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Debt-Ridden Borrowers and Productivity Slowdown

Keiichiro Kobayashi and Daichi Shirai *

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Abstract

Economic growth is known to slow down for an extended period after a financial crisis. We construct a model in which the one-time buildup of debt can depress the economy persistently even when there is no shock on financial technology. We consider the debt dynamics of firms under an endogenous borrowing constraint. The borrowing constraint binds tighter and production inefficiency is higher when the initial amount of debt is larger. Tightening aggregate borrowing constraints lowers aggregate productivity, leading to a persistent recession. This model therefore implies that debt reduction for overly indebted agents may restore economic growth.

JEL code: E30, G01, O40

Keywords: Endogenous borrowing constraint, debt overhang, secular stagnation, labor wedge

1 Introduction

The decade after a financial crisis tends to be associated with low economic growth (Reinhart and Rogoff, 2009; Reinhart and Reinhart, 2010). Growth in total factor productivity (TFP) can slow down or even become negative for a decade (Kehoe and Prescott, 2007). Relatedly, there is growing concern about “secular stagnation” in the aftermath of the Great Recession, namely that the US and/or European economies may stagnate persistently (Summers, 2013; Eggertsson and Mehrotra, 2014). It has also been pointed out that

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financial constraints were tightened during and after the Great Recession of 2007–2009 (e.g., Altavilla et al., 2015). However, which factors tightened these financial constraints and whether their tightening can cause a persistent slowdown in economic growth remain unclear. In this study, we propose a theoretical model in which the buildup of debt tightens borrowing constraints and lowers growth in aggregate productivity persistently.

In existing models, persistent recessions are usually caused by persistent shocks (see, for example, Christiano et al. (2015) for the Great Recession, Cole and Ohanian (2004) for the Great Depression, and Kaihatsu and Kurozumi (2014) for the lost decade of Japan). In this study, a temporary shock changes the economy persistently. We consider that an exogenous shock increases firms’ debt substantially, whereas there is no change in the structural parameters in production or financial technologies. The increase in debt tightens borrowing constraints persistently. The shock we consider can thus be understood as a redistribution shock, which redistributes wealth from borrowers to lenders as a one-time shock. One example of such a redistribution shock is the boom and burst of the asset-price bubble that changes the value of collateral for debt. We consider the debt dynamics in which the stock of debt is repaid in multiple or possibly infinite number of periods.

Our model is a version of those with endogenous borrowing constraints, in which a distinction between inter-period and intra-period loans is made in a similar way as in Jermann and Quadrini (2012). It is shown that the borrowing constraint on intra-period loans is binding, whereas the constraint on inter-period loans is nonbinding. The borrowing constraint binds tighter and production inefficiency is higher as the initial amount of inter-period debt is larger. As the borrowing constraint tightens, firms cannot raise sufficient intra-period debt for working capital and they continue inefficient production. When the initial debt reaches the maximum repayable amount, firms fall into a “debt-ridden” state in which production inefficiency stays highest permanently.

We embed the model with endogenous borrowing constraints into a general equilibrium setting, where the economy grows endogenously because of the productivity growth caused by the firms’ R&D activities. It is shown in the general equilibrium that borrowing constraints become tighter for normal firms when the measure of debt-ridden firms is larger. If a substantial number of firms become debt-ridden, both aggregate borrowing capacity and productivity decline persistently. After the Great Recession of 2007–2009, many authors argued that a shock in the financial sector can cause a severe recession (e.g., a risk shock in Christiano et al. 2014 and a financial shock in Jermann and Quadrini 2012). In our model, the emergence of a substantial number of debt-ridden firms manifests itself as a tightening of the aggregate borrowing constraint, which can be interpreted as a financial shock. Tighter borrowing constraints discourage R&D activity by firms and make productivity growth persistently low. They also diminish the labor wedge persistently. These features of our model seem to be consistent with the facts observed in persistent

recessions after financial crises (see Section 2).

Our contribution is to show that the buildup of debt can persistently tighten the borrowing constraint and thus cause aggregate inefficiency, even if there is no technological shock. This feature of our model implies a policy recommendation distinct from those of the models of exogenous financial shocks that debt restructuring or debt forgiveness for overly indebted borrowers may restore aggregate efficiency and enhance economic growth. If, on the contrary, the risk shock or financial shock were exogenous, debt restructuring would not have any aggregate effects.

Related literature.— Our theory is related to the literature on debt overhang. Myers (1977) pointed out the suboptimality of debt in the corporate finance literature and Lamont (1995) applied the notion of debt overhang in macroeconomics.¹ The debt overhang problem typically causes inefficiency in the short-run. In this study, inefficiency can continue permanently. This study is also close to Caballero et al. (2008). They analyzed “zombie lending,” defined as the provision of a de facto subsidy to unproductive firms from banks. They argued that congesting zombie firms hinder the entry of highly productive firms and lower aggregate productivity. In this study, we make a complementary point to their argument: even an intrinsically productive firm can become unproductive when it is debt-ridden. In the macroeconomic literature, endogenous borrowing constraints were introduced by the seminal work of Kiyotaki and Moore (1997).² Endogenous borrowing constraints in the economy where intra-period and inter-period loans exist are analyzed by Albuquerque and Hopenhayn (2004), Cooley et al. (2004), and Jermann and Quadrini (2006, 2007, 2012). The modeling method in this study is closest to that of Jermann and Quadrini (2012), while the difference is that they consider the borrowing constraint on the total loan, whereas we consider two distinct constraints on the inter-period and intra-period loans. Our model is also closely related to that of Kobayashi and Nakajima (2015), which analyzes endogenous borrowing constraints and nonperforming loans (NPLs). Persistent recession in the aftermath of a financial crisis is explained by Guerron-Quintana and Jinnai (2014). Our model is also close to theirs in that a temporary shock affects productivity growth persistently, although there is a stark difference in policy implications. In our model, the emergence of debt-ridden borrowers because of a temporary redistribution shock causes persistent recession. Thus, debt restructuring (i.e., wealth redistribution from creditors to borrowers) restores aggregate efficiency, whereas debt restructuring has no effect in Guerron-Quintana and Jinnai (2014) because the financial shock is an exogenous technological shock in their model. Another study closely related to ours is Ikeda and Kurozumi (2014). They built a medium scale DSGE model with financial friction

¹See also Krugman (1988) on debt overhang in international finance.

²Kobayashi and Shirai (2016) analyzed the effects of wealth redistribution on the economy by using a similar borrowing constraint model.

à la Jermann and Quadrini (2012) and endogenous productivity growth à la Comin and Gertler (2006). The distinction from ours is that Ikeda and Kurozumi (2014) also posited a financial crisis as an exogenous technological shock.

The remainder of this paper is organized as follows. In the next section, we review the facts on persistent recessions after financial crises. Section 3 presents the partial equilibrium model of the lender–borrower relationship and analyzes the debt dynamics. In Section 4, we construct the full model by embedding the model of the previous section into an endogenous growth model, showing that stagnation can continue persistently when a substantial number of debt-ridden borrowers emerge. Section 5 presents our concluding remarks.

2 Facts on persistent recessions after financial crises

Numerous examples of decade-long stagnation after a financial crisis have been observed. The most notable episode was the Great Depression in the 1930s in the United States and the similar depressions in that period in other major nations. Ohanian (2001) highlighted the large productivity decline during the US Great Depression that is unexplained by capital utilization or labor hoarding. Kehoe and Prescott (2007) drew our attention to the fact that many countries have experienced decade-long recessions, which they called the “great depressions” of the 20th century. The studies presented in their book unanimously emphasized that declines in the growth rate of TFP were the primary cause of these great depressions.

Another example of a decade-long recession was the 1990s in Japan. The growth rates of GDP and TFP in the 1990s were both lower than those in the 1980s. The kink at the beginning of the 1990s is apparent, when huge asset-price bubbles burst in the stock and real estate markets. Table 1 presents various estimates of the TFP growth rate in Japan. Hayashi and Prescott (2002) emphasized that growth in TFP slowed in the 1990s. Fukao and Miyagawa (2008) estimated TFP by using a microeconomic dataset called the Japan Industrial Productivity (JIP) database and confirmed the substantial TFP slowdown in the 1990s.

One notable feature in the 1990s in Japan was the significant decrease in entries and increase in exits of firms. Figure 1 compares the entry and exit of firms in Japan and the United States. In the literature, the procyclicality of net entry is well known (Bilbiie et al., 2012). Net entry also contributes significantly to TFP growth for US manufacturing establishments (Bartelsman and Doms, 2000).³

Another characteristic of Japan in the 1990s that may be related to such a productivity

³Nishimura et al. (2005) argued that the malfunctioning entries and exits contributed substantially to the fall in Japan’s TFP in the late 1990s.

	HP	KI	JIP2014
1971–1980	0.83		2.04
1981–1990	1.93	2.06	2.02
1991–2000	0.36	0.35	0.03
2001–2005		0.71	1.39
2006–2011			-0.28

Table 1: TFP growth rate in Japan

Note: HP, KI, and JIP2014 are from updated versions of Hayashi and Prescott (2002), Kobayashi and Inaba (2006), and Fukao and Miyagawa (2008), respectively.

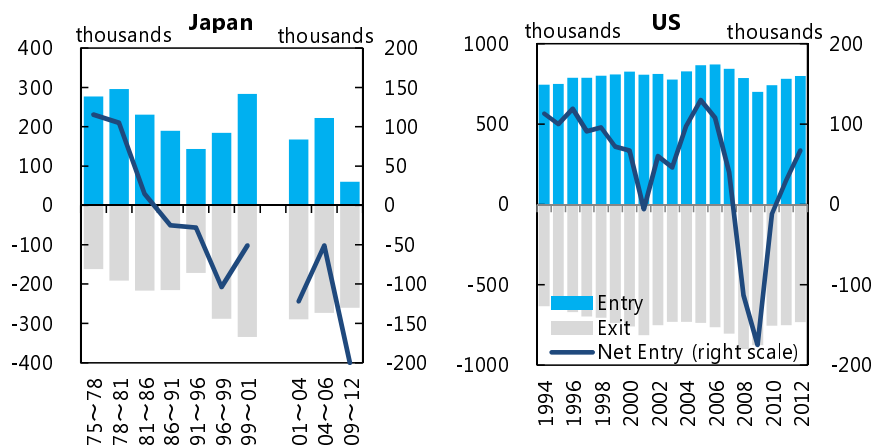


Figure 1: Entry and exit of private sector establishments: United States and Japan

Note: Japan’s figures after 2001 are based on 1993-basis industry classification.

Sources: (Japan) Ministry of Internal Affairs and Communications, “Establishment and Enterprise Census”; (US) Bureau of Labor Statistics, “Business Employment Dynamics.”

slowdown was the persistently lingering NPLs in the banking sector. NPLs represent the excess debt of nonfinancial firms, mainly in the real estate, wholesale, retail, and construction sectors. Figure 2 shows NPLs in Japan from 1992 to 2009. The delayed disposal of huge NPLs was seen as a de facto subsidy to nonviable firms (i.e., zombie lending). This zombie lending has also been considered to be the cause of Japan’s persistent recession (Peek and Rosengren, 2005; Caballero et al., 2008).

Recently, a growing literature on business cycle accounting (Chari et al., 2007) has analyzed various episodes of business fluctuations including decade-long stagnations. Business cycle accounting focuses on four wedges as the driving forces of business cycles: the efficiency wedge, labor wedge, investment wedge, and government wedge. The efficiency wedge is the observed TFP; labor wedge is MRS/MPL , where MRS is the marginal rate of substitution between consumption and leisure and MPL is the marginal product of labor; investment wedge is the wedge between the market rate of interest and stochastic discount

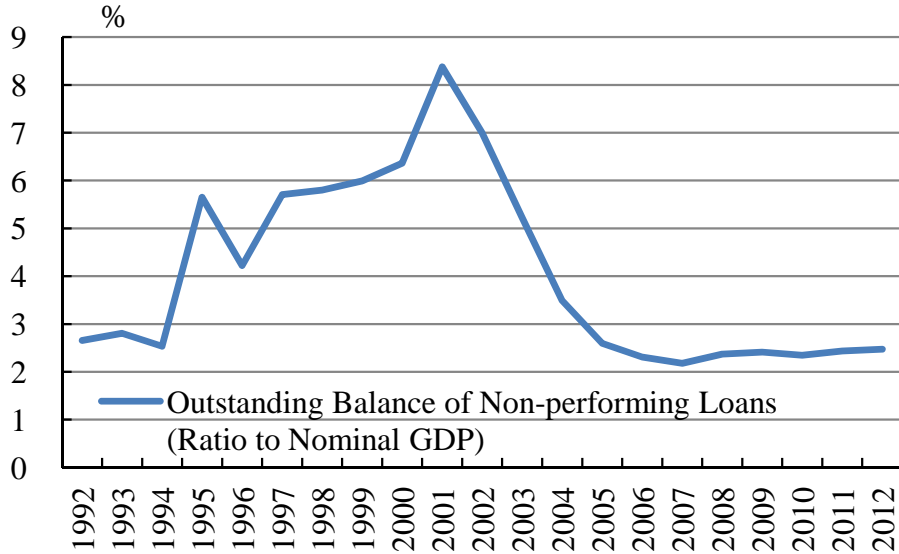


Figure 2: Development of NPLs

Note: The NPLs are the risk management loans (RMLs) defined in the Banking Act in Japan. These consist of loans to bankrupt borrowers, delayed loans, three-month overdue loans, and loans with modified terms and conditions. RMLs do not include securitized loans.

