the effort level is an increasing function of the nominal interest rate i as long as i is small.

Proposition 2 $\left.\frac{d\sigma}{di}\right|_{i=0} > 0.$

Proof. See the Appendix. ■

A higher nominal interest rate reduces the quantity q due to more severe monetary friction. A smaller q makes the buyer's surplus in the DM-C, $S(q) = u(q) - c'(q^s)q$, larger as long as the nominal interest rate is small. This is because under the Friedman rule, trade is efficient (i.e., $u'(q) = c'(q^s)$); then, for a given σ, the surplus satisfies

$$\frac{\partial S}{\partial q} = \{u(q) - c'(2\sigma q)q\}' = -2\sigma c''(q^s)q < 0.$$

The larger surplus makes entering the DM-C more attractive, and buyers expend more effort. Thus $\frac{d\sigma}{di}\Big|_{i=0} > 0$.

Stationary welfare W depends on σ and q:

$$W(\sigma, q) = -g(\sigma) + \zeta(1-\sigma, 1/2)S^* + \sigma u(q) - (1/2)c(2\sigma q).$$

In the following, we evaluate the sign of the term $\frac{dW}{di} = \frac{\partial W}{\partial \sigma}\frac{d\sigma}{di} + \frac{\partial W}{\partial q}\frac{dq}{di}$ under the Friedman rule. From Eq. (21),

$$\begin{aligned} \frac{\partial W}{\partial \sigma}\Big|_{i=0} &= -g'(\sigma^F) - \zeta_1(1-\sigma^F, 1/2)S^* + u(q^F) - c'(2\sigma^F q^F) \\ &= \left\{\frac{\zeta(1-\sigma^F, 1/2)}{1-\sigma^F} - \zeta_1(1-\sigma^F, 1/2)\right\} S^* > 0. \end{aligned}$$

The inequality holds because ζ is concave and $\zeta(0, \mu^s) = 0$; then

$$\frac{\zeta(\mu^b, \mu^s)}{\mu^b} > \zeta_1(\mu^b, \mu^s), \tag{25}$$

for any $\mu^b > 0$ and $\mu^s > 0$. [6] This means that under the Friedman rule, welfare W is *not* maximized with respect to the effort level, and the equilibrium effort level is insufficiently

[6] Let $f^{(1)}(\mu^b) = \zeta(\mu^b, \mu^s)$. The function $F(x) = f^{(1)\prime}(\mu^b)(x - \mu^b) + f^{(1)}(\mu^b) - f^{(1)}(x)$ satisfies $F'(x) < 0$ for $x < \mu^b$ and $F(\mu^b) = 0$. Thus, $F(0) = \mu^b f^{(1)\prime}(\mu^b) - f^{(1)}(\mu^b) > F(\mu^b) = 0$.

low. On the other hand, W is maximized with respect to q because

$$\left.\frac{\partial W}{\partial q}\right|_{i=0} = \sigma^F\{u'(q^F) - c'(2\sigma^F q^F)\} = 0.$$

Therefore, $\left.\frac{dW}{di}\right|_{i=0} = \left.\frac{\partial W}{\partial \sigma}\right|_{i=0} \left.\frac{d\sigma}{di}\right|_{i=0} > 0$, meaning that the Friedman rule is not optimal. Thus, we have proven the following proposition.

Proposition 3 *The Friedman rule is not optimal if Eq. (22) holds.*

Eq. (25) shows the existence of the following search externality in DM-S. Suppose that the number of buyers in the DM-S decreases infinitesimally from $1-\sigma$ to $1-(\sigma+d\sigma)$. On the one hand, buyers recognize $\frac{\zeta(1-\sigma,1/2)}{1-\sigma}S^* d\sigma$ as the private cost of exiting the DM-S. On the other hand, the social cost of buyers exiting the DM-S is $\zeta_1(1-\sigma, 1/2)S^* d\sigma$. Eq. (25) says that the private cost is larger than the social cost. This externality makes the effort level under the Friedman rule strictly lower than the socially optimal level. Thus, an increase in the effort level improves social welfare. Therefore, deviation from the Friedman rule increases the amount of effort and, in turn, increases social welfare. This result implies that the externality associated with the extensive margin of exiting the DM-S is crucial for the sub-optimality of the Friedman rule in our model.

3 Model: Only sellers choose the market

So far, we assume that buyers choose the markets. In this section, we describe a case in which sellers choose the market and show that the non-optimality of the Friedman rule continues to hold. We also show that in this case, output can also increase if monetary policy deviates from the Friedman rule.

