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Debt-Ridden Borrowers and Productivity Slowdown *

Keiichiro Kobayashi† and Daichi Shirai‡

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Abstract

Many authors argue that financial constraints have been tightened in several countries since the Great Recession in 2007–2009. To explain this, we construct a model in which borrowing constraints for firms are tightened as a result of mass default due to a bubble collapse. In Jermann and Quadrini's (2012) model, a defaulting firm either goes back to being a normal firm by (partially) repaying its debt or is liquidated. We assume that there is an intermediate status: a "debt-ridden" firm, defined as a firm whose lender retains the right to liquidate it. The lender allows the debt-ridden firm to continue if it pays continuation fee. In our model, debt forgiveness is infeasible: once a firm defaults on the debt, it is either liquidated or kept as a debt-ridden firm. The defaulter cannot go back to being a normal firm, unless it repays all its debt. Prohibition of debt forgiveness can be justified as a collective choice of the society, in order to expand the borrowing limit for normal firms.

It is shown that borrowing constraints are tighter for debt-ridden than for normal firms. This implies that the emergence of a large mass of debt-ridden borrowers may be a cause of the "financial shocks" discussed in recent macroeconomic literature. Tightened borrowing constraints due to the emergence of debt-ridden firms lower the aggregate productivity. This negative effect on productivity can be permanent. In a version of the model with endogenous growth, the growth rate of aggregate productivity becomes zero if the number of debt-ridden firms exceeds a certain threshold.

Keywords: Borrowing constraint, working capital, financial shocks, productivity slowdown.
JEL Classification numbers: E30, G01, O40.

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1 Introduction

The decade after a financial crisis tends to be associated with low economic growth (Reinhart and Rogoff 2009, Reinhart and Reinhart 2010). It is also known that the growth of total factor productivity slows down or even becomes negative for a decade (Kehoe and Prescott 2007). Why does productivity growth slow down persistently after a financial crisis? Many authors argue that financial constraints were tightened during and after the Great Recession in 2007–2009. Why are financial constraints tightened after a financial crisis or the collapse of an asset-price bubble? Can tightening of financial constraints cause a persistent slowdown in economic growth? We consider these questions in this paper and propose a theoretical model, in which the emergence of debt-ridden borrowers lowers the aggregate productivity persistently through tightening financial constraints.

We construct a general equilibrium model, in which an exogenous shock makes a substantial number of firms default on their debt. The model is based on Jermann and Quadrini's (2006, 2012) models, which explicitly consider the bargaining process after a default to derive a borrowing constraint, à la Kiyotaki and Moore (1997). In Jermann and Quadrini, firms' ability to borrow is bounded by the limited enforceability of the debt contract. The borrowing firm can default on the debt obligation and renegotiate repayment. Borrowing is limited such that the amount borrowed is renegotiation-proof. In the hypothetical renegotiation in Jermann and Quadrini, the defaulting firm can go back to being a normal firm if the lender and the firm agree on repayment; otherwise, the firm is liquidated if the renegotiation breaks down. Thus, there are two statuses for a firm: being a normal firm and being liquidated.

A novel feature of our model is that we assume an intermediate status: being a "debt-ridden" firm. We define a debt-ridden firm as one whose lender retains the right to liquidate it. The lender allows the debt-ridden firm to continue operations, if she agrees to the amount of continuation fee to be paid by the firm. We assume that once a firm defaults on its debt, it can never go back to being normal, unless it repays all of the original debt. The defaulting firm is either liquidated or kept as a debt-ridden firm.

We analyze the borrowing constraint for debt-ridden firms and show that it is tighter than for normal firms in the case of working capital loans. This result seems counterintuitive, because debt-ridden firms are under the control of their lenders, whereas normal firms are not. The reason for this result is that in our model, if a normal firm defaults on its debt, it inevitably becomes a debt-ridden firm unless it repays all of the original debt. Thus, if a normal firm defaults, it loses its status of a normal firm, whereas if a debt-ridden firm defaults, it continues as a debt-ridden firm after renegotiation. Therefore, a normal firm loses more by defaulting than a debt-ridden firm does. Because of this, the incentive-compatibility condition (or no-default condition) implies that a normal firm can borrow more than a debt-ridden firm can.

Tighter borrowing constraints for working capital loans of debt-ridden firms makes their production activity inefficient. If a substantial number of firms become debt-ridden, both the aggregate borrowing capacity and productivity decline. This implies that the emergence of debt-ridden borrowers may be a cause of the "financial shocks" discussed in the recent macroeconomic literature. After the Great Recession in 2007–2009, many authors argue that a shock in the financial sector can cause a severe recession (e.g., a risk shock in Christiano, Motto, and Rostagno 2011, and a financial shock in Jermann and Quadrini 2012). In our model, the emergence of a substantial number of debt-ridden firms manifests itself as a tightening of the aggregate borrowing constraint, which can be interpreted as a financial shock.

We also show that the emergence of debt-ridden firms has a persistent negative effect on productivity. Higher inefficiency due to tighter borrowing constraints lowers the value of new entry to the market for a potential entrant, and discourages R&D activity. The decrease in R&D activity leads the economy into a steady state with low productivity. We then consider a modified version of the model where the economy grows endogenously, and show in a numerical example that the growth rate of aggregate productivity may become zero permanently, if the number of debt-ridden firms exceeds a certain threshold. This result seems consistent with the facts observed in persistent recessions after financial crises (see Section 2).

Related literature Our theory is related to the literature on debt overhang. Myers (1977) pointed out the suboptimality of debt in the corporate finance literature and Lamont (1995) applied the notion of debt overhang in macroeconomics.[1] The debt overhang problem typically occurs when a firm cannot borrow new money for productive projects because its existing debt is too large. Debt overhang arises if the existing debt holder is

[1] See also Krugman (1988) on debt overhang in international finance. See Moyen (2007) and Chen and Manso (2010), for example.

different from the potential lender who would lend new money. In this paper, we take a small step forward by proposing a new theory that inefficiency can arise even if the lender of new money *is* the existing debt holder. This paper is also close to Caballero, Hoshi, and Kashyap (2008). They analyze "zombie lending," defined as a de facto subsidy to unproductive firms from banks. They argue that congesting zombie firms hinder the entry of highly productive firms and lower aggregate productivity. In this paper, we make a complementary point to their argument: even an intrinsically productive firm can become unproductive when it becomes debt-ridden. This is because a debt-ridden firm is subject to tighter borrowing constraints for working capital loans.

This paper is organized as follows. In the next section, we review the facts on persistent recessions after financial crises. Section 3 proposes and analyzes the model. We modify the model to an endogenous growth model in Section 3.4. Section 4 presents our concluding remarks.

2 Facts on persistent recessions after financial crises

Numerous examples of productivity slowdown after a financial crisis have been observed. The most notable is the Great Depression in the 1930s in the US and similar depressions in that period in the major nations. Ohanian (2001) shows that there was a large productivity decline during the US Great Depression that is unexplained by capital utilization or labor hoarding. Kehoe and Prescott (2007) drew our attention to the fact that many countries experienced decade-long severe recessions, which they call the "great depressions" of the twentieth century. Papers in their book unanimously emphasize that declines in the growth rate of total factor productivity were the primary cause of these great depressions.

Another example of a decade-long recession after a financial crisis is the 1990s in Japan. The growth rates of gross domestic product (GDP) and total factor productivity (TFP) in the 1990s were both lower than in the 1980s. Figure 1 shows the GDP and the potential capacity in Japan. The kink in the beginning of the 1990s is apparent, when huge asset-price bubbles burst in the stock and real estate markets. See Figure 2 for asset prices in Japan in the 1990s. Table 1 shows various estimates of the TFP growth rate in Japan. Hayashi and Prescott (2002) emphasize that the growth of TFP slowed down in the 1990s in Japan.[2] One notable feature in the 1990s is the significant decrease in entries and

[2]There is substantial debate on whether the TFP slowdown in Japan is truly a slowdown of technical

increase in exits of firms. See Figure 3 for a comparison of entry and exit of firms between Japan and the US. In the literature, the procyclicality of net entry is well known (Bilbiie, Ghironi, and Melitz 2007). Net entry also contributes significantly to TFP growth for US manufacturing establishments (Bartelsman and Doms 2000). Nishimura, Nakajima, and Kiyota (2005) argue that the malfunctioning of entry and exit contributes substantially to a fall in Japan's TFP in the late 1990s. Another characteristic of Japan in the 1990s was the persistently lingering nonperforming loans (NPLs) in the banking sector. The NPLs were the excess debt of nonfinancial firms, mainly in real estate, wholesale, retail, and construction. Figure 4 shows the amount of NPLs in Japan from 1992 through 2009. The delayed disposal of huge NPLs was seen as a de facto subsidy to nonviable firms ("zombie lending"). This zombie lending has also been named as the cause of Japan's persistent recession (Peek and Rosengren 2005, and Caballero, Hoshi, and Kashyap 2008).

3 Model

We consider a closed economy in which the final good is produced competitively from labor input and varieties of intermediate goods. The firms are monopolistic competitors and produce respective varieties of intermediate goods from material input, which is the final good, and capital input. In our model, when a firm defaults on its debt, the lender cannot forgive the debt. Instead, the lender can choose whether to liquidate the firm or allow it to continue operations as a "debt-ridden firm." In this paper, a debt-ridden firm is one whose lender retains the unilateral discretion to liquidate the firm. We later clarify the difference between normal and debt-ridden firms by formally defining their respective optimization problems. The model is a version of the expanding variety model, in which new entry of firms increases aggregate productivity. The expanding variety model was proposed by Rivera-Batiz and Romer (1991) and simplified by Acemoglu (2009). We follow the latter's exposition. In Sections 3.1–3.3, we analyze the model in which the economy converges to a steady state. In Section 3.4, we introduce a positive externality from the variety of goods to aggregate productivity that enables endogenous growth. We show in Section 3.4 that the growth rate falls to zero if there is a sufficient number of debt-ridden firms.

progress or simply a measurement error (see Kawamoto 2005, Fukao and Miyagawa 2008). The tentative conclusion on this issue in the literature is that there was a slowdown in technical progress in Japan, though it may not be as severe as Hayashi and Prescott originally measured.

3.1 Basic setup

Supplies of capital K and labor L are fixed. Labor is used for producing the final good, whereas capital is used for the production of intermediate goods. A mass of firms, indexed by $i \in [0, N_t]$, produces intermediate goods, where N_t is the measure of the varieties of intermediate goods in period t. The firms are either normal or debt-ridden firms; the status of being "normal" or "debt-ridden" is clarified later. A representative household owns these firms and solves the following problem:

$$
\begin{aligned}
\max_{C_t, I_t, b_{t+1}} \quad & E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln C_t \right], \\
\text{subject to} \quad & C_t + \frac{b_{t+1}}{1+r_t} + I_t \leq w_t L + \int_0^{N_t} \pi_{it} di + \xi_t b_t + d_t, \\
& N_{t+1} = (1-\delta) N_t + \chi I_t, \\
& I_t \geq 0.
\end{aligned}
$$

Here, β is the subjective discount factor, C_t is consumption, L is the fixed amount of labor supply, I_t is the R&D investment, w_t is the wage rate, r_t is the market interest rate, b_t is a bond issued by the normal firms, d_t is payment by the debt-ridden firms, and π_{it} is the profit from firm i, where $i \in [0, N_t]$. The bond is risky debt and ξ_t is its recovery rate, where $\xi_t = 1$ if no firm defaults and $\xi_t < 1$ if some firms default. The cost of R&D investment is measured in units of the final good; one unit of R&D investment creates χ units of a new variety of intermediate goods. The δ fraction of the varieties of intermediate goods depreciates every period. The first-order conditions (FOCs) with respect to I_t and N_{t+1} imply that R&D investment takes place if $\chi V_{nt} = 1$; it does not take place if $\chi V_{nt} < 1$, where V_{nt} is the value of a normal firm. The cost of R&D investment is unity and the expected gain from it is χV_{nt}. The amount of R&D investment is determined such that the cost and gain are equalized: $\chi V_{nt} = 1$.

