Supplemental Material (Technical Appendices)

Appendix H Tax on entrepreneurs

In the main text, we have considered the case that the total bailout money is financed by taxing workers. In this Technical Appendix, we consider a case that in order to finance bailout money, the government taxes not only workers, but also entrepreneurs who do not suffer losses from bubble investments.

In this case, when bubbles collapse at date t, bailout money, $\lambda P_t X$, is financed through aggregate tax revenues from workers, T_t^u, and aggregate tax revenues from entrepreneurs who do not suffer losses, T_t^e:

$$\lambda P_t X = T_t^u + T_t^e,$$

with

$$\begin{aligned} T_t^e &= \tau \left(q_t \alpha^H Z_{t-1}^H - r_{t-1} B_{t-1}^H \right) \\ &= \begin{cases} \tau \dfrac{\alpha^H (1-\theta) p}{(\alpha^L - \theta \alpha^H)[1-\phi(\lambda)] + (\alpha^H - \alpha^L) p} \sigma K_t^\sigma & \text{if } 0 \le \lambda \le \lambda^*, \\ \\ \tau (1-\theta) \sigma K_t^\sigma & \text{if } \lambda^* \le \lambda \le 1, \end{cases} \end{aligned}$$

where τ is a tax rate imposed on the date t net worth of the non–loss-making entrepreneurs, (i.e., H-types in period $t-1$). For technical reasons, i,e., in order to derive entrepreneur's consumption function explicitly, we consider the case that the government taxes entrepreneur's net worth. T_t^e increases with λ in $0 \le \lambda \le \lambda^*$. This means that as λ rises, aggregate H-investments expand during bubbly periods, which increases tax revenues from the non–loss-making entrepreneurs when bubbles collapse. This increase in tax revenues reduces tax burden for workers. When we solve for tax burden per unit of workers, T_t^u (recall that there are workers with unit measure), we learn

$$T_t^u = \lambda P_t X - T_t^e = F(\lambda) \sigma K_t^\sigma, \tag{H.1}$$

with

(H.2)

$$F(\lambda) = \begin{cases} \lambda \dfrac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)} - \tau \dfrac{\alpha^H(1-\theta)p}{(\alpha^L - \theta\alpha^H)[1-\phi(\lambda)] + (\alpha^H - \alpha^L)p} & \text{if } 0 \le \lambda \le \lambda^*, \\ \lambda \dfrac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)} - \tau(1-\theta) & \text{if } \lambda^* \le \lambda \le 1. \end{cases}$$

It follows that T_t^u is a decreasing function of τ.

By using (H.1) and (H.2), $W(\lambda)$ is replaced with

$$\text{(H.3)} \quad M(\lambda) = \log\left[1-\sigma-\sigma F(\lambda)\right] + \frac{\beta\sigma}{1-\beta\sigma}\log\left[1+F(\lambda)\right] + \frac{\beta\sigma}{1-\beta\sigma}\frac{1}{1-\beta}\log\left[\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta\alpha^L\sigma\right] + \frac{\beta}{1-\beta}\log(1-\sigma).$$

From (30) together with (H.3), we see how an increase in τ affects workers' welfare. We learn that (30) is an increasing function of τ, i.e., workers' welfare increases with τ. Intuition is very simple. If the government imposes higher tax rate on the non–loss-making entrepreneurs, tax burden per unit of workers decreases, which increases workers' consumption when bubbles collapse, thereby improving their welfare. We also learn from (30) how an increase in bailout guarantees affects workers' welfare in this case. We find that even in this case, if (A1) holds, then partial bailouts are optimal for workers. Moreover, when we compute (30) with (H.3) under the benchmark parameter case, then we find that λ^{**} increases with τ and approaches λ^*. This means that optimal bailouts for workers approach the bailout level that maximizes ex-ante output efficiency. When the government taxes the non–loss-making entrepreneurs, tax revenues from those entrepreneurs increase together with an increase in λ, since T_t^e is an increasing function of λ. This increase in tax revenues lowers tax burden for workers. As a result, the welfare-enhancing effect captured by the first term of (31) dominates the welfare-reducing effect captured by the second term of (31) even in greater values of $\lambda < \lambda^*$.

