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When is the Laffer curve for consumption tax hump-shaped?*

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Abstract

This paper characterizes the shape of the Laffer curve for consumption tax. It is shown that the Laffer curve for consumption tax can be hump-shaped if the utility function is additively separable in consumption and labor supply. Conversely, it cannot be hump-shaped if the utility function is non-separable as reported by previous researchers. It is also shown that the difference in the utility functions has quantitatively significant effects on the peak tax rates of the Laffer curves for labor and capital income taxes.

Keywords: Laffer curve; tax revenue; consumption tax

JEL classification: E62; H20; H30

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1 Introduction

The main objective of this paper is to investigate the Laffer curve for consumption tax. As in Waninski (1978), Arthur B. Laffer's conjecture is that the Laffer curve is hump-shaped. This is because an increase in a tax rate would have two opposing effects on the tax revenue. In the first effect, the tax revenue would increase as a direct consequence of raising the tax rate. In the second effect, the tax revenue reduces because a high tax rate discourages economic activities of labor supply, capital accumulation, consumption, and output. Contrary to Laffer's conjecture, Trabandt and Uhlig (2011, 2013) recently show that the Laffer curve for consumption tax is monotonically increasing, whereas the Laffer curves for labor and capital income taxes are hump-shaped. It is also found that the monotonically increasing Laffer curve for consumption tax is robust to some variations of the models. However, most of their results are based on numerical analyses, and it is not clear whether the Laffer curve for consumption tax is generally monotonically increasing or not.

This paper characterizes the shape of the Laffer curve for consumption tax both in a simple static general equilibrium model and a standard neoclassical growth model. In a simple static model, output is produced by linear technology of labor, no capital stock, no government consumption, and the tax revenue is used only for the lump-sum transfer. In a neoclassical growth model, capital stock, investment expenditure, government debt, and net imports are introduced to the dynamic setting à la Trabandt and Uhlig (2011). Both of the consumption tax revenue curve and the total tax revenue curve, including labor and capital tax revenues, are considered as the Laffer curves.

It is shown that the Laffer curve for consumption tax can be hump-shaped if the utility function is additively separable in consumption and labor supply, whereas this is not so if the utility function is non-separable. The key parameters for the hump-shaped Laffer curve are the intertemporal elasticity of substitution (hereafter IES), that is, the inverse of the relative risk aversion in our models, and the labor supply elasticity in

the utility function. For the hump-shaped Laffer curve, IES and labor supply elasticity should be sufficiently high. An increase in the consumption tax rate has a negative effect on the tax revenue in that it reduces aggregate labor supply and aggregate consumption. Thus, the parameter of labor supply elasticity in the utility function is important. The aggregate labor supply and aggregate consumption elasticities can be greater than one under sufficiently high values of IES and labor supply elasticity parameters in the case of an additively separable utility function, whereas this cannot be the case for a non-separable utility. It is also shown that the difference in the functional form of the utility has quantitatively significant effects on the peak tax rates of the Laffer curves for labor and capital income taxes. The quantitative impacts of the difference in the utility function on the peak tax rates of the Laffer curves for labor and capital income taxes are about 10% when the Laffer curve for consumption tax is not hump-shaped. They exceed 30% when the Laffer curve is hump-shaped.

Both additively separable and non-separable utility functions are often employed in macroeconomics. For examples, Gali (2008) employs additively separable utility, whereas King and Rebelo (1999) and Trabandt and Uhlig (2011) employ non-separable utility. It is rare to focus on the effect of the difference in utility functions. However, this paper illustrates an example where the difference in the utility functions has a significant effect on the Laffer curves.

The Laffer curve has been investigated by various researchers. Ireland (1994) find that the hump-shaped Laffer curve for capital income tax using an AK model. Schmitt-Grohè and Uribe (1997) derivate the hump-shaped Laffer curve for labor income tax in a neoclassical growth model. Trabandt and Uhlig (2011, 2013) estimate the Laffer curves for consumption, labor, and capital taxes for the U.S. and EU14 using a neoclassical growth model. Nutahara (2015) applies the model of Trabandt and Uhlig (2011) to the Japanese economy. Holter, Krueger, and Stepanchuk (2014) focus on the effect of households' heterogeneity and progressive tax scheme on the peak tax rate of the Laffer

curve for labor income tax using a overlapping generations model.

This paper is closely related to the papers by Trabandt and Uhlig (2011, 2013) and Nutahara (2015), who estimate the Laffer curve for consumption tax. They employ a non-separable utility with constant labor supply elasticity and use numerical analyses to show that the Laffer curve for consumption tax is monotonically increasing. Kobayashi (2014) investigates whether the consumption tax revenue is bounded using a neoclassical growth model with the log utility function. He finds that although the fixed supply of production factor affects the boundness of the consumption tax revenue, the Laffer curve continues to be monotonically increasing in his model. The main contribution of the present paper is to find that the Laffer curve for consumption tax can be hump-shaped if the utility is additively separable.

The remainder of the paper is organized as follows. Section 2 introduces the simple static model and shows the main result. Section 3 extends the result of Section 2 to a dynamic setting à la Trabandt and Uhlig (2011). Section 4 discusses the results. Section 5 concludes.

2 Simple static economy

Assuming a simple static economy, the Laffer curve for consumption tax is characterized in this section.

2.1 Model

The representative households supply labor n to firms and earn wage rate w. They also receive government transfers s. Let τ_t^c denote consumption tax. The budget constraint of households is

$$(1 + \tau^c)\, c \leq wn + s, \tag{1}$$

where c denotes consumption.

The firms are perfectly competitive. Their production function is

$$y = n, \tag{2}$$

where y denotes output.

The government budget constraint is

$$s \leq T, \tag{3}$$

where total tax revenue T is defined by

$$T = \tau^c c. \tag{4}$$

Since there is no investment and government consumption, the resource constraint of this closed economy is

$$y = c. \tag{5}$$

Two types of utility functions are considered. The one is additively separable.

$$U^{AD} = \frac{c^{1-\eta}}{1-\eta} - \kappa \frac{n^{1+\lambda}}{1+\lambda},$$

where η is the relative risk aversion (that is the inverse of the IES under a dynamic setting), and $1/\lambda$ is the labor supply elasticity.[1] This type of utility function is often employed in the literature on the new Keynesian business cycle (see Gali, 2008). The other is a non-separable utility function, such that

$$U^{NS} = \frac{1}{1-\eta}\left\{c^{1-\eta}\left[1 - \kappa(1-\eta)n^{1+\lambda}\right]^{\eta} - 1\right\},$$

which is a static version employed by Trabandt and Uhlig (2011).

[1] In this paper, $1/\lambda$ is called "the labor supply elasticity," but it is often interpreted as "Frisch elasticity" in the literature. A discussion on this topic appears in Section 4.

2.2 Laffer curve for consumption tax in a static economy

First, consider the consumption tax revenue curve as the Laffer curve. The key element here is the elasticity of aggregate consumption to the consumption tax rate. If it is greater than one, an increase in the consumption tax rate increases the consumption tax revenue, and vice versa. In this model, consumption equals labor supply by the resource constraint and production function.