Sources: Financial Services Agency, *Status of Non-Performing Loans*; Cabinet Office, Government of Japan, *Annual Report on National Accounts*.

factor; and government wedge is the deadweight loss, which manifests itself as government consumption in a simple real business cycle model. Chari et al. (2007) noted that reductions in the efficiency wedge and labor wedge were the two primary factors that drove the Great Depression of the 1930s. Kobayashi and Inaba (2006) and Otsu (2011) emphasized the same factors for the lost decade of Japan in the 1990s. The macroeconomic literature has recently focused considerable attention on the effects of a reduction in the labor wedge in recessions (see Mulligan, 2002; Shimer, 2009). A sharp decline in the labor wedge was also observed in the US economy during the Great Recession of 2007–2009 (Kobayashi, 2011; Pescatori and Tasci, 2011).

3 Model of debt dynamics

In this section, we consider the partial equilibrium model of debt contracts. We derive the borrowing constraint and analyze the debt dynamics under the exogenously given prices. We then embed this model into the endogenous growth model in Section 4.

3.1 Setup

Time is discrete and continues from 0 to infinity: $t = 0, 1, 2, \dots, \infty$. There are four agents in this model: two banks (lenders), a firm (borrower), and a household (worker). The

main players are the banks and firm, and the household just supplies labor at the market wage rate, w_t , and buys consumer goods from the firm. The two banks play distinct roles: one bank makes the inter-period loan and the other bank makes the intra-period loan. The details of the loans are specified shortly. We call the former the “inter-period bank” and the latter the “intra-period bank.” Consumer goods are produced by the firm from the labor input. The firm’s gross revenue in period t is given by

$$F_t(k_{t-1}, l_t) = Al_t^\eta + (1 + r_t)k_{t-1},$$

where k_{t-1} is the capital stock purchased in period $t - 1$, l_t is the labor input chosen in period t , r_t is the rate of interest for a safe asset, A is a positive constant, and $0 < \eta < 1$. We focus on the case where there exists the initial debt stock R_0d_{-1} at $t = 0$, where d_{-1} is the amount of inter-period debt at the end of the previous period and R_t is the gross rate of corporate loans, which satisfies (2) below, in equilibrium.

Suppose that the firm holds capital stock k_t and owes debt d_t to the inter-period bank at the end of period t . At the end of period t , a firm dies with probability ρ . When the firm dies, all the remaining debt d_t is automatically defaulted and the remaining capital k_t is eaten by the firm (or, in other words, given to the firm’s owner). Debt evolves from period t to period $t + 1$ at the gross rate R_{t+1} . Thus, debt at the beginning of period $t + 1$ is $R_{t+1}d_t$.

In period $t + 1$, the survived firm employs labor l_{t+1} from the household to produce and sell consumer goods, and earns revenue $F_{t+1}(k_t, l_{t+1}) = Al_{t+1}^\eta + (1 + r_{t+1})k_t$. The cost of the labor input for the firm is $w_{t+1}l_{t+1}$, where w_{t+1} is the market wage rate. The firm needs to borrow $w_{t+1}l_{t+1}$ from the intra-period bank and pay the wage to the worker in advance of production. In addition, at this point the firm borrows b_{t+1} from the intra-period bank to pay the inter-period bank in order to fix the new inter-period debt at

$$d_{t+1} = R_{t+1}d_t - b_{t+1}. \quad (1)$$

Now, we assume that the prices $\{r_t, w_t\}_{t=0}^\infty$ are given. When the firm completes production, it holds revenue $Al_{t+1}^\eta + (1 + r_{t+1})k_t$ and owes inter-period debt d_{t+1} to the inter-period bank and intra-period debt $w_{t+1}l_{t+1} + b_{t+1}$ to the intra-period bank. Intra-period debt is to be repaid in the current period $t + 1$ and inter-period debt is to be carried over to the next period $t + 2$. The risk of sudden death imposes the following constraint on (b_{t+1}, d_{t+1}) because the inter-period bank agrees if and only if the expected repayment value is no less than d_t :

$$d_t \leq \frac{1 - \rho}{1 + r_{t+1}}(b_{t+1} + d_{t+1}),$$

which implies

$$R_{t+1} = \frac{1 + r_{t+1}}{1 - \rho}, \quad (2)$$

in equilibrium. After the new inter-period debt d_{t+1} is fixed, the firm pays $w_{t+1}l_{t+1}+b_{t+1}$ to the intra-period bank. Then, the remaining cash flow, $Al_{t+1}^\eta + (1+r_{t+1})k_t - w_{t+1}l_{t+1} - b_{t+1}$, is paid out as the investment in the new capital k_{t+1} and the dividend to the firm's owner π_{t+1} . Following Jermann and Quadrini (2012), we assume that the gross dividend payout, $\theta(\pi_{t+1})$, consists of the dividend π_{t+1} and adjustment cost, $\frac{1}{2\chi}(\pi_{t+1} - \bar{\pi})^2$:

$$\theta(\pi) = \pi + \frac{1}{2\chi}(\pi - \bar{\pi})^2,$$

where $\bar{\pi}$ is the target dividend. In the general equilibrium setting, in Section 4, we assume that the adjustment cost, $\frac{1}{2\chi}(\pi - \bar{\pi})^2$, is not a deadweight loss, but is given to the household as a lump-sum transfer. The budget constraint for the firm is given by

$$\theta(\pi_{t+1}) + k_{t+1} \leq F_{t+1}(k_t, l_{t+1}) - w_{t+1}l_{t+1} - b_{t+1}. \quad (3)$$

The payment of the intra-period loan $w_{t+1}l_{t+1}+b_{t+1}$ is subject to the borrowing constraint:

$$w_{t+1}l_{t+1} + b_{t+1} \leq \phi Al_{t+1}^\eta + \sigma(1+r_{t+1})k_t, \quad (4)$$

which is derived in the next subsection, where $0 \leq \phi \leq 1$ and $0 \leq \sigma \leq 1$.

Now, we can describe the optimization problem for the firm. Denoting the value of the firm with debt stock d_t and capital stock k_t by $V_t(d_t, k_t)$, the firm's problem is written as the following Bellman equation:

$$V_t(d_t, k_t) = \max_{l_{t+1}, b_{t+1}, k_{t+1}} \rho k_t + \frac{1-\rho}{1+r_{t+1}} [\pi_{t+1} + V_{t+1}(d_{t+1}, k_{t+1})], \quad (5)$$

subject to the budget constraint (3), the law of motion for debt (1), the borrowing constraint (4), and the limited liability constraint⁴

$$\pi_{t+1} \geq 0. \quad (6)$$

The usual arguments of dynamic programming ensure the existence of the value function $V_t(d_t, k_t)$.

3.2 Derivation of the borrowing constraint

In this subsection, we describe the counterfactual defaults and derive the borrowing constraint (4).

3.2.1 Counterfactual default on the inter-period debt

At the beginning of period $t+1$, the firm owes inter-period debt $R_{t+1}d_t$. At this point, the firm has a chance to default on $R_{t+1}d_t$. If the firm defaults, all the remaining assets

⁴Albuquerque and Hopenhayn (2004) called this the limited liability constraint because it implies the limited liability of the firm's owner.

$(1 + r_{t+1})k_t$ are seized by the inter-period bank and the firm obtains zero as the outside value. Hence, the payoff for the firm (or firm owner) when it defaults on the inter-period debt is zero. In what follows, we focus on the case where d_t is not too large so that $V_t(d_t, k_t)$ satisfies

$$V_t(d_t, k_t) > 0. \quad (7)$$

This inequality implies that the firm never chooses to default on the inter-period debt in equilibrium.

3.2.2 Counterfactual default on the intra-period debt

As described in the previous subsection, the firm borrows intra-period debt $w_{t+1}l_{t+1} + b_{t+1}$ in period $t + 1$. After the firm's revenue is realized, there arrives a chance to default on the intra-period debt. Note that the firm also owes $d_{t+1} = (1 + r_{t+1})d_t - b_{t+1}$ to the inter-period bank and can default only on the intra-period debt at this point. Once the firm defaults, the intra-period bank unilaterally seizes a proportion of the firm's revenue, $\phi Al_{t+1}^\eta + \sigma(1 + r_{t+1})k_t$, where $0 \leq \phi \leq 1$ and $0 \leq \sigma \leq 1$. The intra-period bank can impose no additional penalty on the defaulting firm. Thus, the intra-period bank makes the intra-period loan no greater than $\phi Al_{t+1}^\eta + \sigma(1 + r_{t+1})k_t$, implying the constraint (4). The defaulting firm can continue operating with inter-period debt d_{t+1} , which it owes to the inter-period bank. This assumption is used to show that (4) is also the no-default condition, as we see below. The payoff for the firm if it defaults is the solution to

$$\max_{\pi, k} \pi + V(k, d_{t+1}),$$

subject to

$$\theta(\pi) + k \leq (1 - \phi)Al_{t+1}^\eta + (1 - \sigma)(1 + r_{t+1})k_t,$$

$k \geq 0$, and $\pi \geq 0$. The firm's payoff if it does not default is the solution to

$$\max_{\pi, k} \pi + V(k, d_{t+1}),$$

subject to

$$\theta(\pi) + k \leq F_{t+1}(k_t, l_{t+1}) - w_{t+1}l_{t+1} - b_{t+1},$$

$k \geq 0$, and $\pi \geq 0$. The no-default condition is that the former is no greater than the latter, and implies inequality (4).

Note.— Endogenous borrowing constraints are usually formulated as the participation constraint for the borrower (e.g., Albuquerque and Hopenhayn, 2004), following the spirit of Kehoe and Levine (1993). The difference between the borrowing constraint in our model and in the Albuquerque–Hopenhayn model is as follows. The borrowing constraint in the Albuquerque–Hopenhayn model is derived from the participation constraint with

respect to total debt (i.e., the sum of intra- and inter-period debt), while in our model we distinguish between the participation constraint with respect to the intra-period debt and that with respect to the inter-period debt. In our model, the former provides the borrowing constraint, whereas the latter is basically nonbinding. The difference between the two constraints is caused by the difference in financial technology between the inter-period and intra-period banks. That is, the inter-period bank can destroy all the firm's future dividends when it defaults, whereas the intra-period bank can only seize collateral and cannot impose any further penalty on the defaulter. This technological difference seems a realistic setting that reflects, for example, the differences in organizational structures and agency problems in short-term and long-term lenders in reality.

3.3 Characterization of the firm with no debt

The optimization problem for a firm with no debt stock is as follows:

$$V_{nt}(k_t) = \max_{k_{t+1}, l_{t+1}} \rho k_t + \frac{1 - \rho}{1 + r_{t+1}} [\pi_{t+1} + V_{nt+1}(k_{t+1})],$$

$$\text{s.t. } \begin{cases} \theta(\pi_{t+1}) + k_{t+1} \leq F_{t+1}(k_t, l_{t+1}) - w_{t+1}l_{t+1}, \\ w_{t+1}l_{t+1} \leq \phi Al_{t+1}^\eta + \sigma(1 + r_{t+1})k_t. \end{cases}$$

It is easily shown that when the borrowing constraint is nonbinding for all $t \geq 0$, production is efficient:

$$l_{t+1} = l_{nt+1} \equiv \left[\frac{\eta A}{w_t} \right]^{\frac{1}{1-\eta}},$$

and there is no adjustment cost in the dividend payout: $\theta'(\pi_{t+1}) = 1$, which implies $\pi_{t+1} = \bar{\pi}$.⁵

3.4 Debt dynamics and the emergence of debt-ridden firms

We derive the debt dynamics in the case where prices are constant: $w_t = w$ and $r_t = r$ for all t . The results in this section can be easily generalized in the case that prices vary over time. Under time-invariant prices and the nonbinding borrowing constraint, the variables for a firm with no debt are given as follows:

$$l_n = \left[\frac{\eta A}{w} \right]^{\frac{1}{1-\eta}},$$

$$k_n = \frac{\bar{\pi} - (1 - \eta)Al_n^\eta}{r},$$

$$\pi_n = \bar{\pi},$$

$$V_n(k_n) = k_n + \frac{(1 - \rho)(1 - \eta)}{r + \rho} Al_n^\eta.$$

⁵In the numerical example of the general equilibrium model that we analyze in Section 4, the borrowing constraint binds on the balanced growth path (BGP). This is because in the general equilibrium model, firms conduct R&D activity in addition to production, which is subject to the borrowing constraint.

Here, we assume $(1 - \eta)Al_n^\eta \leq \bar{\pi}$. We also assume that the parameters are chosen such that the borrowing constraint is strictly nonbinding:

$$wl_n < \phi Al_n^\eta + \sigma(1 + r)k_n.$$

We analyze the response of the economy to the exogenously given initial debt: $d_{-1} = d > 0$. We assume that the initial capital stock is k_n in all cases that we consider in this section.

$$k_{-1} = k_n.$$

The problem for the firm is written as

$$V(d, k) = \max_{b, k_{+1}, l, \pi} \rho k + \frac{1 - \rho}{1 + r_{t+1}} [\pi + V(d_{+1}, k_{+1})], \quad (8)$$

$$\text{s.t.} \begin{cases} \theta(\pi) + k_{+1} \leq (1 + r_{t+1})k + A_{t+1}l^\eta - w_{t+1}l - b, \\ w_{t+1}l + b \leq \phi A_{t+1}l^\eta + \sigma(1 + r_{t+1})k, \\ \pi \geq 0, \\ d_{+1} = \frac{1+r_{t+1}}{1-\rho}d - b, \\ b \geq 0. \end{cases}$$

where $(r_{t+1}, w_{t+1}, A_{t+1}) = (r, w, A)$.