We can also compute welfare for entrepreneurs in this case. When computing it, we need to take into account the fact that entrepreneurs are taxed when bubbles collapse if they are H-types in one period before bubbles' collapsing (see the Technical Appendix for derivation of the value function in this case). We find that welfare for entrepreneurs monotonically increases with λ even in this case.

Appendix I Derive the demand function for bubble assets of a L-entrepreneur

Each L-entrepreneur chooses optimal amounts of b_t^i, x_t^i, and z_t^i so that the expected marginal utility from investing in three assets is equalized. The first order conditions with respect to x_t^i and b_t^i are

$$(x_t^i): \ \frac{P_t}{c_t^i} = \pi\beta\frac{P_{t+1}}{c_{t+1}^{i,\pi}} + (1-\pi)\lambda\beta\frac{d_{t+1}}{c_{t+1}^{i,(1-\pi)\lambda}}, \tag{I.4}$$

$$(b_t^i): \ \frac{1}{c_t^i} = \pi\beta\frac{r_t}{c_{t+1}^{i,\pi}} + (1-\pi)\lambda\beta\frac{r_t}{c_{t+1}^{i,(1-\pi)\lambda}} + (1-\pi)(1-\lambda)\beta\frac{r_t}{c_{t+1}^{i,(1-\pi)(1-\lambda)}}, \tag{I.5}$$

where $c_{t+1}^{i,\pi} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i + P_{t+1}x_t^i)$, $c_{t+1}^{i,(1-\pi)\lambda} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i + m_{t+1}^i)$, and $c_{t+1}^{i,(1-\pi)(1-\lambda)} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i)$.[24] The RHS of (I.4) is the gain in expected discounted utility from holding one additional unit of bubble assets at date $t+1$. With probability π bubbles survive, in which case the entrepreneur can sell the additional unit at P_{t+1}, but with probability $1-\pi$ bubbles collapse, in which case with probability λ he/she is rescued and receives d_{t+1} units of consumption goods per unit of bubble assets, and with probability $1-\lambda$, he/she is not rescued and receives nothing. The denominators reflect the respective marginal utilities of consumption. The RHS of (I.5) is the gain in expected discounted utility from lending one additional unit. It is similar to the RHS of (I.4), except for the fact that lending yields r_t at date $t+1$, irrespective of whether or not bubbles collapse.

From (17), (I.4), and (I.5), we can derive the demand function for bubble assets of a type i L-entrepreneur in the main text.

Appendix J Aggregation

The great merit of the expressions for each entrepreneur's investment and demand for bubble assets, z_t^i and x_t^i, is that they are linear in period-t net worth, e_t^i. Hence aggregation is easy: we do not need to keep track of the distributions.

[24]Since the entrepreneur consumes a fraction $1-\beta$ of the current net worth in each period, the optimal consumption level at date $t+1$ is independent of the entrepreneur's type at date $t+1$. It only depends on whether bubbles collapse and whether government rescues the entrepreneur.

From (16), we learn the aggregate H-investments:

$$Z_t^H = \frac{\beta p A_t}{1 - \dfrac{\theta q_{t+1}\alpha^H}{r_t}}, \tag{J.6}$$

where $A_t \equiv q_t K_t + P_t X$ is the aggregate wealth of entrepreneurs at date t, and $\sum_{i \in H_t} e_t^i = pA_t$ is the aggregate wealth of H-entrepreneurs at date t. From this investment function, we see that the aggregate H-investments are both history-dependent and forward-looking, because they depend on asset prices, P_t, as well as cash flows from the investment projects in the previous period, $q_t K_t$. In this respect, this investment function is similar to the one in Kiyotaki and Moore (1997). There is a significant difference. In the Kiyotaki-Moore model, the investment function depends on land prices which reflect fundamentals (cash flows from land), while in our model, it depends on bubble prices.