In the case of the additively separable utility function, the optimization condition for the consumption–labor choice is

$$\kappa c^{\eta} n^{\lambda} = \frac{1}{1+\tau^c} w. \tag{6}$$

Solving this condition yields

$$c = n = [\kappa(1+\tau^c)]^{-1/(\eta+\lambda)}, \tag{7}$$

and the elasticity of aggregate consumption to the consumption tax rate is

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{1}{\eta+\lambda} \cdot \frac{\tau^c}{1+\tau^c}. \tag{8}$$

It is easily shown that $\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is increasing in τ^c, $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = 0$ if $\tau^c = 0$, and $\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ converges to $\frac{1}{\eta+\lambda}$ as τ^c approaches infinity. Therefore, the Laffer curve for consumption tax can be hump-shaped if $\frac{1}{\eta+\lambda}$ is greater than one.

The following is a formal statement of a necessary and sufficient condition for a hump-shaped consumption tax revenue curve.

Proposition 1. *Suppose that the utility function is additively separable; U^{AS}. The consumption tax revenue curve is hump-shaped if and only if $\eta + \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta-\lambda}$. Otherwise, the consumption tax revenue curve is monotonically increasing.*

Proof. Note that

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| - 1 = \frac{1}{\eta+\lambda} \cdot \frac{\tau^c}{1+\tau^c} \left[(1-\eta-\lambda)\tau^c - (\eta+\lambda) \right]. \tag{9}$$

Suppose that $\eta + \lambda = 1$. In this case, $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 < 0$ and the consumption tax revenue is monotonically increasing.

Suppose $\eta + \lambda \neq 1$. In this case,

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \left(\frac{1-\eta-\lambda}{\eta+\lambda}\right)\left(\frac{\tau^c}{1+\tau^c}\right)\left(\tau^c - \frac{\eta+\lambda}{1-\eta-\lambda}\right).$$

If $\eta + \lambda \geq 1$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| \leq 1$.

If $\eta + \lambda < 1$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \leq (\eta + \lambda)/(1 - \eta - \lambda)$, and $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| > 1$ for $\tau^c > (\eta + \lambda)/(1 - \eta - \lambda)$. □

The parameters in the utility function, η and λ, should be small because the hump-shaped consumption tax revenue curve can be understood by the optimization condition for the consumption–labor choice (6). The consumption tax revenue curve can be hump-shaped if an increase in the consumption tax rate reduces the labor supply by a sufficient amount. The key parameter is the inverse of λ, that is, the labor supply elasticity to the after-tax wage rate. Then, a low value of λ implies a highly distorted increase in the consumption tax rate. In general equilibrium, consumption c is closely related to the labor supply n through the resource constraint and production function. In the current setting, $c = n$. Then, the parameter η (the inverse of the IES) works as the inverse of the aggregate labor supply elasticity. As a result, the inverse of $\eta + \lambda$ is the elasticity of the aggregate labor supply in general equilibrium as in (7). Then, the inverse of $\eta + \lambda$ is the maximum of the elasticity of consumption since $c = n$.

In the case of the non-separable utility function, the optimization condition for the consumption–labor choice is

$$\eta(1+\lambda)\left(\frac{\kappa c n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}\right) = \frac{1}{1+\tau^c}w. \tag{10}$$

Solving this condition yields

$$c = n = \left[\tau^c \eta\kappa(1+\lambda) + \kappa(\eta\lambda+1)\right]^{-1/(1+\lambda)}, \tag{11}$$

and the elasticity of consumption to the consumption tax rate is

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = \frac{\tau^c \eta\kappa}{\tau^c \eta\kappa\,(1+\lambda) + \kappa(\eta\lambda + 1)}. \tag{12}$$

Contrary to Proposition 1, the non-separable utility function employed by Trabandt and Uhlig (2011) cannot generate a hump-shaped Laffer curve for consumption tax as in Proposition 2, since $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| < 1$ for $\tau^c \geq 0$.

Proposition 2. *Suppose that the utility function is non-separable; U^{NS}. The consumption tax revenue curve is monotonically increasing.*

Proof. It is obvious that

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = \frac{\tau^c \eta\kappa}{\tau^c \eta\kappa\,(1+\lambda) + \kappa(\eta\lambda + 1)} < 1.$$

□

So far, the consumption tax revenue curve is considered to be a Laffer curve. By introducing labor income tax, the Laffer curve refers to the total tax revenue. In this case, the budget constraint of a household becomes

$$(1+\tau^c)\, c \leq (1-\tau^n) wn + s, \tag{13}$$

and the total tax revenue is

$$T_t = \tau^c\, c + \tau^n\, wn. \tag{14}$$

Propositions 3 and 4 are analogues of Propositions 1 and 2.

Proposition 3. *Suppose that the utility function is additively separable; U^{AS}. The total tax revenue curve is hump-shaped if and only if $\tau^n < \eta + \lambda < 1$ and the revenue is maximized at $\tau^c = \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$. If $\eta + \lambda < 1 \leq \tau^n$, the total tax revenue curve is monotonically decreasing. Otherwise, the total tax revenue curve is monotonically increasing.*

Proof. See Appendix A. □

Proposition 4. *Suppose that the utility function is non-separable; U^{NS}. The total tax revenue curve is monotonically increasing.*

Proof. See Appendix B. □

As in the consumption tax revenue curve, the condition $\eta + \lambda < 1$ is necessary for the hump-shaped total tax revenue curve in the case of the additively separable utility function, and in the case of non-separable utility function, the total tax revenue curve is monotonically increasing. Note that the consumption tax revenue curve might be monotonically decreasing. This is because the peak consumption tax rate that maximizes the tax revenue ($\tau^c = \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$) is negative if τ^n is sufficiently high.

3 Dynamic economy à la Trabandt and Uhlig (2011)

In this section, the result of Section 2 is extended to a neoclassical growth model à la Trabandt and Uhlig (2011).

3.1 Model

The representative households hold capital stock k_{t-1} and debt b_{t-1} as assets at the beginning of the period. They supply labor n_t and capital stock k_{t-1} to firms, and earn the wage rate w_t, rental rate of capital d_t, and interest rate on debt R_t^b. They also receive government transfers s_t and transfers from abroad m_t. The latter can be interpreted as net imports as discussed by Trabandt and Uhlig (2011). Let τ_t^c, τ_t^n, and τ_t^k denote the consumption tax, labor tax, and capital tax rates, respectively. The budget constraint of households is

$$(1+\tau_t^c)c_t + x_t + b_t \leq (1-\tau_t^n)w_t n_t + (1-\tau_t^k)(d_t-\delta)k_{t-1} + \delta k_{t-1} + R_t^b b_t + s_t + m_t, \quad (15)$$

where c_t denotes consumption, δ denotes the depreciation rate of capital, and x_t is investment. The capital stock evolves according to the following equation.