Here, we assume that sellers can move from the DM-S to the DM-C with probability σ if they expend effort $h(\sigma)$, which leads to a utility cost of $h(\sigma)$. The function h satisfies $h(0) = h'(0) = 0$, $h' > 0$, $h'' > 0$, and $h'(1) = +\infty$. Buyers are divided among the two markets with equal probability, 1/2; thus, $\mu^b = 1/2$. The buyer's bargaining power in the DM-S is equal to one. The problem in the night market is the same as before. Thus the individual's value function, who holds m units of money at night is $W(m) = \phi m + W(0)$, and (1) holds.

We first study the buyers' problem. In the DM-S, buyers with m units of money who match with the sellers maximize their surplus $\hat{S} = u(\hat{q}) - \phi \hat{d}$ subject to the feasibility constraints on money and the seller's participation constraint:

$$\max_{\hat{q},\hat{d}} [u(\hat{q}) - \phi \hat{d}] \text{ s.t. } \phi \hat{d} = c(\hat{q}) \text{ and } \hat{d} \leq m.$$

If $\phi m \geq c(q^*)$, then $\phi \hat{d} = c(q^*)$ and $\hat{q} = q^*$; otherwise $\phi \hat{d} = \phi m = c(\hat{q})$. In the DM-C, they maximize their surplus $S = u(q) - \phi pq$ subject to the feasibility constraints on money. In the DM-S, buyers match with the sellers with probability $\zeta(\mu^b, \mu^s)/\mu^b = \zeta(0.5, \mu^s)/0.5$. The probability of entering the DM-C and the one of entering the DM-S are equal to 0.5. Thus the buyer's value function is

$$V^b(m) = \zeta(0.5, \mu^s)[u(\hat{q}) - \phi \hat{d}] + 0.5 \max_{pq \leq m} [u(q) - \phi pq] + W(m).$$

Differentiating $V^b(m)$ with m yields

$$\frac{\partial V^b}{\partial m} = \zeta(0.5, \mu^s) \frac{\partial \hat{S}}{\partial m} + 0.5 \frac{\partial S}{\partial m} + \phi, \tag{26}$$

where $\frac{1}{\phi}\frac{\partial S}{\partial m} = \frac{u'(q)}{c'(q^s)} - 1$ and $\frac{1}{\phi}\frac{\partial \hat{S}}{\partial m} = \frac{u'(\hat{q})}{c'(\hat{q})} - 1$.

We next study the sellers' problem. If sellers enter the DM-C, they maximize their surplus $\phi p q^s - c(q^s)$ by choosing q^s. If they enter the DM-S, then they get no surplus. Thus the seller solves

$$V^s(m) = \max_{\sigma}[-h(\sigma) + \sigma \max_{q^s}[\phi p q^s - c(q^s)] + W(m)].$$

The FOCs for $V^s(m)$ are

$$h'(\sigma) = c'(q^s)q^s - c(q^s), \tag{27}$$

$$\phi p = c'(q^s),$$

$$\frac{\partial V^s}{\partial m} = \phi. \tag{28}$$

Eqs. (1), (26) and (28) imply

$$\frac{\phi}{\beta} = 0.5\left(\frac{\partial V^b_{+1}}{\partial m_{+1}} + \frac{\partial V^s_{+1}}{\partial m_{+1}}\right) = \frac{\zeta(0.5,\mu^s)}{2}\frac{\partial \hat{S}}{\partial m} + \frac{1}{4}\frac{\partial S}{\partial m} + \phi_{+1}$$

In a competitive equilibrium, $\mu^s = 1 - \sigma$ and $0.5q = \sigma q^s$. The nominal interest rate $i = \phi/(\phi_{+1}\beta) - 1$ is denoted as

$$i = \frac{\zeta(0.5,\sigma)}{2}\left(\frac{u'(\hat{q})}{c'(\hat{q})} - 1\right) + \frac{1}{4}\left(\frac{u'(2\sigma q^s)}{c'(q^s)} - 1\right).$$

If we denote the allocation under the Friedman rule of setting i to zero as $\{q^{sF}, \hat{q}^F, \sigma^F, \phi^F\}$, we obtain $\hat{q}^F = q^*$, and the variables q^{sF} and σ^F are determined by

$$\begin{aligned} h'(\sigma^F) &= c'(q^{sF})q^{sF} - c(q^{sF}), \\ u'(2\sigma^F q^{sF}) &= c'(q^{sF}) \end{aligned}$$

We assume that the DM-S is efficient and the DM-C is inefficient when the nominal interest rate is low. This holds if $c'(q^{sF})q^{sF} > c(q^*)$. In this case, the FOCs imply

$$i = \frac{1}{4}\left\{\frac{u'(2\sigma q^s)}{c'(q^s)} - 1\right\}. \tag{29}$$

The quantity q^s and the probability σ at the stationary equilibrium are jointly determined by (27) and (29).

