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Abstract

Japan's net government debt is 130% of GDP in 2013. The present paper analyzes the effect of the large government debt on welfare. We use a heterogeneous-agent, incomplete-market model with idiosyncratic wage risk, a borrowing constraint, and endogenous labor supply. We calibrate the model to the Japanese economy using evidence based on macro-level and micro-level data. We find that the optimal level of government debt is –50% of GDP for Japan. The welfare cost of keeping government debt to 130% of GDP rather than the optimal level is 0.19% of consumption.

Keywords: Government debt; welfare; incomplete markets; inequality; uncertainty; Japanese economy.

JEL classification: E62, H63

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1 Introduction

Over the last twenty years, Japan's net government debt increased significantly and it reached 130% of GDP in 2013. The debt-to-GDP ratio is the highest among major developed countries. A large number of papers, including Hoshi and Ito (2014), İmrohoroğlu, Kitao, and Yamada (2016), and Hansen and İmrohoroğlu (2016), analyze Japan's debt problem.[1] However, the effect of the large government debt on welfare has been less understood. Flodén (2001) finds that the optimal level of government debt is 150% of GDP for the United States. Is the optimal level of government debt for Japan similar and should Japan accumulate more debt beyond the current level of 130% of GDP? Or does the current level of debt exceed the optimal level? How much is the welfare cost of maintaining the current amount of debt rather than the optimal quantity?

The present paper examines the welfare implications of government debt for the Japanese economy. We follow Aiyagari and McGrattan (1998) and Flodén (2001), which conduct a similar analysis for the United States, in using an Aiyagari (1994)-style heterogeneous-agent, incomplete-market model with endogenous labor supply. What makes this model distinct from the standard, representative-agent, neoclassical growth model is incomplete insurance against idiosyncratic wage risk. Households use savings and labor supply to self-insure against idiosyncratic risk, which endogenously generates distributions of wealth and labor earnings across households.[2]

We calibrate the model to the Japanese economy for the 1995–2013 period.[3] Although most of the parameters can be set as in existing representative-agent models for Japan (e.g., Hayashi and Prescott (2002)), calibrating idiosyncratic wage risk needs evidence based on micro-level data. Since panel data on individual wages, which are often used for estimating idiosyncratic wage risk, are limited in Japan, we calibrate idiosyncratic wage risk to the

[1] The following subsection reviews the related literature.

[2] This class of models has been widely used for analyzing fiscal policies. Examples include Flodén and Linde (2001) and Alonso-Ortiz and Rogerson (2010).

[3] In Section 3.2, we report how well the model accounts for wealth and income inequality in Japan.

cross-sectional dispersion of (residual) wages documented by Lise, Sudo, Suzuki, Yamada, and Yamada (2014). The result suggests that idiosyncratic wage risk in Japan is smaller than that in the United States assumed by Flodén (2001).[4]

Using the calibrated model, we examine how a change in the amount of government debt affects the Japanese economy. Our welfare measure is utilitarian and it weights all households equally, as in Aiyagari and McGrattan (1998) and Flodén (2001). Government debt affects welfare through several channels in the model and Flodén (2001) decomposes the effect on the utilitarian welfare into three components. First, an increase in government debt crowds out of capital and larger required interest payments increase distortionary taxes, both of which affect the average levels of consumption and leisure.[5] Further, efficiency-weighted and raw labor hours move differently because households with different productivity adjust their labor hours in a different way. This also has the level effect. Second, the return on savings rises, which benefits wealth rich households and widens inequality in consumption and leisure across households. This reduces the utilitarian welfare. Third, the increase in the interest rate reduces the cost of savings to self-insure against idiosyncratic wage risk and it helps households to smooth consumption. Further, the increase in labor income taxes lowers the after-tax wage rate. Both of these reduce uncertainty that households face and improve welfare.[6] The optimal quantity of government debt depends on the relative strengths of these effects.

We find that for our baseline specification, the optimum quantity of government debt is –50% of GDP for Japan. Hence, the current level of 130% of GDP is too high in terms of welfare. The overall welfare cost of holding the current level of debt rather than the optimal level is 0.19% of consumption. As for the welfare decomposition, the reduced uncertainty contributes to a welfare gain of 0.52%, whereas the increased inequality and the reduced

[4] Flodén (2001)'s calibration is based on the estimate by Flodén and Linde (2001). Flodén and Linde (2001) also estimate idiosyncratic wage risk in Sweden. Idiosyncratic risk in Japan assumed here is larger than their estimate for Sweden.

[5] Elmendorf and Mankiw (1999) list these costs as two readily quantifiable costs of government debt.

[6] Gottardi, Kajii, and Nakajima (2016) analyze the distribution and insurance effects of labor and capital income taxes in a two-period model with uninsurable idiosyncratic risk to labor and capital income.

level generate a welfare cost of 0.23% and 0.48%, respectively. Hence, the relatively small total cost is a result of these offsetting forces.

Further analyses identify two reasons why the optimal amount of government debt for Japan is smaller than the optimal debt for the United States found by Flodén (2001). First, as mentioned above, idiosyncratic wage risk is smaller in Japan than in the United States. Second, government transfers, which also provide insurance against idiosyncratic wage risk, are larger in Japan than in the United States. Hence, there is less room for government debt to reduce uncertainty and improve welfare in Japan compared to the United States. As a result, the optimal level of debt is lower in Japan than in the United States.

We also conduct several sensitivity analyses. None of our analyses suggests that the current amount of government debt in Japan is too small in terms of welfare. In particular, if both government debt and public transfers can be set freely, then the optimal level of debt is –120% of GDP.

The rest of the present paper is organized as follows. The following subsection reviews the literature on Japan's fiscal problem. Section 2 describes the model and Section 3 calibrates the model to the Japanese economy. Section 4 presents the results for our baseline specification. Section 5 compares our results for Japan with Flodén (2001)'s result for the United States and conducts sensitivity analyses. Section 6 concludes.

Related Literature

The present paper contributes to the literature on Japan's fiscal problem by analyzing how government debt affects welfare. Previous works analyze various scenarios and options to stabilize government debt in Japan, but there have been few welfare analyses. None of the existing works computes the optimal quantity of government debt for Japan. Our paper is also novel in that we calibrate an Aiyagari (1994)-style model to the Japanese economy and use the model for evaluating the insurance and redistribution effects of government debt. We divide existing works into three groups based on their methodologies.

First, Broda and Weinstein (2005), Doi, Hoshi, and Okimoto (2011), and Hoshi and Ito (2014) directly analyze the government budget constraint. Given the future paths of government expenditures and revenues as well as the interest rates and the growth rates, the future path of government debt can be derived. This approach is simple and useful to analyze various scenarios, but welfare implications are not obtained.