$$k_t = (1-\delta)k_{t-1} + x_t. \tag{16}$$

The firms are perfectly competitive. Their production function is

$$y_t = \xi^t k_{t-1}^{\theta} n_t^{1-\theta}, \tag{17}$$

where ξ denotes the technology growth rate, and θ denotes the capital share in production. The profit maximization problem implies

$$w_t = (1-\theta)\frac{y_t}{n_t} \qquad \text{and} \tag{18}$$

$$d_t = \theta\frac{y_t}{k_{t-1}}. \tag{19}$$

The government budget constraint is

$$g_t + s_t + R_t^b b_{t-1} \leq b_t + T_t, \tag{20}$$

where g_t denotes the government consumption, and the total tax revenue T_t is defined by

$$T_t = \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k (d_t - \delta)k_{t-1}. \tag{21}$$

The resource constraint of this economy is

$$y_t = c_t + x_t + g_t - m_t. \tag{22}$$

The additively separable utility function for this dynamic economy is

$$\mathbb{U}^{AS} = \sum_{t=0}^{\infty}\beta^t\left[\frac{c_t^{1-\eta}}{1-\eta} - \kappa\psi^{t(1-\eta)}\frac{n_t^{1+\lambda}}{1+\lambda} + v(g_t)\right],$$

where $\psi^{t(1-\eta)}$ guarantees the existence of a balanced growth path, and $v(\cdot)$ is an increasing function. The non-separable utility function is

$$\mathbb{U}^{NS} = \sum_{t=0}^{\infty}\beta^t\left[\frac{1}{1-\eta}\left\{c_t^{1-\eta}\left[1-\kappa(1-\eta)n_t^{1+\lambda}\right]^{\eta} - 1\right\} + v(g_t)\right].$$

Following Trabandt and Uhlig (2011), the Laffer curve for consumption tax is given by the relationship between the tax revenue and tax rate on the balanced growth path. Let the growth rate on the balanced growth path be $\psi = \xi^{1/(1-\theta)}$. It is assumed that government debt b_t is on the balanced growth path; $b_{t-1} = \psi^t \bar{b}$. It is also assumed that $g_t = \phi_g y_t$ and $m_t = \phi_m y_t$.[2] The equilibrium system at the balanced growth path is described in Appendix C.

3.2 Laffer curve for consumption tax in the dynamic economy

Propositions 5 and 6 refer to the consumption tax revenue curve in the dynamic economy.

Proposition 5. *Suppose that the utility function is additively separable; $\mathbb{U}^{AS}$. The consumption tax revenue curve is hump-shaped if and only if $\eta + \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta-\lambda}$. Otherwise, the consumption tax revenue curve is monotonically increasing.*

Proof. See Appendix D. □

Proposition 6. *Suppose that the utility function is non-separable; $\mathbb{U}^{NS}$. The consumption tax revenue curve is monotonically increasing.*

Proof. See Appendix E. □

[2] Trabandt and Uhlig (2011) employ alternative assumptions: $g_t = \psi^t \bar{g}$ and $m_t = \psi^t \bar{m}$. The constant steady-state ratio of government consumption to GDP is interpreted as the government controls g_t/y_t as in Hayashi and Prescott (2002). The constant steady-state ratio of net imports to GDP would be interpreted as net imports being closely related to the total income of the home country. These assumptions of constant steady-state ratios are used to prove Propositions 5–8. Under these assumptions, an increase in the consumption tax rate decreases both output and government consumption. This decrease in government consumption implies a positive wealth effect and then consumption increases. Therefore, the Laffer curve for consumption tax is more unlikely to be hump-shaped than those under the assumptions employed by Trabandt and Uhlig (2011).

Note that these two propositions are the same as Propositions 1 and 2, while the dynamic economy has far richer structure (capital, investment, debt evolution, etc.) than the static economy in Section 2.

Propositions 7 and 8 refer to the total tax revenue curve.

Proposition 7. *Suppose that the utility function is additively separable; U^{AS}. The total tax revenue curve is hump-shaped if and only if*

$$\eta + \lambda < 1 \qquad \textit{and} \qquad \left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] < \eta + \lambda,$$

where

$$d = \frac{1}{1-\tau^k}\left[\frac{\psi^\eta}{\beta} - 1 + \delta\right].$$
$$\frac{k}{y} = \frac{\theta}{d},$$
$$\frac{c}{y} = 1 - [\psi - (1-\delta)]\frac{\theta}{d} - \phi_g + \phi_m,$$

and the revenue is maximized at $\tau^c = \frac{1}{1-\eta-\lambda}\left\{(\eta+\lambda) - \left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\right\}$. *Otherwise, the total tax revenue curve is*

- *U-shaped if* $\eta + \lambda > 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] > \eta + \lambda$.
- *monotonically increasing if* $\eta + \lambda > 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] \leq \eta + \lambda$.
- *monotonically increasing if* $\eta + \lambda = 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] < \eta + \lambda$.
- *flat if* $\eta + \lambda = 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] = \eta + \lambda$.
- *monotonically decreasing if* $\eta + \lambda = 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] > \eta + \lambda$.
- *monotonically decreasing if* $\eta + \lambda < 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] \geq \eta + \lambda$.

Proof. See Appendix F. □

Proposition 8. *Suppose that the utility function is non-separable;* $\mathbb{U}^{NS}$. *The total tax revenue curve is monotonically increasing if and only if*

$$\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{k}{y}\right) \leq \frac{1-\eta}{\eta}(1-\theta)(1-\tau^n)+(1+\lambda)\left(\frac{c}{y}\right),$$

where

$$d = \frac{1}{1-\tau^k}\left[\frac{\psi^\eta}{\beta}-1+\delta\right].$$
$$\frac{k}{y} = \frac{\theta}{d},$$
$$\frac{c}{y} = 1-[\psi-(1-\delta)]\frac{\theta}{d}-\phi_g+\phi_m.$$

Otherwise, the total tax revenue curve is U-shaped.

Proof. See Appendix G. □

Propositions 7 and 8 imply that there is a possibility that the total tax revenue curve might be U-shaped under some parameter values. Under this situation, the total tax revenue is decreasing and increasing if the consumption tax rate is low and sufficiently high, respectively. The U-shaped total tax revenue curve is generated when the labor and capital income tax rate are high. The decreases in these tax revenues associated with an increase in the consumption tax rate dominate the increase in consumption tax revenue if the consumption tax rate is low.

4 Discussion

4.1 Likelihood of a hump-shaped Laffer curve for consumption tax

According to Propositions 1, 3, 5, and 7, it is necessary for $\eta + \lambda < 1$ to generate a hump-shaped Laffer curve for consumption tax. For this condition, both η and λ should be less than one. The likelihood of this condition is discussed in this subsection.

The condition $\eta < 1$ might be supported by the empirical findings of Mulligan (2002), Vissing-Jorgensen and Attanasio (2003), Bansal and Yaron (2004), and Gruber (2013), whereas it is standard to set $\eta \geq 1$ in macroeconomics. These papers find that the IES, that is the inverse of η, is greater than one. Kobayashi, Nakajima, and Inaba (2012) find that the IES must be greater than one, and set $\eta = 1/2$ in order to generate a positive response of the asset price to the news shock about future productivity in their theoretical research.