In the steady state, surplus of the seller is $-h(\sigma) + \sigma\{\phi p q^s - c(q^s)\}$, and the one of the buyer is $\zeta(0.5, 1-\sigma)S^* + 0.5\{u(q) - \phi p q\}$. We define welfare as their sum:

$$W = -h(\sigma) + \zeta(0.5, 1-\sigma)S^* + 0.5u(q) - \sigma c(q^s).$$

The next proposition shows that a deviation from the Friedman rule is welfare-improving.

Proposition 4 *The Friedman rule is not optimal, given that $c'(q^{sF})q^{sF} > c(q^*)$, where (σ^F, q^{sF}) is the solution to (27) and (29) with $i = 0$.*

In the following, we show that inflation can increase output. [7] We follow Lagos and Rocheteau (2005) and focus on output in the DMs. In the steady state, the total output in the DMs equals $Y = 1/2\{\sigma q^s + \zeta(1/2, 1-\sigma)q^*\}$. To simplify the analysis, we suppose that $h(\sigma) = \frac{\sigma^2}{2}$, $u(q) = \frac{3a}{2}q^{2/3}$, and $c(q) = \frac{q^2}{2}$ with $a > 0$. The efficient quantity is $q^* = a^{3/4}$. We rearrange Eqs. (27) and (29) as

$$\sigma = \frac{1}{2}(q^s)^2, \tag{30}$$

$$(1+4i)q^s = a(2\sigma q^s)^{-1/3}. \tag{31}$$

Eqs. (30) implies that $2Y = \sqrt{2}\sigma^{\frac{3}{2}} + \zeta(1/2, 1-\sigma)q^*$, and we obtain

$$2\left.\frac{dY}{di}\right|_{i=0} = \left[\frac{3}{2}\sqrt{2\sigma_F} - q^*\zeta_2(1/2, 1-\sigma_F)\right]\left.\frac{d\sigma}{di}\right|_{i=0},$$

where σ_F is the level of σ under the Friedman rule. Eqs. (30) and (31) imply that $\sigma = \frac{a}{2(1+4i)}$. Thus $\frac{d\sigma}{di} < 0$ and $\sigma_F = a/2$. Therefore $\left.\frac{dY}{di}\right|_{i=0} > 0$ if and only if

$$\frac{3}{2}\sqrt{2\sigma_F} - q^*\zeta_2(1/2, 1-\sigma_F) = \frac{3}{2}\sqrt{a} - a^{3/4}\zeta_2(1/2, 1-a/2) < 0. \tag{32}$$

[7] When buyers search the markets, it is not clear whether inflation increases the total output.

Since $\zeta_2(1/2, 0) = \infty$, (32) holds if a is sufficiently close to 2. Therefore we obtain $\frac{dY}{di} > 0$ at $i = 0$.

4 Conclusion

In this study, we investigate a monetary model with two decentralized markets, and each agent chooses which one to participate in by expending effort. In one market, the pricing mechanism is competitive, whereas in the other market, the terms of trade are determined by Nash bargaining. The analysis shows that the optimal monetary policy may deviate from the Friedman rule, even though the search externality is nonexistent in the competitive pricing market. Existing models already show the sub-optimality of the Friedman rule in a competitive market, though the search externality setting in the competitive market is not completely satisfactory. The novelty of our model is that the sub-optimality of the Friedman rule is derived in a more plausible setting for the search externality than those assumed in the existing literature. The intuition for the sub-optimality of the Friedman rule is the following. As the nominal interest rate deviates from zero, buyers expend more effort because a higher interest rate increases their gains from entering the competitive pricing market, while the marginal increase in social welfare by entering the competitive pricing market is also positive.

Appendix

In Appendix A, we provide proofs for the propositions. Appendix B describes a version of the model in which both buyers and sellers choose the market.

A Proofs

A.1 Proof of Lemma 1

Let the function $f^{(3)}$ denote $f^{(3)}(q) = u(q) - u'(q)q$. In the following, the following equations have a unique solution

$$\begin{aligned} \frac{c'(2\sigma q)}{u'(q)} &= 1, && (33) \\ g'(\sigma) + \frac{\zeta(1-\sigma, 0.5)}{1-\sigma} S^* &= f^{(3)}(q), && (34) \end{aligned}$$

The function $\frac{c'(2\sigma q)}{u'(q)}$ is increasing in σ and q. Thus, the first equation shows a negative relationship between σ and q. As q goes to ∞, σ goes to zero. The function $f^{(3)}(q)$ is increasing, and then the second equation shows a positive relationship between σ and q. Since it satisfies $f^{(3)}(0) \leq 0$ and $f^{(3)}(\infty) = \infty$, there is a unique constant $\underline{q}$ satisfying

$$\frac{\zeta(1-\underline{\sigma}, 0.5)}{1-\underline{\sigma}} S^* = f^{(3)}(\underline{q})$$

As q goes to ∞, the σ satisfying the second equation goes to $1 - \underline{\sigma}$ since $g'(1-\underline{\sigma}) = \infty$. Therefore, the two equations have a unique solution when $\frac{c'(2\underline{\sigma}\underline{q})}{u'(\underline{q})} < 1$.