Second, several papers analyze Japan's fiscal problem using a representative-agent neoclassical growth model. For example, İmrohoroğlu and Sudo (2011a) analyze how a rise in consumption tax rate affects primary surplus, whereas İmrohoroğlu and Sudo (2011b) analyze how total factor productivity growth and fiscal adjustments affect the debt-to-GDP ratio. These papers do not conduct welfare analyses. Hansen and İmrohoroğlu (2016) analyze fiscal adjustments and compare welfare under a few long-run levels of government debt, but they do not search for the optimal level. In addition, using a representative-agent model, their study is silent about the insurance and redistribution effects of government debt.[7]

Third, several papers use an overlapping generations model. Arai and Ueda (2013) use a simple overlapping generations model and analyze the level of primary deficits sustainable in the long run. İmrohoroğlu, Kitao, and Yamada (2016) construct a rich overlapping generations model that describes Japan's pension system in detail. They use the model to project the path of government debt. Braun and Joines (2015) also develop a large-scale overlapping generations model that carefully describes the Japanese pension and health care systems. They analyze the future path of the debt-to-GDP ratio under several scenarios concerning fiscal adjustments. Excluding idiosyncratic earnings risk, however, the paper does not examine the insurance effect of government debt. Kitao (2015) constructs a rich life-cycle model that features uninsured idiosyncratic wage risk, intensive and extensive margin adjustments of labor supply, and pension, health care, and long-term care systems. The paper analyzes how the demographic transition in Japan affects the government budget, fixing the debt-to-

[7] Hansen and İmrohoroğlu (2016) fully analyze the transition to a long-run level of debt, whereas following Aiyagari and McGrattan (1998) and Flodén (2001), we focus on a stationary equilibrium where the debt-to-GDP ratio is constant.

GDP ratio. Both Braun and Joines (2015) and Kitao (2015) compare the welfare effects on different cohorts, but they do not analyze the redistribution consequences within cohorts.

2 Model

Our analysis is based on an Aiyagari (1994)-style heterogeneous-agent, incomplete-market model. Households face idiosyncratic wage risk and a borrowing constraint. Labor supply is endogenous. Using the same framework, Aiyagari and McGrattan (1998) and Flodén (2001) analyze the welfare implications of government debt for the United States. We extend their models by incorporating consumption taxes, which generate more than 20% of the total tax revenue in Japan.[8]

2.1 Firms

A representative firm rents capital and labor from households and produces the single good. The production technology is represented by

$$Y = K^{\theta}(zN)^{1-\theta}, \tag{1}$$

where Y is output, K is capital stock, N is labor input, $\theta \in (0,1)$ is the capital share, and z is labor-augmenting productivity. Productivity grows at a constant rate of g or $z' = (1+g)z$, where a variable with a prime indicates its next-period value hereinafter.

Given the return on capital r and the wage rate w, the firm maximizes its profit. The first-order conditions for profit maximization are

$$r = \theta z^{1-\theta} K^{\theta-1} N^{1-\theta} - \delta \tag{2}$$

[8] The data is for the fiscal year 2015 and it is obtained from the Ministry of Internal Affairs and Communications. The consumption tax rate rose from 5% to 8% in April 2014. The share of the revenue from consumption taxes in the total tax revenue is likely to increase further in the near future because the consumption tax rate is scheduled to rise to 10% in October 2019.

and

$$w = (1-\theta)z^{1-\theta}K^{\theta}N^{-\theta}, \tag{3}$$

where $\delta \in (0,1)$ is the capital depreciation rate. In the analysis below, we focus on an equilibrium where output grows at a rate of g. Hence, we rewrite (2) and (3) as

$$r = \frac{\theta}{\tilde{K}} - \delta \tag{4}$$

and

$$\tilde{w} = \frac{(1-\theta)}{N}, \tag{5}$$

where hereinafter a variable with a tilde means its ratio with respect to output (e.g., $\tilde{K} = K/Y$).

2.2 Households

There is a continuum of households of measure one. Households are endowed with one unit of time each period. As in Flodén (2001), the momentary utility function is

$$\begin{aligned} u(c,h) &= \frac{1}{1-\mu}\left\{c^{1-\mu}\exp[-(1-\mu)\kappa h^{1+\frac{1}{\varphi}}]\right\} \text{ if } \mu \neq 1, \mu > 0 \\ &= \ln c - \kappa h^{1+\frac{1}{\varphi}} \text{ if } \mu = 1, \end{aligned} \tag{6}$$

where c is consumption, h is hours worked, μ is the coefficient of relative risk aversion, $\kappa > 0$ governs the disutility of labor, and $\varphi > 0$ is the Frisch labor supply elasticity.

Households differ in their labor productivity. The specification of idiosyncratic productivity is the same as that in Flodén (2001) and Flodén and Linde (2001). Idiosyncratic productivity consists of two components. First, households differ in the permanent level of productivity x. This permanent component is drawn once at the beginning of period zero and it is fixed throughout. The permanent component takes a finite number of values,

$x \in \{x_1, ..., x_{N_x}\}$, which are obtained by discretizing $\ln x \sim N(0, \sigma_x^2)$. Second, households differ in the transitory level of productivity e.[9] This transitory component is a finite-state Markov chain $e \in \{e_1, ..., e_{N_e}\}$. The Markov chain is obtained by approximating an AR(1) process, $\ln e' = \rho \ln e + \varepsilon'$, where ε' is independently distributed as $N(0, \sigma_\varepsilon^2)$, with Tauchen (1986)'s method.[10] A household's labor earnings is $wxeh$.

Asset markets are incomplete and only two risk-free assets, physical capital and government bonds, exist in the economy. These two assets are perfect substitutes for households and they earn the same return r. Households use savings to self-insure against idiosyncratic wage risk. At the beginning of each period, households are distinguished by their total asset holding a, transitory productivity e, and permanent productivity x. There is a borrowing constraint: $a' \geq 0$.

In each period, households choose consumption c, savings a', and labor hours h. The optimization problem for households is written as

$$V(\tilde{a}, e, x) = \max_{\{\tilde{c}, \tilde{a}' h\}} \left\{ u(\tilde{c}, h) + \beta(1+g)^{1-\mu} E[V(\tilde{a}', e', x)|e] \right\} \tag{7}$$

$$\text{subject to } (1+\tau_c)\tilde{c} + (1+g)\tilde{a}' \leq [1 + (1-\tau)r]\tilde{a} + (1-\tau)\tilde{w}xeh + \tilde{T}$$

$$\tilde{c} \geq 0, h \in [0, 1], \tilde{a}' \geq 0,$$

where $V(\tilde{a}, e, x)$ is the value function for households, $\beta \in (0, 1)$ is the discount factor, and E denotes conditional expectation. The second line is the budget constraint: τ_c is the consumption tax rate, τ is the common tax rate on capital and labor income, and T is lump-sum transfers from the government to households (i.e., $\tilde{T} = T/Y$).