The parameter λ should not be not restricted by evidence on the Frisch elasticity as claimed by Christiano, Trabandt, and Walentin (2010), although it is often interpreted as the inverse of Frisch elasticity, and the values are set depending on the estimations using micro data analyses. Empirical evidence from micro data implies that the Frisch elasticity is very small. However, as in the seminal works by Hansen (1985) and Rogerson (1988), even if the individual elasticity of labor supply is zero, the aggregate labor supply can be sensitive to the changes in the real wage rate. Christiano, Trabandt, and Walentin (2010) estimate this parameter for the U.S. economy by using Bayesian impulse response matching, and find that λ is around 0.1.

Therefore, some recent empirical evidence supports small values of η and λ. It would imply that a hump-shaped Laffer curve for consumption tax is possible.

4.2 Numerical results of the Laffer curve for consumption tax

Sections 2 and 3 characterize the shape of the Laffer curve for consumption tax and show that the Laffer curve can be hump-shaped in the case of additively separable utility. This subsection presents some numerical results.

The parameter values are the same as those employed by Trabandt and Uhlig (2011) for the U.S. economy. The capital share in the production function θ is 0.35. The depreciation rate of capital δ is 0.083. The steady-state ratio of debt to output b/y is 0.63. The steady-state ratio of government expenditure to output g/y is 0.08. The steady-state

ratio of transfer from abroad to output m/y is 0.04. The balanced growth parameter ψ is 1.02. The steady-state real interest rate R is 1.04. The steady-state labor supply h is 0.2. The steady-state capital income tax rate is 0.36, labor income tax is 0.28, and consumption tax rate is 0.05.

Figure 1 summarizes the shape of the tax revenue curve for consumption tax. The horizontal axis is η, and the vertical axis is λ. "I" denotes the region of the monotonically increasing total tax revenue curve, "D," the region of the monotonically decreasing curve, "H," the region of the hump-shaped curve, and "U," the region of the U-shaped curve. The panels on the left and right are the cases of additively separable utility and non-separable utility, respectively. The upper panels are the benchmark case with $\tau^n = 0.36$. The middle and lower panels are the cases of $\tau^n = 0.7$ and $\tau^n = 0.9$, respectively.

[Insert Figure 1]

Figure 2 shows a numerical example of the total tax revenue curves and components (consumption tax revenue, labor income tax revenue, and capital income tax revenue). The procedure to calculate the tax revenue curves is described in Appendix H. The circles denote the peak tax rates that maximize the total tax revenues. The vertical dotted lines show the baseline consumption tax rate of 5%. The utility function parameters are set such that $\eta = 1/2$ and $\lambda = 0.1$. The value of η is consistent with Gruber (2013), and that of λ is consistent with the value estimated by Christiano, Trabandt, and Walentin (2010). As already shown in Sections 2 and 3, the total tax revenue curve is hump-shaped in the case of the additively separable utility function, and it is monotonically increasing for the non-separable utility function. It is found that the peak tax rate that maximizes the total tax revenue of the additively separable utility is 45.84%, whereas the consumption tax revenue is maximized at 150%, that is $(\eta + \lambda)/(1 - \eta - \lambda)$.

[Insert Figure 2]

4.3 Quantitative significance of the difference in utility functions on the Laffer curves for labor and capital income taxes

It is shown that the difference in the functional form of the utility has significant effects on the shape of the Laffer curve for consumption tax. The quantitative effects of this difference on the total tax revenue curves for labor and capital income taxes are examined in this subsection.

Figure 3 shows the total tax revenue curves for labor income tax in the cases of the additively separable and non-separable utility functions. The left-hand panel shows the case of $\eta = 2$ and $\lambda = 1$, employed by Trabandt and Uhlig (2011), and the right, of $\eta = 1/2$ and $\lambda = 0.1$, which generate the hump-shaped total tax revenue curves for consumption tax. The other parameter values are the same as in the previous subsection. It is found that the difference in the utility functions has significant effects on the the peak tax rates of the total tax revenue curves. These rates are 71.5% (additively separable utility) and 59.26% (non-separable utility) for $\eta = 2$ and $\lambda = 1$. Notably, the difference in the peak tax rates is more than 10% even for Trabandt and Uhlig's (2011) parameter values. This impact is much strengthened for $\eta = 1/2$ and $\lambda = 0.1$: the peak tax rates are 32.93% (additively separable utility) and 58.99% (non-separable utility).

[Insert Figure 3]

Figure 4 shows the capital income tax analogue of Figure 3. The peak tax rates in the case of $\eta = 2$ and $\lambda = 1$ are 71.11% (additively separable utility) and 59.32% (non-separable utility), and the difference is more than 10% as well. This impact is much strengthened for $\eta = 1/2$ and $\lambda = 0.1$: the peak tax rates are 20.49% (additively separable utility) and 78.8% (non-separable utility).

[Insert Figure 4]

Finally, Figures 3 and 4 show that the difference in the utility functions has quantitatively significant effects on the peak tax rates of the Laffer curves for labor and capital income taxes. Holter, Krueger, and Stepanchuk (2014) find that converting the current U.S. progressive tax code to a flat tax code raises the peak tax rate of the Laffer curve for labor income tax by 6%. Contrary to their finding, the quantitative impacts of the difference in the utility function on the peak tax rates of the Laffer curves for labor and capital income taxes are more than 10%, even while using parameter values where the Laffer curve for consumption tax is not hump-shaped.

5 Concluding remarks

This paper has characterized the shape of the Laffer curve for consumption tax. The Laffer curve for consumption tax can be hump-shaped if the utility function is additively separable in consumption and labor supply. On the other hand, it cannot be hump-shaped if the utility function as employed by Trabandt and Uhlig (2011) is non-separable. This is because the aggregate labor supply and consumption elasticities with respect to the consumption tax rate can be greater than one under sufficiently high parameter values of the IES and labor supply elasticity if the utility is additively separable, whereas the opposite stands when the utility is non-separable.

This paper has also shown that the total tax revenue Laffer curve can be hump-shaped under empirically relevant parameter values. At the same time, the difference in the functional form of the utility has quantitatively significant effects on the peak tax rates of the Laffer curves for labor and capital income taxes.

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Appendix

A Proof of Proposition 3

Proof. The optimization condition for the consumption–labor choice,

$$\kappa c^{\eta} n^{\lambda} = \frac{1-\tau^n}{1+\tau^c} w,$$

indicates that

$$c = \left[\frac{\kappa}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\eta+\lambda}}.$$

Since the total tax revenue is

$$\begin{aligned} T &= \tau^c c + \tau^n w n \\ &= (\tau^c + \tau^n)\left[\frac{\kappa}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\eta+\lambda}}, \end{aligned}$$

then

$$\frac{dT}{d\tau^c} = \left[\frac{\kappa}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\eta+\lambda}-1}\left(\frac{\kappa}{1-\tau^n}\right)\left[\tau^c\left(\frac{\eta+\lambda-1}{\eta+\lambda}\right)+\frac{\eta+\lambda-\tau^n}{\eta+\lambda}\right].$$

Suppose $\eta+\lambda=1$, then $\frac{dT}{d\tau^c}>0$.

Suppose $\eta+\lambda\neq 1$, then

$$\frac{dT}{d\tau^c} = \left[\frac{\kappa}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\eta+\lambda}-1}\left(\frac{\kappa}{1-\tau^n}\right)\left(\frac{\eta+\lambda-1}{\eta+\lambda}\right)\left[\tau^c-\frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}\right].$$

If $\eta+\lambda>1$, then $\frac{dT}{d\tau^c}>0$.