The final good is produced competitively from the intermediate goods x_{it}, $i \in [0, N_t]$ and labor by the following production function:

$$
Y_t = \frac{1}{\eta} \left(\int_0^{N_t} x_{it}^{\eta} di \right) L^{1-\eta},
$$

where $0 < \eta < 1$. Because the final good producer maximizes $Y_t - \int_0^{N_t} p_{it} x_{it} di - w_t L_t$, where p_{it} is the real price of the intermediate good i, perfect competition in the final good market implies that

$$p_{it} = p(x_{it}) = L^{1-\eta} x_{it}^{\eta-1},$$
$$w_t = \frac{1-\eta}{\eta}\frac{Y_t}{L}.$$

Firm i produces the intermediate good i from capital input k_{it} and the material input m_{it}, where m_{it} is the final good, by the following production function:

$$x_{it} = A_{it} k_{it}^{\alpha} m_{it}^{1-\alpha},$$

where A_{it} is the productivity parameter. For simplicity of analysis, we assume that productivity is identical across firms and constant over time, that is,

$$A_{it} = A \quad \text{for all } i \text{ and } t.$$

Firm i needs to buy k_{it} at the price of q_{t-1} in period $t-1$ and m_{it} at the price of unity in period t. We assume for simplicity that on the one hand, there is no financial friction in buying and selling the physical capital k_{it} in the market. Thus, firm i can pay any amount of $q_{t-1}k_{it}$. On the other hand, firm i needs to borrow working capital m_{it} from the representative household to buy the material input; this debt is subject to the financial constraint that we specify below. Firm i incurs both intertemporal debt b'_{it} and intra-temporal debt (working capital) m_{it}. The intertemporal debt contract is such that firm i borrows $\frac{b'_{it}}{1+r_{t-1}}$ at the end of period $t-1$, where r_{t-1} is the market rate. The sole shock in this economy is the idiosyncratic redistribution shock Δ_{it} that changes the amount of the intertemporal debt to be repaid in period t. We define b_{it} by

$$b_{it} \equiv b'_{it} + \Delta_{it}.$$

This is the amount that firm i must repay in period t to the lender of the intertemporal debt. All agents know the probability distribution function of Δ_{it} and take the expectation over the distribution of Δ_{it} when they decide their actions.

The timing of actions of firm i is as follows. At the end of period $t-1$, firm i borrows $\frac{b'_{it}}{1+r_{t-1}}$ and purchases capital stock k_{it} at price q_{t-1}. At the beginning of period t, the redistribution shock Δ_{it} is revealed and the intertemporal debt becomes $b_{it} = b'_{it} + \Delta_{it}$. The firm then borrows working capital m_{it}, purchases the material input, and produces x_{it}. After selling x_{it}, it repays debt $m_{it} + b_{it}$ and sells k_{it} at price q_t. Note that there is no stochastic shock during the short span when the firm borrows the working capital. Therefore, the gross interest rate for the intra-period working capital is 1.

We now specify the borrowing constraint for firm i. In what follows, we omit the subscript i unless there is a risk of confusion. It will be shown that the firm's debt is subject to the following borrowing constraint:

$$m_t + b_t \leq \phi q_t k_t + V_{nt} - V_{zt}, \tag{1}$$

where V_{nt} is the value of a normal firm and V_{zt} is the value of a debt-ridden firm; we will later specify V_{nt} by (2) and V_{zt} by (9). It will be shown that $V_{nt} > V_{zt}$ in equilibrium. The borrowing limit is derived from the limited enforceability of debt contracts, because firms can default on their debt obligations. The basic logic is the same as Jermann and Quadrini (2012). The decision to default arises after the realization of revenues, but before repaying the inter-period and intra-period loans. At this stage, the total liabilities are $m_t + b_t$, that is, the intra-period loan plus the inter-period debt due in period t. As in the Jermann–Quadrini model, the lender and the firm renegotiate on repayment if the firm defaults. If the renegotiation breaks down and the firm is liquidated at this point, the lender obtains $(\phi + \psi)q_t k_t$ by confiscating a part of capital stock (k_t), where $0 < \phi + \psi < 1$. A departure from the Jermann–Quadrini model is the following assumption:

Assumption 1. The institutional environment of this economy is such that a lender cannot forgive the debt of a borrowing firm that defaults. Once the firm defaults, the lender obtains the unilateral discretion to liquidate it. The legal institution in this economy is such that as long as the repayment f_t is strictly less than the original debt ($m_t + b_t$), the lender can retain the right to liquidate the firm. Furthermore, the lender can retain and exercise this right without any penalty even after she receives f_t ($< m_t + b_t$) from the firm. This implies that the lender never waivers the right to liquidate the firm in exchange for any repayment cheaper than the original debt. Instead, the lender either liquidates the defaulting firm immediately or allows it to continue operations as a debt-ridden firm. The debt-ridden firm is one whose lender retains the right to liquidate it and allows it to continue operations if she agrees on the continuation fee that the firm pays to the lender. The lender and the debt-ridden firm negotiate at most twice in each period. The first negotiation arises if the firm defaults on the debt obligation in the middle of the period. In this negotiation, the lender and firm negotiate on the repayment f, whereas the lender obtains $(\phi + \psi)q_t k_t$ if she liquidates the firm at this stage. The second negotiation, which arises at the end of the period, is over the continuation fee d_{t+1} that the firm should pay in the next period. If the lender liquidates the firm in the second negotiation, she obtains only $\psi q_t k_t$.

The difference in the liquidation value of the firm in the first and second negotiations can be justified as follows. The first negotiation takes place in the middle of the period and the second at the end of the period; the firm can conceal or sell off a part of its capital (ϕk_t) during the period between the two negotiations. Given this institutional environment, a normal firm inevitably becomes a debt-ridden firm once it defaults on its debt. The defaulting firm loses $V_{nt} - V_{zt}$ inevitably. Appendix A shows that the defaulting firm and the lender renegotiate on repayment and agree that the firm repays $\phi q_t k_t$ in period t. The incentive-compatibility constraint for a normal firm not to default is that the original debt, $(m_t + b_t)$ is no greater than the value it loses by defaulting, $(\phi q_t k_t + V_{nt} - V_{zt})$. Thus, a normal firm solves the following Bellman equation:

$$V_{nt} = \max_{k,b'} \frac{b'}{1+r_t} - q_t k + E_t \left[\max_m \beta \frac{\lambda_{t+1}}{\lambda_t} \left\{ p(x)x - m - \tilde{b} + q_{t+1}k + \tilde{V}_{t+1} \right\} \right], \tag{2}$$

$$\text{subject to} \quad x = A_{t+1} k^{\alpha} m^{1-\alpha}, \tag{3}$$

$$b = b' + \Delta, \tag{4}$$

$$m \leq \max\{0, \quad \phi q_{t+1}k + V_{nt+1} - V_{zt+1} - b\}, \tag{5}$$

$$\tilde{b} = b \ \text{ and } \ \tilde{V}_{t+1} = V_{nt+1} \quad \text{if } \ b \leq \phi q_{t+1}k + V_{nt+1} - V_{zt+1}, \tag{6}$$

$$\tilde{b} = \phi q_t k_t \ \text{ and } \ \tilde{V}_{t+1} = V_{zt+1} \quad \text{if } \ b > \phi q_{t+1}k + V_{nt+1} - V_{zt+1}, \tag{7}$$

where λ_t is the Lagrange multiplier of the budget constraint in the household's problem and $\tilde{b}$ is the realized repayment of the intertemporal debt b. Constraint (5) says that if the realization of Δ is such that $\phi q_{t+1}k + V_{nt+1} - V_{zt+1} - b < 0$, the firm sets $m = 0$. Because Appendix A shows that the repayment after default is $\phi q_t k_t$, it is easily shown that the payoff of no default, $(q_t k_t + V_{nt+1} - b)$, is strictly less than that of defaulting, $((1-\phi)q_t k_t + V_{zt+1})$. Therefore, the firm defaults on b if $\phi q_{t+1}k + V_{nt+1} - V_{zt+1} - b < 0$.

To exclude the equilibrium in which all firms intentionally default after borrowing too much b_t, we assume that the values of the parameters are chosen such that

$$\forall t, \quad V_{nt} - V_{zt} \geq \psi q_t k_t \tag{8}$$

in the equilibrium path.[3]

Why is debt forgiveness prohibited? Our institutional setting of no debt forgiveness is justified as a collective choice of the society to expand the borrowing limit. Given that the parameter values satisfy (8), the borrowing limit for a normal firm in an economy where debt forgiveness is feasible is $(\phi + \psi)q_t k_t$, as we show in Appendix D. However, the limit is $\phi q_t k_t + V_{nt} - V_{zt}$ in an economy where debt forgiveness is prohibited. (8) implies that the borrowing limit is higher in the economy where debt forgiveness is prohibited than in the economy where it is allowed. Thus, the prohibition of debt forgiveness can be justified as a social choice to expand the borrowing limit.

We now consider the behavior of a debt-ridden firm. We assume the following for the institutional setting surrounding the debt-ridden firm.

Assumption 2. At the end of period t, the lender and the debt-ridden firm bargain over the continuation fee, d_{t+1}, that the latter should pay the former in period $t+1$. If they agree on d_{t+1}, the lender allows the firm to continue operations in period $t+1$ and the firm maximizes its net profit after paying d_{t+1}. The debt-ridden firm cannot accumulate financial assets intertemporally.

The last sentence of this assumption implies that the debt-ridden firm cannot make savings. As we show in Appendix C, this assumption is not necessary for our results in the case of the deterministic equilibrium.

The bargaining over d_{t+1} takes place at the end of period t. We assume that if the bargaining breaks down and the firm is liquidated at this point, the lender obtains only $\psi q_t k_t$ by confiscating a part of the capital stock. We assume that the lender cannot confiscate $(\phi+\psi)q_t k_t$, because $\phi q_t k_t$ has been concealed from her at the end of the period.[4]

Therefore, if the debt-ridden firm continues operations, the lender can obtain $D_t \equiv \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left\{ \tilde{d}_{t+1} + D_{t+1} \right\} \right]$, where $\tilde{d}_t \leq d_t$ is the realized payment, whereas the promised

[3] Note that the lender is willing to lend as long as $m_t + b_t \leq (\phi + \psi)q_t k_t$, because she can obtain $(\phi + \psi)q_t k_t$ by liquidating the firm if it defaults on the debt. Thus, if $V_{nt} - V_{zt} < \psi q_t k_t$, all firms set the highest possible b_t at the end of period $t-1$ such that $\phi q_t k_t + V_{nt} - V_{zt} < m_t + b_t \leq (\phi + \psi)q_t k_t$, and all of them intentionally default in period t. In this case, all firms become debt-ridden firms from period t onward. This equilibrium path is self-consistent but does not seem to be relevant to reality. Condition (8) excludes the possibility of the emergence of this "all default" equilibrium.