3.4.1 Debt dynamics with the initial debt d

In this model, we can show that inefficiency is inevitable when there exists a positive amount of initial debt $d_{-1} = d > 0$.

Lemma 1. *We consider that there exists a positive amount of initial debt $d > 0$ and focus on the equilibrium path that converges to a steady state eventually. Then, there exists $t \geq 0$ such that $\pi_t < \bar{\pi}$ and/or $l_t < l_n$.*

Proof. See Appendix B □

Lemma 2. *Suppose that the initial debt d is positive, $d > 0$, and that the equilibrium path converges to a steady state: $\pi_t \rightarrow \pi_\infty$ and $l_t \rightarrow l_\infty$. Then, $\pi_\infty < \bar{\pi}$ and $l_\infty < l_n$.*

Proof. See Appendix B □

This lemma implies that if there exists the initial debt $d > 0$, then the borrowing constraint binds and production becomes inefficient eventually. When the initial amount of debt is larger, the borrowing constraint binds tighter and production inefficiency is higher in the steady state. This implication that even a small amount of debt has a negative effect permanently is an artifact due to the specific form of the borrowing constraint for intra-period loans. The model can be modified with additional complications in such a way that debt is fully repaid and efficiency is restored in a few periods if the initial debt is

small.⁶ Nevertheless, we stick to the current form of the model for simplicity because we focus on the case where the borrower owes the maximum repayable debt and the borrowing constraint in the modified model ends up in (4) when d is sufficiently large.

3.4.2 Debt-ridden firms and persistence of inefficiency

We define a *debt-ridden firm* as one with the maximum repayable initial debt. Define d_z and l_z by

$$d_z = \frac{(1-\rho)(1-\eta)}{r+\rho} \left\{ \phi + \frac{(1-\rho)(1-\phi)\sigma}{1-(1-\rho)(1-\sigma)} \right\} Al_z^\eta + \Omega_1 k_n - \Omega_2 \theta(0),$$

$$l_z = \left[\left\{ \phi + \frac{(1-\rho)(1-\phi)\sigma}{1-(1-\rho)(1-\sigma)} \right\} \frac{\eta A}{w} \right]^{\frac{1}{1-\eta}},$$

where the derivation of d_z and l_z and the values of Ω_1 and Ω_2 are given in Appendix A. d_z is the maximum repayable debt by the firm.⁷ When a firm becomes debt-ridden, debt stays high at $d_t = d_z$, and production stays inefficient, i.e., $l_t = l_z$, forever. The dividend is zero for the debt-ridden firm:

$$\pi_t = 0, \quad \forall t \geq 0.$$

It is shown that

$$l_z < l_n,$$

if either $\sigma = 0$ or $\rho > 0$. We assume $\sigma > 0$ and $\rho > 0$ in what follows.⁸ Note also that the production inefficiency ($l_t = l_z < l_n$) is not a decline in the borrower's productivity, but rather a reduction in the labor wedge at the firm level.⁹ In the next section, we see that this inefficiency causes a slowdown in productivity growth at the aggregate level.

Our result that inefficiency due to debt stock can continue indefinitely is in stark contrast to the existing literature on financial frictions. In standard models such as Carlstrom and Fuerst (1997) and Bernanke et al. (1999), financial frictions have negative effects only

⁶Suppose that the intra-period bank can liquidate the defaulter and recover the liquidation value, $V_L > 0$, and the inter-period debt d_{t+1} is senior to the intra-period debt. In this case, the similar argument as Jermann and Quadrini (2012) implies that the intra-period borrowing constraint is

$$w_{t+1}l_{t+1} + b_{t+1} \leq \phi Al_{t+1}^\eta + \sigma(1+r_{t+1})k_t + \max \{V_L - d_{t+1}, 0\},$$

instead of (4). With this modified borrowing constraint, it is shown that the debt, d , is fully repaid in a finite period if $d < V_L$.

⁷If the initial debt d is larger than d_z , the bank has no other choice than to write off the excess, $d - d_z$, in this model. It is interesting to analyze the debt dynamics when the bank is allowed to evergreen the debt $d (> d_z)$. Kobayashi and Nakajima (2015) analyzed such an evergreening of debt.

⁸As k_t is used as collateral for the borrowing, k_t relaxes the borrowing constraint if $\sigma > 0$. When $\sigma > 0$ and $\rho > 0$, k_t is not used as collateral with probability ρ . This failure also causes inefficiency.

⁹The labor wedge, $1 - \tau$, can be defined as $1 - \tau = \frac{w}{\eta Al_t^\eta} = \frac{w}{\eta A} l_t^{1-\eta}$ at the firm level. See also footnote 12. The labor wedge reduces as l_t changes from l_n to l_z .

temporarily. In our model, a debt stock has a negative effect on the output of the borrower potentially indefinitely. The persistent effects of debt may enrich the macroeconomic analysis, as we see in the next section.

4 Full model

Now, we embed the partial equilibrium model of the previous section into a general equilibrium model. We consider a closed economy in which the final good is produced competitively from capital input and from varieties of intermediate goods. The firms are monopolistic competitors and they produce their respective varieties of intermediate goods from the labor input. The model is a version of the expanding variety model, in which the new entry of firms increases aggregate productivity (Rivera-Batiz and Romer 1991, Acemoglu 2009). We follow Benassy (1998) in that labor is used to produce the intermediate goods as well as to conduct R&D activities that expand the variety. We assume that the monopolistically competitive firms, which are subject to borrowing constraints, produce the intermediate goods as well as conduct R&D.

4.1 Basic setup

A representative household owns a mass of firms, indexed by $i \in [0, N_t]$, that produce intermediate goods, where N_t is the measure of the varieties of intermediate goods in period t . Firm i produces the variety i monopolistically, and can borrow funds from the bank, which is also owned by the representative household. In what follows, we omit the bank for simplicity and consider the household as the lender. The final good is produced competitively from the intermediate goods x_{it} , $i \in [0, N_t]$, and capital by the following production function:

$$Y_t = \left(\int_0^{N_t} x_{it}^\eta di \right)^{\frac{\alpha}{\eta}} K_{t-1}^{1-\alpha},$$

where $0 < \alpha < 1$ and $0 < \eta < 1$. Because the final good producer maximizes $Y_t - \int_0^{N_t} p_{it} x_{it} di - r_t^K K_{t-1}$, where p_{it} is the real price of the intermediate good i and r_t^K is the rental rate of capital, perfect competition in the final good market implies that

$$\begin{aligned} r_t^K &= (1 - \alpha) \frac{Y_t}{K_{t-1}}, \\ p_{it} &= p(x_{it}) = A_t x_{it}^{\eta-1}, \end{aligned}$$

where

$$A_t \equiv \alpha Y_t^{1-\frac{\eta}{\alpha}} K_{t-1}^{\frac{\eta}{\alpha}-\eta}.$$

Firm i produces the intermediate good i from labor input l_{it} by the following production function:

$$x_{it} = l_{it}.$$

Firm i , where $0 \leq i \leq N_t$, chooses $x_{it}(= l_{it})$ to obtain revenue $p(x_{it})x_{it}$ and pay wages $w_t l_{it}$, where w_t is the wage rate. In this subsection, we only consider the case where firms do not owe any debt.¹⁰ Each firm i employs labor h_{it} , produces intermediate goods $x_{it} = l_{it}$, and conducts R&D with input $h_{it} - l_{it} (\geq 0)$. The labor input $h_{it} - l_{it}$ in the R&D activity creates $\kappa \bar{N}_t \{h_{it} - l_{it}\}$ units of new varieties of intermediate goods, where \bar{N}_t is the social level of the variety, which represents the externality from the stock of knowledge on the R&D activity. This externality ensures the existence of the balanced growth path (BGP). When a new variety is created, a new monopolistic firm that produces the variety is also born. Each new variety is produced by the newborn firm. A newborn firm is given capital stock k_{It} by the parent firm that created the new firm. The parent firm treats newborn firms as members of its own dynasty and decides k_{It} to maximize the value of its dynasty. The ρ proportion of the varieties of intermediate goods and corresponding firms die every period.

The value of the firm is determined by

$$V_{nt}(k_t) = \max_{\substack{h_{t+1}, l_{t+1}, \pi_{t+1}, \\ k_{t+1}, k_{It+1}}} \rho k_t + \frac{1 - \rho}{1 + r_{t+1}} [\pi_{t+1} + V_{nt+1}(k_{t+1}) + \kappa \bar{N}_{t+1} \{h_{t+1} - l_{t+1}\} V_{nt+1}(k_{It+1})], \quad (9)$$

$$\text{s.t.} \quad \begin{cases} \theta_{t+1}(\pi_{t+1}) + k_{t+1} \leq F_{t+1}(k_t, l_{t+1}) - w_{t+1} h_{t+1} - \kappa \bar{N}_{t+1} \{h_{t+1} - l_{t+1}\} k_{It+1}, \\ w_{t+1} h_{t+1} \leq \phi A_{t+1} l_{t+1}^\eta + \sigma(1 + r_{t+1}) k_t, \end{cases}$$

where

$$F_{t+1}(k_t, l_{t+1}) = (1 + r_{t+1})k_t + A_{t+1} l_{t+1}^\eta;$$

and r_{t+1} is the market rate of interest for safe assets. The gross dividend, $\theta_t(\pi_t)$, is defined by

$$\theta_t(\pi_t) = \pi_t + \frac{1}{2\chi_t} (\pi_t - \bar{\pi}_t)^2,$$

where the second term is the adjustment cost transferred to the household as a lump sum. The FOCs imply that the initial capital stock k_{It+1} for a newborn firm in period t is equal to k_{t+1} :

$$k_{It+1} = k_{t+1},$$

and the values of the parent and newborn firms are identical (i.e., $V_{nt+1}(k_{t+1})$).

¹⁰In Section 4.3 we consider the case where some firms with measure Z_t have the maximum repayable debt, meaning that they are debt-ridden, whereas others with measure $N_t - Z_t$ have no debt.

A representative household solves the following problem:

$$\max_{C_t, H_t, K_t^C} \sum_{t=0}^{\infty} \beta^t [\ln C_t + \gamma \ln(1 - H_t)] \quad (10)$$

subject to the budget constraint

$$C_t + K_t^C - (1 - \delta)K_{t-1}^C \leq w_t H_t + r_t^K K_{t-1}^C + \int_0^{N_t} \pi_{it} di + T_t, \quad (11)$$

where β is the subjective discount factor, C_t is consumption, H_t is total labor supply, K_t^C is capital stock, δ is the depreciation rate of capital, π_{it} is the dividend from firm $i \in [0, N_t]$, and T_t is a lump-sum transfer from the firms, which represents the adjustment cost of the dividend and transfer associated with the deaths of firms. Note that K_t^C is less than total capital stock in this economy, K_t , because the firms hold some capital stock. Let λ_t be the Lagrange multiplier associated with the budget constraint for the representative household, which is given by the FOC with respect to C_t :

$$\lambda_t = \frac{1}{C_t}.$$

The FOC with respect to K_t^C and arbitrage between K_t^C and a safe asset imply

$$1 + r_{t+1} = r_{t+1}^K + 1 - \delta = \frac{\lambda_t}{\beta \lambda_{t+1}} = \frac{C_{t+1}}{\beta C_t}.$$

The market clearing conditions are

$$\begin{aligned} K_t &= K_t^C + \int_0^{N_{t+1}} k_{nt} di, \\ C_t + K_t - (1 - \delta)K_{t-1} &= Y_t, \\ \int_0^{N_t} h_t di &= N_t h_t = H_t, \\ \int_0^{N_t} l_t di &= N_t l_t = L_t. \end{aligned}$$

The law of motion for varieties is

$$N_{t+1} = (1 - \rho)[N_t + \kappa \bar{N}_t (H_t - L_t)].$$

The equilibrium conditions are

$$\begin{aligned} \bar{N}_t &= N_t, \\ T_t &= \int_0^{N_t} \{\theta(\pi_{it}) - \pi_{it}\} di + [N_t + \kappa \bar{N}_t (H_t - L_t)] \rho k_{nt}. \end{aligned} \quad (12)$$

4.2 BGP without debt-ridden firms

In this subsection, we characterize the BGP where firms do not owe intertemporal debt.

Competitive equilibrium.— A competitive equilibrium where firms do not owe debt consists of sequences of prices $\{r_t, r_t^K, w_t\}$, household's decisions $\{C_t, H_t, K_t^c\}$, firms' decisions $\{\pi_t, h_t, l_t, k_{nt}\}$, aggregate capital stock K_t , and measure of varieties N_t such that (i) the representative household and firms solve their respective optimization problems, taking prices as given; and (ii) the market clearing conditions, law of motion for varieties, and equilibrium conditions are all satisfied.

On the BGP, labor and the growth rate are constant: $H_t = H$ and $L_t = L$ and $N_{t+1}/N_t = g$. We define g_Y by

$$g_Y = g^{\frac{1-\eta}{\eta}}.$$

We guess that $Y_t = Y \times N_t^{(1-\eta)/\eta}$, $C_t = C \times N_t^{(1-\eta)/\eta}$, $K_t = K \times N_t^{(1-\eta)/\eta}$, $w_t = w \times N_t^{(1-\eta)/\eta}$, $h_t = H/N_t$, $l_t = L/N_t$, $V_t = V \times N_t^{(1-2\eta)/\eta}$, $\pi_t = \pi N_t^{(1-2\eta)/\eta}$, $\theta_t(\pi_t) = \theta(\pi) N_t^{(1-2\eta)/\eta}$, $\chi_t = \chi N_t^{(1-2\eta)/\eta}$, $\bar{\pi}_t = \bar{\pi} N_t^{(1-2\eta)/\eta}$, and $0 < \bar{\pi} \leq \chi$, where

$$\theta(\pi) = \pi + \frac{1}{2\chi}(\pi - \bar{\pi})^2.$$

The FOCs and constraints imply that there exists the unique BGP, which is given in Appendix C.