If $\eta+\lambda<1$, then $\frac{dT}{d\tau^c}>0$ for $\tau^c<\frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$, and $\frac{dT}{d\tau^c}<0$ for $\tau^c>\frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$. □

B Proof of Proposition 4

Proof. By the optimization condition for the consumption–labor choice,

$$\eta(1+\lambda)\left(\frac{\kappa c n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}\right) = \frac{1-\tau^n}{1+\tau^c} w,$$

it follows that

$$c = (1-\tau^n)^{1/(1+\lambda)}\left[\tau^c\eta\kappa(1+\lambda)+\kappa(\eta\lambda+1)-\tau^n\kappa(1-\eta)\right]^{-1/(1+\lambda)}.$$

The total tax revenue is

$$\begin{aligned} T &= \tau^c c + \tau^n wn \\ &= (\tau^c+\tau^n)(1-\tau^n)^{1/(1+\lambda)}\left[\tau^c\eta\kappa(1+\lambda)+\kappa(\eta\lambda+1)-\tau^n\kappa(1-\eta)\right]^{-1/(1+\lambda)}. \end{aligned}$$

Then,

$$\begin{aligned} \frac{dT}{d\tau^c} = (1-\tau^n)^{1/(1+\lambda)}&\left[\tau^c\eta\kappa(1+\lambda)+\kappa(\eta\lambda+1)-\tau^n\kappa(1-\eta)\right]^{-1/(1+\lambda)-1} \\ &\times\left[\tau^c\eta\kappa\lambda+\kappa(\eta\lambda+1-\tau^n)\right]>0. \end{aligned}$$

□

C Equilibrium system of the dynamic model

The equilibrium system of the dynamic model is

$$\begin{aligned} &(1+\tau_t^c)\lambda_t = u_1(c_t,n_t), \\ &\lambda_t(1-\tau_t^n)w_t = -u_2(c_t,n_t), \\ &\lambda_t = \beta E_t\left\{\lambda_{t+1}\left[(1-\delta)+(1-\tau_{t+1}^k)(d_{t+1}-\delta)+\delta\right]\right\}, \\ &\lambda_t = \beta E_t\left[\lambda_{t+1}R_{t+1}^b\right], \\ &k_t = (1-\delta)k_{t-1}+x_t, \\ &y_t = \xi^t\left[k_{t-1}\right]^\theta n_t^{1-\theta}, \\ &w_t = (1-\theta)\frac{y_t}{n_t}, \\ &d_t = \theta\frac{y_t}{k_{t-1}}, \\ &y_t = c_t+x_t+g_t-m_t, \\ &T_t = \tau_t^c c_t+\tau_t^n w_t n_t+\tau_t^k(d_t-\delta)k_{t-1}, \end{aligned}$$

where the marginal utilities are defined by

$$u_1(c_t, n_t) \equiv (c_t)^{-\eta},$$
$$u_2(c_t, n_t) \equiv -\kappa \psi^{t(1-\gamma)} n_t^{\lambda}$$

if the utility function is additively separable $\mathbb{U}^{AS}$, and by

$$u_1(c_t, n_t) \equiv (c_t)^{-\eta} \left[1 - \kappa(1-\eta) n_t^{1+\lambda}\right]^{\eta},$$
$$u_2(c_t, n_t) \equiv -\eta (1+\lambda) \left\{(c_t)^{1-\eta} \left[1 - \kappa(1-\eta) n_t^{1+\lambda}\right]^{\eta-1} \kappa n_t^{\lambda}\right\}$$

if the utility function is non-separable $\mathbb{U}^{NS}$.

Detrend the equilibrium system by $\psi = \xi^{1/(1-\theta)}$, and let $a_t/\xi^t \equiv \tilde{a}_t$ (except for $\tilde{k}_{t-1} \equiv k_{t-1}/\xi^t$ and λ). The detrended equilibrium system is

$$(1+\tau_t^c)\tilde{\lambda}_t = u_1(\tilde{c}_t, n_t),$$
$$\tilde{\lambda}_t(1-\tau_t^n)\tilde{w}_t = -u_2(\tilde{c}_t, n_t),$$
$$\tilde{\lambda}_t = \beta\psi^{-\eta} E_t \left\{\tilde{\lambda}_{t+1}\left[(1-\delta) + (1-\tau_{t+1}^k)(d_{t+1}-\delta) + \delta\right]\right\},$$
$$\tilde{\lambda}_t = \beta\psi^{-\eta} E_t \left[\tilde{\lambda}_{t+1} R_{t+1}^b\right],$$
$$\psi\tilde{k}_t = (1-\delta)\tilde{k}_{t-1} + \tilde{x}_t,$$
$$\tilde{y}_t = \left[\tilde{k}_{t-1}\right]^{\theta} n_t^{1-\theta},$$
$$\tilde{w}_t = (1-\theta)\frac{\tilde{y}_t}{n_t},$$
$$d_t = \theta\frac{\tilde{y}_t}{\tilde{k}_{t-1}}.$$
$$\tilde{y}_t = \tilde{c}_t + \tilde{x}_t + \tilde{g}_t - \tilde{m}_t,$$
$$\tilde{T}_t = \tau_t^c \tilde{c}_t + \tau_t^n \tilde{w}_t n_t + \tau_t^k (d_t - \delta)\tilde{k}_{t-1}.$$

On the balanced growth path, the system becomes

$$
\begin{aligned}
&(1+\tau^{c})\tilde{\lambda} = u_1(\tilde{c}, n),\\
&\tilde{\lambda}(1-\tau^{n})\tilde{w} = -u_2(\tilde{c}, n),\\
&1 = \beta\psi^{-\eta}\left[(1-\delta)+(1-\tau^{k})(d-\delta)+\delta\right],\\
&1 = \beta\psi^{-\eta}R^{b},\\
&\psi\tilde{k} = (1-\delta)\tilde{k}+\tilde{x},\\
&\tilde{y} = \left[\tilde{k}\right]^{\theta} n^{1-\theta},\\
&\tilde{w} = (1-\theta)\frac{\tilde{y}}{n},\\
&d = \theta\frac{\tilde{y}}{\tilde{k}}.\\
&\tilde{y} = \tilde{c}+\tilde{x}+\tilde{g}-\tilde{m}_t,\\
&\tilde{T} = \tau_t^{c}\tilde{c}+\tau^{n}\tilde{w}n+\tau^{k}(d-\delta)\tilde{k}.
\end{aligned}
$$

The balanced growth path values are obtained by

$$
\begin{aligned}
&R^{b} = \frac{\psi^{\eta}}{\beta},\\
&d = \frac{1}{1-\tau^{k}}\left[R^{b}-1+\delta\right].\\
&\frac{\tilde{k}}{\tilde{y}} = \frac{\theta}{d},\\
&\frac{\tilde{x}}{\tilde{y}} = \left[\psi-(1-\delta)\right]\frac{\tilde{k}}{\tilde{y}},\\
&\frac{\tilde{c}}{\tilde{y}} = 1-\frac{\tilde{x}}{\tilde{y}}-\frac{\tilde{g}}{\tilde{y}}+\frac{\tilde{m}}{\tilde{y}},\\
&\frac{n}{\tilde{y}} = \left[\frac{\tilde{y}}{\tilde{k}}\right]^{\theta/(1-\theta)},\\
&\tilde{w} = (1-\theta)\frac{\tilde{y}}{\tilde{n}},
\end{aligned}
$$

given $\tilde{g}/\tilde{y} = \phi_g$ and $\tilde{m}/\tilde{y} = \phi_m$. From this system, the following lemma is obtained from the balanced growth path equilibrium system.