A.2 Proof of Lemma 2

If Eq. (18) continues to hold as i converges to zero, then under the Friedman rule, the money balances in the DM-C satisfy $\phi^F M = c'(2\sigma^F q^F)q^F$. Similarly, if Eq. (19) continues to hold as i converges to zero, then under the Friedman rule, $\phi^F M = c(q^*)$. In the DM-C, trades are efficient, and then $c'(2\sigma^F q^F) = u'(q^F)$. Therefore, Case 1 occurs only if $c(q^*) = u'(q^F)q^F$.

A.3 Proof of Proposition 1

We have $c'(2\sigma q) = (2\sigma q)^\eta$ and $q^* = 1$. The first order conditions imply

$$
\begin{aligned}
q^{1-\alpha} &= h(\sigma) \equiv \frac{1}{2^{\frac{\eta(1-\alpha)}{\alpha+\eta}}(2i+\sigma)^{\frac{1-\alpha}{\alpha+\eta}}\sigma^{\frac{\eta-1}{\alpha+\eta}(1-\alpha)}},\\
q^{1-\alpha} &= k(\sigma) \equiv \left(\frac{1}{1-\alpha} - \frac{\sigma}{2i+\sigma}\right)^{-1}\left[g'(\sigma) + \frac{\zeta(1-\sigma, 0.5)}{1-\sigma}S^*\right].
\end{aligned}
$$

The function h is decreasing and satisfies $h(\infty) = 0$, while k is increasing and satisfies $k(1-\underline{\sigma}) = \infty$. Thus there exists a unique equilibrium if and only if $h(\underline{\sigma}) > k(\underline{\sigma})$. Since $0 < \alpha < 1$ and $\eta > 1$ by assumption and $S^* < \frac{1}{1-\alpha}$, we have the following inequalties for $i \leq 0.5\underline{\sigma}$:

$$
\begin{aligned}
h(\underline{\sigma}) &= \frac{1}{2^{\frac{\eta(1-\alpha)}{\alpha+\eta}}(2\underline{\sigma})^{\frac{1-\alpha}{\alpha+\eta}}\underline{\sigma}^{\frac{\eta-1}{\alpha+\eta}(1-\alpha)}} = \frac{1}{2^{\frac{1-\alpha}{\alpha+\eta}}(2\underline{\sigma})^{\frac{\eta}{\alpha+\eta}(1-\alpha)}} > \frac{1}{2(2\underline{\sigma})^{\frac{\eta}{\alpha+\eta}(1-\alpha)}} > \frac{1}{2(2\underline{\sigma})^{\frac{1-\alpha}{\alpha+1}}} > \frac{1}{4\underline{\sigma}^{\frac{1-\alpha}{2}}},\\
k(\underline{\sigma}) &= \left(\frac{1}{1-\alpha} - \frac{\underline{\sigma}}{2i+\underline{\sigma}}\right)^{-1}\frac{\zeta(1-\underline{\sigma}, 0.5)}{1-\underline{\sigma}}S^* < \left(\frac{1}{1-\alpha} - 1\right)^{-1}S^* < \frac{1}{\alpha},
\end{aligned}
$$

Thus $h(\underline{\sigma}) > k(\underline{\sigma})$ if $4\underline{\sigma}^{\frac{1-\alpha}{2}} < \alpha$ or equivalently $\underline{\sigma} < (\alpha/4)^{2/(1-\alpha)}$. Therefore the equilibrium is uniquely determined if $i \leq 0.5\underline{\sigma}$

Under the Friedman rule, we have

$$
(q^F)^{1-\alpha} = \frac{1-\alpha}{\alpha}\left[g'(\sigma^F) + \frac{\zeta(1-\sigma^F, 0.5)}{1-\sigma^F}S^*\right].
$$

We have

$$
g'(\sigma^F) + \frac{\zeta(1-\sigma^F, 0.5)}{1-\sigma^F}S^* > \zeta(1, 0.5)\left(\frac{1}{1-\alpha} - \frac{1}{1+\eta}\right) = \frac{\zeta(1, 0.5)(\alpha+\eta)}{(1-\alpha)(1+\eta)}. \tag{35}
$$

Case 1 holds if $u'(q^F)q^F = (q^F)^{1-\alpha} > c(q^*) = \frac{1}{1+\eta}$, or equivalently

$$
g'(\sigma^F) + \frac{\zeta(1-\sigma^F, 0.5)}{1-\sigma^F}S^* > \frac{\alpha}{(1-\alpha)(1+\eta)}
$$

Inequality (35) implies that this condition holds if $\zeta(1,0.5)(\alpha+\eta) > \alpha$ or

$$\eta > \alpha\left(\frac{1}{\zeta(1,0.5)} - 1\right).$$

It holds if $\eta > \frac{1}{\zeta(1,0.5)}$.