For the numerical simulation, we set the parameter values, which are given in Table 2, according to the method described in Appendix F. Note that we set $\beta = 0.98$ as it is an annual model.¹¹ This set of parameter values implies that the borrowing constraint binds on the BGP, that is, the value of the Lagrange multiplier associated with the borrowing constraint, μ , is positive on the BGP: $\mu = 0.1$.

4.3 Low growth equilibrium with debt-ridden firms

Now, we consider the equilibrium where some firms owe the maximum repayable debt to the representative household. We assume that firms $i \in [0, Z_t]$ have identical debt stock d_t and firms $i \in (Z_t, N_t]$ have no debt. Owing to symmetry, we can assume without loss of generality that all firms with debt choose an identical labor input $l_{it} = l_{zt}$ and repayment $b_{it} = b_{zt}$ for $i \in [0, Z_t]$ and that all firms with no debt also choose an identical labor input $l_{it} = l_{nt}$ for $i \in (Z_t, N_t]$. We assume that the initial value of the measure of debt-ridden firms, Z_0 , is given exogenously and Z_t evolves by

$$Z_t = (1 - \rho)Z_{t-1}, \quad \text{for } t \geq 1.$$

¹¹Hayashi and Prescott (2002) estimated $\beta = 0.976$ and Sugo and Ueda (2008) used $\beta = 0.98$ for the annual discount rate of the Japanese economy.

Common parameters

Parameter	Economic interpretation	value
β	the subjective discount factor	0.98
δ	the depreciation rate of capital	0.06
η	the parameter for the aggregation function	0.7
ρ	the exit rate	0.06

Country-specific parameters

Parameter	Economic interpretation	Japan	United States	EU
α	the share of labor in production	0.69	0.67	0.63
χ	the inverse of the adjustment cost of dividends	0.018	0.018	0.025
γ	the inverse of the elasticity of labor supply	1.58	2.01	2.08
κ	the efficiency of R&D	1.47	1.67	1.51
ϕ	the collateral ratio of revenue	0.95	0.94	0.93
$\bar{\pi}$	the target level of dividends	0.018	0.018	0.025
σ	the collateral ratio of safe assets	0.11	0.15	0.18
H	total labor supply at the steady state	0.36	0.30	0.28
g_{TFP}	the growth rate of TFP at the steady state	1.020	1.017	1.011
z_{10}	the ratio of debt-ridden firms in period 10	0.560	0.504	0.478

Table 2: Parameter settings

In this case, the representative household solves (10) subject to the following budget constraint

$$C_t + K_t^C - (1 - \delta)K_{t-1}^C \leq w_t H_t + r_t^K K_{t-1}^C + \int_0^{Z_t} \pi_{zt} di + \int_{Z_t}^{N_t} \pi_{nt} di + \int_0^{Z_t} b_{zt} di + T_t,$$

instead of (11). The borrowing constraint for a firm with debt stock d_t and capital stock k_t is derived from the same argument as presented in Section 3:

$$w_{t+1} h_{t+1} + b_{t+1} \leq \phi A_{t+1} l_{t+1}^\eta + \frac{\lambda_t}{\beta \lambda_{t+1}} k_t,$$

where $d_t = \frac{(1-\rho)\beta\lambda_{t+1}}{\lambda_t}(b_{t+1} + d_{t+1})$. The problem for the owner of a debt-ridden firm, who owes the maximum repayable debt, d_{zt} , is given as follows:

$$V_{zt}(d_{zt}, k_{zt}) = \max_{\substack{\pi_{zt+1}, h_{zt+1}, l_{zt+1}, \\ b_{zt+1}, k_{zt+1}, k_{zIt+1}}} \rho k_{zt} + \frac{(1-\rho)\beta\lambda_{t+1}}{\lambda_t} [\pi_{zt+1} + V_{zt+1}(d_{zt+1}, k_{zt+1})] \quad (13) \\ + \kappa \bar{N}_{t+1} \{h_{zt+1} - l_{zt+1}\} V_{nt+1}(k_{zIt+1}),$$

subject to

$$\left\{ \begin{array}{l} b_{zt+1} = \frac{\lambda_t}{(1-\rho)\beta\lambda_{t+1}}d_{zt} - d_{zt+1}, \\ \theta_{t+1}(\pi_{zt+1}) + k_{zt+1} \leq F_{t+1}(k_{zt}, l_{zt+1}) - w_{t+1}h_{zt+1} - b_{zt+1} \\ \quad - \kappa\bar{N}_{t+1}\{h_{zt+1} - l_{zt+1}\}k_{zt+1}, \\ w_{t+1}h_{zt+1} + b_{zt+1} \leq \phi A_{t+1}l_{zt+1}^\eta + \sigma \frac{\lambda_t}{\beta\lambda_{t+1}}k_{zt}, \\ l_{zt+1} \leq h_{zt+1}, \\ \pi_{zt+1} \geq 0. \end{array} \right.$$

We choose parameter values such that (D3) in Appendix D is satisfied so that $h_{zt} = l_{zt}$ when d_{zt} is the maximum repayable debt. Given that $h_{zt} = l_{zt}$, d_{zt} is given as the solution to the following Ramsey problem for the lender.

$$d_{zt} = \max_{b_{zt+1}, k_{zt+1}, l_{zt+1}} \frac{(1-\rho)\beta C_t}{C_{t+1}} [b_{zt+1} + d_{zt+1}],$$

subject to

$$\left\{ \begin{array}{l} \theta(\pi_{zt+1}) + k_{zt+1} \leq F_{t+1}(k_{zt}, l_{zt+1}) - w_{t+1}l_{zt+1} - b_{zt+1}, \\ w_{t+1}l_{zt+1} + b_{zt+1} \leq \phi A_{t+1}l_{zt+1}^\eta + \sigma \frac{C_{t+1}}{\beta C_t}k_{zt}, \\ \pi_{zt+1} \geq 0. \end{array} \right.$$

The following solution is derived from a similar argument to that in Appendix A:

$$l_{zt} = x_{zt} = \left\{ \left[\phi + \frac{(1-\rho)(1-\phi)\sigma}{1-(1-\rho)(1-\sigma)} \right] \eta \frac{A_t}{w_t} \right\}^{\frac{1}{1-\eta}},$$

$$\pi_{zt} = 0.$$

The normal firms solve (9).

Competitive equilibrium.— A competitive equilibrium with normal and debt-ridden firms consists of sequences of prices $\{r_t, r_t^K, w_t\}$, household's decisions $\{C_t, H_t, K_t^c\}$, firms' decisions $\{\pi_{nt}, \pi_{zt}, b_{zt}, h_{nt}, h_{zt}, l_{nt}, k_{nt}, k_{zt}\}$, the aggregate capital stock K_t , and measures of varieties $\{N_t, Z_t\}$ such that (i) the representative household and normal and debt-ridden firms solve their respective optimization problems, taking prices as given; and (ii) the market clearing conditions, laws of motion for varieties, and equilibrium conditions are all satisfied, where one of the equilibrium conditions (12) is replaced to

$$T_t = \int_0^{N_t} \{\theta(\pi_{it}) - \pi_{it}\} di + [N_t - Z_t + \kappa\bar{N}_t(H_t - L_t)]\rho k_{nt} + Z_t\rho k_{zt}.$$

Numerical experiment: We can calculate the equilibrium dynamics numerically by using a full nonlinear method. Linearization is not necessary for the deterministic simulation (see Appendix D for detrending and Appendix E for the details of the dynamics). Figure 3 shows the results of the numerical simulation in which the economy is initially on the BGP, where $Z_t = 0$, and an unexpected redistribution shock hits the economy in period 10 that makes $z_{10} = 0.56$, where z_t is defined by $z_t \equiv Z_t/N_t$. That is, 56% of all

firms become debt-ridden in period 10. The parameter values are given as those for Japan in Table 2 and calibrated in Appendix F. The features of the equilibrium path shown in Figure 3 are as follows:

- Productivity slowdown: Borrowing constraints are tighter not only for debt-ridden firms but also for normal firms after the buildup of debt. Thus, the aggregate labor input for R&D is lowered and productivity growth slows for an extended period.
- Decrease in net entry: The growth rate of the number of firms, $g_t = N_{t+1}/N_t$, decreases. This feature is consistent with observations on the entry and exit of firms in Japan in the 1990s.
- Buildup of NPLs: In this example, there are Z_t debt-ridden firms and their debt stays at an inefficiently high level. This feature is consistent with the historical episodes of persistent stagnation with overly indebted firms and/or households, such as Japan in the 1990s.
- Labor-wedge reduction: In this example, the labor wedge, $1 - \tau$, diminishes persistently as a direct consequence of the tightening of the aggregate borrowing constraint on working capital loans for wage payments. This tighter borrowing constraint creates a larger gap between the wage rate and marginal product of labor. This gap is measured by τ . In this way, the persistent reduction in the labor wedge observed in the aftermath of a financial crisis can be accounted for by the emergence of debt-ridden firms.¹²

Note that immediately after the exogenous shock hits the economy, the labor reallocation from the R&D sector to the production sector results in a spike in L that causes spikes in the macroeconomic variables. This labor reallocation is caused by the buildup of debt that makes the debt-ridden firms unable to conduct R&D. The counterfactual spikes may lower if realistic frictions such as the imperfect mobility of labor are introduced into our model.

¹² As Chari et al. (2007) posited, the labor wedge, $1 - \tau_t$, is defined by $1 - \tau_t = \frac{MRS_t}{MPL_t}$, where $MRS_t = \frac{\gamma C_t}{1 - H_t} = w_t$ and $MPL_t = \frac{\alpha Y_t}{H_t}$ in our model. Thus, the labor wedge can be calculated by $1 - \tau_t = \frac{w_t H_t}{\alpha Y_t}$. In our model, the labor wedge $1 - \tau$ is proportional to the labor share. Thus, both the productivity slowdown and the shrinkage of labor share because of the buildup of debt are observed simultaneously in our model. This feature of our model is in stark contrast to the countercyclicality of the labor share in business cycle frequencies (Schneider, 2011). Our model seems, however, compatible with countercyclicality in the short run. In our model, the buildup of debt causes the long-term variations in the labor wedge, whereas short-run countercyclicality can be caused by factors such as productivity shocks and redistributive shocks in the business cycle frequencies (Ríos-Rull and Santaella-Llopis, 2010).

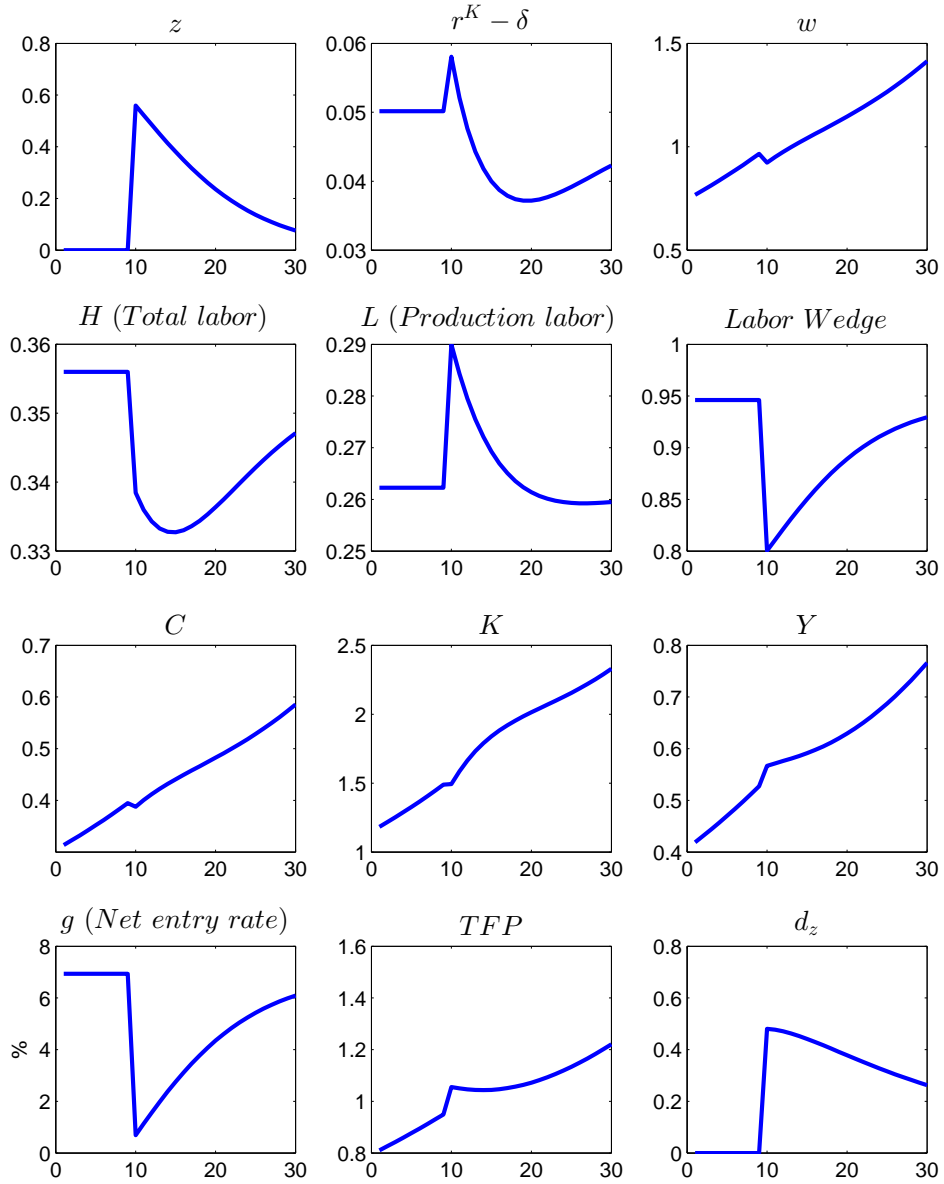


Figure 3: Responses to a buildup of debt (Japan)

Next, we calibrate and conduct numerical simulations for the United States and European Union (EU).¹³ The parameter values are also given in Table 2. Figure 4 compares the observed TFP of the numerical experiment with the actual TFP in Japan, the United States, and the EU. Similarly, Figure 5 compares the output in the numerical experiment with actual GDP. We assume that the unexpected shock hits the economy in period 10 of the simulation, which corresponds to the asset-price bubble collapse in 1991 in Japan and the housing-price collapse in 2006 in the United States. The figures show that the model fits the data fairly well. The conspicuous spikes in the growth rates of TFP and output are caused by labor reallocations from R&D to production, whereas they could have been mitigated if we assumed realistic rigidities in labor reallocations.