Lemma 1. *On the balanced growth path, the dividend (d), capital–output ratio ($k/y = \tilde{k}/\tilde{y}$), investment–output ratio ($x/y = \tilde{x}/\tilde{y}$), consumption–output ratio ($c/y = \tilde{c}/\tilde{y}$), and labor–output ratio ($n/\tilde{y}$) are independent from the consumption tax rate (τ^c).*

D Proof of Proposition 5

Proof. By the optimization condition for the consumption–labor choice,

$$\kappa \tilde{c}^{\eta} n^{\lambda} = \frac{1-\tau^n}{1+\tau^c}\tilde{w},$$

it follows that

$$\tilde{y} = (1+\tau^c)^{-1/(\eta+\lambda)} \left[\frac{1-\theta}{\kappa}(1-\tau^n) \left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta} \left(\frac{n}{\tilde{y}}\right)^{-1-\lambda} \right]^{1/(\eta+\lambda)}.$$

Since $\tilde{c}/\tilde{y} = c/y$ and $h/\tilde{y}$ are independent of τ^c as in Lemma 1 of Appendix C, it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} = \frac{d\tilde{y}/\tilde{y}}{d\tau^c/\tau^c} = -\frac{1}{\eta+\lambda} \cdot \frac{\tau^c}{1+\tau^c}.$$

Then,

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 = \frac{1}{\eta+\lambda} \cdot \frac{\tau^c}{1+\tau^c}\Big\{(1-\eta-\lambda)\tau^c - (\eta+\lambda)\Big\}.$$

Suppose $\eta + \lambda = 1$. In this case, $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 < 0$.

Suppose $\eta + \lambda \neq 1$. In this case,

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 = \frac{1-\eta-\lambda}{\eta+\lambda} \cdot \frac{\tau^c}{1+\tau^c}\left\{\tau^c - \frac{\eta+\lambda}{1-\eta-\lambda}\right\}.$$

If $\eta + \lambda \geq 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \geq 0$.

If $\eta + \lambda < 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \leq (\eta+\lambda)/(1-\eta-\lambda)$, and $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| > 1$ for $\tau^c > (\eta+\lambda)/(1-\eta-\lambda)$. □

E Proof of Proposition 6

Proof. The optimization condition for the consumption–labor choice,

$$\eta(1+\lambda)\left\{\frac{\kappa\tilde{c}n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}\right\}=\frac{1-\tau^{n}}{1+\tau^{c}}(1-\theta)\frac{\tilde{y}}{h},$$

yields that

$$\tilde{y}=\left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1+\tau^{c}}{1-\tau^{n}}\right]^{-1/(1+\lambda)}.$$

Since $\tilde{y}>0$ for $\tau^{c}\geq 0$,

$$(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^{n}}>0.$$

Since $\tilde{c}/\tilde{y}=c/y$ is independent of τ^{c} as in Lemma 1 of Appendix C, it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^{c}/\tau^{c}}=\frac{d\tilde{y}/\tilde{y}}{d\tau^{c}/\tau^{c}}=-\frac{\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\frac{\tau^{c}}{1-\tau^{n}}}{(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\left(\frac{1+\tau^{c}}{1-\tau^{n}}\right)(1+\lambda)}.$$

Letting

$$\Psi=(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\left(\frac{1+\tau^{c}}{1-\tau^{n}}\right)(1+\lambda)>0,$$

it follows that

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^{c}/\tau^{c}}\right|-1=-\frac{1}{\Psi}\left\{(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^{n}}+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\frac{\tau^{c}}{1-\tau^{n}}\lambda\right\}<0.$$

□

F Proof of Proposition 7

Proof. The total tax revenue is

$$\begin{aligned}\tilde{T}&=\tau^{c}\tilde{c}+\tau^{n}\tilde{w}n+\tau^{k}(d-\delta)\tilde{k}\\&=\left[\tau^{c}\left(\frac{\tilde{c}}{\tilde{y}}\right)+\tau^{n}(1-\theta)+\tau^{k}(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\tilde{y}.\end{aligned}$$

Since $\tilde{c}/\tilde{y} = c/y$ and $\tilde{k}/\tilde{y} = k/y$ are independent of τ^c as in Lemma 1 of Appendix C, the first-order derivative is

$$\frac{d\tilde{T}}{d\tau^c} = \left(\frac{\tilde{c}}{\tilde{y}}\right)\tilde{y} + \left[\tau^c\left(\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\frac{d\tilde{y}}{d\tau^c}.$$

Since

$$\tilde{y} = (1+\tau^c)^{-1/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)},$$

then

$$\frac{d\tilde{T}}{d\tau^c} = (1+\tau^c)^{-1/(\eta+\lambda)-1}\left[\frac{1-\theta}{\kappa}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)} \times \left(\frac{\tilde{c}}{\tilde{y}}\right)\left\{1+\tau^c\left(\frac{\eta+\lambda-1}{\eta+\lambda}\right) - \frac{1}{\eta+\lambda}\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\right\}.$$

Suppose that $\eta + \lambda = 1$. Then,

$$\frac{d\tilde{T}}{d\tau^c} = (1+\tau^c)^{-1/(\eta+\lambda)-1}\left[\frac{1-\theta}{\kappa}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)} \times \left(\frac{\tilde{c}}{\tilde{y}}\right)\frac{1}{\eta+\lambda}\left\{\eta+\lambda - \left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\right\}.$$

If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] < \eta+\lambda$, then, $\frac{d\tilde{T}}{d\tau^c} > 0$.
If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] > \eta+\lambda$, then $\frac{d\tilde{T}}{d\tau^c} > 0$.
If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] = \eta+\lambda$, then $\frac{d\tilde{T}}{d\tau^c} = 0$.

Suppose that $\eta + \lambda \neq 1$. It follows that

$$\frac{d\tilde{T}}{d\tau^c} = (1+\tau^c)^{-1/(\eta+\lambda)-1}\left[\frac{1-\theta}{\kappa}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}\left(\frac{\tilde{c}}{\tilde{y}}\right)\left(\frac{\eta+\lambda-1}{\eta+\lambda}\right)\times \left[\tau^c - \frac{1}{\eta+\lambda-1}\left\{\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] - (\eta+\lambda)\right\}\right].$$

Suppose that $\eta + \lambda > 1$.

If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] \leq \eta+\lambda$, then $\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c \geq 0$.
If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] > \eta+\lambda$,

then $\frac{d\tilde{T}}{d\tau^c} < 0$ for $\tau^c < \frac{1}{\eta+\lambda-1}\left\{\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]-(\eta+\lambda)\right\}$,
and $\frac{d\tilde{T}}{d\tau^c} \geq 0$ for $\tau^c > \frac{1}{\eta+\lambda-1}\left\{\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]-(\eta+\lambda)\right\}$.