[9] Aiyagari and McGrattan (1998) only consider the transitory component of idiosyncratic productivity.

[10] The transitory productivity process is approximated with nine states ($N_e = 9$), whereas the permanent component is approximated with five states ($N_x = 5$).

2.3 Government

The government finances its consumption and lump-sum transfers to households through debt and taxes. The budget constraint is

$$G + T + rB = B^{'} - B + \tau[wN + r(K + B)] + \tau_c C, \tag{8}$$

where G is government consumption, B is government debt, and C is aggregate private consumption. Dividing (8) by Y, we obtain the following:

$$\tilde{G} + \tilde{T} + r\tilde{B} = (1 + g)\tilde{B}^{'} - \tilde{B} + \tau[\tilde{w}N + r(\tilde{K} + \tilde{B})] + \tau_c \tilde{C}. \tag{9}$$

In the baseline exercise, we vary $\tilde{B}$ (note that $\tilde{B} = \tilde{B}^{'}$ in a stationary equilibrium) and change τ so that (9) holds, while fixing $(\tilde{G}, \tilde{T}, \tau_c)$ at their baseline value and changing $(r, \tilde{w}, N, \tilde{K}, \tilde{C})$ endogenously. This is the same exercise done by Aiyagari and McGrattan (1998) and Flodén (2001).

2.4 Recursive Equilibrium

We focus on a stationary equilibrium in which output grows at a constant rate of g.[11] Let $\Gamma(\tilde{a}, e, x)$ be the stationary distribution of households over wealth, transitory productivity, and permanent productivity. Given the government policy $(\tilde{G}, \tilde{T}, \tilde{B}, \tau_c, \tau)$, a stationary competitive equilibrium $(\tilde{w}, r, V, \tilde{c}, \tilde{a}^{'}, h, \tilde{K}, N, \tilde{C}, \Gamma)$ satisfies the following conditions:

1. Households' optimization:

   $V(\tilde{a}, e, x)$ satisfies (7), while $\tilde{c}(\tilde{a}, e, x), \tilde{a}^{'}(\tilde{a}, e, x)$, and $h(\tilde{a}, e, x)$ are the associated policy functions.

2. Firms' optimization:

[11] The solution method for solving a stationary equilibrium is similar to the method used by Aiyagari and McGrattan (1998) and Flodén (2001). In particular, we solve for households' optimization problem and the stationary distribution of households in a way similar to that in Takahashi (2015).

The representative firm chooses $\tilde{K}$ and N to satisfy (4) and (5).

3. Labor market clearing:

$$N = \int xh(\tilde{a}, e, x)d\Gamma$$

4. Asset market clearing:

$$\tilde{K}' + \tilde{B}' = \int \tilde{a}'(\tilde{a}, e, x)d\Gamma$$

5. Goods market clearing:

$$\tilde{C} + \tilde{G} + (g + \delta)\tilde{K} = 1, \text{ with } \tilde{C} = \int \tilde{c}(\tilde{a}, e, x)d\Gamma.$$

6. Government budget constraint:

The government budget constraint holds as in (9).

7. Stationary household distribution:

The saving decision of households and the transition probabilities for idiosyncratic productivity generate the stationary distribution of households $\Gamma(\tilde{a}, e, x)$. Specifically, for all $D \subseteq [0, \bar{a}]$,

$$\Gamma(D, e', x) = \int_{\{(\tilde{a},e,x)|\tilde{a}'(\tilde{a},e,x)\in D\}} \pi_e(e'|e)d\Gamma,$$

where $\bar{a}$ is the upper bound for asset holding and $\pi_e(e'|e)$ is the transition probability from e to e'.

3 Parameter Values and Inequality

This section determines parameter values for the above model. We then compare wealth and income inequality in the model with those in Japan.

Symbol	Meaning	Value
δ	Capital depreciation rate	0.06
θ	Capital share	0.37
μ	Relative risk aversion	1.5
φ	Frisch labor supply elasticity	0.5
g	Growth rate	0.009
$\tilde{G}$	Government consumption-to-output ratio	0.180
$\tilde{T}$	Transfer-to-output ratio	0.141
$\tilde{B}$	Debt-to-output ratio	0.78
τ_c	Consumption tax rate	0.05
ρ	Persistence of idiosyncratic productivity	0.90
σ_ε	Volatility of idiosyncratic productivity shocks	0.1754
σ_x	Dispersion of permanent productivity	0.2530
κ	Disutility of labor	22.05
β	Discount factor	0.9929

Table 1: Parameter values.

3.1 Baseline Parameter Values

We calibrate the model to the Japanese economy for the 1995–2013 period. Table 1 lists the parameter values. One period corresponds to one year. The capital depreciation rate δ is 0.06 and the capital share θ is 0.37. The coefficient of relative risk aversion μ is 1.5, whereas the Frisch labor supply elasticity φ is 0.5, both of which are the same as in Flodén (2001). These four parameter values are also similar to those used in existing models for the Japanese economy, such as Hayashi and Prescott (2002), Sugo and Ueda (2008), and Nutahara (2015).

We use the SNA93 and compute the average of the growth rate and fiscal policy variables for the 1995–2013 period. The growth rate of real GDP g is 0.9%. The share of government consumption in GDP $\tilde{G}$ is 0.180. The share of government transfers in GDP $\tilde{T}$ is 0.141.[12] The data on net government debt is obtained from the IMF World Economic Outlook October 2014 and the debt-to-GDP ratio $\tilde{B}$ is 0.78. The consumption tax rate τ_c is set to 5%.[13]

Next, we determine parameters concerning idiosyncratic productivity $(\rho, \sigma_\varepsilon, \sigma_x)$. These

[12] We use the data on 'Social benefits other than social transfers in kind.'