[4] The debt-ridden firm and the lender can determine the value of d_{t+1} in the first negotiation in the middle of period t. However, because they have a second chance to renegotiate at the end of period t, the value of d_{t+1} must be identical to the value that is determined in the second renegotiation. Otherwise, d_{t+1} is revised in the second renegotiation at the end of period t.

amount is d_t; and D_t is the present value of the flow of payments from the firm. On the other hand, the lender can obtain $\psi q_t k_t$ if she liquidates the firm immediately. As we see in Appendix B, the bargaining outcome d_{t+1} is determined by

$$D_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left\{ \tilde{d}_{t+1} + D_{t+1} \right\} \right] = \psi q_t k_t.$$

The lender and the firm determine d_{t+1}, taking $\{k_{t+j}\}_{j=1}^{\infty}$ as given. See Appendix B for a detailed description of the values of d_{t+1} and $\tilde{d}_{t+1}$. After d_{t+1} is agreed upon at the end of period t, the debt-ridden firm is allowed to operate in period $t+1$. The debt-ridden firm's actions are as follows. At the end of period t, after d_{t+1} is agreed upon, it sells k_t and buys k_{t+1} in the market. At the beginning of period $t+1$, after realization of A_{t+1}, it borrows working capital m_{t+1} to purchase the material input (the final good). It produces the intermediate good $A_{t+1} k_{t+1}^{\alpha} m_{zt+1}^{1-\alpha}$ and sells it in the monopolistically competitive market. After it receives the proceeds of sales, it repays the debt $m_{zt+1} + d_{t+1}$.

The debt-ridden firm's ability to borrow the working capital m_{zt+1} is bounded by the limited enforceability of debt contracts. After realization of revenues and before repaying m_{zt+1}, the debt-ridden firm has a chance to default on its debt $(m_{zt+1} + d_{t+1})$. If the firm defaults, the firm and the lender renegotiate on the repayment f as described in Appendix B. If they agree on f, the lender obtains $f + D_{t+1}$, whereas she obtains $(\phi + \psi) q_{t+1} k_{t+1}$ by liquidating the firm if the bargaining breaks down. Therefore, $f + D_{t+1} = (\phi + \psi) q_{t+1} k_{t+1}$ and the no-default condition ($m \leq \max\{0, f - d_{t+1}\}$) imply that

$$m_{zt+1} \leq \max\{0,\ (\phi + \psi) q_{t+1} k_{t+1} - d_{t+1} - D_{t+1}\}.$$

Because it is shown that $D_{t+1} = \psi q_{t+1} k_{t+1}$ in Appendix B, the borrowing constraint for m_{zt+1} is $m_{zt+1} \leq \max\{0,\ \phi q_{t+1} k_{t+1} - d_{t+1}\}$. The debt-ridden firm maximizes its own value V_{zt}, defined by the following Bellman equation. Given $\{d_{t+j}\}_{j=1}^{\infty}$, the debt-ridden firm solves

$$V_{zt} = \max_{k_{t+1}} -q_t k_{t+1} + E_t \left[\max_{m_{t+1}} \beta \frac{\lambda_{t+1}}{\lambda_t} \{ p(x) x - m_{t+1} - d_{t+1} + q_{t+1} k_{t+1} + V_{zt+1} \} \right], \quad (9)$$

$$\text{subject to} \quad x = A_{t+1} k_{t+1}^{\alpha} m_{t+1}^{1-\alpha}, \quad (10)$$

$$m_{t+1} \leq \max\{0,\ \ \phi q_{t+1} k_{t+1} - d_{t+1}\}. \quad (11)$$

Note that d_{t+1} does not depend on the firm's choice of k_{t+1}, because the lender and the firm agree on d_{t+1} at the end of t, taking the expected value of k_{t+1} as given.

One may suspect that prohibiting corporate savings by debt-ridden firms in Assumption 2 is crucial in deriving a persistent inefficiency due to the binding borrowing constraint (11). If the firm can accumulate financial assets, it may be possible to relax the borrowing constraint eventually. However, we show in Appendix C that at least in the deterministic equilibrium, the borrowing constraint is identical to (11), even if the debt-ridden firm can save, under the assumption that the lender can confiscate the savings when she liquidates the firm. Thus, there is no incentive for the debt-ridden firm to save in the deterministic case.

3.2 Equilibrium without debt-ridden firms

In this subsection, we characterize the deterministic equilibrium in which $\Delta_{it} = 0$ with probability one. We also assume that $\delta = 0$, that is, a variety of good is not depleted once it is created.

We consider the economy in which all firms are normal and there is no debt-ridden firm. The resource constraints of the economy are

$$C_t + I_t + \int_0^{N_t} m_{it} = Y_t,$$
$$K = \int_0^{N_t} k_{it} di.$$

The economy converges to a stationary equilibrium in which the number of firms $N_t = N$ is time-invariant and determined by

$$V_{nt} = \frac{1}{\chi}.$$

In the symmetric stationary equilibrium, there is no R&D investment ($I_t = 0$) and the resource constraints are $C = Y - Nm$ and $K = Nk$, where k and m are the capital and material inputs, respectively, for one firm. We focus on the parameter values that make the borrowing constraint (5) non-binding in the stationary equilibrium. In this case, $m = \hat{m}$, where $\hat{m} = \arg\max_m p(A_{it+1} k_{t+1}^{\alpha} m^{1-\alpha}) A_{it+1} k_{t+1}^{\alpha} m^{1-\alpha} - m$. That the borrowing constraint is non-binding implies that

$$\hat{m} \leq \phi q_{t+1} k_{t+1} + V_{nt+1} - V_{zt+1}.$$

In this case, b_{t+1} is indeterminate. However, we assume an infinitesimally small tax benefit for issuing intertemporal debt, such that firms are willing to borrow intertemporal debt up to the borrowing limit. In the case of a deterministic equilibrium, the amount of b_{t+1} is determined by $b_{t+1} = \phi q_{t+1}k_{t+1} - \hat{m} + V_{nt+1} - V_{zt+1}$. Note that V_{nt} does not depend on b_{t+1}, because the tax benefit is infinitesimal, and that the loan rate and market rate are equal and satisfy

$$\frac{b_{t+1}}{1+r_t} = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right] b_{t+1}.$$

3.3 Equilibrium with debt-ridden firms

We assume that the economy is initially in the symmetric stationary equilibrium where all firms are normal, $I_t = 0$, and $N_t = N$, where N is determined so that $V_n = \frac{1}{\chi}$. We assume that in period τ, the economy is hit by redistribution shocks $\{\Delta_{i\tau}\}_{i\in[0,Z]}$ that make $b_{i\tau} > \phi q_\tau k_\tau + V_{n\tau} - V_{z\tau}$ for $i \in [0, Z]$ and $b_{i\tau} \leq \phi q_\tau k_\tau + V_{n\tau} - V_{z\tau}$ for $i \in (Z, N]$. Thus, firm $i \in [0, Z]$ chooses to default on $b_{i\tau}$ and pays $\phi q_t k_t$, whereas firm $i \in (Z, N]$ does not default on its debt. At the end of period τ, the firms in $[0, Z]$ and their lenders negotiate on the amount of the continuation fee $d_{\tau+1}$. We conjecture and verify later that $V_{nt} < \frac{1}{\chi}$ along the equilibrium path. Since $V_{nt} < \frac{1}{\chi}$, no R&D investment takes place and N_t does not change from N. Since N, K, and L are all constant over time, the equilibrium is the steady state. We focus on the equilibrium where the borrowing constraint (5) for normal firms is not binding and (11) for debt-ridden firms is binding. Denoting the variables for debt-ridden and normal firms with the subscripts z and n, respectively, the equilibrium is described by the following system of equations:

$$1 = (1-\alpha)\eta L^{1-\eta} A^{\eta} k_n^{\alpha\eta} m_n^{(1-\alpha)\eta - 1}, \tag{12}$$

$$1 + \mu_z = (1-\alpha)\eta L^{1-\eta} A^{\eta} k_z^{\alpha\eta} m_z^{(1-\alpha)\eta - 1}, \tag{13}$$

$$q = \beta \left[\alpha\eta L^{1-\eta} \frac{\left(A k_n^{\alpha} m_n^{1-\alpha}\right)^{\eta}}{k_n} + q \right], \tag{14}$$

$$q = \beta \left[\alpha\eta L^{1-\eta} \frac{\left(A k_z^{\alpha} m_z^{1-\alpha}\right)^{\eta}}{k_z} + (1 + \phi\mu_z) q \right], \tag{15}$$

$$Y = \frac{1}{\eta} A^{\eta} L^{1-\eta} \{ Z (k_z^{\alpha} m_z^{1-\alpha})^{\eta} + (N - Z)(k_n^{\alpha} m_n^{1-\alpha})^{\eta} \}, \tag{16}$$

$$C = Y - (N - Z) m_n - Z m_z, \tag{17}$$

$$K = Z k_z + (N - Z) k_n, \tag{18}$$

$$m_n < \phi q k_n + V_n - V_z, \tag{19}$$

$$d = (\beta^{-1} - 1)\psi q k_z, \tag{20}$$

$$m_z + d = \phi q k_z, \tag{21}$$

$$V_n = -q k_n + \beta \{ L^{1-\alpha} (A k_n^{\alpha} m_n^{1-\alpha})^{\eta} - m_n + q k_n + V_n \}, \tag{22}$$

$$V_z = -q k_z + \beta \{ L^{1-\alpha} (A k_z^{\alpha} m_z^{1-\alpha})^{\eta} - m_z - d + q k_z + V_z \}, \tag{23}$$

where μ_z is the Lagrange multiplier for (11). This system of equations is solved as follows. First, (13), (15), (20), and (21) imply that

$$\mu_z = \frac{1 - \beta - [\phi - (\beta^{-1} - 1)\psi] \frac{\alpha\beta}{1-\alpha}}{\phi\beta + [\phi - (\beta^{-1} - 1)\psi] \frac{\alpha\beta}{1-\alpha.}} \tag{24}$$

Given μ_z, we define Λ by

$$\Lambda = \left[\frac{1 - \beta}{1 - (1 + \phi\mu_z)\beta} \right]^{\frac{1-(1-\alpha)\eta}{1-\eta}} (1 + \mu_z)^{-\frac{(1-\alpha)\eta}{1-\eta}}. \tag{25}$$

The macroeconomic variables are given by

$$k_n = \frac{K}{N + (\Lambda - 1)Z}, \tag{26}$$

$$k_z = \frac{\Lambda K}{N + (\Lambda - 1)Z}, \tag{27}$$

$$m_n = \left[(1-\alpha)\eta L^{1-\eta} A^{\eta} k_n^{\alpha\eta}\right]^{\frac{1}{1-(1-\alpha)\eta}}, \tag{28}$$

$$m_z = \left(\frac{\Lambda^{\alpha\eta}}{1+\mu_z}\right)^{\frac{1}{1-(1-\alpha)\eta}} m_n, \tag{29}$$

$$q = \frac{\alpha\beta}{(1-\alpha)(1-\beta)} \frac{m_n}{k_n}, \tag{30}$$

$$d = (\beta^{-1} - 1)\psi q k_z \tag{31}$$

$$V_n = \frac{\beta}{(1-\alpha)(1-\beta)}(\eta^{-1} - 1)m_n, \tag{32}$$

$$V_z = \frac{\beta}{1-\beta}\left\{\frac{(1+\mu_z)}{(1-\alpha)\eta} m_z - m_z - d\right\} - q k_z. \tag{33}$$

These variables must satisfy (8) and (19), which we check numerically in the example of Figure 5.