A.4 Proof of Proposition 2

We express Eqs. (12) as

$$\frac{2i}{\sigma} = \frac{u'(q)}{c'(2q\sigma)} - 1. \tag{36}$$

Differentiating Eqs. (12) and (13) with respect to i under the Friedman rule, we obtain

$$\begin{aligned} a_1\frac{d\sigma}{di} &= -a_2\frac{dq}{di}, \\ 2 &= -a_3\frac{d\sigma}{di} - a_4\frac{dq}{di}, \end{aligned}$$

where

$$\begin{aligned} a_1 &= g''(\sigma) + \frac{\zeta(1-\sigma,1/2)}{(1-\sigma)^2}S^* + \frac{\zeta_1(1-\sigma,1/2)}{1-\sigma}S^* + 2(q)^2c'' > 0, \\ a_2 &= 2\sigma q c'' > 0, \\ a_3 &= \frac{2\sigma q u' c''}{(c')^2}, \\ a_4 &= \frac{2(\sigma)^2 u' c''}{(c')^2} - \frac{\sigma u''(q)}{c'}. \end{aligned}$$

In this case, we obtain $\frac{d\sigma}{di} = \frac{2a_2}{a_1a_4 - a_2a_3}$, and the numerator a_2 is also positive. We express its denominator as

$$\begin{aligned} a_1a_4 - a_2a_3 \quad = \quad & \left[g'' + \frac{\zeta S^*}{(1-\sigma)^2} + \frac{\zeta_1 S^*}{1-\sigma}\right]\frac{2(\sigma)^2u'c''}{(c')^2} \\ & + \left[g'' + \frac{\zeta S^*}{(1-\sigma)^2} + \frac{\zeta_1 S^*}{1-\sigma} + 2(q)^2c''\right]\frac{-\sigma u''}{c'}, \end{aligned}$$

which is also positive. Therefore, we obtain $\frac{d\sigma}{di} > 0$ and $\frac{dq}{di} = -\frac{a_1}{a_2}\frac{d\sigma}{di} < 0$ under the Friedman rule. ■

A.5 Proof of Proposition 4

If $i = 0$, we obtain $u'(q) = c'(q^s)$. Eq. (27) determines σ as a function of q^s. If we let $\sigma(q) = h'^{(-1)}(c'(q)q - c(q))$, $\sigma'(q) > 0$. We can also express Eq. (29) as $4ic'(q^s) = u'(2\sigma(q^s)q^s) - c'(q^s)$. Differentiating both sides of the equation by q^s, we get

$$4\frac{di}{dq^s}c'(q^s) = 2\{q^s\sigma(q^s)\}'u''(2\sigma(q^s)q^s) - c''(q^s) - 4ic''(q^s). \tag{37}$$

The right hand side of Eq. (37) is negative; thus, $\left.\frac{dq^s}{di}\right|_{i=0} < 0$ and

$$\left.\frac{d\sigma}{di}\right|_{i=0} = \sigma'(q^s)\left.\frac{dq^s}{di}\right|_{i=0} < 0. \tag{38}$$

Using Eq. (27), we obtain

$$\frac{dW}{dq^s} = -\sigma'(q^s)\zeta_2 S^* + \sigma\{u'(2\sigma q^s) - c'(q^s)\}.$$

Under the Friedman rule, $u'(2\sigma q^s) = c'(q^s)$ and $\left.\frac{dW}{dq^s}\right|_{i=0} = -\zeta_2 S^* \sigma'(q^s) < 0$; therefore, $\left.\frac{dW}{di}\right|_{i=0} = \left.\frac{dq^s}{di}\frac{dW}{dq^s}\right|_{i=0} > 0$ and a deviation from the Friedman rule improves welfare. ■

B Case in which both buyers and sellers choose the market

In this section, we consider a case in which both buyers and sellers choose the market in which to participate. In the following, we suppose that $\zeta(\mu^b, \mu^s) = z(\mu^b)^\alpha(\mu^s)^{1-\alpha}$ with $\alpha \in (0, 1)$.

The seller can enter the DM-C with probability σ^s if the seller expends effort σ^s. The utility cost of the seller's effort $h(\sigma)$ satisfies $h(0) = h'(0) = 0$, $h'(\sigma) > 0$, and $h''(\sigma) > 0$ if $\sigma > 0$. The matching function is still given by $\zeta(\mu^b, \mu^s)$ and the probability of meeting a buyer is $(1-\sigma^s)\frac{\zeta(\mu^b,\mu^s)}{\mu^s}$. In the competitive equilibrium, $\mu^s = 1 - \sigma^s$, where σ^s is the seller's average search intensity.