Aggregate L-investments depend on the level of the interest rate:

$$Z_t^L = \begin{cases} \beta A_t - \frac{\beta p A_t}{1 - \frac{\theta \alpha^H}{\alpha^L}} - P_t X & \text{if } r_t = q_{t+1}\alpha^L, \\ \\ 0 & \text{if } r_t > q_{t+1}\alpha^L. \end{cases} \tag{J.7}$$

When $r_t = q_{t+1}\alpha^L$, L-entrepreneurs may invest positive amount. In this case, we know from (19) that aggregate L-investments are equal to aggregate savings of the economy minus aggregate H-investments minus aggregate value of bubbles. When $r_t > q_{t+1}\alpha^L$, L-entrepreneurs do not invest.

The aggregate counterpart to (18) is

$$P_t X_t = \frac{\delta(\lambda)\frac{P_{t+1}}{P_t} - r_t}{\frac{P_{t+1}}{P_t} - r_t} \beta(1-p)A_t, \tag{J.8}$$

where $\sum_{i \in L_t} e_t^i = (1-p)A_t$ is the aggregate net worth of L-entrepreneurs at date t. (J.8) is the aggregate demand function for bubble assets at date t.

Appendix K Worker's Behavior

We verify that workers do not save nor buy asset bubbles in equilibrium. First, we verify that workers do not save. When the borrowing constrained binds, workers do not save. The condition that the borrowing constraint binds is

$$\frac{1}{c_t^u} > \pi\beta\frac{r_t}{c_{t+1}^{u,\pi}} + (1-\pi)\beta\frac{r_t}{c_{t+1}^{u,1-\pi}}.$$

We know that $c_t^u = w_t$ and $c_{t+1}^{u,\pi} = w_{t+1}$ if workers do not save nor buy bubble assets. Then, the above can be written as

$$1 > \left[\pi + (1-\pi)\frac{1-\sigma}{1-\sigma-\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma}\right]\beta\frac{K_t^\sigma}{K_{t+1}^\sigma}r_t. \tag{K.9}$$

When $r_t = q_{t+1}\alpha^L$, (K.9) can be written as

$$\frac{H(\lambda)}{\beta\sigma\alpha^L} > \pi + (1-\pi)\frac{1-\sigma}{1-\sigma-\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma}. \tag{K.10}$$

Since $H(\lambda)/\beta\sigma\alpha^L > 1$ in the bubble regions and the right hand side of (K.10) is an increasing function of σ and equals one with $\sigma = 0$, (K.10) holds if σ is sufficiently small.

When $r_t = \theta q_{t+1}\alpha^H[1-\beta\phi(\lambda)]/[1-p-\phi(\lambda)]$, (K.10) can be written as

$$\frac{H(\lambda)[1-p-\phi(\lambda)]}{\theta\beta\sigma\alpha^H[1-\phi(\lambda)]} > \pi + (1-\pi)\frac{1-\sigma}{1-\sigma-\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma}. \tag{K.11}$$

Since $H(\lambda)[1-p-\phi(\lambda)]/\theta\beta\sigma\alpha^H[1-\phi(\lambda)] > 1$ in the bubble regions and the right hand side of (K.10) is an increasing function of σ and equals one with $\sigma = 0$, (K.11) holds if σ is sufficiently small. Under the reasonable parameter values in our numerical examples, both (K.10) and (K.11) hold.

Next, we verify that workers do not buy bubble assets. When the short sale constraint binds, workers do not buy bubble assets. The condition that the short sale constraint binds is

$$\frac{1}{c_t^u} > \pi\beta\frac{1}{c_{t+1}^{u,\pi}}\frac{P_{t+1}}{P_t}.$$

We know $c_t^u = w_t$ and $c_{t+1}^{u,\pi} = w_{t+1}$ if workers do not save nor buy bubble assets.

Then, the above can be written as

$$1 > \pi\beta \frac{w_t}{w_{t+1}} \frac{P_{t+1}}{P_t} = \pi\beta,$$

which is true.