Figure 5 shows the variables in the steady state equilibrium as functions of Z. The parameter values are given in the caption of Figure 5. We set χ such that the value of N equals unity in the stationary equilibrium, where $Z = 0$. The value of χ is, therefore, given by $V_{SS} = \chi_{SS}^{-1} = 3.4234$, where V_{SS} is the value of the normal firm in the stationary equilibrium where $Z = 0$. The aggregate productivity of the economy is proportional to

$$\frac{Y - (N - Z)m_n - Z m_z}{K^{\theta} L^{1-\theta}},$$

where $\theta = \frac{\alpha\eta}{1-(1-\alpha)\eta}$. $C = Y - (N - Z)m_n - Z m_z$ decreases as Z increases in this figure. Therefore, the aggregate productivity is decreasing in Z. Figure 5 also justifies our conjecture that $V_n < \chi_{SS}^{-1}$ $(= V_{SS})$ in the equilibrium where $Z > 0$. This is because V_n is decreasing in Z, as Figure 5 shows. Why does V_n decrease as the number of debt-ridden firms increases? Because the borrowing constraint is tight for debt-ridden firms, the value of capital stock k_t as a collateral asset for these firms is higher and they purchase more capital than the normal firms. As a result, the price of capital increases and the increased cost of capital pushes V_n down. This mechanism is similar to the congestion effect of zombie firms in Caballero, Hoshi, and Kashyap's (2008) model. Because of this congestion effect, the decline in productivity is permanent. This result forms a striking contrast to the equilibrium outcome in the case where debt forgiveness is feasible, which is described in Appendix D. We see in Appendix D that aggregate productivity comes back to the normal level immediately after a mass default, if the lenders and firms can agree on debt forgiveness.

Next, we consider the endogenous growth model and show that the congestion effect can lower the growth rate to zero permanently, because the raised cost of capital due to the emergence of debt-ridden firms discourages R&D investment.

3.4 Endogenous growth and the zero growth path

To enable endogenous growth in our model, we need the following externality:

Assumption 3. There is a positive externality from the growth in varieties of intermediate goods to productivity: $A_t = \hat{A}N_t^{\alpha}$, where $\hat{A}$ is an exogenous parameter. All agents take A_t as given and households do not recognize the effect of their choice of N_t on A_t.

The externality from expanding variety enables endogenous growth in this economy. It is shown below that if Z is small, the economy converges to the balanced growth path (BGP), in which productivity grows at a constant rate. However, if Z is large, the economy falls into the zero growth path (ZGP), in which the growth rate is zero.

The BGP in this economy is an equilibrium path in which all firms are normal and the economy grows at a constant rate, that is, $N_{t+1}/N_t = Y_{t+1}/Y_t = C_{t+1}/C_t = 1+g$, where $g > 0$. Because the capital K and labor L are fixed in supply, the expansion of variety N_t is the only source of economic growth in this model. We assume that the borrowing constraint (5) does not bind in the BGP. The price of capital in the BGP is $N_t q$. The variables (m, q, g) in the BGP are specified by

$$
\begin{aligned}
m &= [(1-\alpha)\eta L^{1-\eta}(\hat{A}K^{\alpha})^{\eta}]^{\frac{1}{1-(1-\alpha)\eta}},\\
q &= \frac{\alpha\beta}{(1-\alpha)(1-\beta)}\frac{m}{K},\\
\frac{1}{\chi} &= \left[\frac{\beta}{1+g-\beta}\left\{\frac{1}{(1-\alpha)\eta}-1\right\} - \frac{\alpha\beta}{(1-\alpha)(1-\beta)}\right] m.
\end{aligned}
$$

We consider a numerical example, the parameter values of which are the same as those in the example in the previous subsection, except for χ. We set the value of χ such that the growth rate of the BGP equals 1%. The value of χ is, therefore, given by $V_{BGP} = \chi_{BGP}^{-1} = 2.9307$. In this example, the variables are given by $m = 0.6657$, $q = 1.997$, and $g = 0.01$.

We are interested in whether the emergence of debt-ridden firms makes the economy follow the ZGP, in which R&D investment does not take place and the economy does not grow. Suppose that $N_t = 1$ and there emerge debt-ridden firms, whose measure is Z, in period t. Given Z, the ZGP is specified by the system of equations in the previous subsection (12)–(23). We consider the same numerical example as in the previous subsection, except that we change the value of χ from χ_{SS} to χ_{BGP}. If V_n calculated by (22) satisfies $V_n < \chi_{BGP}^{-1}$, then R&D investment does not take place and N_{t+j} stays at 1 for $j \geq 1$. In this case, the economy stays at a steady state where the productivity stays constant, which is the ZGP. As Figure 5 shows, $V_n > \chi_{BGP}^{-1}$ for $0 \leq Z < 0.5439$ and $V_n < \chi_{BGP}^{-1}$ for $0.5439 < Z \leq 1$. The economy falls into the ZGP if Z is larger than 0.5439.

In this example, V_n calculated by (22) is larger than χ_{BGP}^{-1} for $Z \in [0, 0.4647)$. In this case, the equilibrium is *not* given by (12)–(23); for if the equilibrium were the ZGP, the value of new entry V_{nt}, given by (22), would be strictly greater than the cost, χ_{BGP}^{-1}, and R&D investment would take place, leading to $N_{t+1} > N_t$, a contradiction. The economy grows such that the value of new entry V_{nt} is equal to χ_{BGP}^{-1} and converges to the BGP eventually. We describe the dynamics of the economy for a small $Z \in [0, 0.5439)$ in Appendix E.

Another numerical example is shown in Figure 6. In this example, we set $\phi = 0.13$, instead of 0.15 as in the example of Figure 5. Figure 6 shows the variables in the ZGP. With $\phi = 0.13$, the value of a debt-ridden firm along the BGP is negative: $V_{z,BGP} = -0.5505$, whereas it is positive along the ZGP, as the panel of V_z in Figure 6 shows. This means that there exist multiple equilibria, the BGP and the ZGP, in this economy for a large initial value of Z. First, for any value of Z, the BGP can be an equilibrium. Suppose that macroeconomic expectations prevail in the initial period that the economy would follow the BGP. Then, all Z firms that defaulted in the initial period exit immediately, anticipating their negative payoff, $V_z = -0.5505$, and only the normal firms remain in the market. Because of the exit of debt-ridden firms, the capital price q_t becomes sufficiently cheap such that R&D investment takes place and the economy grows at the BGP rate. Second, for $Z > 0.4651$, the ZGP can be an equilibrium. Suppose that macroeconomic expectations prevail in the initial period that the economy would follow the ZGP. Then, all the Z firms that defaulted in the initial period stay in the market, anticipating their positive payoff, $V_z > 0$. Figure 6 shows that $\chi_{BGP} V_{n,ZGP} < 1$ for $Z > 0.4651$, where $V_{n,ZGP}$ is the value of a new firm in the ZGP. In this case, the gains from R&D investment are strictly less than its cost along the ZGP. Therefore, if $Z > 0.4651$ and the macroeconomic expectations are that the economy would follow the ZGP, then no households conduct R&D investment and the economy stays at the ZGP.[5]

These examples in Figures 5 and 6 show that a mass default may shift the equilibrium from a growth path to the ZGP. This shift occurs because debt-ridden firms facing tighter borrowing constraints purchase collateral assets, that is, physical capital in our model, aggressively, leading to a rise in the cost of capital. The higher capital cost reduces the expected gains from R&D investment and discourages entry of new firms. If there emerge a sufficient number of debt-ridden firms, the expected gains from R&D fall below its cost and no one undertakes R&D, leading to zero growth in aggregate productivity. This negative effect of higher capital cost on firms' entry is similar to the negative congestion effect of zombie lending on productivity in Caballero, Hoshi, and Kashyap (2008).

4 Conclusion

Decade-long recessions with low productivity growth are often observed after financial crises. We proposed a hypothesis that the emergence of debt-ridden borrowers causes a persistent productivity slowdown. Economic agents become overly indebted as a result of the boom and bust of asset-price bubbles. If debt reduction or debt forgiveness is not easily implemented owing to rigidities in the market institution, the borrowers become debt-ridden. Analyzing the bargaining after default, we show that debt-ridden borrowers are subject to tighter borrowing constraints than normal firms, although they are under their lenders' control. The emergence of a substantial number of debt-ridden borrowers lowers the aggregate productivity through tightening the aggregate borrowing constraint. The emergence of debt-ridden firms also has a congestion effect, as discussed in Caballero, Hoshi, and Kashyap (2008), because these firms purchase the collateral asset aggressively and push its price up, discouraging the entry of new firms. In a version of our model in which the economy grows endogenously, we also show that the congestion effect lowers the growth rate of aggregate productivity to zero, if the measure of debt-ridden firms exceeds a certain threshold level and pessimistic expectations prevail.

[5] Note that the ZGP cannot exist in this economy if $Z \leq 0.4651$. This is because $\chi_{BGP} V_{n,ZGP} \geq 1$ for $Z \leq 0.4651$, implying that the gains from R&D investment are greater than or equal to its cost along the ZGP. Because R&D is conducted and N_t increases if the initial value of Z is no greater than 0.4651, the economy inevitably grows and the ZGP cannot be an equilibrium.

Tightening of aggregate borrowing constraints owing to the emergence of many debt-ridden borrowers may manifest itself as a “financial shock” during or after a financial crisis. The mechanism of tightening of the borrowing constraint in this model is simple. We can easily embed this model into a standard dynamic stochastic general equilibrium model and assess, qualitatively or quantitatively, whether the emergence of debt-ridden borrowers is a primary cause of the financial shocks in recent macroeconomic events. We leave this topic for future research.

Appendix A: Derivation of borrowing constraint for normal firms

The borrowing constraint is derived from the argument of what happens if a firm defaults on its debt. The firm and the lender have the following three options after default.

- Liquidation: The firm manager refuses to pay and walks away. The lender confiscates a part of the firm's assets.
- Making the firm debt-ridden: In exchange for a continuation fee, the lender allows the firm to continue operations but retains the right to liquidate it.
- Debt forgiveness: In exchange for a partial or zero repayment, the lender releases the firm and waivers the right to liquidate it.

We need to specify the details of the institutional setting for bargaining after default. For this, we make the following detailed assumption instead of Assumptions 1 and 2.

Assumption 4. In period t, firm i can default on the debt obligation of $m_t + b_t$ after the firm obtains the proceeds $p(x_t)x_t$. If firm i defaults, then the firm and the lender can renegotiate on the amount of repayment f_t.

1. Once firm i defaults, the lender obtains the unilateral discretion to liquidate it. The legal institution in this economy is such that as long as the repayment f_t is strictly less than the original debt $(m_t + b_t)$, the lender can retain the right to liquidate firm i. Furthermore, the lender can retain and exercise this right without any penalty even after she receives f_t $(< m_t + b_t)$ from firm i, implying that the lender never sells the right to liquidate firm i at any price cheaper than the original debt.

2. If the lender liquidates the firm in the middle of the period, she obtains a part of the physical capital $(\phi + \psi)q_t k_t$, where $0 < \phi + \psi < 1$. When firm i is liquidated, the variety i of the intermediate goods disappears from this economy.

3. If the lender and the firm agree on f_t that is equal to or greater than the original debt, the firm regains the right to choose whether or not to continue its own business. Both the lender and the firm can verify that the lender loses the right to liquidate the firm the moment she receives f_t ($\geq m_t + b_t$).

4. If the lender and the firm agree on f_t that is strictly less than the original debt, the lender retains the right to liquidate the firm. In this case, at the end of period t, the lender and the firm negotiate over the continuation fee d_{t+1} that the firm must pay in period $t+1$ (in addition to the working capital m_{t+1}).