The problem of the night market is the same as before. The buyer and seller enter the DM-S with probability $1 - \sigma^b$ and $1 - \sigma^s$, respectively. In the DM-S, the buyer and seller trade bilaterally with Nash bargaining as follows:

$$\max_{d \leq m}[u(\hat{q}) - \phi d]^{\theta}[\phi d - c(\hat{q})]^{1-\theta}, \tag{39}$$

where $\theta \in (0, 1]$ denotes the bargaining power of the buyer and $(\hat{q}, d)$ represents the terms of trade.

The value function of the buyer holding m dollars is

$$\begin{aligned} V^b(m) \quad = \quad & \max_{\sigma^b}[-g(\sigma^b) + \sigma^b \max_{pq^b \leq m}\{u(q^b) - \phi p q^b\} \\ & +(1-\sigma^b)\frac{\zeta(\mu^b, \mu^s)}{\mu^b}\{u(\hat{q}) - \phi d\} + W(m)]. \end{aligned}$$

We denote the buyer's surplus in the DM-C, $u(q^b) - \phi p q^b$, as s^b, and that in the DM-S as $u(\hat{q}) - \phi d$ as $\hat{s}^b$. The FOCs for $V^b(m)$ are

$$\begin{aligned} & u'(q^b) \geq \phi p, \\ & g'(\sigma^b) = s^b - \frac{\zeta(\mu^b, \mu^s)}{\mu^b}\hat{s}^b, \\ & \frac{\partial V^b}{\partial m} = \sigma^b \frac{\partial s^b}{\partial m} + (1-\sigma^b)\frac{\partial \hat{s}^b}{\partial m} + \phi. \end{aligned}$$

The value function of a seller holding m dollars is

$$\begin{aligned} V^s(m) &= \max_{\sigma^s}[-h(\sigma^s) + \sigma^s \max_{q^s}[\phi p q^s - c(q^s)] \\ &\quad +(1-\sigma^s)\frac{\zeta(\mu^b,\mu^s)}{\mu^s}[\phi \hat{d} - c(\hat{q})] + W(m)] \end{aligned}$$

The FOCs for $V^s(m)$ are

$$\begin{aligned} &\phi p = c'(q^s), \\ &h'(\sigma^s) = \phi p q^s - c(q^s) - \frac{\zeta(\mu^b,\mu^s)}{\mu^s}\hat{s}^s \\ &\frac{\partial V^s}{\partial m} = \phi. \end{aligned}$$

We again focus on the case in which the DM-S is efficient when the nominal interest rate is close to zero. We denote the buyer's bargaining power as θ. The stationary equilibrium allocation $\{q, q^s, \sigma, \sigma^s, \phi\}$ is determined by

$$g'(\sigma) = u(q) - c'(q^s)q - \frac{\theta S^*}{1-\sigma}\zeta(1-\sigma, 1-\sigma^s), \tag{40}$$

$$h'(\sigma^s) = c'(q^s)q^s - c(q^s) - \frac{(1-\theta)S^*}{1-\sigma^s}\zeta(1-\sigma, 1-\sigma^s), \tag{41}$$

$$\phi M = c'(q^s)q \; > \theta c(q^*) + (1-\theta)u(q^*), \tag{42}$$

$$\sigma^s q^s = \sigma q, \tag{43}$$

and Eq. (23). The stationary welfare is

$$W = -g(\sigma) - h(\sigma^s) + \zeta(1-\sigma, 1-\sigma^s)S^* + \sigma u(q) - \sigma^s c(q^s).$$

If $\frac{dW}{di} > 0$ at $i = 0$, then the Friedman rule is not optimal. We can show the following proposition.

Proposition 5 *Suppose that $\alpha < \theta$, and that when $i = 0$,*

$$g'' + \frac{\zeta\theta S^*}{(1-\sigma)^2} - \frac{\zeta_1\theta S^*}{1-\sigma} - \frac{\sigma^s\zeta_1(1-\theta)S^*}{\sigma(1-\sigma^s)} > 0, \tag{44}$$

$$h'' + \frac{\zeta(1-\theta)S^*}{(1-\sigma^s)^2} - \frac{\zeta_2(1-\theta)S^*}{1-\sigma^s} - \frac{\sigma\zeta_2\theta S^*}{\sigma^s(1-\sigma)} > 0, \tag{45}$$

where $\zeta = \zeta(1-\sigma, 1-\sigma^s)$. In this case, the Friedman rule is not optimal.