[13] The consumption tax rate was 5% for almost the entire 1995–2013 period. The consumption tax rate was 3% until March 1997. The tax rate was 5% between April 1997 and March 2014.

parameters are usually estimated using panel data on individual wages. Since such panel data is limited in Japan, we choose those parameter values by targeting the cross-sectional dispersion of wages.[14] In particular, we use the results in Lise, Sudo, Suzuki, Yamada, and Yamada (2014). First, the variance of log of *residual* wages is 0.162.[15] We assume that this residual wage dispersion is generated by the transitory component of idiosyncratic productivity: $\sigma_\varepsilon^2/(1-\rho^2) = 0.162$. The lack of panel data implies a lack of separate estimates for ρ and σ_ε. Existing analyses using other countries' data find that the transitory component of idiosyncratic productivity is quite persistent. For example, Flodén and Linde (2001) find that $\rho = 0.914$ for the United States and $\rho = 0.814$ for Sweden, whereas Alonso-Ortiz and Rogerson (2010) use $\rho = 0.94$ for the United States based on the estimates by various studies. Hence, we set $\rho = 0.90$.[16] The corresponding σ_ε is 0.1754. Second, the variance of log of *raw* wages is 0.226. We choose σ_x so that the overall wage dispersions match between the model and the Japanese economies: $\sigma_\varepsilon^2/(1-\rho^2) + \sigma_x^2 = 0.226$ or $\sigma_x = 0.2530$.

These parameter values for Japan lie between the estimates for the United States and Sweden by Flodén and Linde (2001). The standard deviation of the permanent component σ_x is 0.2530 in Japan, 0.3428 in the United States, and 0.2161 in Sweden. The wage dispersion generated by the transitory component $\sigma_\varepsilon^2/(1-\rho^2)$ is 0.162 in Japan, 0.2588 in the United States, and 0.0966 in Sweden. Hence, both the permanent and transitory components in Japan are between those in the United States and Sweden.

Lastly, we choose the discount factor β and the disutility for labor κ so that the after-tax

[14] We thank Michio Suzuki for the suggestion. The Japanese Panel Survey of Consumers, which is available since the 1990s, focus on young women.

[15] Lise, Sudo, Suzuki, Yamada, and Yamada (2014) analyze the Basic Survey on Wage Structure until 2008. We use their result for the 1995–2008 period. Residual wages are computed by controlling for demographic variables, such as age and education. We use the result for men. The overall wage dispersion is similar between men and women, but the residual wage dispersion is smaller for women than for men. The result of Section 5.1 suggests that the optimal quantity of government debt decreases as idiosyncratic risk is reduced. Hence, calibrating idiosyncratic wage risk to the residual wage dispersion for women would decrease the optimal quantity of debt for Japan.

[16] We conduct sensitivity analyses with respect to ρ, while adjusting σ_ε in order to maintain $\sigma_\varepsilon^2/(1-\rho^2) = 0.162$. We consider $\rho = 0.80$ and 0.94. The optimal quantity of government debt is –110% and –40% of GDP, respectively. Hence, the result that the current debt-to-GDP ratio of 1.3 exceeds the optimal level does not change for the plausible values for ρ.

return on savings $(1-\tau)r$ is 1.81% and the average hours worked $H \equiv \int h(\tilde{a}, e, x) d\Gamma$ is 0.207. The target for the after-tax return is computed in a way similar to Braun and Joines (2015). We obtain the return on existing government bonds from the Ministry of Finance. The after-tax return is computed by applying 20% taxes to interests on government bonds. We then compute the real return using the CPI inflation rate obtained from the Statistics Bureau, the Ministry of Internal Affairs and Communications.[17] The return targeted here is a bit lower than the return used by Braun and Joines (2015), who calibrate their model to the 2000–2006 period, because the return on government bonds decreased substantially in more recent years. The target for the average hours worked is computed in a way similar to Nutahara (2015). It is computed by the number of employees times hours worked per employee divided by the working age population and then divided by 5,760. The data on the number of employees and hours worked per employee are taken from the SNA93. The data on the working age population is computed by the OECD and it is taken from the FRED database at the Federal Reserve Bank of St. Louis. The results are $\beta = 0.9929$ and $\kappa = 22.05$.

3.2 Wealth and Earnings Inequality

Since the model endogenously generates distributions of wealth and earnings, it is important to check whether the model's inequality is close to the inequality in Japan. The exercise is interesting beyond the present paper because it serves as a test examining how well an Aiyagari (1994)-type model, when the process for idiosyncratic wage risk is calibrated to micro-level evidence, can account for wealth and labor earnings inequality in Japan.

Both inequality in wealth and labor earnings are large in Japan. We use the results documented by Lise, Sudo, Suzuki, Yamada, and Yamada (2014).[18] The Gini coefficient for net financial wealth is 0.62, whereas the top 10% share in net financial wealth is 0.43. As

[17] Using the CPI inflation rate excluding fresh food does not change the result.

[18] The results reported here are their results based on the National Survey of Family Income and Expenditures for the years of 1994, 1999, and 2004.

for labor earnings, the Gini coefficient is 0.26, whereas the top 10% share is 0.21.

The calibrated model also generates substantial inequality in wealth and labor earnings. The Gini coefficient for wealth is 0.50, while the top 10% share is 0.30. The Gini coefficient for labor earnings is 0.36, whereas the top 10% share is 0.26. While the model's inequality is similar to that in the Japanese data, comparing the two needs a care. First, the Japanese result is based only on financial wealth. Second, the Japanese inequality is for equivalized wealth and labor earnings based on the OECD weights and for households whose heads are between 25 and 59 years old.

Another interesting measure for inequality is the fraction of households below the poverty line. As consistent with the OECD's and Japanese government's definition, we define the poverty line as the 50% of the median earnings, where earnings consist of the after-tax labor income, the after-tax capital income, and public transfers.[19] For our baseline calibration, 3.7% of households are below the poverty line. In Japan, the share of individuals below the poverty line is 9.7% for the 1999–2014 period.[20] It is not surprising that the model's share is lower than the share in Japan. The Japanese share is based on equivalized earnings and it counts individuals including retirees and children instead of households.

Overall, the results of this subsection suggest that our model accounts well for wealth and income inequality in Japan. However, some gaps remain. An important future task would be to construct a model that more closely matches wealth and earnings inequality in the actual data.

4 Results

We compare welfare under different debt-to-GDP ratios using the utilitarian welfare measure that weights all households equally, as in Aiyagari and McGrattan (1998) and Flodén (2001).

[19] The amount of transfers is 27.3% of the median earnings in the model.

[20] The result is based on the National Survey of Family Income and Expenditures for the years of 1999, 2004, 2009, and 2014 and it is taken from the Statistics Bureau, the Ministry of Internal Affairs and Communications.