   (a) If they agree on d_{t+1}, the lender allows the firm to continue operating in the next period.

   (b) If they do not agree on d_{t+1}, the lender liquidates the firm at the end of period t. Because the bargaining over d_{t+1} takes place at the end of period t when a part of capital $\phi q_t k_t$ has already been concealed from the lender, the lender can confiscate only $\psi q_t k_t$ by liquidating the firm at this stage.

   (c) The debt-ridden firm cannot make savings intertemporally.

5. Suppose that a debt-ridden firm and the lender agree on d_{t+1} in period t and the firm continues operating in period $t+1$. The firm's unpaid debt evolves to $B_{t+j+1} = (1 + r_{t+j})\{m_{t+j} + B_{t+j} - f_{t+j}\}$ in period $t+j+1$ for $j \geq 0$ with $B_t = b_t$, where f_{t+j} is the realized repayment in period $t+j$. As long as $B_{t+j} + m_{t+j} > f_{t+j}$, the lender retains the right to liquidate the firm. If $f_{t+1} \geq m_{t+1} + B_{t+1}$, the firm returns to a normal firm and the lender loses the right to liquidate it.

Assumption 4–1 implies that debt forgiveness is infeasible, because as long as repayment is strictly less than the original debt, the lender chooses to retain or exercise the right to liquidate it. Thus, after default, the two possibilities for the firm are either liquidation or continuation as a debt-ridden firm.

Suppose that firm i defaults on the debt $m_t + b_t$. After default, the firm and the lender negotiate over repayment f_t. If they do not reach an agreement and the lender liquidates the firm, the lender obtains $(\phi+\psi)q_tk_t$, while the firm obtains $p(x_t)x_t+(1-\phi-\psi)q_tk_t$. If they agree on repayment f_t $(\leq m_t+b_t)$, firm i continues operations as a debt-ridden firm. If firm i becomes a debt-ridden firm by paying f_t, it obtains $p(x_t)x_t+q_tk_t-f_t+V_{zt}$ and the lender obtains f_t+D_t, where D_t is the present value of the expected cash flow that the lender can receive from a debt-ridden firm from period $t+1$ onward. The bargaining over f_t after firm i defaults on m_t+b_t is described as a Nash bargaining as follows:

$$\max_{f_t}[(\phi+\psi)q_tk_t+V_{zt}-f_t]^{\sigma}[f_t+D_t-(\phi+\psi)q_tk_t]^{1-\sigma}.$$

For simplicity of analysis, we assume that the firm has all the bargaining power, that is, $\sigma=1$. Thus, the bargaining outcome is $f_t=(\phi+\psi)q_tk_t-D_t$. Here, we use $D_t=\psi q_tk_t$, shown in Appendix B, to have $f_t=\phi q_tk_t$. Therefore, if firm i defaults on m_t+b_t, the lender and the firm will agree on the repayment $f_t=\phi q_tk_t$. Thus, firm i continues as a debt-ridden firm as a result of the bargaining.

We now specify the condition for firm i not to default on m_t+b_t. After receiving the proceeds, $p(x_t)x_t$, if firm i does not default, it obtains $p(x_t)x_t+q_tk_t-m_t-b_t+V_{nt}$. On the other hand, if it defaults, the firm and the lender bargain over repayment f_t. This leads to the agreement $f_t=\phi q_tk_t$ and firm i obtains $p(x_t)x_t+(1-\phi)q_tk_t+V_{zt}$. The no-default condition for firm i is $p(x_t)x_t+q_tk_t-m_t-b_t+V_{nt}\geq p(x_t)x_t+(1-\phi)q_tk_t+V_{zt}$. This can be rewritten as

$$m_t+b_t\leq\phi q_tk_t+V_{nt}-V_{zt}.$$

Appendix B: Derivation of borrowing constraint for debt-ridden firms

We consider the bargaining between a debt-ridden firm and the lender over the fee d_{t+1} for continuation in the next period. The bargaining takes place at the end of period t. If the debt-ridden firm and the lender agree on d_{t+1}, the firm obtains $V_{zt}(d_{t+1})$, which depends on d_{t+1}, and the lender obtains $\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left\{\tilde{d}_{t+1}+D_{t+1}\right\}\right]$, where $\tilde{d}_{t+1}$ is the realized payment.[6] Note that the lender and the firm take D_{t+1} as given in period t, because D_{t+1} is determined by the bargaining at the end of period $t+1$ and they have no ability to pre-commit to the outcome of future bargaining. If the firm and the lender

[6]Note that here, $V_{zt}(d_{t+1})$ is not V_{zt} in the Bellman equation (9), but the sum of V_{zt} and the value of the capital stock as follows: $V_{zt}(d_{t+1})=V_{zt}+q_tk_t$.

do not agree on d_{t+1}, then the firm obtains $(1-\psi)q_t k_t$ and the lender obtains only $\psi q_t k_t$ by liquidating the firm. This is because of Assumption 4–4–(b). The Nash bargaining between a debt-ridden firm and the lender is, therefore,

$$\max_{d_{t+1}} \{V_{zt}(d_{t+1}) - (1-\psi)q_t k_t\}^{\sigma} \left\{\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left\{\tilde{d}_{t+1} + D_{t+1}\right\}\right] - \psi q_t k_t\right\}^{1-\sigma}.$$

With $\sigma = 1$, the bargaining outcome is

$$\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left\{\tilde{d}_{t+1} + D_{t+1}\right\}\right] = \psi q_t k_t. \tag{34}$$

We will shortly describe how d_{t+1} and $\tilde{d}_{t+1}$ are determined.

After d_{t+1} is agreed upon at the end of period t, the debt-ridden firm is allowed to operate in period $t+1$. The actions of the debt-ridden firm are as follows. At the end of period t, it sells k_t and buys k_{t+1}. At the beginning of period $t+1$, it borrows working capital m_{zt+1}, where m_{zt+1} is the material input (the consumer good) for the debt-ridden firm. It produces the intermediate good $A_{t+1}k_{t+1}^{\alpha}m_{zt+1}^{1-\alpha}$ and sells it in the monopolistically competitive market. After it receives the proceeds of sales, it repays the debt $m_{zt+1} + d_{t+1}$.[7] The debt-ridden firm can default on the debt $(m_{zt+1} + d_{t+1})$ after production. If the firm defaults on the debt, the lender and the firm renegotiate over repayment f. The renegotiation on f is as follows. If the debt-ridden firm and the lender reach an agreement, the debt-ridden firm obtains $p(x_{t+1})x_{t+1} + q_{t+1}k_{t+1} + V_{zt+1} - f$ and the lender obtains $f + D_{t+1}$. If there is no agreement, the lender liquidates the firm. Thus, the debt-ridden firm obtains $p(x_{t+1})x_{t+1} + (1-\phi-\psi)q_{t+1}k_{t+1}$ and exits the market, and the lender obtains $(\phi+\psi)q_{t+1}k_{t+1}$. Because the bargaining power of the debt-ridden firm is 1 and that of the lender is zero, we have

$$f = (\phi+\psi)q_{t+1}k_{t+1} - D_{t+1}.$$

The no-renegotiation condition for m_{zt+1} implies that $m_{zt+1} \leq \max\{0,\ f - d_{t+1}\}$. Therefore, the borrowing constraint that m_{zt+1} must satisfy is

$$m_{zt+1} \leq \max\{0,\ (\phi+\psi)q_{t+1}k_{t+1} - d_{t+1} - D_{t+1}\}.$$

After the bargaining on d_{t+1} ends at the end of period t, the firm purchases k_{t+1}, knowing that its own choice of k_{t+1} directly changes $D_{t+1} = \psi q_{t+1}k_{t+1}$. Therefore, the borrowing

[7] We assume for simplicity that the redistribution shock Δ_{it} does not hit the debt-ridden firm.

constraint imposed on the working capital loan m_{zt+1} at the beginning of period $t+1$ is rewritten as

$$m_{zt+1} \leq \max\{\phi q_{t+1} k_{t+1} - d_{t+1},\ 0\}. \tag{35}$$

Now we specify how d_{t+1} and $\tilde{d}_{t+1}$ are determined. The borrowing constraint (35) implies that if $\phi q_{t+1} k_{t+1} < d_{t+1}$, the firm cannot borrow working capital and is forced to set $m_{zt+1} = 0$ and $\tilde{d} = \phi q_{t+1} k_{t+1}$. Although the firm defaults, the lender allows the firm to continue as a debt-ridden firm from the next period onward. On the other hand, if $\phi q_{t+1} k_{t+1} \geq d_{t+1}$, then $\tilde{d}_{t+1} = d_{t+1}$. Thus d_{t+1} is the solution to

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left\{ \tilde{d}_{t+1} + \psi q_{t+1} \overline{k}_{t+1} \right\} \right] = \psi q_t k_t,$$

where $\tilde{d}_{t+1} = q_{t+1} \overline{k}_{t+1}$ if $\phi q_{t+1} \overline{k}_{t+1} < d_{t+1}$ and $\tilde{d}_{t+1} = d_{t+1}$ otherwise; $\overline{k}_{t+1}$ is the expected amount of capital stock that the debt-ridden firm purchases at the end of period t after the agreement on d_{t+1} is reached. We denote it by $\overline{k}_{t+1}$, not k_{t+1}, because the firm and the lender take it as given when they bargain over d_{t+1}. Furthermore, neither the firm nor the lender can commit to the amount of capital stock (k_{t+1}) that is to be purchased after the bargaining.

In the deterministic case where $\tilde{d}_{t+1} = d_{t+1}$, the value of d_{t+1} is given by the following equation:

$$d_{t+1} = \frac{\psi q_t k_t}{\beta \frac{\lambda_{t+1}}{\lambda_t}} - \psi q_{t+1} \overline{k}_{t+1}. \tag{36}$$

This is because $D_{t+1} = \psi q_{t+1} \overline{k}_{t+1}$.

Appendix C: Neutrality of corporate savings in the deterministic case

In our basic model, firms cannot make savings (Assumption 2 or Assumption 4–4–(c) in Appendix A). Here, we consider the case where debt-ridden firms can accumulate financial assets. In this appendix, we focus on the deterministic case. First, we assume that only if the lender liquidates the firm at the end of period t, she obtains $\psi q_t k_t + s_t$, where s_t is the risk-free bond that the firm buys in period t. Suppose that the debt-ridden firm chooses a positive amount of savings, $s_t > 0$. The bargaining over d_{t+1} at the end of period t leads to $D_t = \beta \frac{\lambda_{t+1}}{\lambda_t} \{d_{t+1} + D_{t+1}\} = \psi q_t k_t + s_t$. We define the market interest rate r_t by

$(1+r_t)^{-1} = \beta\frac{\lambda_{t+1}}{\lambda_t}$. Thus, we have

$$d_{t+1} = (1+r_t)(\psi q_t k_t + s_t) - D_{t+1}. \tag{37}$$

At the beginning of period $t+1$, the firm obtains $(1+r_t)s_t$ from its asset and borrows $m_{t+1} - (1+r_t)s_t$ as the working capital. After production, the firm has to repay $m_{t+1} + d_{t+1} - (1+r_t)s_t$. Then, the firm and the lender renegotiate over the repayment f. Note that at this stage of bargaining, the firm does not own the financial asset s_{t+1} yet. It buys s_{t+1} after the repayment is done. The bargaining then proceeds as follows. If they agree on f, the lender obtains $f + \hat{D}_{t+1}$, where $\hat{D}_{t+1} = \psi q_{t+1}k_{t+1} + \overline{s}_{t+1}$ and $\overline{s}_{t+1}$ is the expected value of s_{t+1} at this stage of bargaining, whereas she obtains $(\phi+\psi)q_{t+1}k_{t+1}$ if they do not agree on f. Because the bargaining power of the firm is 1, the bargaining outcome is $f = \phi q_{t+1}k_{t+1} - \overline{s}_{t+1}$. This implies the following borrowing constraint for working capital:

$$m_{t+1} + d_{t+1} - (1+r_t)s_t \leq \phi q_{t+1}k_{t+1} - \overline{s}_{t+1}. \tag{38}$$

Equations (37) and (38) imply that

$$m_{t+1} + (1+r_t)\psi q_t k_t + \overline{s}_{t+1} - D_{t+1} \leq \phi q_{t+1}k_{t+1}. \tag{39}$$

Because $(1+r_t)^{-1} = \beta\frac{\lambda_{t+1}}{\lambda_t}$ and $D_{t+1} = \psi q_{t+1}\overline{k}_{t+1} + \overline{s}_{t+1}$, the borrowing constraint (39) reduces to (11). That is, d_{t+1} in (11) is rewritten as $(1+r_t)\psi q_t k_t - \psi q_{t+1}\overline{k}_{t+1}$, as (36) implies. Note that s_t does not appear in this constraint, implying that there is no incentive for the debt-ridden firm to accumulate s_t. Thus, we have shown that in the deterministic equilibrium, corporate savings s_t do not affect equilibrium prices and allocations.