4.4 Policy implications

The policy implications of the results presented in this paper have significant importance from a practical point of view. The shocks that cause persistent stagnations are considered to be exogenous in the existing literature. In our model, the one-time buildup of debt tightens the borrowing constraint and causes a persistent slowdown in economic growth. Thus, our model implies that reducing overly accumulated debt can restore economic growth. As described in Sections 3 and 4, lenders are content to keep borrowers debt-ridden forever. Thus, policy interventions by the government that foster wealth redistribution from lenders to borrowers can improve social welfare. The policy measures may include regulatory reforms to make bankruptcy procedures less costly and debt-for-equity swaps easier as well as subsidies to banks that forgive debt and write off NPLs. This policy implication is straightforward and robust in our model and seems reasonable from our experience of recent financial crises, whereas existing models may not clearly imply that the reduction in excessively accumulated debt is good for a crisis-hit economy.

5 Conclusion

Decade-long recessions with low productivity growth are often observed after financial crises. In particular, the “secular stagnation” hypothesis has drawn much research attention since the Great Recession. In this paper, we hypothesized that the emergence of debt-ridden borrowers causes a persistent productivity slowdown. Economic agents become overly indebted, sometimes for reasons such as the boom and burst of asset-price bubbles. By analyzing the endogenous borrowing constraint, we showed that borrowers

¹³The EU comprises the following 28 countries: Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, the Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, and the United Kingdom.

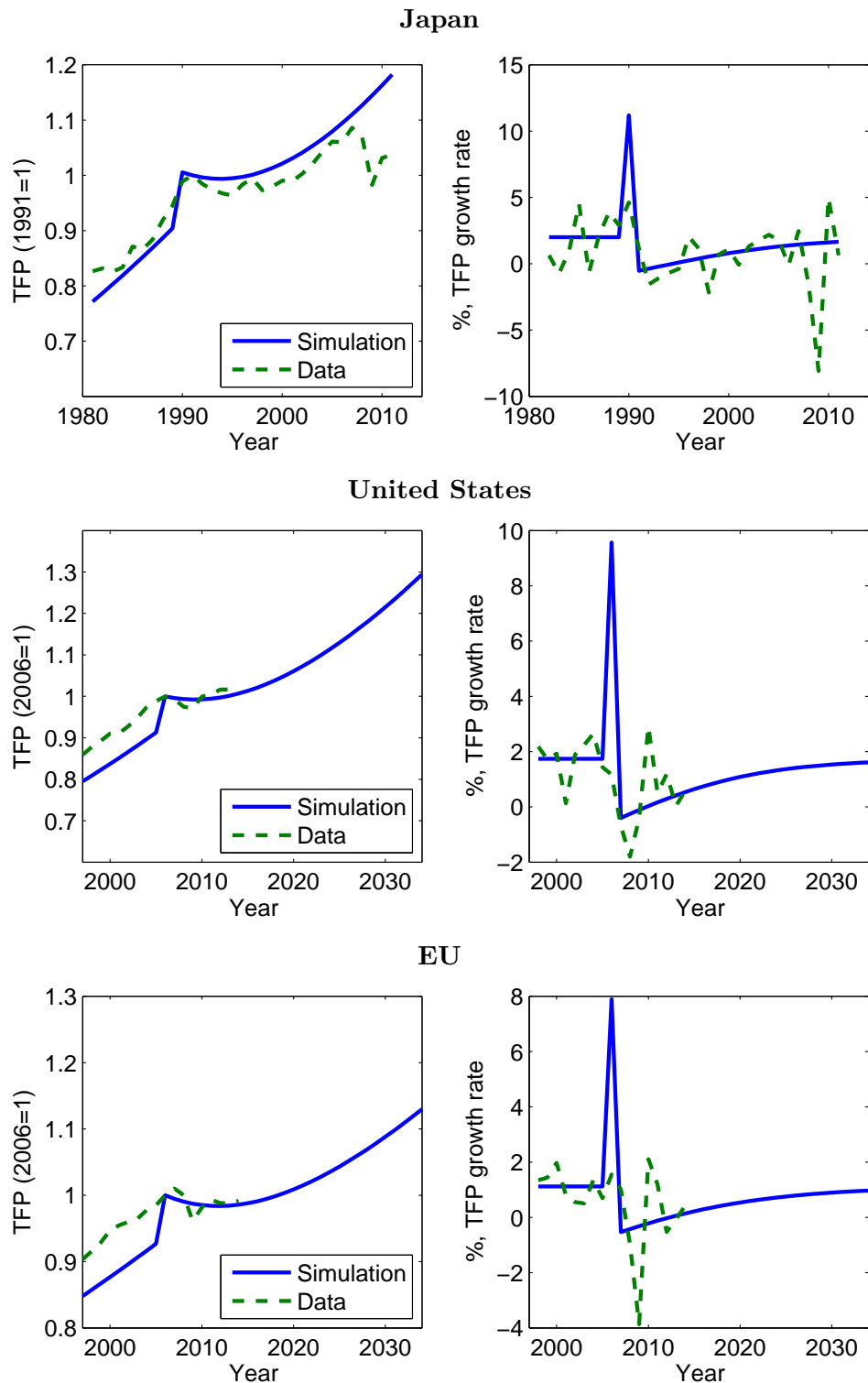


Figure 4: TFP for Japan, the United States, and the EU: Comparison between the data and simulation

Note: The classification of TFP in Japan is the “market economy” sectors that excludes education, medical services, government activities, and imputed house rent.

Sources: Our calculation; The Research Institute of Economy, Trade and Industry, *JIP 2014 database*; Fernald (2012); European Commission, *AMECO*

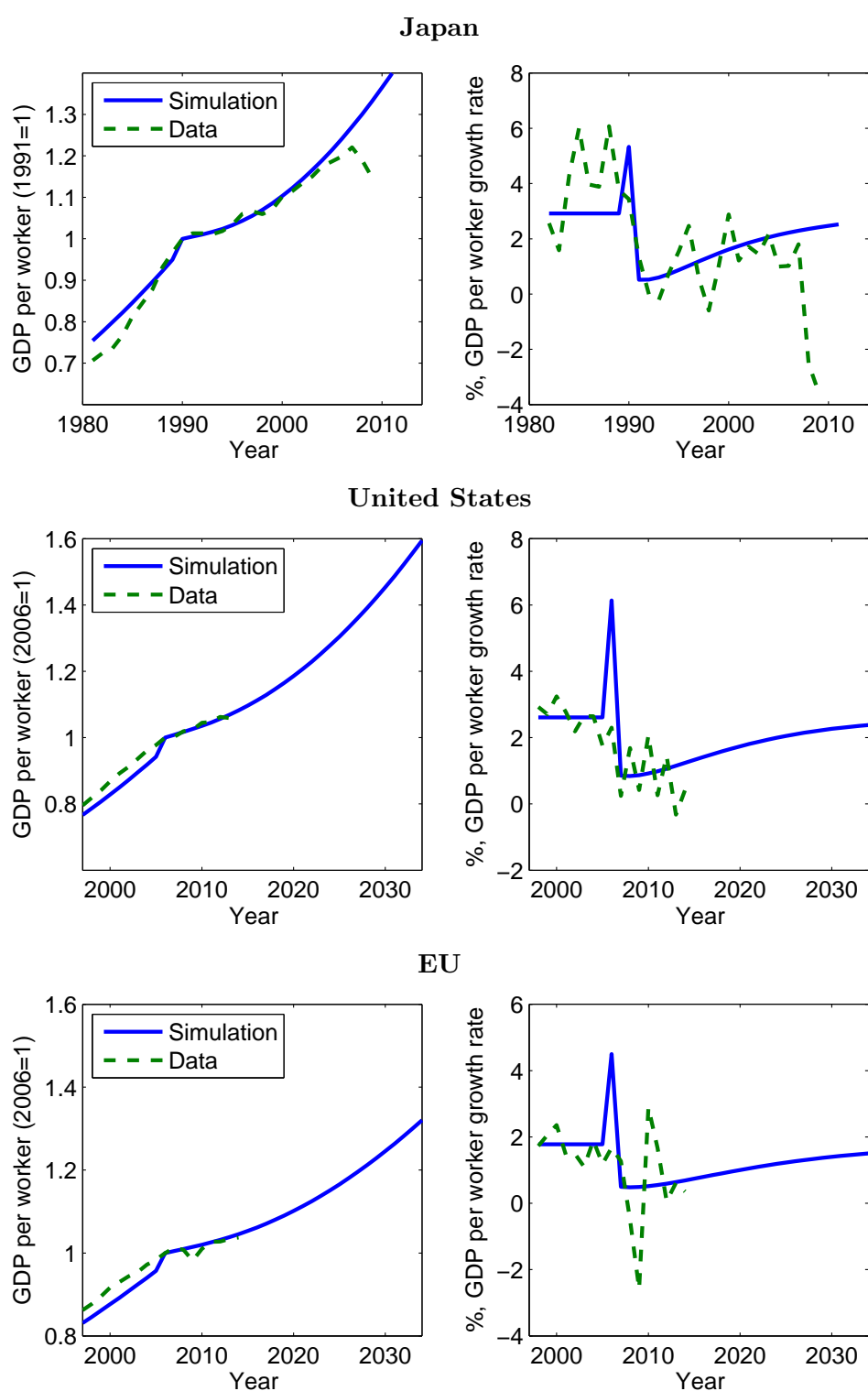


Figure 5: GDP for Japan, the United States, and the EU: Comparison between the data and simulation

Sources: Our calculation; Cabinet Office, Government of Japan, *Annual Report on National Accounts*; Fernald (2012); European Commission, *AMECO*

with the maximum repayable debt fall into a debt-ridden state, where they are subject to tighter borrowing constraints than those in normal times and continue inefficient production persistently.

The emergence of a substantial number of debt-ridden borrowers lowers aggregate productivity by tightening the aggregate borrowing constraint. This tightening of aggregate borrowing constraints owing to the mass emergence of debt-ridden borrowers may manifest itself as a “financial shock” during or after a financial crisis. We also showed that the growth rate of aggregate productivity lowers persistently if the measure of debt-ridden firms is large. This result has a significant policy implication that governmental intervention to facilitate mass debt restructuring for overly indebted borrowers may enhance economic growth when the economy falls into persistent stagnation in the aftermath of a financial crisis.

The endogenous borrowing constraint of this study has a unique feature that debt tightens the constraint and generates inefficiency permanently, and therefore may serve as a useful building block for business cycle models, thereby enriching aggregate dynamics. Broader applications are left for future research.

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A Appendix A: Derivation of l_z and d_z

We derive the labor input and debt stock for the debt-ridden firm, which solves the following problem. Here, we allow (A_t, w_t, r_t) to vary over time. The firm with debt d solves (8).

We assume and verify later that $\pi_t = 0$ and the borrowing constraint binds for all t . Under this assumption,

$$\begin{aligned} k_{t+1} &= (1 - \phi)A_{t+1}l_{t+1}^\eta + (1 - \sigma)(1 + r_{t+1})k_t - \theta(0), \\ b_{t+1} &= \phi A_{t+1}l_{t+1}^\eta + \sigma(1 + r_{t+1})k_t - w_{t+1}l_{t+1}, \end{aligned}$$

with $k_{-1} = k_n$. Debt d_t is defined recursively by

$$d_t = \frac{1 - \rho}{1 + r_{t+1}} [b_{t+1} + d_{t+1}] = \sum_{j=1}^{\infty} \frac{(1 - \rho)^j}{\prod_{i=1}^j (1 + r_{t+i})} b_{t+j}.$$

The terms in d_t that include l_{t+1} are written as follows:

$$\begin{aligned}
\frac{1+r_{t+1}}{1-\rho}d_t &= b_{t+1} + \frac{(1-\rho)}{(1+r_{t+2})}b_{t+2} + \frac{(1-\rho)^2}{(1+r_{t+2})(1+r_{t+3})}b_{t+3} + \dots \\
&= -w_{t+1}l_{t+1} + \phi A_{t+1}l_{t+1}^\eta + \frac{(1-\rho)}{(1+r_{t+2})}\sigma(1+r_{t+2})(1-\phi)A_{t+1}l_{t+1}^\eta \\
&\quad + \frac{(1-\rho)^2}{(1+r_{t+2})(1+r_{t+3})}\sigma(1+r_{t+2})(1-\sigma)(1+r_{t+3})(1-\phi)A_{t+1}l_{t+1}^\eta + \dots \\
&= -w_{t+1}l_{t+1} + [\phi + (1-\rho)\sigma(1-\phi) + (1-\rho)^2\sigma(1-\sigma)(1-\phi) + \dots] A_{t+1}l_{t+1}^\eta + \dots \\
&= \left\{ \phi + \frac{(1-\rho)(1-\phi)\sigma}{1-(1-\rho)(1-\sigma)} \right\} A_{t+1}l_{t+1}^\eta - w_{t+1}l_{t+1} + \dots
\end{aligned}$$

This equation implies that if

$$l_{t+1} = l_{zt+1} \equiv \left[\left\{ \phi + \frac{(1-\rho)(1-\phi)\sigma}{1-(1-\rho)(1-\sigma)} \right\} \frac{\eta A_{t+1}}{w_{t+1}} \right]^{\frac{1}{1-\eta}},$$

then d_t is maximized. Note that the value of l_{t+1} that maximizes d_t does not depend on the prices and revenue parameters (w_{t+j} and A_{t+j} for $j \geq 2$) in future periods but only on the current price, w_{t+1} , and current parameter, A_{t+1} .