Suppose that $\eta+\lambda<1$.
If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] \geq \eta+\lambda$, then $\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c \geq 0$.
If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] < \eta+\lambda$,
then $\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c < \frac{1}{1-\eta-\lambda}\left\{(\eta+\lambda)-\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\right\}$,
and $\frac{d\tilde{T}}{d\tau^c} < 0$ for $\tau^c > \frac{1}{1-\eta-\lambda}\left\{(\eta+\lambda)-\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\right\}$. □

G Proof of Proposition 8

Proof. The total tax revenue is

$$\tilde{T} = \tau^c\tilde{c} + \tau^n\tilde{w}n + \tau^k(d-\delta)\tilde{k} = \left[\tau^c\left(\frac{\tilde{c}}{\tilde{y}}\right)+\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\tilde{y}.$$

Since $\tilde{c}/\tilde{y} = c/y$ and $\tilde{k}/\tilde{y} = k/y$ are independent of τ^c as in Lemma 1 of Appendix C, the first-order derivative is

$$\frac{d\tilde{T}}{d\tau^c} = \left(\frac{\tilde{c}}{\tilde{y}}\right)\tilde{y} + \left[\tau^c\left(\frac{\tilde{c}}{\tilde{y}}\right)+\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\frac{d\tilde{y}}{d\tau^c}.$$

Since

$$\tilde{y} = \left(\frac{\tilde{y}}{n}\right)\kappa^{-1/(1+\lambda)}\left[(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\left(\frac{1+\tau^c}{1-\tau^n}\right)\right]^{-1/(1+\lambda)},$$

it follows that

$$\frac{d\tilde{y}}{d\tau^c} = \left(\frac{\tilde{c}}{n}\right)\kappa^{-1/(1+\lambda)}\left[(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)(1+\tau^c)\right]^{-1/(1+\lambda)-1}\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\left(\frac{1}{1-\tau^n}\right)\eta\times$$
$$\left\{\tau^c\lambda-\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)-\frac{1-\eta}{\eta}(1-\theta)(1-\tau^n)-(1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)\right]\right\}.$$

If $\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right) \leq \frac{1-\eta}{\eta}(1-\theta)(1-\tau^n)+(1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)$, then $\frac{d\tilde{T}}{d\tau^c} \geq 0$.
If $\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right) > \frac{1-\eta}{\eta}(1-\theta)(1-\tau^n)+(1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)$,

then $\frac{d\tilde{T}}{d\tau^c} < 0$ for $\tau^c < \frac{1}{\lambda}\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)-\frac{1-\eta}{\eta}(1-\theta)(1-\tau^n)-(1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)\right]$, and $\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c > \frac{1}{\lambda}\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta)+\tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)-\frac{1-\eta}{\eta}(1-\theta)(1-\tau^n)-(1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)\right]$.

□

H Procedure for the numerical calculations

Given the steady-state labor supply $n = 0.2$, the parameter of disutility of labor, κ, is calibrated as follows. First, the steady-state values are calculated by

$$
\begin{aligned}
d &= \frac{1}{1-\tau^k}\left[\frac{\psi^\eta}{\beta}-1\right]+\delta,\\
\frac{\tilde{k}}{\tilde{y}} &= \frac{\theta}{d},\\
\frac{\tilde{x}}{\tilde{y}} &= [\psi-(1-\delta)]\frac{\tilde{k}}{\tilde{y}},\\
\frac{\tilde{c}}{\tilde{y}} &= 1-\frac{\tilde{x}}{\tilde{y}}-\frac{\tilde{g}}{\tilde{y}}+\frac{\tilde{m}}{\tilde{y}},\\
\frac{n}{\tilde{y}} &= \left[\frac{\tilde{k}}{\tilde{y}}\right]^{-\theta/(1-\theta)},\\
\tilde{y} &= n\times\left(\frac{n}{\tilde{y}}\right)^{-1}.
\end{aligned}
$$

If the utility is additively separable $\mathbb{U}^{AD}$, κ is given by

$$
\kappa = \frac{1-\theta}{\tilde{y}^{\eta+\lambda}}\frac{1-\tau^n}{1+\tau^c}\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}.
$$

If the utility is non-separable $\mathbb{U}^{ND}$, κ is given by

$$
\kappa = \tilde{y}^{-(1+\lambda)}\left(\frac{\tilde{y}}{n}\right)^{1+\lambda}\left[(1-\eta)+\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1+\tau^c}{1-\tau^n}\right]^{-1}.
$$

Given the value of κ, the output is given by

$$
\tilde{y} = (1+\tau^c)^{-1/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)},
$$

if the utility is additively separable $\mathbb{U}^{AD}$. If the utility is non-separable $\mathbb{U}^{ND}$, the output is given by

$$\tilde{y} = \left(\frac{\tilde{y}}{n}\right)\kappa^{-1/(1+\lambda)}\left[(1-\eta) + \frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1+\tau^c}{1-\tau^n}\right]^{-1/(1+\lambda)}.$$

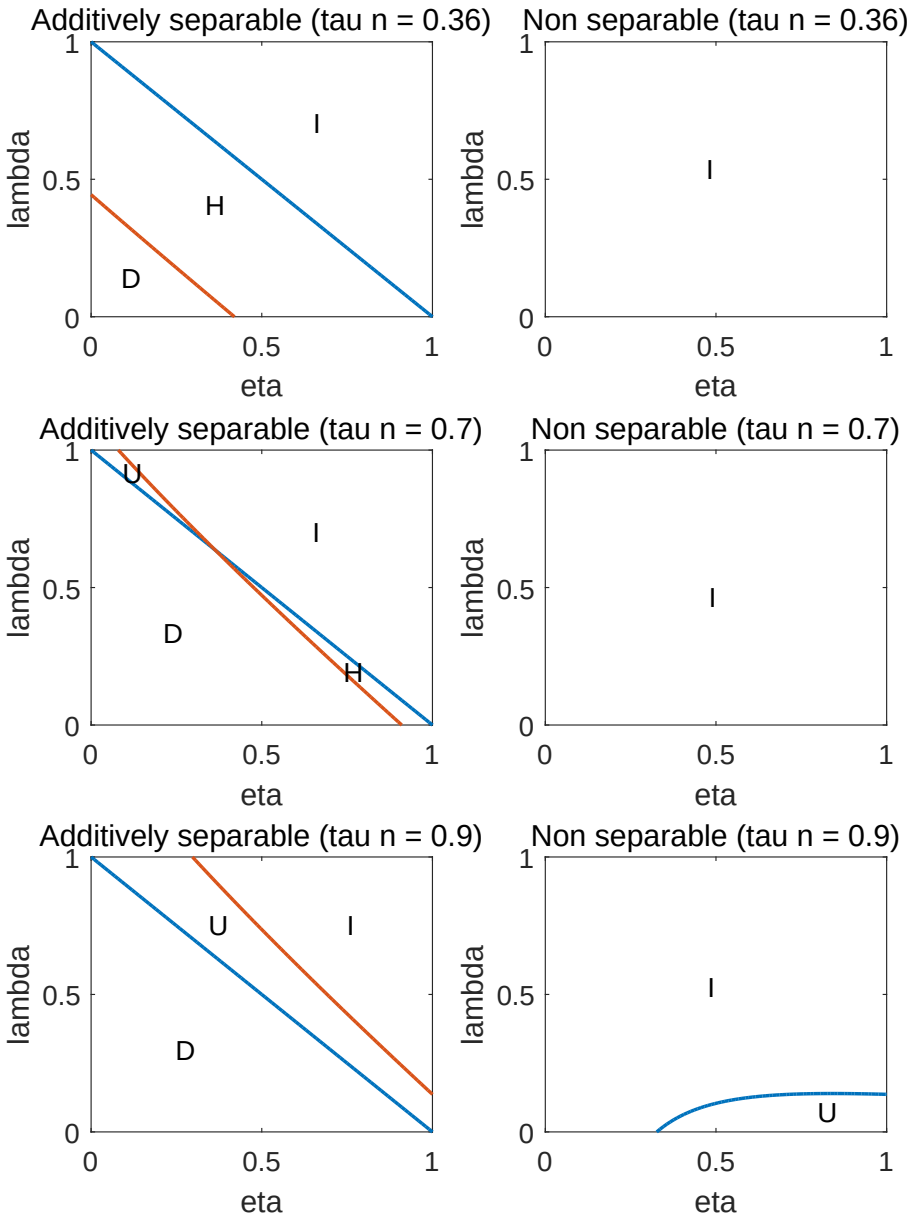
The associated capital stock and consumption are