Appendix D: Equilibrium with debt forgiveness

In this appendix, we assume that the lender can commit to debt forgiveness. We then describe the equilibrium after a mass default.

In this appendix, we eliminate Assumption 4–1 and assume that the lender can forgive the debt of defaulters. We consider the following two-stage bargaining when the firm defaults on its debt:

- The lender and the firm negotiate whether or not to forgive the debt. If they agree on repayment f, the firm continues as a normal firm after repaying f.

- If they do not agree on debt forgiveness, they bargain whether to allow the firm to continue as a debt-ridden firm or to liquidate it.

We show in what follows that the borrowing constraint for a normal firm is tighter when debt forgiveness is feasible. Furthermore, if the mass default described in Section 3.3 occurs in period t, all defaulters return to normal firms in period $t+1$ through debt forgiveness.

Lemma 1. *If debt forgiveness is feasible, the borrowing constraint for a normal firm is*

$$m_t \leq \max\{0, \quad (\phi + \psi)q_t k_t - b_t\}.$$

(Proof) Suppose that the firm defaults on the debt $m_t + b_t$ in period t after receiving the proceeds $p(x_t)x_t$. The outcome of the second-stage bargaining, that is, whether to make the firm debt-ridden or to liquidate it, is the same as in Appendix B. The firm pays $\phi q_t k_t$ to the lender and becomes a debt-ridden firm, whereas the present value of the expected cash flow that the lender can receive from the debt-ridden firm from period $t+1$ onward is $D_t = \psi q_t k_t$.

We now consider the first-stage bargaining. If the firm and the lender agree on repayment f, the firm continues as a normal firm. Thus, if the agreement is reached, the firm obtains $p(x_t)x_t - f + q_t k_t + V_n$ and the lender obtains f. If they do not agree on f, they move to the second stage of bargaining, in which the firm obtains $p(x_t)x_t + (1-\phi)q_t k_t + V_{zt}$ and the lender obtains $\phi q_t k_t + D_t = (\phi+\psi)q_t k_t$. Therefore, the first-stage bargaining is expressed as the following Nash bargaining:

$$\max_f [\phi q_t k_t + V_{nt} - V_{zt} - f]^\sigma [f - (\phi + \psi)q_t k_t]^{1-\sigma},$$

with $\sigma = 1$, which implies that $f = (\phi + \psi)q_t k_t$. The borrowing constraint for m_t is derived from the no-default condition, $m_t \leq \max\{0, \; f - b_t\}$. (End of proof)

Note that the parameters must be chosen such that (8) holds, in order to avoid the "all default" equilibrium (see footnote 3). Thus, the borrowing constraint for normal firms in the case where debt forgiveness is feasible is tighter than in the case where it is not.

Next, we briefly describe what happens in the case of a mass default in an economy where debt forgiveness is feasible. Suppose that the redistribution shock Δ_{it} is such that $b_{it} > (\phi + \psi)q_t k_t$ for $i \in [0, Z]$. Firm $i \in [0, Z]$ cannot obtain working capital and cannot produce anything in period t. At the end of period t, the firms default on b_t and start bargaining on repayment f; this process is the same as above. The bargaining outcome is that the firms pay $f = (\phi+\psi)q_t k_t$; the lenders forgive the remaining debt $b_t - (\phi+\psi)q_t k_t$ and waiver the right to liquidate the firms. The defaulting firms return to normal in period $t+1$ and the mass default does not generate inefficiency from period $t+1$ onward.

This result presents a stark contrast to the persistent productivity decline in the case where debt forgiveness is infeasible (Section 3.3).

Appendix E: Dynamics when the number of debt-ridden firms is small

We consider only the deterministic equilibrium. Suppose that $N_0 = 1$ and there emerge debt-ridden firms, whose measure is Z, in period 0. For a small Z, the economy grows and eventually converges to the BGP. The equilibrium path in which R&D investment takes place and N_t grows is described by the following system of equations. Note that $V_{nt} = \chi^{-1}$ in this equilibrium.

$$1 = (1-\alpha)\eta L^{1-\eta}\hat{A}^{\eta}(N_t k_{nt})^{\alpha\eta} m_{nt}^{(1-\alpha)\eta-1},$$
$$1 + \mu_{zt} = (1-\alpha)\eta L^{1-\eta}\hat{A}^{\eta}(N_t k_{zt})^{\alpha\eta} m_{zt}^{(1-\alpha)\eta-1},$$
$$q_t = \beta\frac{C_t}{C_{t+1}}\left[\alpha\eta L^{1-\eta}\frac{\hat{A}^{\eta}(N_{t+1}k_{nt+1})^{\alpha\eta} m_{nt+1}^{(1-\alpha)\eta}}{k_{nt+1}} + q_{t+1}\right],$$
$$q_t = \beta\frac{C_t}{C_{t+1}}\left[\alpha\eta L^{1-\eta}\frac{\hat{A}^{\eta}(N_{t+1}k_{zt+1})^{\alpha\eta} m_{zt+1}^{(1-\alpha)\eta}}{k_{zt+1}} + (1+\phi\mu_{zt+1})q_{t+1}\right],$$
$$Y_t = \frac{\hat{A}^{\eta}N_t^{\alpha\eta}}{\eta}L^{1-\eta}\left\{Z(k_{zt}^{\alpha}m_{zt}^{1-\alpha})^{\eta} + (N_t - Z)(k_{nt}^{\alpha}m_{nt}^{1-\alpha})^{\eta}\right\},$$
$$C_t + I_t = Y_t - (N_t - Z)m_{nt} - Zm_{zt},$$
$$K = Zk_{zt} + (N_t - Z)k_{nt},$$
$$d_t = \frac{\psi}{\beta}\frac{C_{t+1}}{C_t}q_t k_{zt} - \psi q_{t+1}k_{zt+1},$$
$$m_{zt} = \phi q_t k_{zt} - d_t,$$
$$\chi^{-1} = -q_t k_{nt+1} + \beta\frac{C_t}{C_{t+1}}\left\{L^{1-\eta}(\hat{A}N_{t+1}^{\alpha}k_{nt+1}^{\alpha}m_{nt+1}^{1-\alpha})^{\eta} - m_{nt+1} + q_{t+1}k_{nt+1} + \chi^{-1}\right\},$$
$$V_{zt} = -q_t k_{zt+1} + \beta\frac{C_t}{C_{t+1}}\left\{L^{1-\eta}(\hat{A}N_{t+1}^{\alpha}k_{zt+1}^{\alpha}m_{zt+1}^{1-\alpha})^{\eta} - m_{zt+1} - d_{t+1} + q_{t+1}k_{zt+1} + V_{zt+1}\right\},$$
$$N_{t+1} = N_t + \chi I_t.$$

The following inequality must be satisfied:

$$m_{nt} < \phi q_t k_t + V_{nt} - V_{zt},$$
$$V_{zt} \geq 0.$$

The first inequality is necessary because we assumed that the borrowing constraint for normal firms is not binding in equilibrium. The second inequality is necessary, because otherwise, the debt-ridden firm exits because liquidation is preferable to continuing as a debt-ridden firm. As this equilibrium converges to the BGP, it is convenient to define and analyze the following detrended variables: $c_t = C_t/N_t$, $i_t = I_t/N_t$, $y_t = Y_t/N_t$, $\tilde{q}_t =$

q_t/N_t, $K_{nt} = N_t k_{nt}$, $K_{zt} = N_t k_{zt}$, $z_t = Z/N_t$, $G_t = N_{t+1}/N_t$. The above equations are rewritten as the following system of equations of the detrended variables.

$$
\begin{aligned}
&1 = (1-\alpha)\eta L^{1-\eta}\hat{A}^{\eta}K_{nt}^{\alpha\eta}m_{nt}^{(1-\alpha)\eta-1}, \\
&1+\mu_{zt} = (1-\alpha)\eta L^{1-\eta}\hat{A}^{\eta}K_{zt}^{\alpha\eta}m_{zt}^{(1-\alpha)\eta-1}, \\
&\tilde{q}_t = \beta\frac{c_t}{c_{t+1}}\left[\alpha\eta L^{1-\eta}\frac{\hat{A}^{\eta}K_{nt+1}^{\alpha\eta}m_{nt+1}^{(1-\alpha)\eta}}{K_{nt+1}} + \tilde{q}_{t+1}\right], \\
&\tilde{q}_t = \beta\frac{c_t}{c_{t+1}}\left[\alpha\eta L^{1-\eta}\frac{\hat{A}^{\eta}K_{zt+1}^{\alpha\eta}m_{zt+1}^{(1-\alpha)\eta}}{K_{zt+1}} + (1+\phi\mu_{zt+1})\tilde{q}_{t+1}\right], \\
&y_t = \frac{\hat{A}^{\eta}}{\eta}L^{1-\eta}\left\{z_t(K_{zt}^{\alpha}m_{zt}^{1-\alpha})^{\eta} + (1-z_t)(K_{nt}^{\alpha}m_{nt}^{1-\alpha})^{\eta}\right\}, \\
&c_t + i_t = y_t - (1-z_t)m_{nt} - z_t m_{zt}, \\
&K = z_t K_{zt} + (1-z_t)K_{nt}, \\
&d_t = \frac{\psi}{\beta}\frac{c_{t+1}}{c_t}G_t\tilde{q}_t K_{zt} - \psi\tilde{q}_{t+1}K_{zt+1}, \\
&m_{zt} = \phi\tilde{q}_t K_{zt} - d_t, \\
&\chi^{-1} = -\tilde{q}_t K_{nt+1} + \frac{\beta}{G_t}\frac{c_t}{c_{t+1}}\left\{L^{1-\eta}(\hat{A}K_{nt+1}^{\alpha}m_{nt+1}^{1-\alpha})^{\eta} - m_{nt+1} + \tilde{q}_{t+1}K_{nt+1} + \chi^{-1}\right\}, \\
&V_{zt} = -\tilde{q}_t K_{zt+1} + \frac{\beta}{G_t}\frac{c_t}{c_{t+1}}\left\{L^{1-\eta}(\hat{A}K_{zt+1}^{\alpha}m_{zt+1}^{1-\alpha})^{\eta} - m_{zt+1} - d_{t+1} + \tilde{q}_{t+1}K_{zt+1} + V_{zt+1}\right\}, \\
&G_t = 1 + \chi i_t, \\
&z_{t+1} = z_t/G_t.
\end{aligned}
$$