Proof. We reduce the system to the following three equations for the three unknowns $\{\sigma, \sigma^s, q\}$, given i:

$$2i = \sigma\left(\frac{u'(q)}{c'(\sigma q/\sigma^s)} - 1\right), \tag{46}$$

$$g'(\sigma) = u(q) - c'\left(\frac{\sigma q}{\sigma^s}\right)q - \frac{\zeta(1-\sigma, 1-\sigma^s)}{1-\sigma}\theta S^*, \tag{47}$$

$$h'(\sigma^s) = c'\left(\frac{\sigma q}{\sigma^s}\right)\frac{\sigma q}{\sigma^s} - c\left(\frac{\sigma q}{\sigma^s}\right) - \frac{\zeta(1-\sigma, 1-\sigma^s)}{1-\sigma^s}(1-\theta)S^*, \tag{48}$$

Stationary welfare depends on three unknowns: σ, σ^s, and q.

$$W(\sigma, \sigma^s, q) = -g(\sigma) - h(\sigma^s) + \zeta(1-\sigma, 1-\sigma^s)S^* + \sigma u(q) - \sigma^s c(\sigma q/\sigma^s)$$

Thus, we obtain $\frac{dW}{di} = \frac{\partial W}{\partial\sigma}\frac{d\sigma}{di} + \frac{\partial W}{\partial\sigma^s}\frac{d\sigma^s}{di} + \frac{\partial W}{\partial q}\frac{dq}{di}$. In the following, we let $\tilde\theta = 1-\theta$, $\mu^b = 1-\sigma$, and $\mu^s = 1-\sigma^s$. Differentiating Eqs. (46), (47), and (48) by i, we obtain

$$2 = -\omega_1\frac{d\sigma}{di} + \omega_2\frac{d\sigma^s}{di} - \omega_3\frac{dq}{di},$$
$$\omega_4\frac{d\sigma}{di} = -\omega_5\frac{dq}{di} + \omega_6\frac{d\sigma^s}{di},$$
$$\omega_7\frac{d\sigma^s}{di} = \omega_8\frac{dq}{di} + \omega_9\frac{d\sigma}{di},$$

where $\omega_1 = \frac{\sigma q u' c''}{(c')^2\sigma^s}$, $\omega_2 = \frac{(\sigma)^2 q u' c''}{(\sigma^s)^2(c')^2}$, $\omega_3 = \sigma(\frac{u'c''\sigma}{(c')^2\sigma^s} - \frac{u''}{c'})$, $\omega_4 = g'' + \frac{(q)^2 c''}{\sigma^s} + \frac{\zeta\theta S^*}{(\mu^b)^2} - \frac{\zeta_1\theta S^*}{\mu^b}$, $\omega_5 = \frac{c''\sigma q}{\sigma^s}$, $\omega_6 = \frac{(q)^2\sigma c''}{(\sigma^s)^2} + \frac{\zeta_2\theta S^*}{\mu^b}$, $\omega_7 = h'' + \frac{(q)^2(\sigma)^2 c''}{(\sigma^s)^3} + \frac{\zeta\tilde\theta S^*}{(\mu^s)^2} - \frac{\zeta_2\tilde\theta S^*}{\mu^s}$, $\omega_8 = \frac{c''(\sigma)^2 q}{(\sigma^s)^2}$ and $\omega_9 = \frac{(q)^2\sigma c''}{(\sigma^s)^2} + \frac{\zeta_1\tilde\theta S^*}{\mu^s}$.

We obtain

$$\frac{d\sigma^s}{di} = -\frac{\omega_4\omega_8 - \omega_5\omega_9}{\omega_5\omega_7 - \omega_6\omega_8}\frac{d\sigma}{di},$$
$$\frac{d\sigma}{di} = \frac{2\omega_5(\omega_5\omega_7 - \omega_6\omega_8)}{\Delta},$$

where $\Delta = (\omega_2\omega_9 - \omega_1\omega_7)\omega_5^2 + (\omega_1\omega_6 - \omega_2\omega_4)\omega_5\omega_8 + (\omega_4\omega_7 - \omega_6\omega_9)\omega_3\omega_5$. We show that

$$\omega_4\omega_8 - \omega_5\omega_9 = \frac{(\sigma)^2 c'' q}{(\sigma^s)^2}\left[\omega_{10} - \frac{\sigma^s}{\sigma}\frac{\zeta_1}{\mu^s}\tilde{\theta}S^*\right],$$
$$\omega_5\omega_7 - \omega_6\omega_8 = \frac{c''\sigma q}{\sigma^s}\left[\omega_{11} - \frac{\sigma}{\sigma^s}\frac{\zeta_2}{\mu^b}\theta S^*\right],$$

where $\omega_{10} = g'' + \frac{\zeta}{(\mu^b)^2}\theta S^* - \frac{\zeta_1}{\mu^b}\theta S^*$ and $\omega_{11} = h'' + \frac{\zeta}{(\mu^s)^2}\tilde{\theta}S^* - \frac{\zeta_2}{\mu^s}\tilde{\theta}S^*$. In addition, we show that