Denoting $\Gamma \equiv \left\{ \phi + \frac{(1-\rho)(1-\phi)\sigma}{1-(1-\rho)(1-\sigma)} \right\}$, the terms of d_{-1} that includes $l_0, l_1, l_2, l_3, \dots$ are written as follows:

$$d_{-1} = \sum_{t=0}^{\infty} \frac{(1-\rho)^{t+1}}{\prod_{i=0}^t (1+r_i)} [\Gamma A_t l_t^\eta - w_t l_t] + \text{terms that include } k_n \text{ and } \theta(0).$$

In the steady state in which $A_t = A$, $r_t = r$, and $w_t = w$, the maximum repayable debt, d_z , is given by $l_t = l_z$. As $w l_z = \eta \Gamma A l_z^\eta$, $d_{-1} = d_z$ is expressed as

$$\begin{aligned}
d_z &= \frac{1-\rho}{r+\rho} [\Gamma A l_z^\eta - w l_z] + \Omega_1 k_n - \Omega_2 \theta(0) \\
&= \frac{(1-\rho)(1-\eta)}{r+\rho} \Gamma A l_z^\eta + \Omega_1 k_n - \Omega_2 \theta(0),
\end{aligned}$$

where Ω_1 and Ω_2 are given below.

Now, we show that $l_t = l_z$ for all $t \geq 0$ is attained as the solution to the Ramsey problem for the bank. The Ramsey problem is for the bank to maximize the initial debt d_{-1} so that the firm chooses l_z and repayment plan $\{b_t\}_{t=0}^{\infty}$ by solving its problem. The FOCs for the firm's problem implies that

$$l_{t+1} = \left\{ \frac{1-\nu_{t+1} + \phi \mu_{t+1} \theta'(\pi_{t+1})}{1-\nu_{t+1} + \mu_{t+1} \theta'(\pi_{t+1})} \frac{\eta A_{t+1}}{w_{t+1}} \right\}^{\frac{1}{1-\eta}},$$

where μ_{t+1} is the Lagrange multiplier for the borrowing constraint and ν_{t+1} is the Lagrange multiplier for the limited liability constraint ($\pi_{t+1} \geq 0$). The solution to the firm's problem includes $l_{t+1} = l_{zt+1}$ if

$$\begin{aligned}
\frac{\mu_{t+1} \theta'(\pi_{t+1})}{1-\nu_{t+1}} &= \frac{\rho}{(1-\rho)\sigma}, \\
\pi_{t+1} &= 0,
\end{aligned}$$

and k_{t+1} is given by the budget constraint. These variables satisfy all constraints and the FOCs for the firm's problem. Therefore, if the bank sets the initial debt $d_{-1} = d_z$, then the firm optimally chooses $l_t = l_{zt}$ for all t .

The values of Ω_1 and Ω_2 are given by similar arguments to those above. By rearranging the terms of the initial debt d_{-1} that include $k_{-1} = k_n$ and $\theta(0)$, we obtain

$$\Omega_1 = \frac{(1 - \rho)\sigma}{1 - (1 - \rho)(1 - \sigma)},$$

$$\Omega_2 = \frac{(1 - \rho)^2\sigma}{[1 - (1 - \rho)(1 - \sigma)](r + \rho)}.$$

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B Appendix B: Proofs of Lemma 1 and Lemma 2

The FOCs and constraints for the problem (8) imply that the dynamics are given by

$$\begin{aligned}\theta(\pi_{t+1}) + k_{t+1} &\leq Al_{t+1}^\eta + (1 + r)k_t - wl_{t+1} - b_{t+1}, \\ wl_{t+1} + b_{t+1} &\leq \phi Al_{t+1}^\eta + \sigma(1 + r)k_t, \\ 1 &= \theta'(\pi_{t+1})\lambda_{t+1}, \\ \lambda_{t+1} + \mu_{t+1} + \nu_{t+1} - \xi_{t+1} &= -V_d(d_{t+1}, k_{t+1}), \\ \lambda_{t+1} &= V_k(d_{t+1}, k_{t+1}), \\ \lambda_{t+1}[\eta Al_{t+1}^{\eta-1} - w] + \mu_{t+1}[\phi Al_{t+1}^\eta - w] &= 0, \\ V_d(d_t, k_t) &= V_d(d_{t+1}, k_{t+1}) + \nu_{t+1}, \\ V_k(d_t, k_t) &= \rho + (1 - \rho)(\lambda_{t+1} + \sigma\mu_{t+1}),\end{aligned}$$

where λ_t , μ_t , ν_t , and ξ_t are the Lagrange multipliers for the budget constraint, borrowing constraint, nonnegativity constraint for d_{t+1} ($d_{t+1} \geq 0$), and nonnegativity constraint for b_{t+1} ($b_{t+1} \geq 0$).

Proof of Lemma 1.— Proof is by contradiction. Suppose that $\pi_t = \bar{\pi}$, $l_t = l_n$, and $k_t = k_n$ for all $t \geq 0$. Then, the budget constraint implies that $b_t = 0$ for all t and so $d > 0$ is not repayable. It is a contradiction. Therefore, there must exist t such that $\pi_t < \bar{\pi}$ and/or $l_t < l_n$ and/or $k_t < k_n$. Now, suppose that $\pi_t = \bar{\pi}$ and $l_t = l_n$ for all $t \geq 0$. For b_t to be positive, it must be the case that $k_t < k_n$. Then, the budget constraint implies that $k_{t+1} < k_t$ for any $b_{t+1} \geq 0$ and it is easily shown that $k_t < 0$ eventually. This contradicts the condition that $k_t \geq 0$. Therefore, there must exist t such that $\pi_t < \bar{\pi}$ and/or $l_t < l_n$.

Without loss of generality, Lemma 1 implies that $\pi_0 < \bar{\pi}$ and/or $l_0 < l_n$ in period 0, given that $d_{-1} = d > 0$. That $\pi_0 < \bar{\pi}$ and $l_0 < l_n$ imply that $\lambda_0 = \frac{1}{\theta'(\pi_0)} > 1$ and $\mu_0 > 0$, respectively. Therefore, $\lambda_0 + \mu_0 > 1$. Obviously, it must be the case that $b_0 > 0$ and so $\xi_0 = 0$. Thus, we have shown that $\lambda_0 + \mu_0 - \xi_0 > 1$.

Proof of Lemma 2.— We can show by induction that $\lambda_t + \mu_t - \xi_t \geq \lambda_0 + \mu_0 - \xi_0 > 1$ for all $t \geq 0$. We know $\lambda_0 + \mu_0 - \xi_0 > 1$. Suppose that $\lambda_t + \mu_t - \xi_t \geq \lambda_0 + \mu_0 - \xi_0 > 1$. The FOC and envelope condition for d_t imply that

$$\lambda_t + \mu_t - \xi_t + \nu_t = \lambda_{t+1} + \mu_{t+1} - \xi_{t+1}.$$

As $\nu_t \geq 0$ by definition, it is clear that $\lambda_{t+1} + \mu_{t+1} - \xi_{t+1} \geq \lambda_t + \mu_t - \xi_t \geq \lambda_0 + \mu_0 - \xi_0 > 1$. Thus, we have shown $\lambda_t + \mu_t - \xi_t \geq \lambda_0 + \mu_0 - \xi_0 > 1$ for all $t \geq 0$. As we assume that the equilibrium converges to the steady state, λ_t and μ_t also converge to constants, namely λ_∞ and μ_∞ , respectively. The above inequality implies that

$$\lambda_\infty + \mu_\infty \geq \lambda_0 + \mu_0 > 1.$$

The FOC and envelope condition for k_t imply that

$$\lambda_\infty = 1 + \frac{(1 - \rho)\sigma}{\rho} \mu_\infty.$$

These two results directly imply that $\lambda_\infty > 1$ and $\mu_\infty > 0$. Thus, it follows that $\pi_\infty < \bar{\pi}$ and $l_\infty < l_n$.

C Appendix C: The BGP

We choose the parameter values such that the borrowing constraint is binding on the BGP. The detrended variables of the BGP, $\{g, g_Y, Y, K, C, H, L, w, r^K, V, k, \pi, \theta, \mu\}$, are

determined by the following system of equations:

$$\begin{aligned}
g_Y &= g^{\frac{1-\eta}{\eta}}, \\
Y &= K^{1-\alpha} L^\alpha, \\
C + (g_Y - 1 + \delta)K &= Y, \\
w &= \frac{\gamma C}{1 - H}, \\
r^K &= (1 - \alpha) \frac{Y}{K}, \\
g_Y &= \beta(r^K + 1 - \delta), \\
g &= (1 - \rho)[1 + \kappa(H - L)], \\
V &= \rho k + \frac{(1 - \rho)\beta}{g} \{ \pi + [1 + \kappa(H - L)]V \}, \\
k &= \frac{\beta}{\sigma g} [wH - \phi \alpha K^{1-\alpha} L^\alpha] > 0, \\
\theta(\pi) &= (1 - \phi)\alpha K^{1-\alpha} L^\alpha + (1 - \sigma) \frac{g}{\beta} k - [1 + \kappa(H - L)]k, \\
w &= \frac{\kappa(V\theta'(\pi) - k)}{1 + \mu\theta'(\pi)}, \\
\kappa(V\theta'(\pi) - k) &= [1 + \phi\mu\theta'(\pi)]\eta\alpha K^{1-\alpha} L^{\alpha-1}, \\
\frac{1}{\theta'(\pi)} &= \rho + \frac{(1 - \rho)[1 + \sigma\mu\theta'(\pi)]}{\theta'(\pi)}, \\
\theta(\pi) &= \pi + \frac{1}{2\chi}(\pi - \bar{\pi})^2.
\end{aligned}$$

D Appendix D: Detrending for firm's problems

We change the variables as follows: $V_{zt} = \tilde{V}_{zt} N_t^{(1-2\eta)/\eta}$, $A_t = \tilde{A}_t N_t^{(1-\eta)^2/\eta}$, $Y_t = \tilde{Y}_t N_t^{(1-\eta)/\eta}$, $C_t = \tilde{C}_t N_t^{(1-\eta)/\eta}$, $K_t = \tilde{K}_t N_t^{(1-\eta)/\eta}$, $x_{zt} = \tilde{x}_{zt}/N_t$, $x_{nt} = \tilde{x}_{nt}/N_t$, $l_{zt} = \tilde{l}_{zt}/N_t$, $l_{nt} = \tilde{l}_{nt}/N_t$, $h_{zt} = \tilde{h}_{zt}/N_t$, $h_{nt} = \tilde{h}_{nt}/N_t$, $w_t = \tilde{w}_t N_t^{\frac{1-\eta}{\eta}}$, $b_{zt} = \tilde{b}_{zt} N_t^{(1-2\eta)/\eta}$, $d_{zt} = \tilde{d}_{zt} N_t^{(1-2\eta)/\eta}$, $\pi_t = \tilde{\pi}_t N_t^{(1-2\eta)/\eta}$, $\theta_t(\pi_t) = \tilde{\theta}(\tilde{\pi}_t) N_t^{(1-2\eta)/\eta}$, $z_t = Z_t/N_t$, and $g_t = N_{t+1}/N_t$. Then, the firm's problem (13) can be rewritten as follows. Given \tilde{d}_{zt} ,

$$\begin{aligned}
\tilde{V}_{zt}(\tilde{d}_{zt}, \tilde{k}_{zt}) &= \max \rho \tilde{k}_{zt} + \frac{(1 - \rho)\beta \tilde{C}_t}{g_t \tilde{C}_{t+1}} \left[\tilde{\pi}_{zt+1} + \tilde{V}_{zt+1}(\tilde{d}_{zt+1}, \tilde{k}_{zt+1}) \right. \\
&\quad \left. + \kappa \{ \tilde{h}_{zt+1} - \tilde{l}_{zt+1} \} \tilde{V}_{nt+1}(\tilde{k}_{zIt+1}) \right],
\end{aligned}$$

subject to

$$\left\{ \begin{aligned}
\tilde{b}_{zt+1} &= \frac{g_t \tilde{C}_{t+1}}{(1-\rho)\beta \tilde{C}_t} \tilde{d}_{zt} - \tilde{d}_{zt+1}, \\
\tilde{\theta}(\tilde{\pi}_{zt+1}) + \tilde{k}_{zt+1} &\leq \tilde{F}_{t+1}(\tilde{k}_{zt}, \tilde{l}_{zt+1}) - \tilde{w}_{t+1} \tilde{h}_{zt+1} - \tilde{b}_{zt+1} - \kappa \{ \tilde{h}_{zt+1} - \tilde{l}_{zt+1} \} \tilde{k}_{zIt+1}, \\
\tilde{w}_{t+1} \tilde{h}_{zt+1} + \tilde{b}_{zt+1} &\leq \phi \tilde{A}_{t+1} \tilde{l}_{zt+1}^\eta + \sigma \frac{g_t \tilde{C}_{t+1}}{\beta \tilde{C}_t} \tilde{k}_{zt}, \\
\tilde{l}_{zt+1} &\leq \tilde{h}_{zt+1}, \\
\tilde{\pi}_{zt+1} &\geq 0.
\end{aligned} \right.$$

The FOCs imply that if for any $\tilde{k}_{zIt+1} \geq 0$,

$$\mu_{zt+1}\tilde{\theta}'(\tilde{\pi}_{zt+1}) > \frac{\kappa\{\tilde{V}_{nt+1}(\tilde{k}_{zIt+1})\tilde{\theta}'(\tilde{\pi}_{zt+1}) - \tilde{k}_{zIt+1}\}}{\tilde{w}_{t+1}} - 1, \quad (14)$$

then,

$$\begin{aligned}\tilde{h}_{zt+1} &= \tilde{l}_{zt+1}, \\ \tilde{k}_{zIt+1} &= 0,\end{aligned}$$

where μ_{zt+1} is the Lagrange multiplier for the borrowing constraint. Inequality (14) implies that the marginal cost of hiring labor for the R&D activity, $\{1 + \mu_{zt+1}\tilde{\theta}'(\tilde{\pi}_{zt+1})\}\tilde{w}_{t+1}$, is strictly larger than the marginal benefit of R&D, $\kappa\{\tilde{V}_{nt+1}(\tilde{k}_{zIt+1})\tilde{\theta}'(\tilde{\pi}_{zt+1}) - \tilde{k}_{zIt+1}\}$, for a debt-ridden firm. We choose the parameters such that inequality (14) holds in our numerical simulation.