The detrended variables must satisfy

$$
\begin{aligned}
&m_{nt} < \phi\tilde{q}_t K_{nt} + V_{nt} - V_{zt}, \\
&V_{zt} \geq 0.
\end{aligned}
$$

After linearizing around the BGP (steady state), we can solve for the variables $\{c_t,\ i_t,\ y_t,\ d_t,\ m_{nt},\ m_{zt},\ \mu_{zt},\ \tilde{q}_t,\ K_{nt},\ K_{zt},\ z_t,\ G_t,\ V_{zt}\}$ as linear functions of the states, z_t and d_t. The parameters are set at the same values as in the text. The impulse response to the productivity shock is shown in Figures 7 and 8. Figure 7 shows the IRF for 1,000 periods, while Figure 8 shows the same for 40 periods. According to both figures, most of the macroeconomic variables converge to the BGP monotonically, except for the continuation fee d_t and the present value of its flow, D_t. These variables become higher and then lower than their BGP values after the shock. This non-monotonicity can be explained as follows: $\beta\frac{\lambda_{t+1}}{\lambda_t}$ declines largely on impact and then monotonically converges to the BGP value. Because $\psi q_t k_{zt} = D_t = \sum_{i=0}^{\infty} \beta^i (\lambda_{t+i}/\lambda_t)\ d_{t+i}$, we can interpret $\beta^i \frac{\lambda_t+i}{\lambda_t}$ as the shadow price of d_{t+i} for the debt-ridden firm. Because the total value of D_t is fixed at $\psi q_t k_{zt}$ and the shadow price $\beta^i \frac{\lambda_t+i}{\lambda_t}$ is lower for smaller i than their BGP values, the debt-ridden firm increases d_{t+i} for smaller i and decreases it for larger i.

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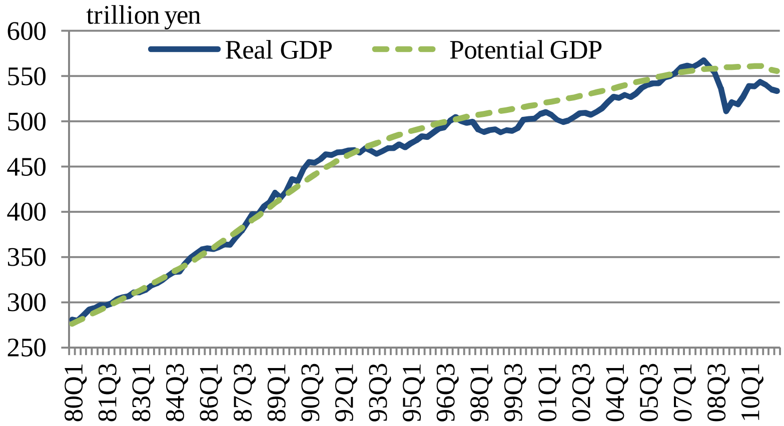


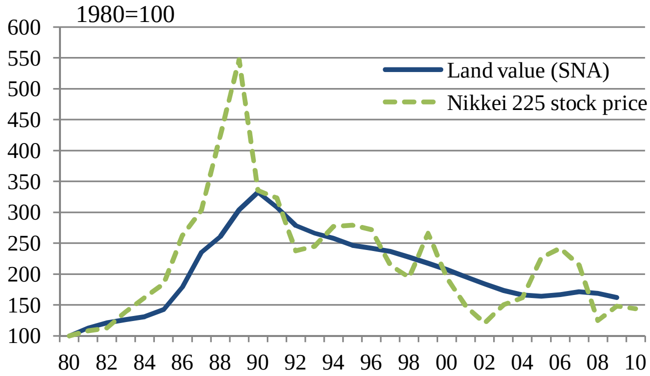
Source: Cabinet Office, Government of Japan, "Annual Report on National Accounts"

Figure 1: Real and potential GDP in Japan

	HP	KI	JIP2011
1971–80	0.83		1.68
1981–90	1.93	2.06	1.39
1991–2000	0.36	0.35	0.04
2001–2007		0.48	1.13

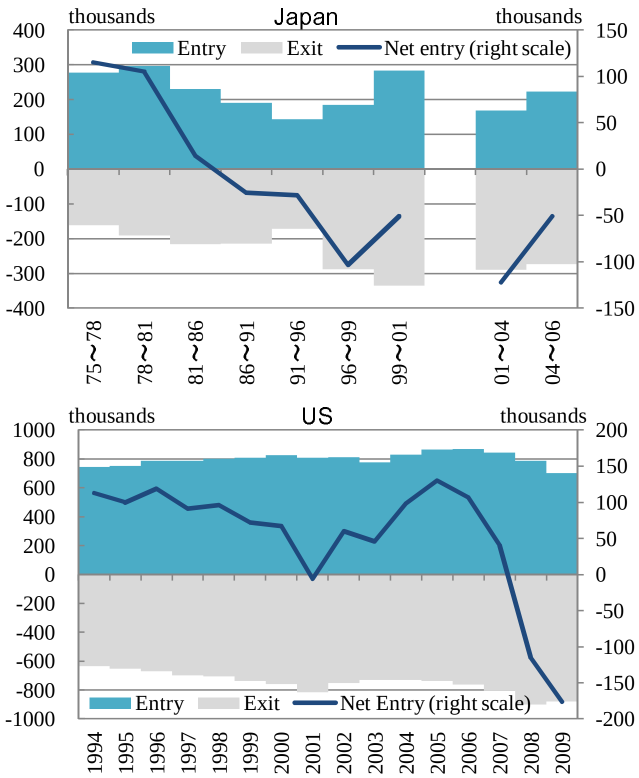
Note: HP, KI, JIP2011 are from updated versions of Hayashi and Prescott (2002), Kobayashi and Inaba (2006), and Fukao and Miyagawa (2008), respectively.

Table 1: TFP growth rate in Japan



Sources: Cabinet Office, Government of Japan; Nihon Keizai Shimbun.

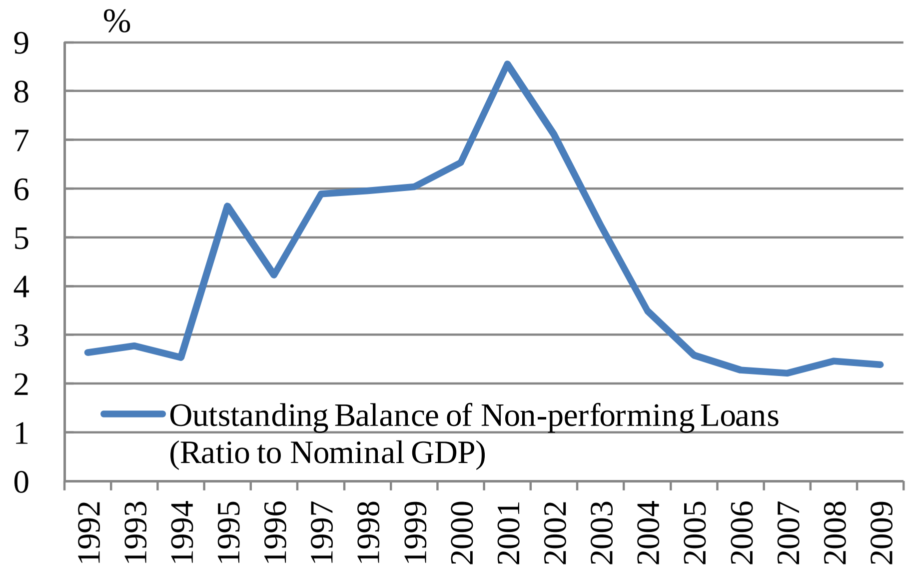
Figure 2: Land value and stock prices in Japan



Note: Japan's figures after 2001 are based on 1993-basis industry classification.

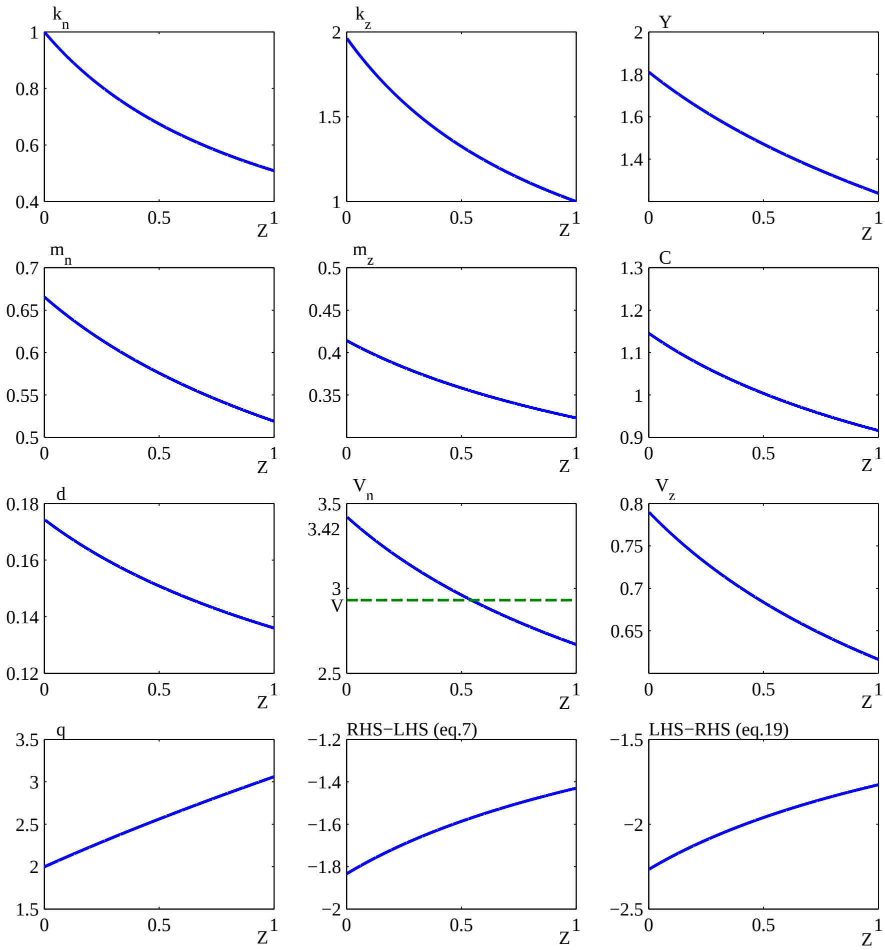
Sources: (Japan) Ministry of Internal Affairs and Communications, "Establishment and Enterprise Census"; (US) Bureau of Labor Statistics, "Business Employment Dynamics."