Appendix L Behavior of H-types

We verify that H-types do not buy bubble assets in equilibrium. When the short sale constraint binds, H-types do not buy bubble assets. In order that the short sale constraint binds, the following condition must hold:

$$\text{(L.12)} \qquad \frac{1}{c_t^i} > \beta E_t \left[\frac{1}{c_{t+1}^i} \frac{P_{t+1}}{P_t} \right].$$

Since the borrowing constraint is binding for H-types, we have

$$\text{(L.13)} \qquad \frac{1}{c_t^i} = \beta E_t \left[\frac{r_t}{c_{t+1}^i} \frac{q_{t+1}\alpha^H(1-\theta)}{r_t - \theta q_{t+1}\alpha^H} \right].$$

We also know that $c_{t+1}^i = (1-\beta)\left[\frac{r_t\alpha^H(1-\theta)}{r_t - \theta q_{t+1}\alpha^H}\right]$ if (L.12) is true. Inserting (L.13) into (L.12) yields

$$\text{(L.14)} \qquad \beta \frac{1}{c_{t+1}^i} \frac{r_t\left[q_{t+1}\alpha^H - \delta(\lambda)\frac{P_{t+1}}{P_t}\right] + \theta q_{t+1}\alpha^H\left[\delta(\lambda)\frac{P_{t+1}}{P_t} - r_t\right]}{r_t - \theta q_{t+1}\alpha^H} > 0.$$

If (L.14) holds, then the short sale constraint binds. We see that the second term in the numerator is positive as long as $\phi > 0$ and we know that $\phi > 0$ on the saddle path. Thus, if the first term is positive, (L.14) holds. The condition that the first term is positive is

$$q_{t+1}\alpha^H > \delta(\lambda)\frac{P_{t+1}}{P_t}.$$

On the saddle path, since P_t follows according to

$$\text{(L.15)} \qquad P_t = \frac{\beta\phi(\lambda)}{X[1-\beta\phi(\lambda)]}\sigma K_t^{\sigma},$$

Using (L.15), the above inequality condition can be written as

$$\sigma \alpha^H K_t^\sigma > \delta(\lambda) K_{t+1}. \tag{L.16}$$

First, we show that (L.16) holds in $0 \leq \lambda \leq \lambda^*$. In $0 \leq \lambda \leq \lambda^*$, aggregate capital stock follows (28). Thus, (L.16) can be written as

$$\alpha^H > \delta(\lambda) \left[(1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta \alpha^H} p) \beta \alpha^L - \beta \alpha^L \phi(\lambda) \right] / \left[1 - \beta \phi(\lambda) \right], \tag{L.17}$$

which is equivalent to

$$\alpha^H > \delta(\lambda) \left[(1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta \alpha^H} p) \beta \alpha^L - \beta \alpha^L (1-p) \right] / \left[1 - \delta(\lambda) \beta (1-p) \right]. \tag{L.18}$$

The right hand side of (L.18) is an increasing and convex function of λ in $0 \leq \lambda \leq \lambda^*$. Thus (L.16) holds in $0 \leq \lambda \leq \lambda^*$ if (L.16) is true at $\lambda = \lambda^*$. At $\lambda = \lambda^*$, we know $\phi = [\alpha^L(1-p) - \theta \alpha^H]/(\alpha^L - \theta \alpha^H)$. Inserting this relation into (L.17) yields

$$\alpha^H (1-\beta) + \frac{\alpha^L p \beta \alpha^H}{\alpha^L - \theta \alpha^H} [1 - \delta(\lambda^*)] > 0,$$

which is true.

Next, we show that (L.16) holds in $\lambda^* \leq \lambda \leq 1$. In $\lambda^* \leq \lambda \leq 1$, aggregate capital stock follows (28). Thus, (L.16) can be written as

$$1 - \beta \phi > \delta(\lambda) \beta (1 - \phi),$$

which is true, since $1 - \beta\phi > 1 - \phi$ and $\delta(\lambda)\beta < 1$.