The welfare measure also indicates the expected utility of a household before drawing their initial state from the stationary distribution. When $\mu \neq 1$, the welfare measure is computed by

$$U = Y_0^{1-\mu} \int V(\tilde{a}, e, x) d\Gamma, \tag{10}$$

where Y_0 is the output at the initial date whose productivity is normalized to 1.0 (i.e., $Y_0 = \tilde{K}^{\theta} N^{1-\theta}$).[21] This part is affected by the level of government debt through its effect on the capital-to-output ratio and aggregate labor input. The rest is the utilitarian welfare for the stationary equilibrium computed above and it of course depends on the level of debt.

We quantify the welfare effect of government debt in consumption units. The welfare gain of having a certain level of debt $\tilde{B}$, instead of the benchmark level of debt $\tilde{B}^0$, shows how much consumption at $\tilde{B}^0$ must increase at all states and dates in order to achieve the utilitarian welfare under $\tilde{B}$. We use two benchmark levels of debt. One is the optimal level or $\tilde{B}^0 = \tilde{B}^*$. The optimal level of debt varies with specifications, and it is not necessarily the most convenient benchmark to compare the welfare costs of government debt under different specifications. For this reason, we also use the level of government debt for the 1995–2013 period as our benchmark ($\tilde{B}^0 = 0.78$).

When $\mu \neq 1$, the welfare gain of holding a certain level of government debt $\tilde{B}$ rather than the benchmark level $\tilde{B}^0$, ω_U, is computed by

$$\omega_U = \left(\frac{U}{U^0} \right)^{\frac{1}{1-\mu}} - 1, \tag{11}$$

where U and U^0 are the utilitarian welfare at $\tilde{B}$ and $\tilde{B}^0$, respectively.

As discussed by Aiyagari and McGrattan (1998) and Flodén (2001), a change in government debt affects welfare through several channels. Toward better understandings for these

[21] See Flodén (2001) for the derivation of (10), (11), and (12). In the case of $\mu = 1$, $U = \frac{\ln Y_0}{1-\beta} + \frac{\beta \ln(1+g)}{(1-\beta)^2} + \int V(\tilde{a}, e, x) d\Gamma$, whereas $\omega_U = \exp[(1-\beta)(U - U^0)]$.

different effects, we employ the method of Flodén (2001) and decompose the utilitarian welfare gain ω_U into gains related to changes in the average levels of consumption and leisure, changes in uncertainty that households face, and changes in inequality in consumption and leisure across households:

$$\omega_U = (1 + \omega_{lev})(1 + \omega_{unc})(1 + \omega_{ine}) - 1. \tag{12}$$

The level gain ω_{lev} is the gain arising from a change in aggregate private consumption, when a change in aggregate leisure is compensated. The further decomposition into the uncertainty gain ω_{unc} and the inequality gain ω_{ine} is done as follows. For each household state, we compute certainty-equivalent consumption and leisure.[22] The uncertainty gain ω_{unc} is a gain in the uncertainty cost. The uncertainty cost for each level of debt is computed by the welfare difference between having the average consumption/leisure and having the average certainty-equivalent consumption/leisure. Hence, the uncertainty cost is the welfare loss arising from uncertainty faced by households. In contrast, the inequality gain ω_{ine} is a change in the inequality cost. The inequality cost for a certain level of debt is computed by the difference between the welfare of having the average certainty-equivalent consumption/leisure and the average welfare of having certainty-equivalent consumption/leisure. Therefore, the inequality cost is the welfare loss arising from the dispersions in consumption and leisure across households.

The results are summarized in Table 2. For the baseline specification, the optimum quantity of government debt is –50% of GDP for Japan. Hence, the current level of 130% of GDP is too high in terms of welfare. Relative to the optimal level, the current level of debt generates a 0.19% cost in consumption units. The upper-left panel of Figure 1 shows the total welfare cost for the debt-to-GDP ratios between –2.0 and 2.0. For this figure, we set the benchmark level of debt to the optimal level ($\tilde{B}^0 = \tilde{B}^* = -0.5$). The overall welfare

[22] The pair of certainty-equivalent consumption and leisure is not unique. Following Flodén (2001), the result reported here is based on the assumption that certainty-equivalent leisure is the current leisure choice.

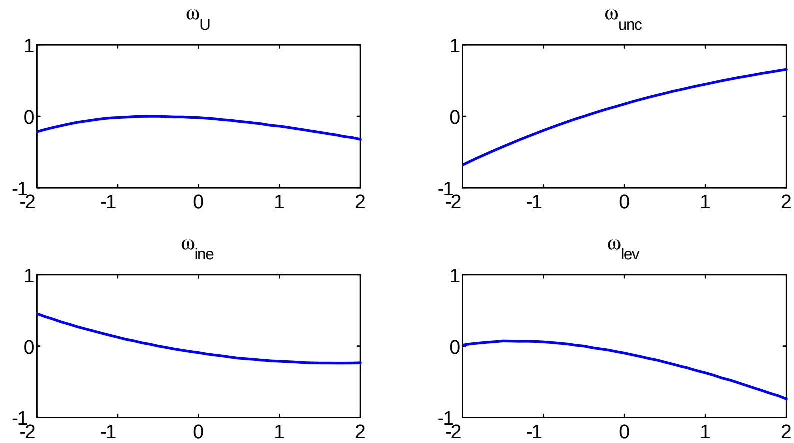

Figure 1: Welfare: Baseline. Horizontal axis: Debt-to-GDP ratio. The welfare measures are multiplied by 100 and expressed as a percent of consumption.

cost is relatively insensitive to the debt-to-GDP ratio.

Next, we look at how government debt affects the level, uncertainty and inequality gains. As shown in the rest of the panels of Figure 1, a change in government debt affects the three gains differently. The inequality gain decreases monotonically with the debt-to-GDP ratio, while the uncertainty gain increases. The level gain first increases and then decreases. At the current debt-to-GDP ratio of 1.3, the level cost relative to the optimal level is 0.48%, while the inequality cost is 0.23%. The uncertainty gain is 0.52%. Hence, the uncertainty gain largely offsets the level and inequality costs, leading to the mild total welfare cost.