$$\begin{aligned}
(\omega_2\omega_9 - \omega_1\omega_7)\omega_5^2 =& \frac{(q)^3(c'')^3\left(\sigma\right)^3 u'}{(\sigma^s)^3(c')^2}\left[-\omega_{11} + \frac{\sigma}{\sigma^s}\frac{\zeta_1}{\mu^s}\tilde{\theta}S^*\right],\\
(\omega_1\omega_6 - \omega_2\omega_4)\omega_5\omega_8 =& \frac{(q)^3(c'')^3(\sigma)^5 u'}{(\sigma^s)^5(c')^2}\left[-\omega_{10} + \frac{\sigma^s}{\sigma}\frac{\zeta_2}{\mu^b}\theta S^*\right],\\
(\omega_4\omega_7 - \omega_6\omega_9)\omega_3\omega_5 =& \omega_3\omega_5\left(\omega_{10}\omega_{11} - \frac{\zeta_1\zeta_2\theta\tilde{\theta}(S^*)^2}{\mu^b\mu^s}\right)\\
&+ \omega_{12}\frac{(q)^2 c''}{\sigma^s}\left(\omega_{11} - \frac{\sigma\zeta_2\theta S^*}{\sigma^s\mu^b}\right)\\
&+ \omega_{12}\frac{(q)^2(\sigma)^2 c''}{(\sigma^s)^3}\left(\omega_{10} - \frac{\sigma^s\zeta_1\tilde{\theta}S^*}{\sigma\mu^s}\right)\\
&- (\omega_2\omega_9 - \omega_1\omega_7)\omega_5^2 - (\omega_1\omega_6 - \omega_2\omega_4)\omega_5\omega_8,
\end{aligned}$$

where $\omega_{12} = (-\frac{u''}{c'})(\frac{c''(\sigma)^2 q}{\sigma^s})$. Therefore, the denominator of $d\sigma/di$, Δ, satisfies

$$\begin{aligned}
\Delta = {} & \omega_3\omega_5 \left(\omega_{10}\omega_{11} - \frac{\zeta_1\zeta_2\theta\tilde{\theta}(S^*)^2}{\mu^b\mu^s}\right) + \omega_{12}\frac{(q)^2 c''}{\sigma^s}\left(\omega_{11} - \frac{\sigma\zeta_2\theta S^*}{\sigma^s\mu^b}\right) \\
& + \omega_{12}\frac{(q)^2(\sigma)^2 c''}{(\sigma^s)^3}\left(\omega_{10} - \frac{\sigma^s\zeta_1\tilde{\theta}S^*}{\sigma\mu^s}\right) \\
> {} & \omega_3\omega_5\left(\omega_{10} - \frac{\sigma^s\zeta_1\tilde{\theta}S^*}{\sigma\mu^s}\right)\left(\omega_{11} - \frac{\sigma\zeta_2\theta S^*}{\sigma^s\mu^b}\right) + \omega_{12}\frac{(q)^2 c''}{\sigma^s}\left(\omega_{11} - \frac{\sigma\zeta_2\theta S^*}{\sigma^s\mu^b}\right) \\
& + \omega_{12}\frac{(q)^2(\sigma)^2 c''}{(\sigma^s)^3}\left(\omega_{10} - \frac{\sigma^s\zeta_1\tilde{\theta}S^*}{\sigma\mu^s}\right),
\end{aligned}$$

where the inequality follows from $\omega Y - xy > (\omega - x)(Y - y)$ for $\omega > x > 0$ and $Y > y > 0$.

Since $\theta > \alpha$, we obtain

$$\begin{aligned}
\frac{\partial W}{\partial q} &= \sigma[u'(q) - c'(q^s)] = 0, \\
\frac{\partial W}{\partial \sigma} &= -g' - \zeta_1 S^* + u(q) - qc'(q^s) = \frac{\zeta}{\mu^b}\theta S^* - \zeta_1 S^* \; > 0, \\
\frac{\partial W}{\partial \sigma^s} &= -h' - \zeta_2 S^* - c(q^s) + c'(q^s)q^s = \frac{\zeta}{\mu^s}\tilde{\theta} S^* - \zeta_2 S^* \; < 0.
\end{aligned}$$

The assumptions directly lead to $\frac{d\sigma}{di} > 0$ and $\frac{d\sigma^s}{di} < 0$. Therefore, $\frac{dW}{di} = \frac{\partial W}{\partial \sigma}\frac{d\sigma}{di} + \frac{\partial W}{\partial \sigma^s}\frac{d\sigma^s}{di} > 0$.

This inequality implies that raising the nominal interest rate from $i = 0$ can improve welfare.

■

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