Appendix M Derivation of taxpayer's value function

Suppose that at date t, bubbles collapse. After the date t, the economy is in the bubbleless economy. Let V_t^{BL} be the value function of taxpayers at date t when bubbles collapse and the government bails out entrepreneurs. First, we solve V_{t+1}^{BL}.

Given the optimal decision rules, the Bellman equation can be written as

$$V_{t+1}^{BL}(K_{t+1}) = \log c_{t+1} + \beta V_{t+2}^{BL}(K_{t+2}), \quad \text{after date } t+1, \tag{M.19}$$

with

$$\begin{cases} c_{t+1} = w_{t+1} & \text{after date } t+1, \\ K_{t+2} = \left[1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} p\right] \beta\alpha^L \sigma K_{t+1}^{\sigma} & \text{after date } t+1. \end{cases} \tag{M.20}$$

We guess that the value function is a linear function of $\log K$:

$$V_{t+1}^{BL}(K_{t+1}) = f + g \log K_{t+1} \quad \text{after date } t+1. \tag{M.21}$$

From (M.19)-(M.21), applying the method of undetermined coefficients yields

$$f = \frac{1}{1-\beta} \log(1-\sigma) + \frac{1}{1-\beta} \frac{\beta\sigma}{1-\beta\sigma} \log\left[\left(1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} p\right) \beta\alpha^L \sigma\right],$$

$$g = \frac{\sigma}{1-\beta\sigma}.$$

Thus, we have

$$\begin{aligned} V_{t+1}^{BL}(K_{t+1}) \quad = \quad & \frac{1}{1-\beta} \log(1-\sigma) + \frac{1}{1-\beta} \frac{\beta\sigma}{1-\beta\sigma} \log\left[\left(1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} p\right) \beta\alpha^L \sigma\right] \\ & + \frac{\sigma}{1-\beta\sigma} \log K_{t+1}, \ \text{after date } t+1. \end{aligned} \tag{M.22}$$

Next, we derive the value function of taxpayers at date t when bubbles collapse and the government bails out entrepreneur by taking into account the effects of bailouts on the date t consumption and the date $t+1$ aggregate capital stock. The value function of taxpayers at date t satisfies

$$V_t^{BL}(K_t) = \log c_t + \beta V_{t+1}^{BL}(K_{t+1}), \tag{M.23}$$

with

$$
\text{(M.24)} \qquad \begin{cases} c_t = w_t - \lambda P_t X = w_t - \lambda \frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)} \sigma K_t^{\sigma}, \\ \\ K_{t+1} = \left[1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} p\right] \beta \alpha^L \sigma \left[1 + \lambda \frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\right] K_t^{\sigma}. \end{cases}
$$

From (M.22), (M.23), and (M.24), we have (29) in the text.

Now, we are in a position to derive the value function at any date t in the bubble economy. Let $V_t^{BB}(K_t)$ be the value function of taxpayers at date t in the bubble economy. Given optimal decision rules, the Bellman equation can be written as

$$
\text{(M.25)} \qquad V_t^{BB}(K_t) = \log c_t + \beta \left[\pi V_{t+1}^{BB}(K_{t+1}) + (1-\pi) V_{t+1}^{BL}(K_{t+1})\right].
$$

with the optimal decision rule of aggregate capital stock until bubbles collapse:

$$
\text{(M.26)} \qquad K_{t+1} = H(\lambda) K_t^{\sigma},
$$

We guess that the value function is a linear function of $\log K$:

$$
\text{(M.27)} \qquad V_t^{BB}(K_t) = s + Q \log K_t,
$$

From (29), and (M.25)-(M.26), applying the method of undetermined coefficients yields

$$
s = \frac{1}{1-\beta\pi} \log(1-\sigma) + \frac{\beta(1-\pi)}{1-\beta\pi} M(\lambda) + \frac{1}{1-\beta\pi} \frac{\beta\sigma}{1-\beta\sigma} \log H(\lambda),
$$

$$
Q = \frac{\sigma}{1-\beta\sigma}.
$$

Thus, we have (30) in the text.