Detrended problem for normal firms without debt is written as follows.

$$\tilde{V}_{nt}(\tilde{k}_{nt}) = \max \rho\tilde{k}_{nt} + \frac{(1-\rho)\beta\tilde{C}_t}{g_t\tilde{C}_{t+1}} \left[\tilde{\pi}_{nt+1} + \{1 + \kappa(\tilde{h}_{nt+1} - \tilde{l}_{nt+1})\}\tilde{V}_{nt+1}(\tilde{k}_{nt+1}) \right], \quad (15)$$

subject to

$$\begin{cases} \tilde{\theta}(\tilde{\pi}_{nt+1}) + \tilde{k}_{nt+1} \leq \tilde{F}_{t+1}(\tilde{k}_{nt}, \tilde{l}_{nt+1}) - \tilde{w}_{t+1}\tilde{h}_{nt+1} - \kappa\{\tilde{h}_{nt+1} - \tilde{l}_{nt+1}\}\tilde{k}_{nt+1}, \\ \tilde{w}_{t+1}\tilde{h}_{nt+1} \leq \phi\tilde{A}_{t+1}\tilde{l}_{nt+1}^\eta + \sigma\frac{g_t\tilde{C}_{t+1}}{\beta\tilde{C}_t}\tilde{k}_{nt}, \\ \tilde{l}_{nt+1} \leq \tilde{h}_{nt+1}, \\ \tilde{\pi}_{nt+1} \geq 0. \end{cases}$$

The FOCs for normal firms imply that

$$\mu_{nt+1}\tilde{\theta}'(\tilde{\pi}_{nt+1}) = \frac{\kappa\{\tilde{V}_{nt+1}(\tilde{k}_{nt+1})\tilde{\theta}'(\tilde{\pi}_{nt+1}) - \tilde{k}_{nt+1}\}}{\tilde{w}_{t+1}} - 1,$$

where we denote the Lagrange multiplier associated with the borrowing constraint by μ_{nt+1} . Given this equation, we obtain the sufficient condition for condition (14), which is used in our numerical experiment as a condition for selecting the parameter values.

Lemma 3. *The sufficient condition for (14) is given by*

$$\mu_{zt+1}\tilde{\theta}'(\tilde{\pi}_{zt+1}) > \mu_{nt+1}\tilde{\theta}'(\tilde{\pi}_{nt+1}). \quad (16)$$

Proof. As $\tilde{\pi}_{zt+1} \leq \tilde{\pi}_{nt+1}$, it must be the case that $0 \leq \tilde{\theta}'(\tilde{\pi}_{zt+1}) \leq \tilde{\theta}'(\tilde{\pi}_{nt+1})$. Then, the right-hand side of (14) satisfies

$$\begin{aligned} & \frac{\kappa\{\tilde{V}_{nt+1}(\tilde{k}_{zIt+1})\tilde{\theta}'(\tilde{\pi}_{zt+1}) - \tilde{k}_{zIt+1}\}}{\tilde{w}_{t+1}} - 1 \\ & \leq \frac{\kappa\{\tilde{V}_{nt+1}(\tilde{k}_{zIt+1})\tilde{\theta}'(\tilde{\pi}_{nt+1}) - \tilde{k}_{zIt+1}\}}{\tilde{w}_{t+1}} - 1 \\ & \leq \frac{\kappa\{\tilde{V}_{nt+1}(\tilde{k}_{nt+1})\tilde{\theta}'(\tilde{\pi}_{nt+1}) - \tilde{k}_{nt+1}\}}{\tilde{w}_{t+1}} - 1 = \mu_{nt+1}\tilde{\theta}'(\tilde{\pi}_{nt+1}), \end{aligned}$$

where the last inequality is due to $\tilde{k}_{nt+1} = \arg \max_k \tilde{V}_{nt+1}(k)\tilde{\theta}'(\tilde{\pi}_{nt+1}) - k$. This inequality implies that (16) is the sufficient condition for (14). \square

E Appendix E: Transition dynamics with $z_{10} > 0$

In this appendix, we describe the transition dynamics in the case where the economy is initially on the BGP and the z_{10} proportion of firms are suddenly imposed the maximum debt d_z at time 10, where $0 < z_{10} \leq 1$. The economy eventually converges to the BGP. All agents have the perfect foresight on the paths after the one-time buildup of debt. In this setting, we can apply a deterministic simulation with an occasionally binding borrowing constraint by using Dynare (see Adjemian et al., 2011). In our numerical experiments, we set parameter values such that the borrowing constraint is always binding both in transition and in the BGP. This approach can solve a full nonlinear system of simultaneous equations using a modified Newton–Raphson algorithm. The details of the algorithm can be found in Juillard (1996). This algorithm solves $n \times T$ simultaneous equations, where n is the number of endogenous variables and T is the number of simulation periods. We set the number of periods of the simulation to 300, i.e., $T = 300$ and our model has 25 endogenous variables, i.e., $n = 25$. The Lagrange multiplier for the borrowing constraint for normal firms is denoted by μ_{nt} . Altogether, 25 variables— $\{\tilde{A}_t, \tilde{Y}_t, \tilde{K}_t, \tilde{C}_t, H_t, L_t, \tilde{w}_t, \tilde{h}_{nt}, \tilde{h}_{zt}, \tilde{l}_{zt}, \tilde{l}_{nt}, \tilde{x}_{zt}, \tilde{x}_{nt}, g_t, r_t^K, z_t,$

$\tilde{V}_{nt}, \tilde{k}_{nt}, \tilde{\pi}_{nt}, \tilde{\theta}(\tilde{\pi}_{nt}), g_{Yt}, \mu_{nt}, \tilde{k}_{zt}, \tilde{b}_{zt}, \tilde{d}_{zt}$ —are calculated from the following 25 equations:¹⁴

$$\begin{aligned}
\tilde{A}_t &\equiv \alpha \tilde{Y}_t^{1-\frac{\eta}{\alpha}} \tilde{K}_{t-1}^{(1-\alpha)\frac{\eta}{\alpha}}, \\
\tilde{x}_{nt} &= \tilde{l}_{nt}, \\
\tilde{x}_{zt} &= \tilde{l}_{zt}, \\
\tilde{l}_{zt} &= \tilde{h}_{zt}, \\
\tilde{h}_{zt} &= \left\{ \left[\phi + \frac{(1-\rho)(1-\phi)\sigma}{1-(1-\rho)(1-\sigma)} \right] \eta \frac{\tilde{A}_t}{\tilde{w}_t} \right\}^{\frac{1}{1-\eta}}, \\
\tilde{Y}_t &= \{z_t \tilde{x}_{zt}^\eta + (1-z_t) \tilde{x}_{nt}^\eta\}^{\frac{\alpha}{\eta}} \tilde{K}_{t-1}^{1-\alpha}, \\
L_t &= z_t \tilde{l}_{zt} + (1-z_t) \tilde{l}_{nt}, \\
\tilde{C}_t + g_t \frac{1-\eta}{\eta} \tilde{K}_t - (1-\delta) \tilde{K}_{t-1} &= \tilde{Y}_t, \\
\tilde{w}_t &= \frac{\gamma \tilde{C}_t}{1-H_t}, \\
r_t^K &= (1-\alpha) \frac{\tilde{Y}_t}{\tilde{K}_{t-1}}, \\
1 &= \frac{\beta \tilde{C}_t}{g_t \frac{1-\eta}{\eta} \tilde{C}_{t+1}} [r_{t+1}^K + 1 - \delta], \\
g_t &= (1-\rho)[1 + \kappa(H_t - L_t)], \\
(1-\rho)z_t &= g_t z_{t+1}, \\
\tilde{V}_{nt} &= \rho \tilde{k}_{nt} + \frac{(1-\rho)\beta \tilde{C}_t}{g_t \tilde{C}_{t+1}} \left[\pi_{nt+1} + \{1 + \kappa(\tilde{h}_{nt+1} - \tilde{l}_{nt+1})\} \tilde{V}_{nt+1} \right], \\
\mu_{nt} \tilde{\theta}'(\tilde{\pi}_{nt}) &= \frac{\kappa \{ \tilde{V}_{nt} \tilde{\theta}'(\tilde{\pi}_{nt}) - \tilde{k}_{nt} \}}{\tilde{w}_t} - 1, \\
\tilde{w}_t \tilde{h}_{nt} &= \phi \tilde{A}_t \tilde{l}_{nt}^\eta + \sigma \frac{g_{t-1} \tilde{C}_t}{\beta \tilde{C}_{t-1}} \tilde{k}_{nt-1}, \\
\tilde{\theta}(\tilde{\pi}_{nt}) &= \tilde{\pi}_{nt} + \frac{1}{2\chi} (\tilde{\pi}_{nt} - \bar{\pi})^2, \\
\tilde{k}_{nt} &= \max \left\{ \frac{\beta \tilde{C}_t}{g_t \tilde{C}_{t+1}} \left[\{1 + \kappa(\tilde{h}_{nt+1} - \tilde{l}_{nt+1})\} \tilde{k}_{nt+1} \right. \right. \\
&\quad \left. \left. - \tilde{A}_{t+1} \tilde{l}_{nt+1}^\eta + \tilde{w}_{t+1} \tilde{h}_{nt+1} + \tilde{\theta}(\tilde{\pi}_{nt+1}) \right], 0 \right\}, \\
\kappa \{ \tilde{V}_{nt} \tilde{\theta}'(\tilde{\pi}_{nt}) - \tilde{k}_{nt} \} &= (1 + \phi \mu_{nt} \tilde{\theta}'(\tilde{\pi}_{nt})) \eta \alpha \tilde{Y}_t^{1-\frac{\eta}{\alpha}} \tilde{K}_t^{(1-\alpha)\frac{\eta}{\alpha}} \tilde{x}_{nt}^{\eta-1}, \\
H_t &= (1-z_t) \tilde{h}_{nt} + z_t \tilde{h}_{zt}, \\
g_{Yt} &= g_t \frac{1-\eta}{\eta} \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t},
\end{aligned}$$

¹⁴More precisely, there are 27 equations because we distinguish $\{\tilde{k}_{z0}, \tilde{b}_{z0}\}$ from $\{\tilde{k}_{zt}, \tilde{b}_{zt}\}$.

$$\begin{cases}
\tilde{k}_{z0} = (1 - \phi)\tilde{A}_0\tilde{l}_{z0}^\eta + (1 - \sigma)\frac{g_{-1}\tilde{C}_0}{\beta C}k - \theta(0), & \text{where } C \text{ and } k \text{ are the values on the BGP,} \\
\tilde{b}_{z0} = \phi\tilde{A}_0\tilde{l}_{z0}^\eta - \tilde{w}_0\tilde{l}_{z0} + \sigma\frac{g_{-1}\tilde{C}_0}{\beta C}k, & \text{where } C \text{ and } k \text{ are the values on the BGP,} \\
\tilde{k}_{zt} = (1 - \phi)\tilde{A}_t\tilde{l}_{zt}^\eta + (1 - \sigma)\frac{g_{t-1}\tilde{C}_t}{\beta\tilde{C}_{t-1}}\tilde{k}_{zt-1} - \theta(0), \\
\tilde{b}_{zt} = \phi\tilde{A}_t\tilde{l}_{zt}^\eta - \tilde{w}_t\tilde{l}_{zt} + \sigma\frac{g_{t-1}\tilde{C}_t}{\beta\tilde{C}_{t-1}}\tilde{k}_{zt-1}, \\
\tilde{d}_{zt} = \frac{(1 - \rho)\beta\tilde{C}_t}{g_t\tilde{C}_{t+1}}[\tilde{b}_{zt+1} + \tilde{d}_{zt+1}], \\
\frac{1}{\tilde{\theta}'(\tilde{\pi}_{nt})} = \rho + \frac{(1 - \rho)[1 + \sigma\mu_{nt+1}\tilde{\theta}'(\tilde{\pi}_{nt+1})]}{\tilde{\theta}'(\tilde{\pi}_{nt+1})}.
\end{cases}$$

This system of equations decides the equilibrium dynamics.