In order to understand these welfare results, we examine how aggregate variables change as the debt-to-GDP ratio changes (Figure 2). Increasing government debt lowers the capital-to-GDP ratio, whereas the after-tax return on savings rises. The total asset in the economy, the sum of capital and debt relative to output, increases. Aggregate hours worked and efficiency-weighted labor both decrease, while the before-tax wage rate rises (not shown in the figure). Further, when government debt is low, an increase in debt leads to an increase in the tax base, output minus capital depreciation for the factor income taxes, and this outweighs an increase in the interest payments (or outweighs the decrease in the interest

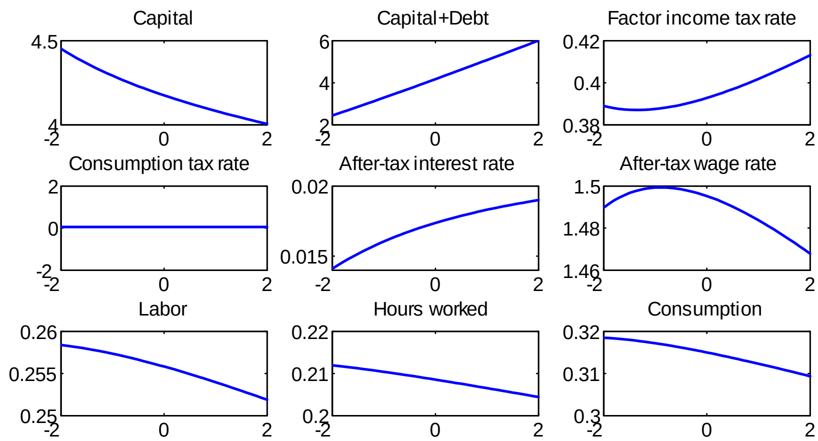

Figure 2: Aggregate variables: Baseline. Horizontal axis: Debt-to-GDP ratio. Capital, Capital+Debt, and After-tax wage rate are their ratio relative to output. Consumption is the level at the initial period whose productivity is normalized to 1.0.

	Baseline	US Transfer	US Risk	US Return	Ctax	SOE
$\tilde{B}^*$	–0.5	2.0	0.5	–2.0	–0.6	–2.0
$\tilde{B}^0 = \tilde{B}^*$						
ω_U	–0.19	–0.05	–0.05	–2.37	–0.15	–14.85
ω_{unc}	0.52	–0.18	0.27	1.58	0.51	0.28
ω_{ine}	–0.23	0.00	–0.09	–1.31	–0.28	0.21
ω_{lev}	–0.48	0.13	–0.23	–2.62	–0.37	–15.27
$\tilde{B}^0 = 0.78$						
ω_U	–0.08	0.06	–0.04	–0.48	–0.06	–2.74
ω_{unc}	0.12	0.16	0.17	0.17	0.11	0.04
ω_{ine}	–0.03	–0.04	–0.04	–0.12	–0.04	0.05
ω_{lev}	–0.17	–0.06	–0.16	–0.52	–0.12	–2.83

Table 2: Summary of the welfare results. The welfare measures are for the debt-to-GDP ratio of 1.3. The measures are multiplied by 100 and expressed as a percent of consumption.

receipts). Thus, the factor income tax rate decreases slightly before rising. Hence, the after-tax wage rate initially rises before falling. Aggregate private consumption decreases.

These movements of aggregate variables explain the welfare effects of government debt as follows. Increasing debt reduces uncertainty faced by households. Households hold larger wealth and hence they are better insured against idiosyncratic risk. In addition, the after-tax wage rate eventually decreases, which also reduces earnings risk. These increase the uncertainty gain. In contrast, the combination of the higher return on savings and the lower wage rate is bad for small-wealth households and good for large-wealth households. Hence, an increase in debt widens inequality in consumption and leisure across households, raising the inequality cost. The level gain is not monotone. When government debt is sufficiently low, capital overaccumulation is severe. Low-productivity households also work inefficiently long because they hold low wealth under the low return on savings. An increase in debt mitigates these problems. The distortionary taxes also initially decrease. Hence, the level gain initially increases. As debt increases further, the positive effects become smaller and the distortionary taxes start to increase. As a result, the level cost starts to increase.

5 Sensitivity Analyses

This section compares our result for Japan with the result for the United States by Flodén (2001). Various sensitivity analyses are also conducted.[23]

5.1 Comparison with Flodén (2001)

Flodén (2001) finds that the optimal level of government debt is 150% of GDP for the United States. We analyze what accounts for the difference between Flodén (2001)'s result for the

[23] In addition to the sensitivity analyses presented below, we also examine a version of our model with indivisible labor. The optimum amount of government debt is similar to that in our baseline case with divisible labor and the optimal level of debt is –80% of GDP. In Nakajima and Takahashi (2016), we analyze the insurance and redistribution effects of *a consumption tax and public transfer program* in a model similar to the present one. In that paper, we find that the indivisibility of labor substantially weakens the effectiveness of the transfer program as insurance and redistribution against idiosyncratic wage risk.

United States and our result for Japan. Three factors are considered: public transfers, idiosyncratic wage risk, and the return on savings. These are the main differences in calibration between the United States and Japan. Further, Aiyagari and McGrattan (1998) and Flodén (2001) find that these are important determinants for the optimal quantity of government debt.

Following Aiyagari and McGrattan (1998), Flodén (2001) assumes that the level of transfers is 8.2% of GDP, which is the post-war average in the U.S. While the transfer-to-GDP ratio in Japan is relatively low in the 1990s, the U.S. level is even lower than the average for the 1995–1999 period in Japan, which is 0.116. When we assume the U.S. level of transfers, the optimal amount of government debt for Japan becomes much larger than the baseline specification and it is the highest level of 200% of GDP.[24] In the present model, public transfers guarantee the same amount of income for all states and hence they work as insurance against idiosyncratic risk. When the amount of transfers is smaller, the insurance benefit of government debt is more important and the optimal amount of debt increases. Increasing government debt also more substantially improves capital overaccumulation and the allocation of labor hours across households under the low level of transfers. Hence, the level cost decreases from 0.17% in the baseline specification to 0.06% with the U.S. transfers.

Second, we set idiosyncratic wage risk to the U.S. level used by Flodén (2001): $\rho = 0.90$ and $\sigma = 0.21$. In this case, the optimal level of government debt becomes larger than our baseline case and it is 50% of GDP. Since households face greater wage risk and the amount of transfers is fixed, the insurance role of government debt increases. Hence, the optimal quantity of debt becomes larger.

Lastly, we set the before-tax return on savings at the 1995–2013 average debt ($\tilde{B} = 0.78$) to the return assumed by Flodén (2001), which is 4.50%, by lowering the discount factor ($\beta = 0.9826$). This substantially reduces the optimal level of government debt and the optimal debt becomes the lowest level of –200% of GDP.[25] The main reason is the level cost.

[24] The amount of transfers is 16.6% of the median earnings in this case.