Appendix N Derivation of entrepreneur's value function

Appendix N.1 the case where the government does not tax entrepreneurs

Suppose that at date t, bubbles collapse. After the date t, the economy is in the bubbleless economy. Let $W_t^{BL}(e_t, K_t)$ be the value function of the entrepreneur at date t who holds the net worth, e_t, at the beginning of the period t before knowing his/her type of the period t. First, we solve $W_{t+1}^{BL}(e_t, K_t)$. Given the optimal decision rules, the Bellman equation can be written as

(N.28)

$$W_{t+1}^{BL}(e_{t+1}, K_{t+1}) = \log c_{t+1}^i + \beta \begin{bmatrix} pW_{t+2}^{BL}(R_{t+1}'^H \beta e_{t+1}, K_{t+2}) \\ +(1-p)W_{t+2}^{BL}(R_{t+1}'^L \beta e_{t+1}, K_{t+2}) \end{bmatrix} \text{ after date } t+1,$$

where $R_{t+1}'^H \beta e_{t+1}$ and $R_{t+1}'^L \beta e_{t+1}$ are the date $t+2$ net worth of the entrepreneur when he/she was H-type and L-type at date $t+1$, respectively. $R_{t+1}'^H$ and $R_{t+1}'^L$ are realized rate of return per unit of saving from date $t+1$ to date $t+2$ in the bubbleless economy, and they satisfy

$$\text{(N.29)} \qquad \begin{cases} R_t'^H = \frac{q_{t+1}\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}} & \text{after date } t, \\ \\ R_t'^L = q_{t+1}\alpha^L & \text{after date } t. \end{cases}$$

Aggregate capital stock follows:

$$\text{(N.30)} \qquad K_{t+2} = (1 + \tfrac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} p)\beta\alpha^L \sigma K_{t+1}^{\sigma} \quad \text{after date } t+1.$$

We guess that the value function are linear functions of $\log K$ and $\log e$:

$$\text{(N.31)} \qquad W_{t+1}^{BL}(e_{t+1}, K_{t+1}) = f_1 + g_1 \log K_{t+1} + h_1 \log e_{t+1}$$

From (N.28)-(N.31), applying the method of undetermined coefficients yields

$$
\begin{aligned}
\text{(N.32)} \qquad f_1 \;=\;& \frac{1}{1-\beta}\log(1-\beta)+\frac{\beta}{(1-\beta)^2}\log(1-\beta)+\frac{\beta}{(1-\beta)^2}\log\sigma \\
&+\frac{\beta}{(1-\beta)^2}\left[p\log\frac{\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}}+(1-p)\log\alpha^L\right] \\
&+\frac{\beta(\sigma-1)}{(1-\beta)^2}\frac{1}{1-\beta\sigma}\log\left[(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p)\beta\alpha^L\sigma\right],
\end{aligned}
$$

$$
\text{(N.33)} \qquad g_1=\frac{\beta\sigma}{1-\beta\sigma}\frac{\sigma-1}{1-\beta},
$$

$$
\text{(N.34)} \qquad h_1=\frac{1}{1-\beta}.
$$

Next, we derive the value function at date t when bubbles collapse and the government bails out entrepreneurs by taking into account the effects of bailouts on the date $t+1$ aggregate capital stock. Given the optimal decision rules, the value function at date t satisfies

(N.35)

$$
W_t^{BL}(e_t,K_t)=\log c_t+\beta\left[pW_{t+1}^{BL}(R_t'^H\beta e_t,K_{t+1})+(1-p)W_{t+1}^{BL}(R_t'^L\beta e_t,K_{t+1})\right],
$$

with

$$
\text{(N.36)} \qquad K_{t+1}=\left[1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right]\beta\alpha^L\sigma\left[1+\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\right]K_t^\sigma.
$$

From (N.31)-(N.36), we obtain

(N.37)

$$
W_t^{BL}(e_t,K_t)=f_1+\frac{\beta(\sigma-1)}{1-\beta}\frac{1}{1-\beta\sigma}\log\left[1+\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\right]+\frac{\beta\sigma}{1-\beta\sigma}\frac{\sigma-1}{1-\beta}\log K_t+\frac{1}{1-\beta}\log e_t.
$$