Figure 3: Entry and exit of private sector establishments: US and Japan



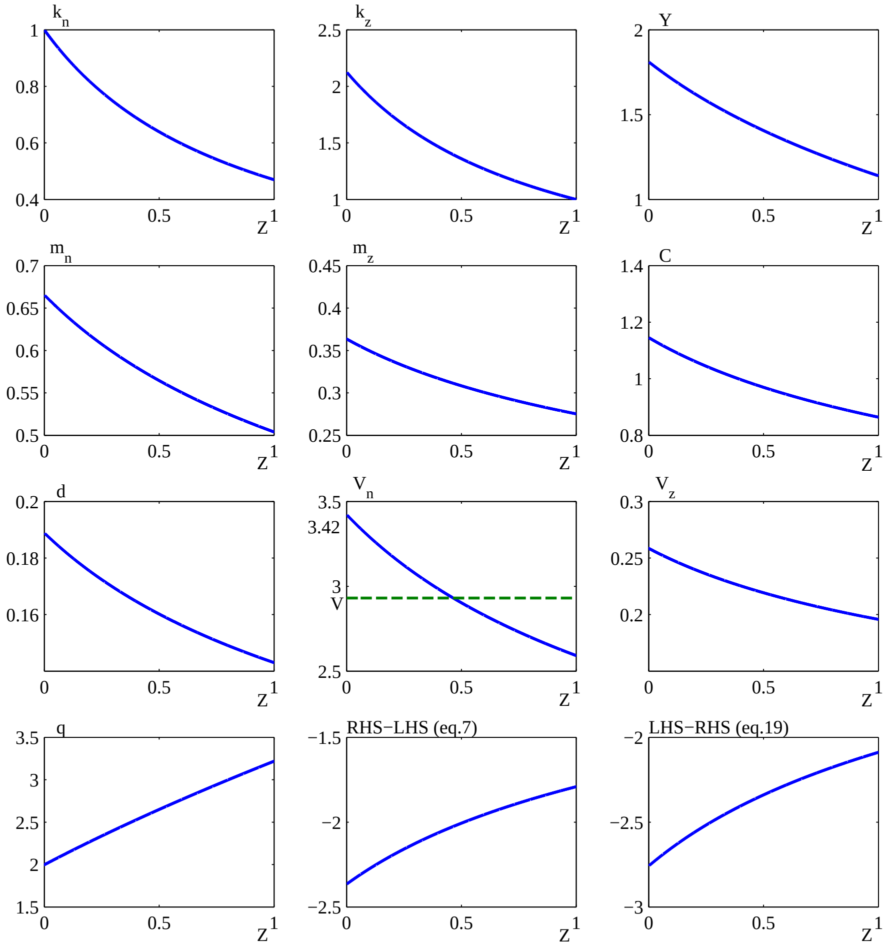
Note: The non-performing loans are the Risk Management Loans (RMLs) defined in the Banking Act in Japan. These consist of loans to bankrupt borrowers, delayed loans, three-month overdue loans, and loans with modified terms and conditions. RMLs do not include securitized loans.
Sources: Financial Services Agency,*Status of Non-Performing Loans*; Cabinet Office, Government of Japan,*Annual Report on National Accounts*.

Figure 4: Development of non-performing loans



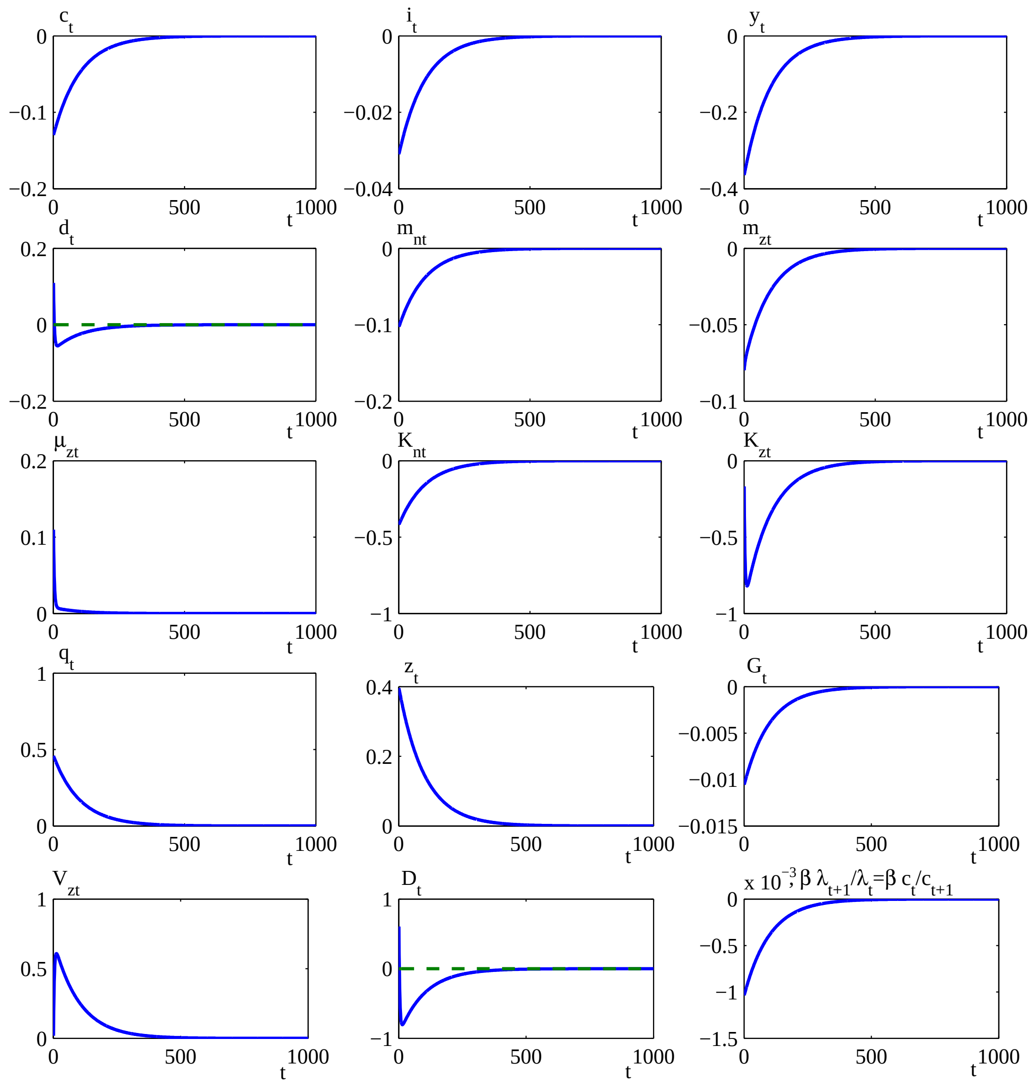
Parameters: $\alpha = 0.25, \quad \beta = 0.9, \quad \phi = 0.15, \quad \eta = 0.7, \quad A = 1.9048,$
$K = 1, \quad L = 1, \quad \psi = 0.4, \quad \chi_{SS}^{-1} = 3.4234, \quad V = \chi_{BGP}^{-1} = 2.9307.$
Results: $\mu_z = 0.41, \quad \Lambda = 1.9648, \quad m = 0.6657, \quad g = 0.01$. If $z \geq 0.5439$, $Vn < \chi_{BGP}^{-1}$.

Figure 5: Equilibrium with debt-ridden firms



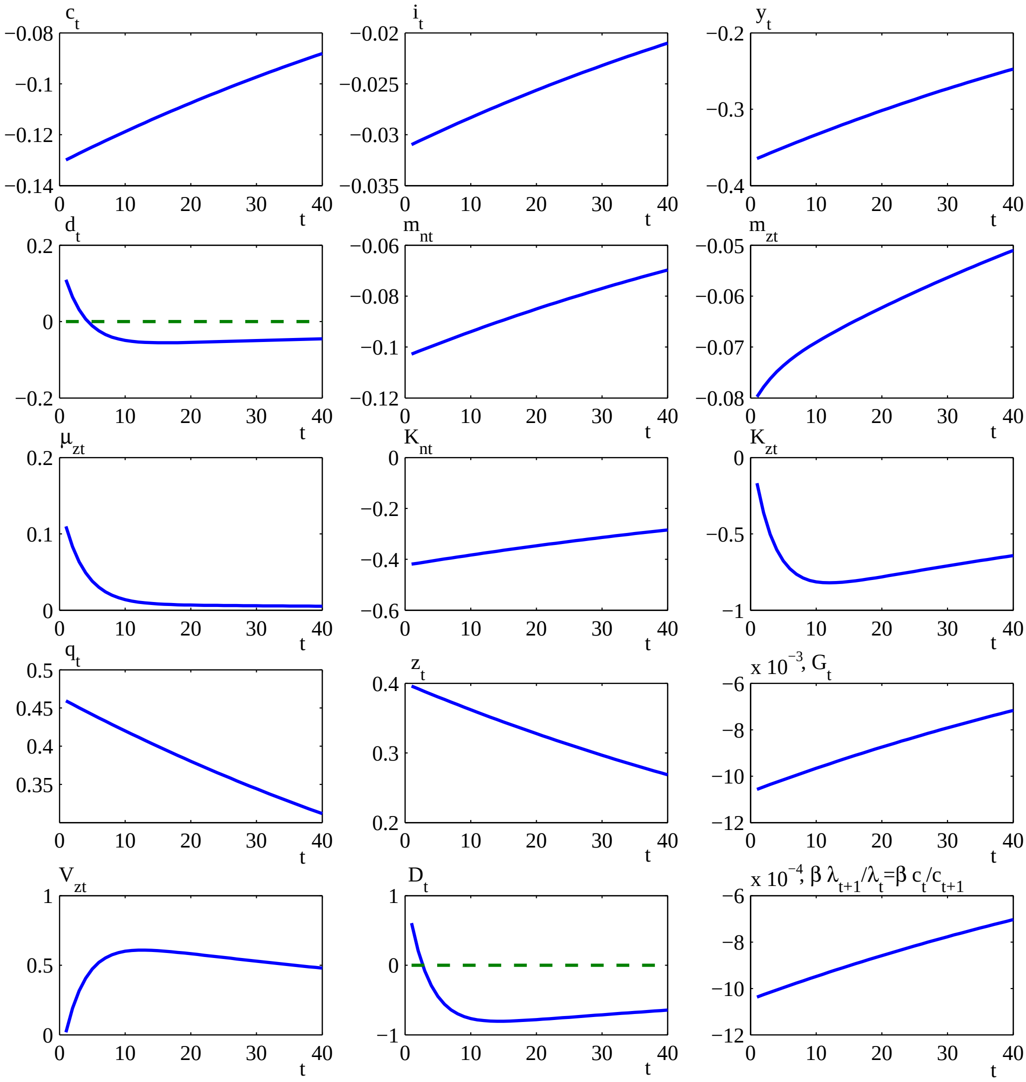
Parameters: $\alpha = 0.25, \quad \beta = 0.9, \quad \phi = 0.13, \quad \eta = 0.7, \quad A = 1.9048,$
$K = 1, \quad L = 1, \quad \psi = 0.4, \quad \chi_{SS}^{-1} = 3.4234, \quad V = \chi_{BGP}^{-1} = 2.9307.$
Results: $\mu_z = 0.5210, \quad \Lambda = 2.1283, \quad m = 0.6657, \quad g = 0.01$. If $z \geq 0.4651, \; Vn < \chi_{BGP}^{-1}$.

Figure 6: Equilibrium with debt-ridden firms: Multiple equilibria



Parameters: $\alpha = 0.25$, $\beta = 0.9$, $\phi = 0.13$, $\eta = 0.7$, $A = 1.9048$,
$K = 1$, $L = 1$, $\psi = 0.4$, $\chi_{SS}^{-1} = 3.4234$, $V = \chi_{BGP}^{-1} = 2.9307$.

Figure 7: Impulse responses to the productivity shock



Parameters: $\alpha = 0.25$, $\beta = 0.9$, $\phi = 0.13$, $\eta = 0.7$, $A = 1.9048$,
$K = 1$, $L = 1$, $\psi = 0.4$, $\chi_{SS}^{-1} = 3.4234$, $V = \chi_{BGP}^{-1} = 2.9307$.

Figure 8: Impulse responses to the productivity shock