[25] The Japanese tax rate is higher than the U.S. tax rate. Hence, if we target the U.S. after-tax return on

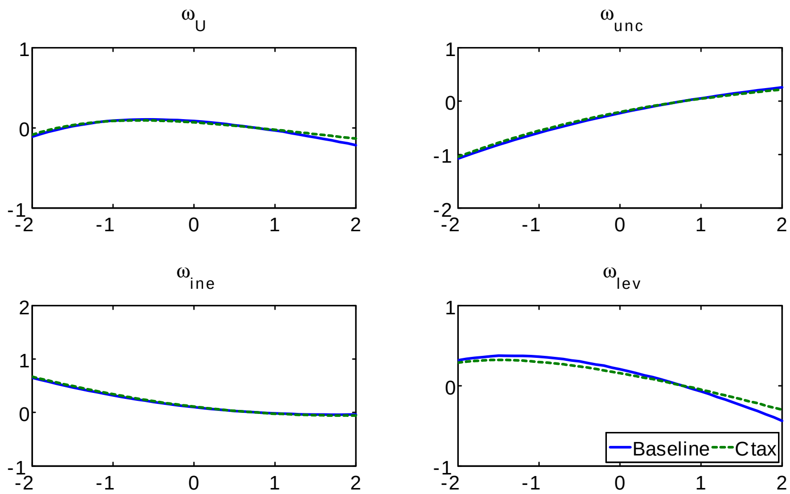

Figure 3: Welfare: Factor income taxes (Baseline) versus Consumption taxes (Ctax). Horizontal axis: Debt-to-GDP ratio. The welfare measures are multiplied by 100 and expressed as a percent of consumption.

The higher return on savings implies that interest payments on government debt increase more quickly with government debt. Hence, an increase in government debt raises the factor income tax rate more significantly. As a result, the level cost increases and it is optimal to hold a very low level of government debt.

To summarize, the optimal quantity of government debt for Japan is smaller than that for the United States because the insurance role of government debt is less important. Japanese households face less labor earnings risk and they are more insured with a larger amount of public transfers. The lower return on savings reduces the cost of government debt and tends to raise the optimal amount of government debt. However, this effect only partially offsets the other two forces.

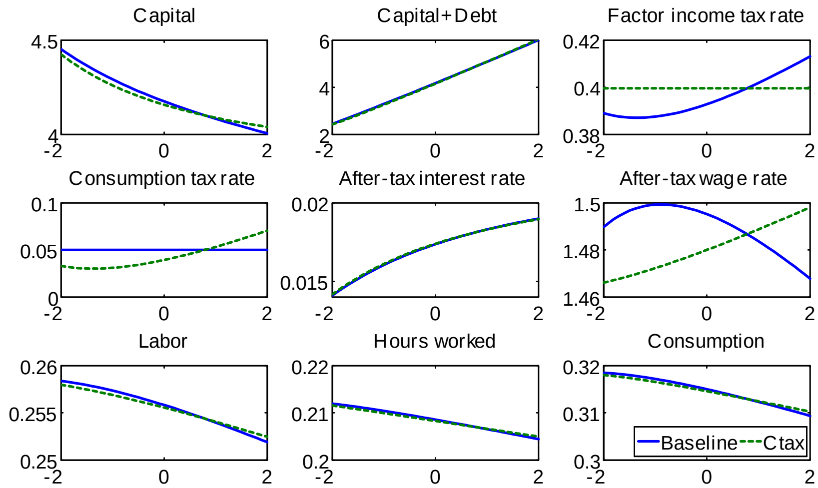

Figure 4: Aggregate variables: Factor income taxes (Baseline) versus Consumption taxes. Horizontal axis: Debt-to-GDP ratio. Capital, Capital+Debt, and After-tax wage rate are their ratio relative to output. Consumption is the level at the initial period whose productivity is normalized to 1.0.

5.2 Adjustments through Consumption Taxes

As in Aiyagari and McGrattan (1998) and Flodén (2001), the analyses so far have varied the debt-to-GDP ratio and adjusted the factor income taxes in order to satisfy the government budget constraint, while fixing other fiscal policy variables to their baseline value. In this subsection, we vary the debt-to-GDP ratio and adjust consumption taxes, while fixing other fiscal variables, including the factor income tax rate, at the baseline value. This exercise would be interesting because in a representative-agent model (Hansen and İmrohoroğlu (2016)) and an overlapping generations model (Kitao (2015)), consumption taxes are found to be less distortionary than labor income taxes to address Japan's fiscal problem.

When consumption taxes are adjusted, the optimal level of government debt is –60% of GDP, as shown in Table 2. Hence, using consumption taxes does not rationalize the current debt-to-GDP ratio of 1.3. Relative to the optimal level, the current debt generates a welfare cost of 0.15% in consumption units.

savings, then we need to assume even a higher before-tax return on savings than 4.50%, which should reduce the optimal amount of government debt further.

For comparison with the baseline specification, it is convenient to set the benchmark level of government debt to the 1995–2013 level because the optimal levels of debt are different between the two specifications. Figure 3 shows the comparison, whereas the bottom section of Table 2 summarizes the results. The overall welfare cost is reduced from 0.08% for the baseline specification where the factor income taxes are adjusted to 0.06% for the case where consumption taxes are adjusted. Consumption taxes are less distortionary than factor income taxes. The level cost decreases from 0.17% under the baseline specification to 0.12% and the reduction in the level cost is the most responsible for the decrease in the total welfare cost. In contrast, the uncertainty and inequality effects are not very much dependent on which taxes are adjusted to satisfy the government budget constraint.

5.3 Small Open Economy

Our analyses so far have assumed that Japan is a closed economy. The assumption is natural as the baseline specification. However, the closed economy setting may overstate the crowding-out effect of government debt and hence might provide a biased estimate for the optimal quantity of debt. We address this concern here by assuming that Japan is a small open economy. We set the world interest rate to 3.02%, which is the before-tax return on savings in our baseline specification. We also fix the capital income tax rate and hence the after-tax return on savings at their baseline levels of 40.0% and 1.81%, respectively.[26] We only adjust labor income taxes to satisfy the government budget constraint.

We find that increasing the openness of the economy substantially reduces the optimal level of government debt. In particular, the optimal level of debt is –200% of GDP in the small open economy. The welfare loss of maintaining the current level of debt rather than the optimal level is significant and it is 14.9% in consumption units.

Figure 5 compares the welfare implications between the closed and small open economies.

[26]The interest rate under incomplete markets cannot be higher than the interest rate under complete markets. Otherwise, savings diverge. For our baseline calibration, the after-tax return under complete markets is 2.08% ($(1+g)^{\mu}/\beta - 1 = 0.0208$). Hence, we cannot set the after-tax return to the U.S. level assumed by Flodén (2001) (2.82%).