Now, we are in a position to derive the value function at any date t in the bubble economy. $W_t^{BB}(e_t,K_t)$ is the value function of the entrepreneur at any date t in the bubble economy who holds the net worth, e_t, at the beginning of the period t before knowing his/her type of the period t. Given optimal decision rules, the Bellman equation can be written as

(N.38)

$$W_t^{BB}(e_t, K_t) = \log c_t + \beta\pi\left[pW_{t+1}^{BB}(R_t^H \beta e_t, K_{t+1}) + (1-p)W_{t+1}^{BB}(R_t^L \beta e_t, K_{t+1})\right] + \beta(1-\pi)\begin{bmatrix} pW_{t+1}^{BL}(R_t^H \beta e_t, K_{t+1}) + (1-p)\lambda W_{t+1}^{BL}(R_t^L \beta e_t, K_{t+1}) \\ +(1-p)(1-\lambda)W_{t+1}^{BL}(R_t^{LL} \beta e_t, K_{t+1}) \end{bmatrix},$$

where $R_t^H \beta e_t$, $R_t^L \beta e_t$, and $R_t^{LL} \beta e_t$ are the date $t+1$ net worth of the entrepreneur in each state. R_t^H, R_t^L, and R_t^{LL} are realized rate of return per unit of saving from date t to date $t+1$, and in $0 \leq \lambda \leq \lambda^*$, they satisfy

$$\text{(N.39)} \qquad \begin{cases} R_t^H = \frac{q_{t+1}\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}}, \\ R_t^L = \delta(\lambda)\frac{P_{t+1}}{P_t} = \delta(\lambda)\frac{q_{t+1}\alpha^L[1-p-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}, \\ R_t^{LL} = \frac{q_{t+1}\alpha^L[1-p-\phi(\lambda)]}{1-p}. \end{cases}$$

and in $\lambda^* \leq \lambda \leq 1$, they satisfy

$$\text{(N.40)} \qquad \begin{cases} R_t^H = \frac{q_{t+1}\alpha^H(1-\theta)[1-\phi(\lambda)]}{p}, \\ R_t^L = \delta(\lambda)\frac{P_{t+1}}{P_t} = \delta(\lambda)\frac{q_{t+1}\theta\alpha^H[1-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}, \\ R_t^{LL} = \frac{q_{t+1}\theta\alpha^H[1-\phi(\lambda)]}{1-p}. \end{cases}$$

Aggregate capital stock until bubbles collapse follows:

$$\text{(N.41)} \qquad K_{t+1} = H(\lambda)K_t^\sigma.$$

We guess that the value function are linear functions of $\log K$ and $\log e$:

$$\text{(N.42)} \qquad W_t^{BB}(e_t, K_t) = m + l\log K_t + n\log e_t.$$

From (N.37)-(N.42), and (N.38), applying the method of undetermined coefficients yields

$$
\begin{aligned}
m &= \frac{1}{1-\beta\pi}\log(1-\beta)+\frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\log\beta+\frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\log\sigma \\
&\quad+\frac{\beta(\sigma-1)}{1-\beta\pi}\frac{1}{1-\beta\sigma}\frac{1}{1-\beta}\log H(\lambda) \\
&\quad+\frac{\beta(1-\pi)}{1-\beta\pi}\left\{f_1+\frac{\beta(\sigma-1)}{1-\beta}\frac{1}{1-\beta\sigma}\log\left[1+\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\right]\right\} \\
&\quad+\frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\left[\pi J_1+(1-\pi)J_2\right],
\end{aligned}
$$

$$
l=\frac{\beta\sigma(\sigma-1)}{1-\beta\sigma}\frac{1}{1-\beta},
$$

$$
n=\frac{1}{1-\beta},
$$

where in $0\le\lambda\le\lambda^*$,

$$
J_1=p\log\frac{\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}}+(1-p)\log\left[\delta(\lambda)\frac{\alpha^L[1-p-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right],
$$