The total factor productivity, TFP_t , and the labor wedge, LW_t , are calculated by

$$\begin{aligned}
TFP_t &= N_t^{\frac{(1-\eta)\alpha}{\eta}} \tilde{TFP}_t = N_t^{\frac{(1-\eta)\alpha}{\eta}} \frac{\tilde{Y}_t}{\tilde{K}_t^{1-\alpha} H_t^\alpha}, \\
LW_t &= \frac{\tilde{w}_t H_t}{\alpha \tilde{Y}_t}.
\end{aligned}$$

These variables must satisfy the following conditions for all t to constitute the equilibrium path:

$$\begin{aligned}
0 &< \mu_{nt+1}\tilde{\theta}'(\tilde{\pi}_{nt+1}) < \mu_{zt+1}\tilde{\theta}'(\tilde{\pi}_{zt+1}), \\
0 &< \tilde{\pi}_{nt} < \bar{\pi}, \\
\tilde{k}_{nt} &\geq 0, \\
\tilde{k}_{zt} &\geq 0, \\
\tilde{d}_{zt} &< \tilde{V}_{nt}, \\
\tilde{b}_{zt} &\geq 0.
\end{aligned}$$

F Appendix F: Calibration and Data

Table 2 reports the values of the calibrated parameters. First, the parameters β , δ , η , and ρ are common values to all countries. We set the discount factor β to 0.98, depreciation rate δ to 0.06, parameter for the aggregation function η to 0.7, and exit rate ρ to 0.1. These are standard settings in the literature. In addition, we assume that the borrowing constraint is always binding in our numerical simulation and set the Lagrange multiplier of the borrowing constraint μ to 0.1 on the BGP.

Second, we calibrate the country-specific parameters and some BGP values. The share of labor in production (α), total labor supply on the BGP (H), and growth rate of TFP on the BGP (g_{TFP}) are set by the data. g_{TFP} is defined by $g_{TFP} = g_Y^\alpha = g^{\frac{(1-\eta)\alpha}{\eta}}$. H is set to the ratio of average annual hours worked per person employed to total hours. We

assume that the economy is on the BGP before a financial crisis. In the case of Japan, the financial crisis starts in 1991, while in the case of the United States and EU, it starts in 2006. α , H , and g_{TFP} in Japan are taken as the average during 1982–1990 from the JIP database, and for the United States and EU they are taken as the average during 1997–2005. For the United States, H is from the Penn World Tables and the others are from Fernald (2012). For the EU, H and g_{TFP} are from the European Commission’s Annual macro-economic database (AMECO) constructed by Havik et al. (2014), and the value of α is taken from Havik et al. (2014). When we calculate the variables on the BGP, H and g_{TFP} are given by the data and μ is exogenously set to 0.1. Hence, the inverse of the elasticity of labor supply γ , efficiency of R&D κ , and target level of dividend $\bar{\pi}$ are endogenously determined in the system of the BGP, which is given in Appendix C. We also assume that

$$\chi = \bar{\pi}.$$

Lastly, the collateral ratio (ϕ , σ) is identified by using a grid search method following the simulated least squares criterion:

$$\min_{\phi, \sigma} (\mathbf{X}_t - \hat{\mathbf{X}}_t)' (\mathbf{X}_t - \hat{\mathbf{X}}_t) \quad (17)$$

subject to

$$0 < \phi < 1, \quad 0 < \sigma < 1, \quad (18)$$

$$0 < \mu_{nt+1} \tilde{\theta}'(\tilde{\pi}_{nt+1}) < \mu_{zt+1} \tilde{\theta}'(\tilde{\pi}_{zt+1}), \quad (18)$$

$$0 < \tilde{\pi}_{nt} < \bar{\pi}, \quad (19)$$

$$\tilde{k}_{nt} \geq 0, \quad (20)$$

$$\tilde{k}_{zt} \geq 0, \quad (21)$$

$$\tilde{d}_{zt} < \tilde{V}_{nt}, \quad (22)$$

$$\tilde{b}_{zt} \geq 0. \quad (23)$$

where \mathbf{X}_t is the observed variables vector and $\hat{\mathbf{X}}_t$ is the simulation-generated variables vector. $\mathbf{X}_t = [\text{TFP growth rate}; \text{per capita real output growth rate}]$. The simulation-generated variables, $\hat{\mathbf{X}}_t = [TFP_t/TFP_{t-1} - 1; Y_t/Y_{t-1} - 1]$, are calculated by using the method presented in Appendix E, taking $(\gamma, \kappa, \phi, \eta, \sigma, z_{10})$ as the given exogenous parameters. The sample period is from 1991 to 2010 in Japan and from 2006 to 2014 in the United States and EU.

To calibrate the parameters, we choose the optimum parameters to minimize the distance between the simulation implied by our model and the actual data. This procedure is similar to impulse response matching, as described in, for example, Rotemberg and Woodford (1997), which chooses the parameters to minimize the distance between the impulse

responses implied by a reduced-form VAR and implied by a DSGE model.¹⁵

We perform a grid search for only two parameters (ϕ, σ) and these parameters are a ratio that can take values in a limited interval $(0, 1)$. Hence, we can find the global minimum of the objective function (17).

Lastly, we summarize our calibration procedure:

Step 1. Set the parameter values, $(\alpha, \beta, \delta, \eta, \rho, H, g_{TFP})$, exogenously.

Step 2. Set the value $\underline{\phi} = \underline{\sigma} = 0.01$, $\bar{\phi} = \bar{\sigma} = 0.99$ and $\phi^0 = \sigma^0 = 0.5$

Step 3. Compute the BGP sequentially on a grid $(\phi_{(i)}, \sigma_{(j)})$ of 99×99 equally spaced points over the square $[\underline{\phi}; \bar{\phi}] \times [\underline{\sigma}; \bar{\sigma}]$, where $i = 1, 2, \dots, 99$, $j = 1, 2, \dots, 99$. Tentatively, (κ, π_n) are taken as given.

$$\begin{aligned}
z &= 0, \\
g_Y &= g_{TFP}^{\frac{1}{\alpha}}, \\
g &= g_Y^{\frac{\eta}{1-\eta}}, \\
r &= \frac{g_Y}{\beta} - 1 + \delta, \\
\mu_n &= 0.1, \\
L &= H + \frac{1}{\kappa} \left(1 - \frac{g}{1-\rho} \right) \\
K &= \left(\frac{1-\alpha}{r} \right)^{\frac{1}{\alpha}} L, \\
Y &= K^{1-\alpha} L^{\alpha}, \\
A &= \alpha Y^{1-\frac{\eta}{\alpha}} K^{(1-\alpha)\frac{\eta}{\alpha}}, \\
\bar{\pi} &= \pi_n + \frac{(1-\rho)\chi\sigma\mu}{\rho + (1-\rho)\sigma\mu} = \frac{[\rho + (1-\rho)\sigma\mu]\pi_n}{\rho}, \\
\theta'(\pi) &= 1 + \frac{\pi_n - \bar{\pi}}{\chi}, \\
\theta &= \pi_n + \frac{1}{2\chi} (\pi_n - \bar{\pi})^2, \\
J &\equiv \beta[1 + \kappa(H - L)] - (1 - \sigma)g, \\
V &= \rho k + \frac{(1-\rho)\beta}{g} \{ \pi + [1 + \kappa(H - L)]V \} \\
&= \frac{\beta\rho g[(1-\phi)AL^{\eta} - \theta] + J\beta\pi_n(1-\rho)}{J\{g - (1-\rho)\beta[1 + \kappa(H - L)]\}}, \\
k &= \frac{\beta}{J} \{ (1-\phi)AL^{\eta} - \theta \}, \\
w &= \frac{\kappa[V\theta'(\pi) - k]}{1 + \mu\theta'(\pi)}, \\
\gamma &= \frac{(1-H)w}{C}.
\end{aligned}$$

¹⁵Our method can be regarded as a variant of impulse response matching if the actual data are interpreted as a response to a one-time shock to the debt buildup.

Solve the simultaneous equations for (κ, π_n) ,

$$\begin{aligned}\kappa[V\theta'(\pi_n) - k] &= [1 + \phi\mu\theta'(\pi_n)]\eta\alpha K^{1-\alpha}L^{\alpha-1}, \\ k &= \frac{\beta}{\sigma g}[wH - \phi\alpha K^{1-\alpha}L^\alpha].\end{aligned}$$

- Step 4. Compute the transition dynamics based on Appendix E, on a grid of 99×99 equally spaced points over the square $[\underline{\phi}; \bar{\phi}] \times [\underline{\sigma}; \bar{\sigma}]$. Given the collateral ratio (ϕ, σ) , z_{10} is determined to minimize (17) for each grid.
- Step 5. Choose $(\phi_{(i^*)}, \sigma_{(j^*)})$ to minimize (17) over the two dimensions $[\underline{\phi}; \bar{\phi}] \times [\underline{\sigma}; \bar{\sigma}]$ where (i^*, j^*) is the optimal index.
- Step 6. Set $(\phi^1, \sigma^1) = (\phi_{(i^*)}, \sigma_{(j^*)})$. If $|\phi^1 - \phi^0| < 1e-05$ and $|\sigma^1 - \sigma^0| < 1e-05$ stop, else set the value $\underline{\phi} = \phi_{(i^*-2)}$, $\underline{\sigma} = \sigma_{(j^*-2)}$, $\bar{\phi} = \phi_{(i^*+2)}^*$, $\bar{\sigma} = \sigma_{(j^*+2)}$, $\phi^0 = \phi^1$, $\sigma^0 = \sigma^1$ and return to step 3.

Figure 6 shows the results of the grid search for Japan, the United States, and the EU. The value of this figure represents the residuals of (17). A cold (warm) hue of contour lines shows a small (long) distance between the simulation and actual data. In the white areas of the parameter space, conditions (18)–(23) are not satisfied. This figure implies that parameters (ϕ, σ) are uniquely determined because (17) does not have local minima.

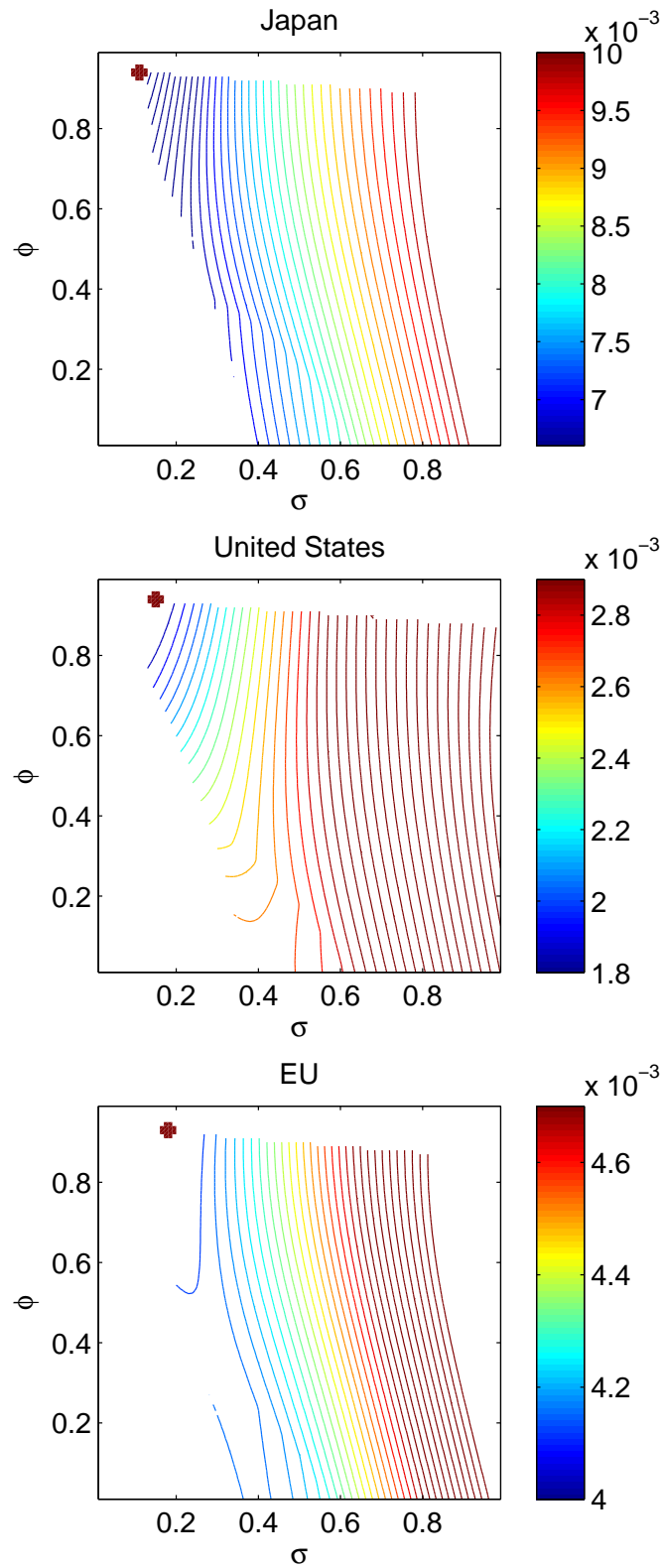


Figure 6: Grid search

Note: A contour line represents the residuals of (17) and a red cross means the global minimum of the residuals.