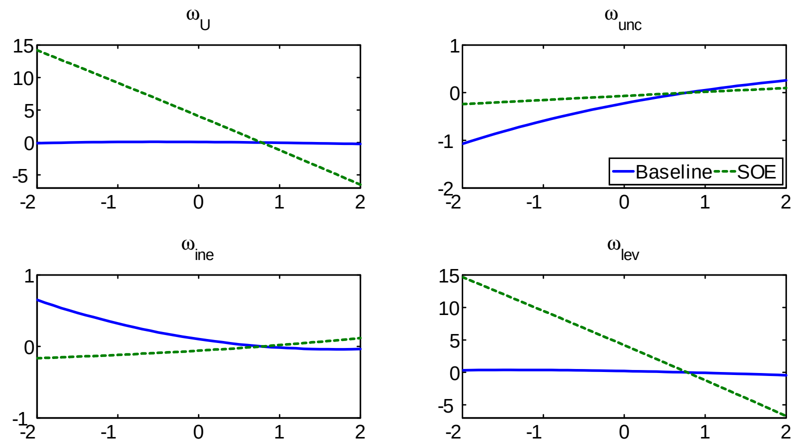

Figure 5: Welfare: Closed economy (Baseline) versus Small open economy (SOE). Horizontal axis: Debt-to-GDP ratio. The welfare measures are multiplied by 100 and expressed as a percent of consumption.

The current debt generates a much larger welfare cost when the economy is small open than it is closed. The level effect is dominant. As government debt increases, the amount of assets held by the foreign sector increases. A part of interest payments goes to the foreign sector and cannot be taxed. Hence, the labor income tax rate must increase substantially in order to meet the government budget constraint. Moreover, since a fall in the after-tax wage rate decreases earnings risk, households reduce precautionary savings. This further increases the amount of assets held by the foreign sector and leads to increases in the distortionary taxes. As a result, the level cost increases substantially.

5.4 Optimal Transfers and Debt

Lastly, we jointly analyze government debt and transfers, searching for the optimal combination of the two. The analysis also shows how the welfare implications of government debt depend on the level of transfers. To save computational time, we only consider the transfer-to-GDP ratio of 0.08, 0.10,..., and 0.18.

When both government debt and public transfers are chosen freely, the optimal level of government debt is –120% of GDP. Hence, the current level of 130% of GDP exceeds

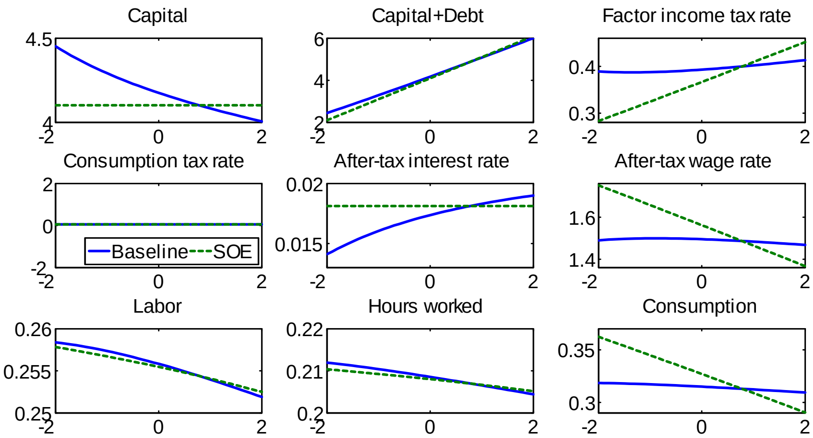

Figure 6: Aggregate variables: Closed economy (Baseline) versus Small open economy (SOE). Horizontal axis: Debt-to-GDP ratio. Capital, Capital+Debt, and After-tax wage rate are their ratio relative to output. Consumption is the level at the initial period whose productivity is normalized to 1.0.

the optimal level. In contrast, the optimal level of public transfers is 16% of GDP. The result is similar to that for the United States. Flodén (2001) finds that for the United States, the optimal level of transfers is 23% of GDP, whereas the optimal level of government debt is –100% of GDP, which is the lowest level that he considers. As shown in Figure 7, for any level of transfers, raising government debt increases the uncertainty gain, while reducing the inequality gain. In contrast, as is consistent with the finding by Flodén (2001), public transfers improve both uncertainty and inequality for any level of government debt. However, increasing transfers raises the level cost.[27] Reducing government debt decreases interest payments (or increases interest receipts) and decreases the distortionary taxes, which reduces the level cost. Hence, it is optimal to have a relatively high level of transfers and a low level of government debt. A further reduction in debt does not improve welfare, though. As debt decreases further, the distortionary taxes must start to increase. The reason is that the capital-to-output ratio increases and as a result the tax base decreases. Hence, a further

[27] Flodén (2001) shows that when the levels of public transfers and government debt are both low, increasing transfers improves the level effect. The reason is that in such a situation, an increase in transfers mitigates overaccumulation of capital and improves the allocation of labor hours across households.

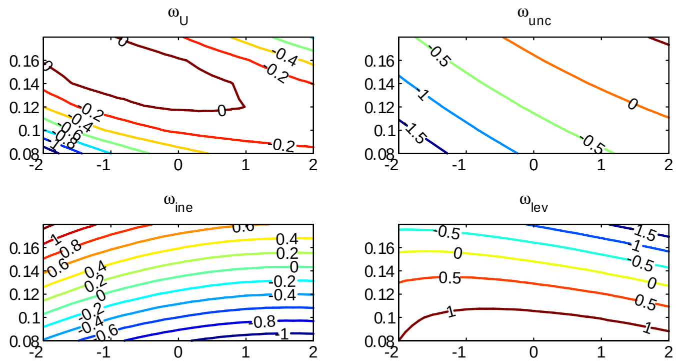

Figure 7: Welfare: Debt and Transfers. Horizontal axis: Debt-to-GDP ratio. Vertical axis: Transfer-to-GDP ratio. The welfare measures are multiplied by 100 and expressed as a percent of consumption.

reduction in government debt does not increase the level gain significantly and an increase in the uncertainty cost dominates. Hence, the total welfare deteriorates.

6 Conclusion

Japan has accumulated a large amount of government debt over the last two decades. In 2013, Japan's net government debt is 130% of GDP and the debt-to-GDP ratio is the highest among major countries. We have analyzed the welfare implications of Japan's large government debt. Our framework is a neoclassical growth model with uninsured idiosyncratic wage risk and the model includes various effects of government debt. Calibrating the model to the Japanese economy, we have found that the current level of debt exceeds the optimal level. Maintaining the current debt rather than the optimal quantity generates a non-negligible welfare cost.

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