$$
\begin{aligned}
J_2 &= p\log\frac{\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}}+(1-p)\lambda\log\left[\delta(\lambda)\frac{\alpha^L[1-p-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right] \\
&\quad+(1-p)(1-\lambda)\log\left[\frac{\alpha^L[1-p-\phi(\lambda)]}{1-p}\right].
\end{aligned}
$$

and in $\lambda^*\le\lambda\le 1$,

$$
J_1=p\log\frac{\alpha^H(1-\theta)[1-\phi(\lambda)]}{p}+(1-p)\log\left[\delta(\lambda)\frac{\theta\alpha^H[1-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right],
$$

$$
\begin{aligned}
J_2 &= p\log\frac{\alpha^H(1-\theta)[1-\phi(\lambda)]}{p}+(1-p)\lambda\log\left[\delta(\lambda)\frac{\theta\alpha^H[1-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right] \\
&\quad+(1-p)(1-\lambda)\log\left[\frac{\theta\alpha^H[1-\phi(\lambda)]}{1-p}\right].
\end{aligned}
$$

Thus, we have (32) in the text.

Appendix N.2 the case where the government taxes entrepreneurs

When the government taxes entrepreneurs who do not suffer losses from bubble investments, m and J_2 change as follows:

$$\begin{aligned}
m &= \frac{1}{1-\beta\pi}\log(1-\beta)+\frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\log\beta+\frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\log\sigma \\
&+\frac{\beta(\sigma-1)}{1-\beta\pi}\frac{1}{1-\beta\sigma}\frac{1}{1-\beta}\log H(\lambda) \\
&+\frac{\beta(1-\pi)}{1-\beta\pi}\left\{f_1+\frac{\beta(\sigma-1)}{1-\beta}\frac{1}{1-\beta\sigma}\log\left[1+F(\lambda)\right]\right\} \\
&+\frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\left[\pi J_1+(1-\pi)J_2\right],
\end{aligned}$$

in $0 \leq \lambda \leq \lambda^*$,

$$\begin{aligned}
J_2 &= p\log\frac{(1-\tau)\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}}+(1-p)\lambda\log\left[\delta(\lambda)\frac{\alpha^L[1-p-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right] \\
&+(1-p)(1-\lambda)\log\left[\frac{\alpha^L[1-p-\phi(\lambda)]}{1-p}\right].
\end{aligned}$$

in $\lambda^* \leq \lambda \leq 1$,

$$\begin{aligned}
J_2 &= p\log\frac{(1-\tau)\alpha^H(1-\theta)[1-\phi(\lambda)]}{p}+(1-p)\lambda\log\left[\delta(\lambda)\frac{\theta\alpha^H[1-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right] \\
&+(1-p)(1-\lambda)\log\left[\frac{\theta\alpha^H[1-\phi(\lambda)]}{1-p}\right].
\end{aligned}$$

Appendix O Procedures to derive numerical examples of entrepreneur's welfare

When we compute (32), we make the following assumptions: aggregate capital stock in the initial period is set to the steady-state value of the bubbleless economy; population measure of entrepreneurs is assumed to be equal to one; in the initial period, each entrepreneur is endowed with the same amount of capital, $k_t^i = k_t$, and one unit of bubble assets, and owes no debt. Under these assumptions, all entrepreneurs hold the same amount of net worth in the initial period, i.e., $e_0 = q_0 k_0 + P_0$. By using determination of equilibrium bubble prices (L.15), e_0 can be written as

$$e_0(\lambda) = \frac{1}{1 - \beta\phi(\lambda)} \sigma K_0^{\sigma}.$$

Inserting the above relation into (32) yields

$$W_0^{BB}(K_0) = m(\lambda) + \frac{1}{1-\beta} \log \sigma + \frac{1}{1-\beta\sigma} \log K_0 - \frac{1}{1-\beta} \log \left[1 - \beta\phi(\lambda)\right].$$

Figure 4 describes the relationship between W_0^{BB} and λ.