$$\tilde{k} = \frac{\tilde{k}}{\tilde{y}} \times \tilde{y},$$
$$\tilde{c} = \frac{\tilde{c}}{\tilde{y}} \times \tilde{y},$$

respectively. Finally, the total tax revenue is given by

$$\begin{aligned} T &= \tau^c\tilde{c} + \tau^n\tilde{w}n + \tau^k(d-\delta)\tilde{k} \\ &= \tau^c\tilde{c} + \tau^n(1-\theta)\tilde{y} + \tau^k(d-\delta)\tilde{k}. \end{aligned}$$

Figure 1: Shape of the total tax revenue curve for consumption tax



Note - I: monotonically increasing, H: hump-shaped, D: monotonically decreasing, U: U-shaped

Figure 2: Total tax revenue curve for consumption tax: $\eta = 1/2$ and $\lambda = 1/10$

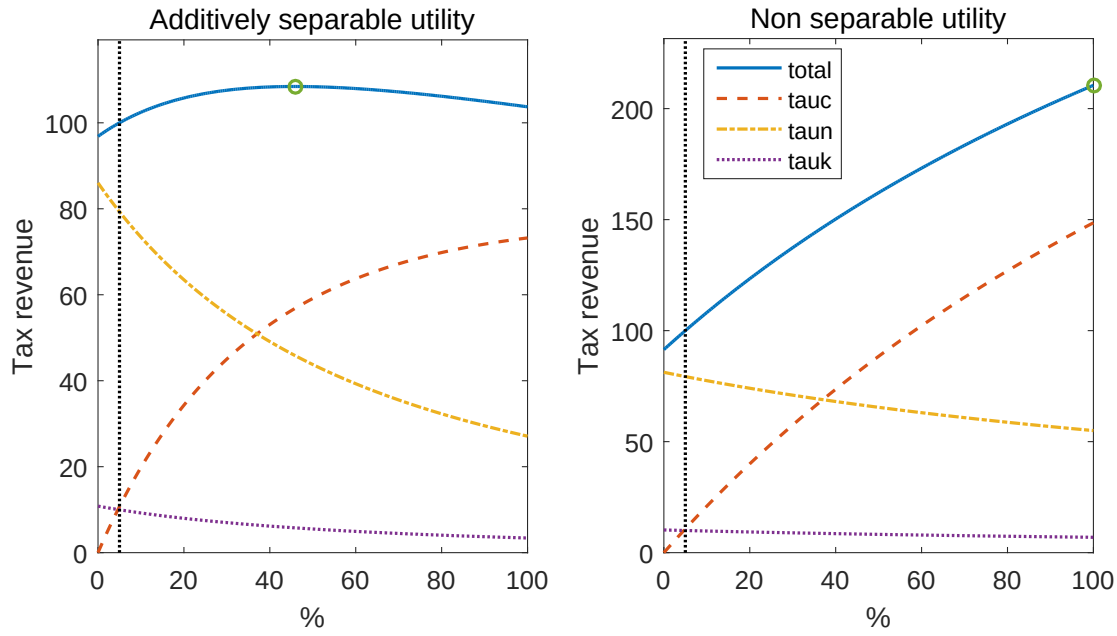


Figure 3: Total tax revenue curves for labor income tax

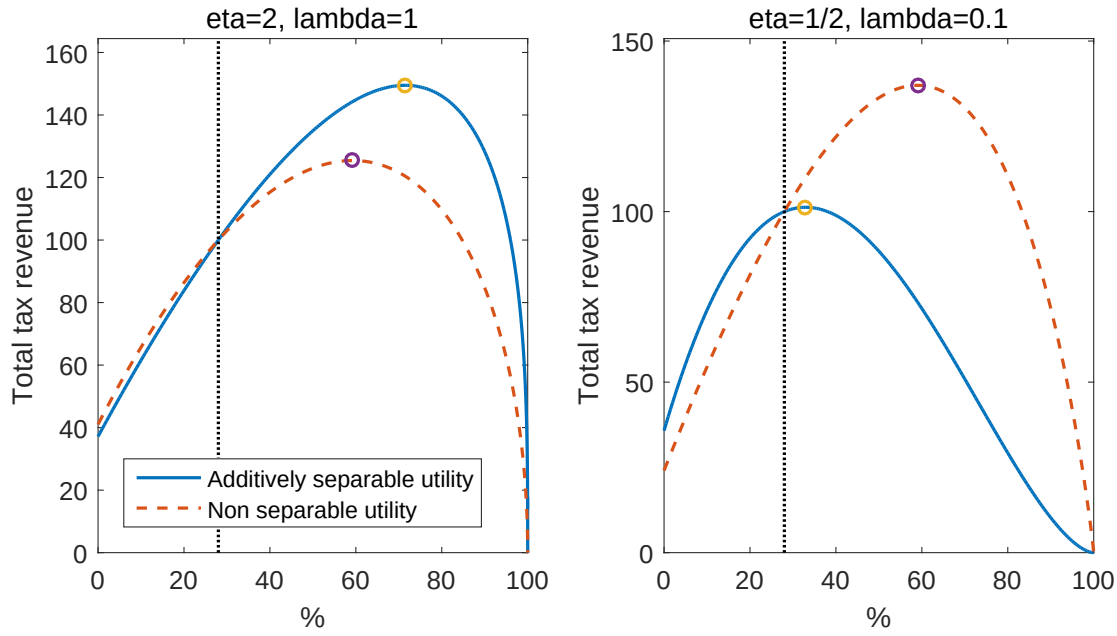


Figure 4: Total tax revenue curves for capital income tax